K^+N scattering and in-medium strange quark condensate

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- in-medium quark condensate
  correlation function approach

- K^+ nucleon scattering revisited

- possible broad resonance in KN channel

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in-medium quark condensate

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

- correlation function approach

  a correlation function (axial vector)in medium

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\, \partial^\mu \langle \Omega | \mathrm{T}[A_\mu^a(x) \phi_5^b(0) | \Omega \rangle$$

axial current $A_\mu^a = \frac{1}{2}\bar{q}\gamma_\mu\gamma_5\tau^a q$, pseudoscalar field $\phi_5^a = \bar{q}i\gamma_5\tau^a q$

in soft limit, according to chiral Ward identity

$$\Pi_5^{ab}(0) = \langle \Omega | [Q_5^a, \phi_5^b] | \Omega \rangle = -i\delta^{ab}\langle \bar{q}q \rangle^*$$

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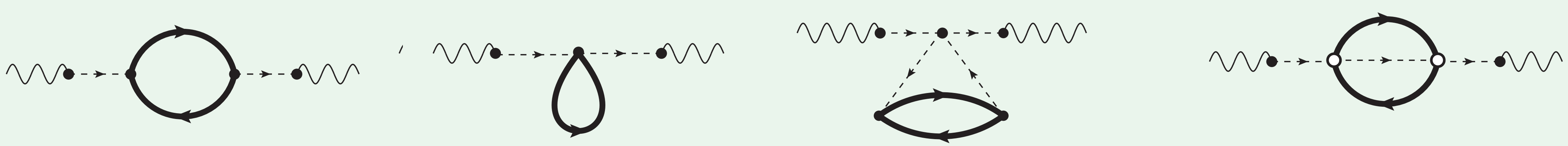
- if we calculate the correlation function in medium and take soft limit, we obtain the quark condensate in nuclear medium
- in-medium chiral perturbation theory provides diagrammatical calculation



etc

Goda, DJ, PRC 88 (2013), 065204

Hübsch, DJ, PRC 104 (2021), 015202.

in-medium quark condensate

- ChPT at NNLO, density up to k_F^5, without N-N correlation

Hübsch, DJ, PRC 104 (2021), 015202.

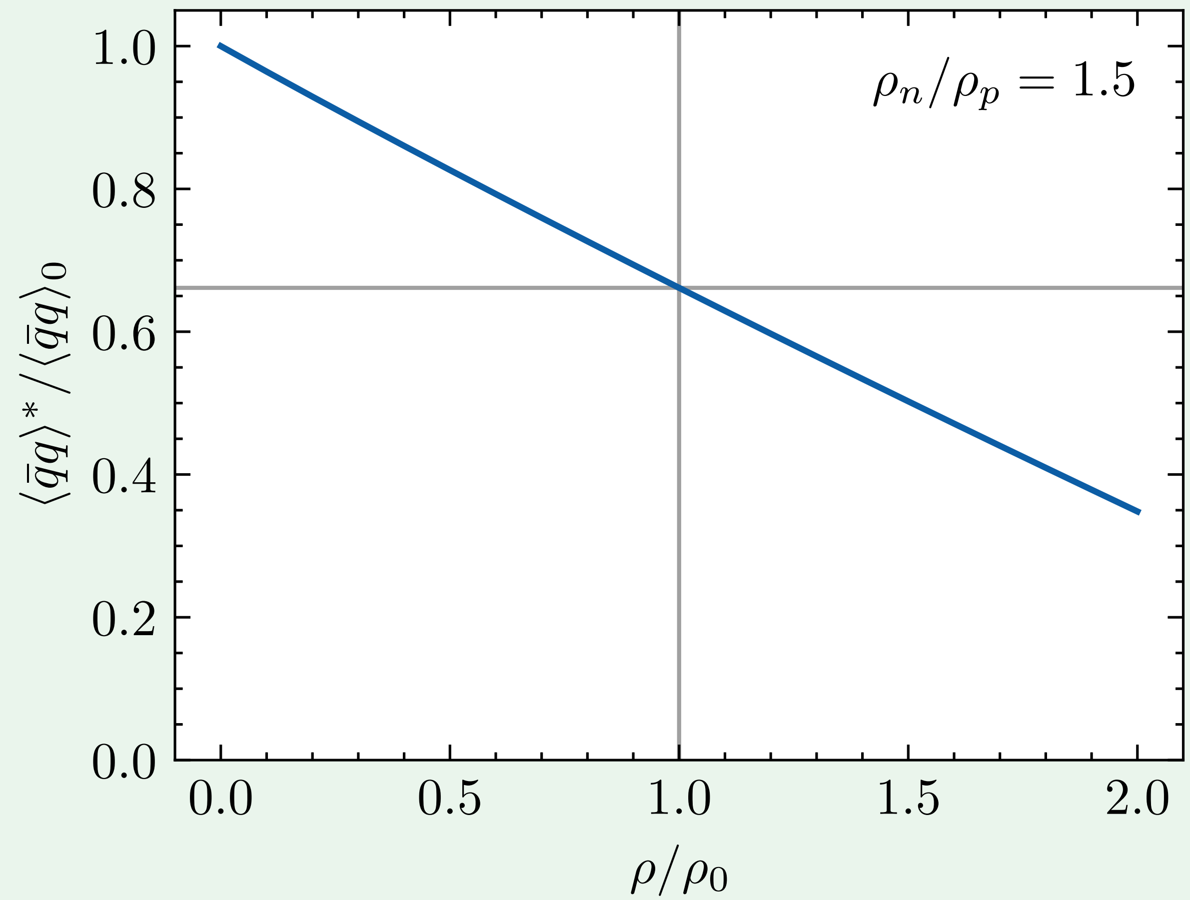


FIG. 1. Density dependence of the in-medium quark condensate, normalized to the vacuum condensate. The ratio of neutrons to protons is given by ρ_n/ρ_p. At normal nuclear density, $\rho = \rho_0$, the quark condensate is reduced by about 35% compared to its vacuum value.

See also, Kaiser, Homont, Weise, PRC 77 (2008), 025204.

in-medium strange quark condensate

- *extended to SU(3), we can calculate the strange quark condensate*

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\,\partial^\mu \langle \Omega \,|\, \mathrm{T}[A_\mu^a(x)\phi_5^b(0) \,|\, \Omega \rangle$$

taking a,b = 4,5 for kaon sector

$$\Pi_5^{4+i5,4-i5}(0) = -\,i\langle \bar{u}u + \bar{s}s \rangle^*$$

S. Hübsch,
PhD Thesis (2021)

Iizawa, Hübsch, DJ, in preparation

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Iizawa, Hübsch, DJ, in preparation

- at leading order (SU(3) symmetric)

$$\frac{\langle \bar{s}s\rangle^*}{\langle \bar{s}s\rangle_0} = \frac{4\rho}{f^2}(b_0 + b_D - b_F)$$

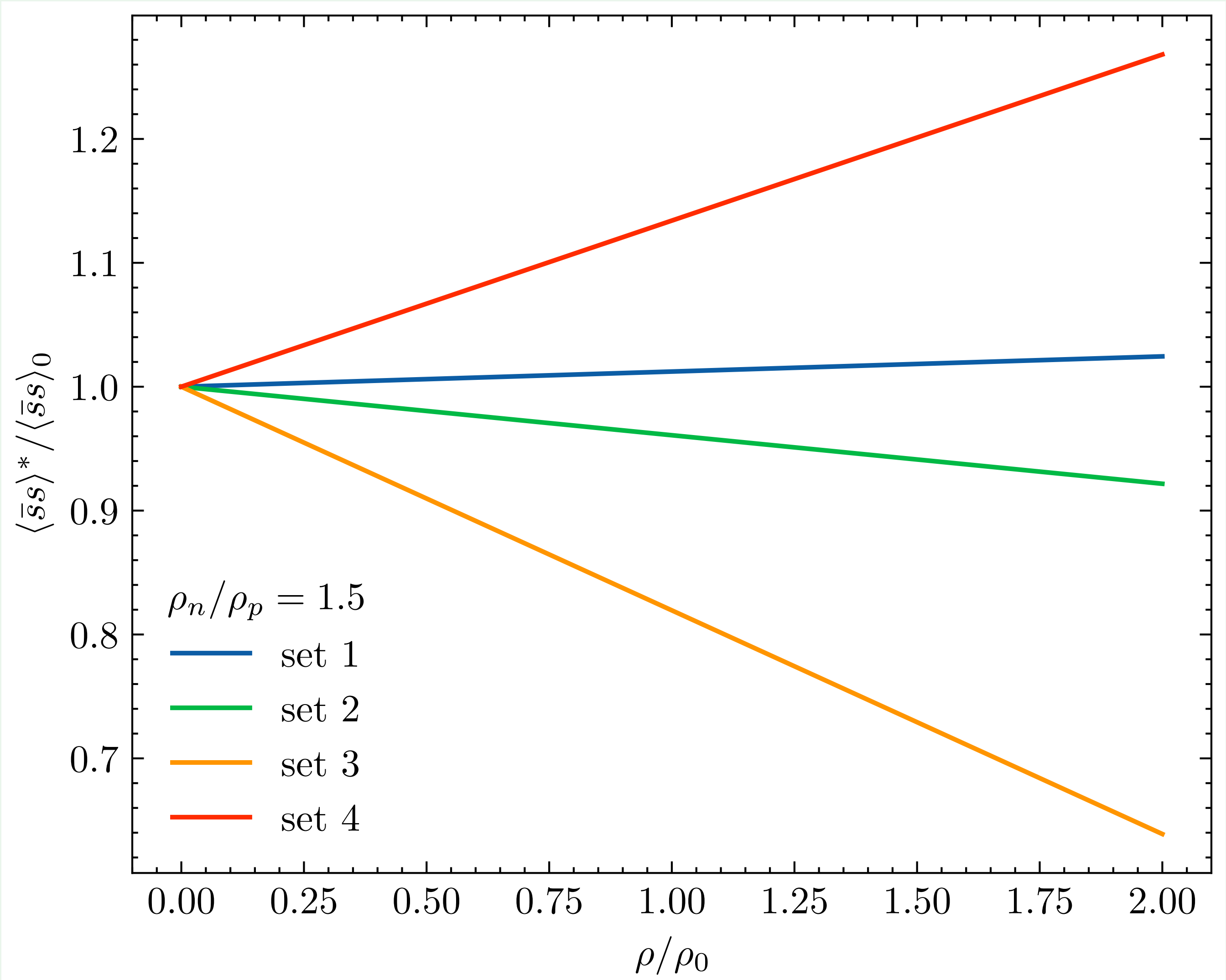
SU(3) chiral Lagrangian

$$\mathcal{L}_{MB}^{(2)} = b_D \,\mathrm{Tr}\left\{\bar{B}\{\chi_+, B\}\right\} + b_F \,\mathrm{Tr}\left\{\bar{B}[\chi_+, B]\right\} + b_0 \,\mathrm{Tr}\left\{\bar{B}B\right\}\mathrm{Tr}\left\{\chi_+\right\} + \ldots$$

it provides baryon mass terms and meson-baryon contact interactions

in-medium strange quark condensate

S. Hübsch, PhD Thesis (2021)



set 4 determined by KN scattering (incomplete NLO)

set 1,2 determined by baryon mass (NNNLO) + lattice QCD

set 3 determined by baryon mass (LO)

Iizawa, Hübsch, DJ, in preparation

- Set 1, 2: X. L. Ren, L. S. Geng, J. M. Camalich, J. Meng, and H. Toki, *Octet baryon masses in next-to-next-to-next-to-leading order covariant baryon chiral perturbation theory*, JHEP **2012**, 73 (2012)
- Set 3: *B. Kubis and U. G. Meißner, Baryon form factors in chiral perturbation theory, Eur. Phys. J. C18, 747 (2001)*
- Set 4: K. Aoki and D. Jido, K^+–*nucleus elastic scattering revisited from the perspective of partial restoration of chiral symmetry*, PTEP **2017**, 103D01 (2017)

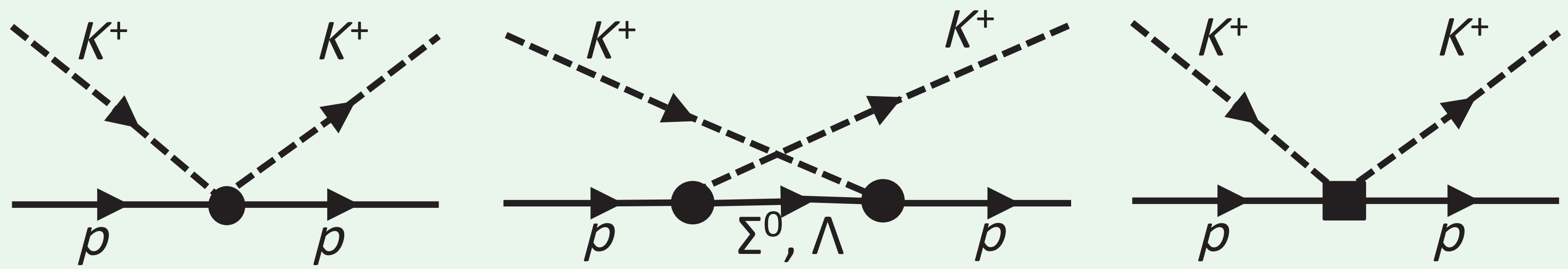
K^+ nucleon elastic scattering revisited

KN scattering in ChPT

Aoki, DJ, PTEP2017,103D01(17)

- KN elastic scattering in chiral perturbation theory

  LO + NLO, 4 LECs, (incomplete, some terms missing)



experimental data

K^+p elastic differential cross sections, I=0 total cross section

I=0 LECS determined with baryon masses

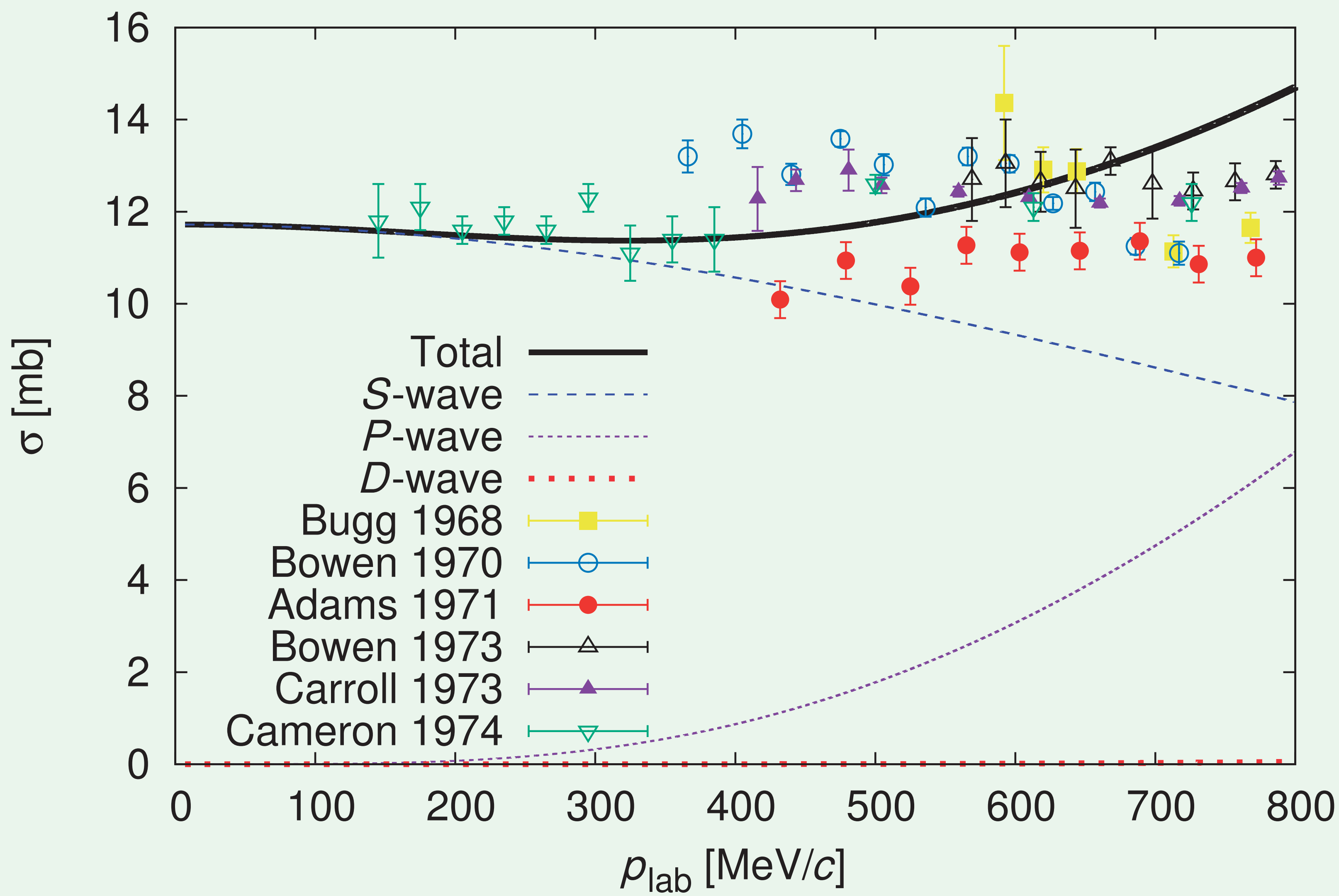
Table 1. Determined parameters for the tree-level amplitude. $b^{I=0}$ can be fixed by $b^{I=1}$, b_D, and b_F.

$b^{I=1}$	1.01×10^{-5}	MeV^{-1}	$\chi^2/N = 0.83$
$d^{I=1}$	4.33×10^{-4}	MeV^{-1}	
$b^{I=0}$	1.55×10^{-4}	MeV^{-1}	
$d^{I=0}$	3.91×10^{-4}	MeV^{-1}	$\chi^2/N = 12.0$

KN scattering in ChPT

Aoki, DJ, PTEP2017,103D01(17)

- I=1 (K^+p) cross sections are reproduced well



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PTEP2017,103D01(17)

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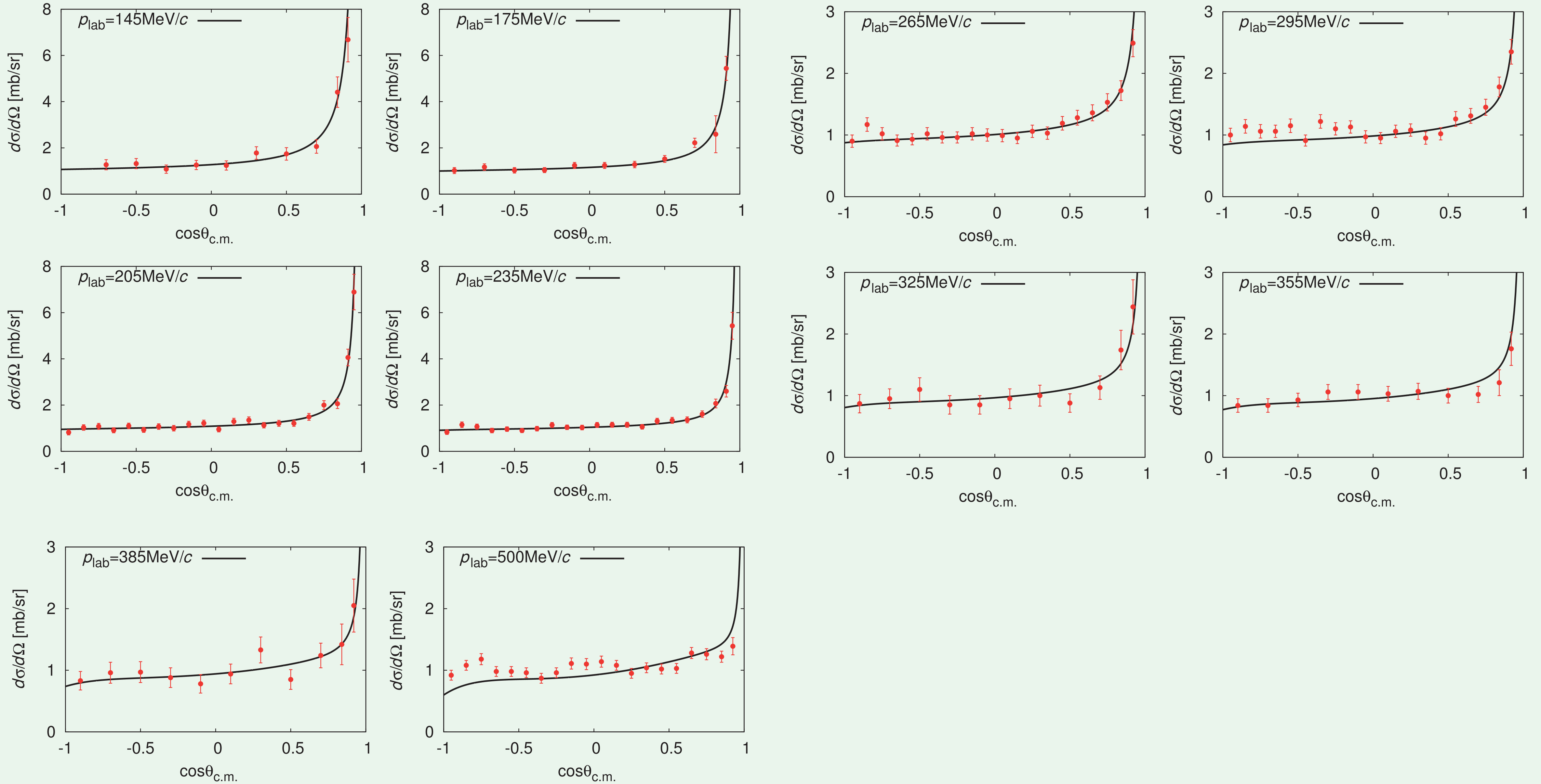
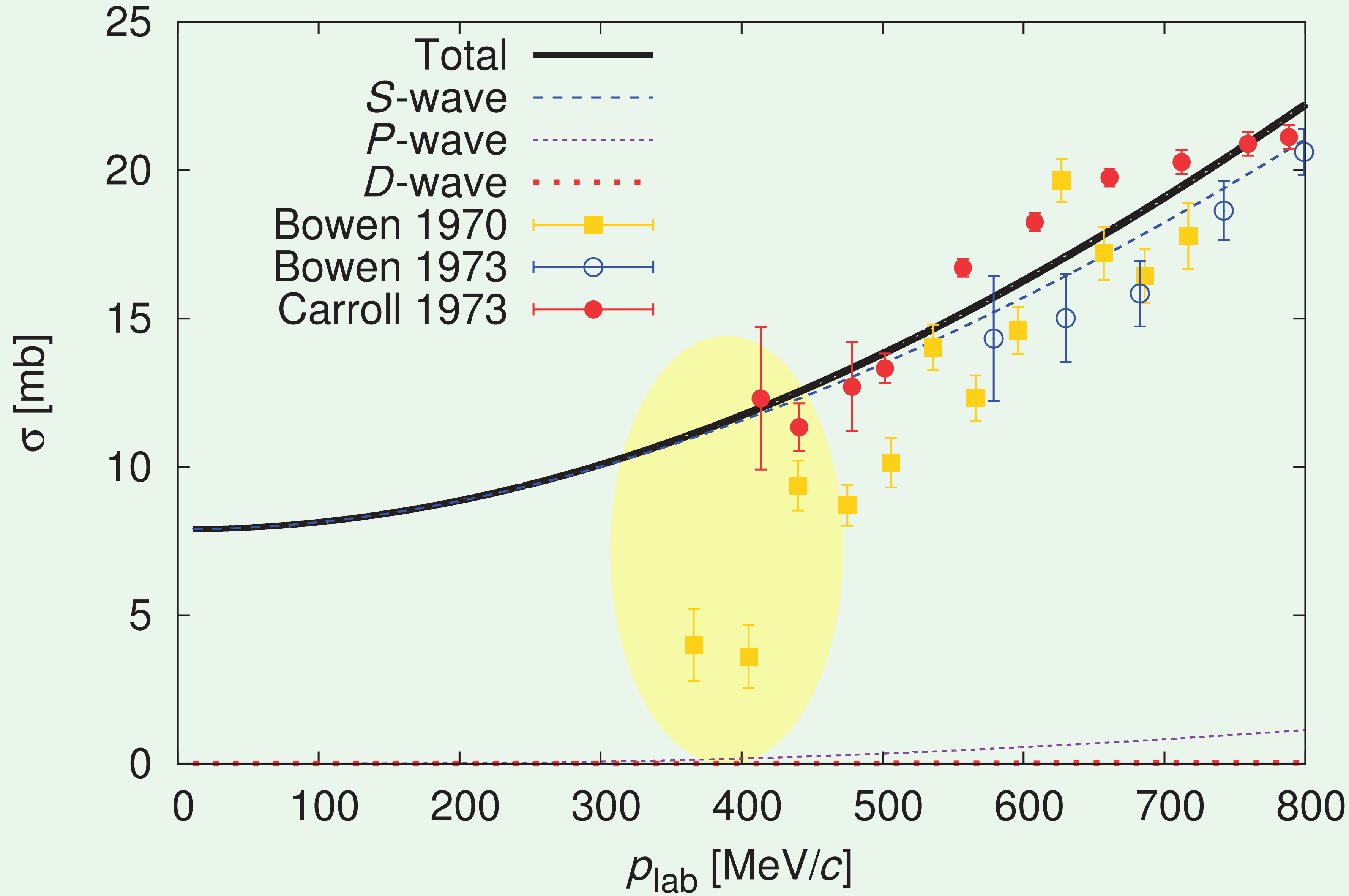


KN scattering in ChPT

Aoki, DJ, PTEP2017,103D01(17)

- I=0 cross sections are not reproduced so well at low energies



KN scattering in ChPT

Aoki, DJ,
PTEP2017,103D01(17)

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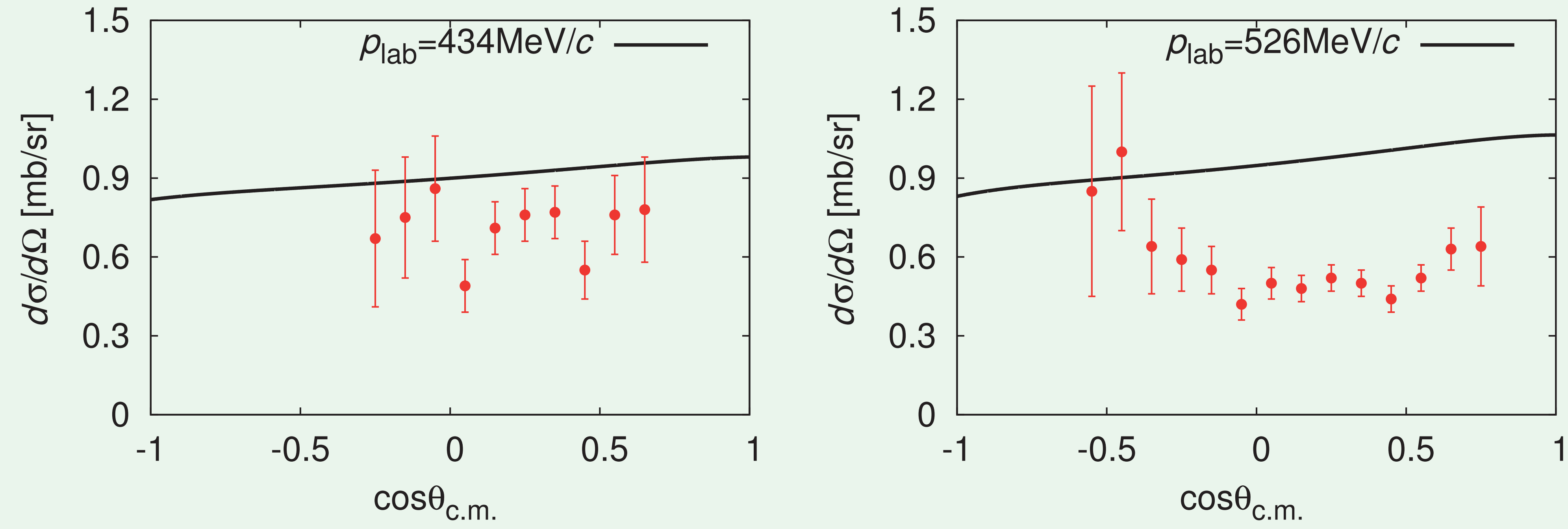


K^+ - nucleus scattering revisited

- **self-energy of K^+ in Fermi gas approximation**

$$\Pi(\omega, \vec{p}) = 4 \int^{k_F} \frac{d^3q}{(2\pi)^3} T_{K^+N}(p,q)$$

- **wavefunction renormalization**

can be one of the corrections beyond linear density

- energy dependence of self-energy provides WFR

$$\Pi = 2m_{K^+}V_{\rm opt} \simeq -Z\rho T_{K^+N}$$

Kolomeitsev, Kaiser, Weise, PRL90, 092501 (03)

wavefunction renormalization $Z \equiv 1 + \left.\frac{\partial \Pi}{\partial \omega^2}\right|_{\omega = m_{K^+}}$

ChPT leading order calculation of KN scattering amplitude

Aoki, DJ, PTEP2017,103D01(17)

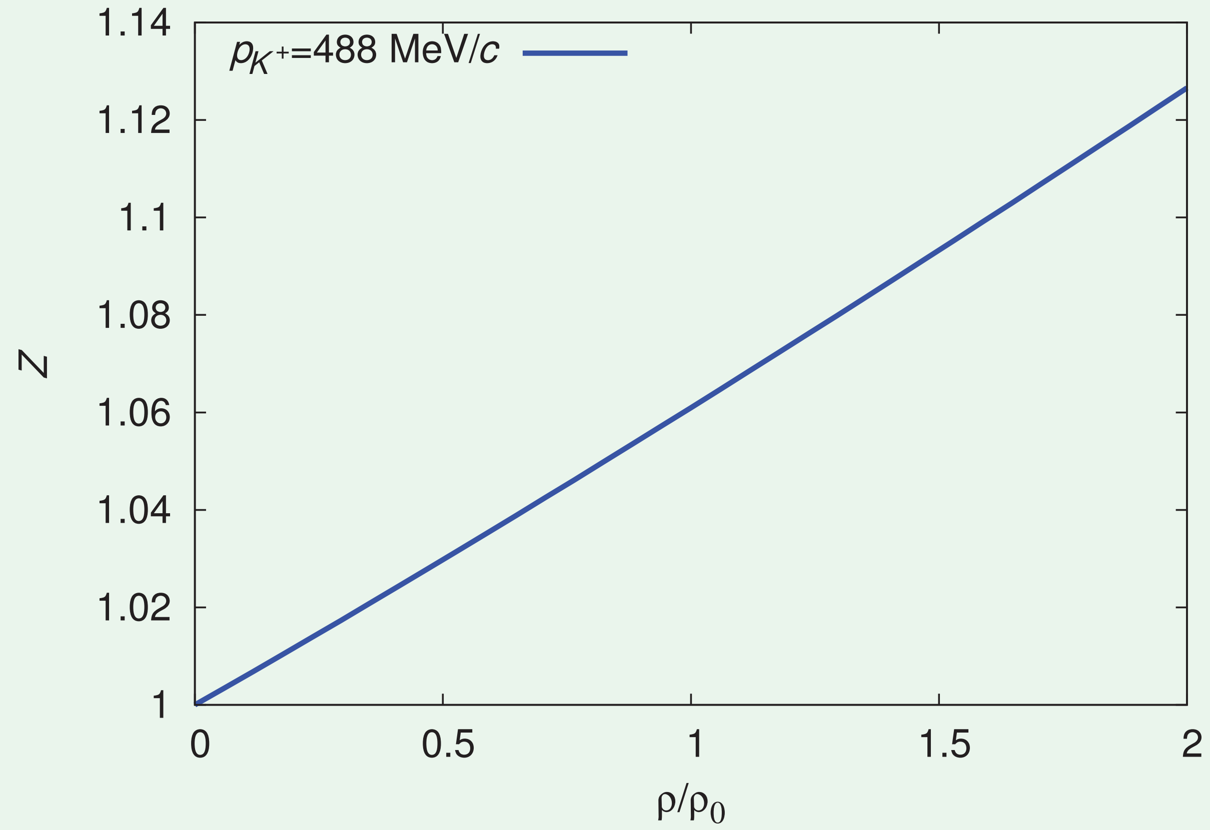
wavefunction renormalization

Aoki, DJ, PTEP2017,103D01(17)

- leading order (Weinberg-Tomozawa term)

$$Z = 1 + \frac{3\rho_0}{8M_K f_K^2}\frac{\rho}{\rho_0} = 1 + 0.082\frac{\rho}{\rho_0}, \qquad \text{8\% enhancement at } \rho = \rho_0$$

- + next-to-leading order (without medium modification on kaon)

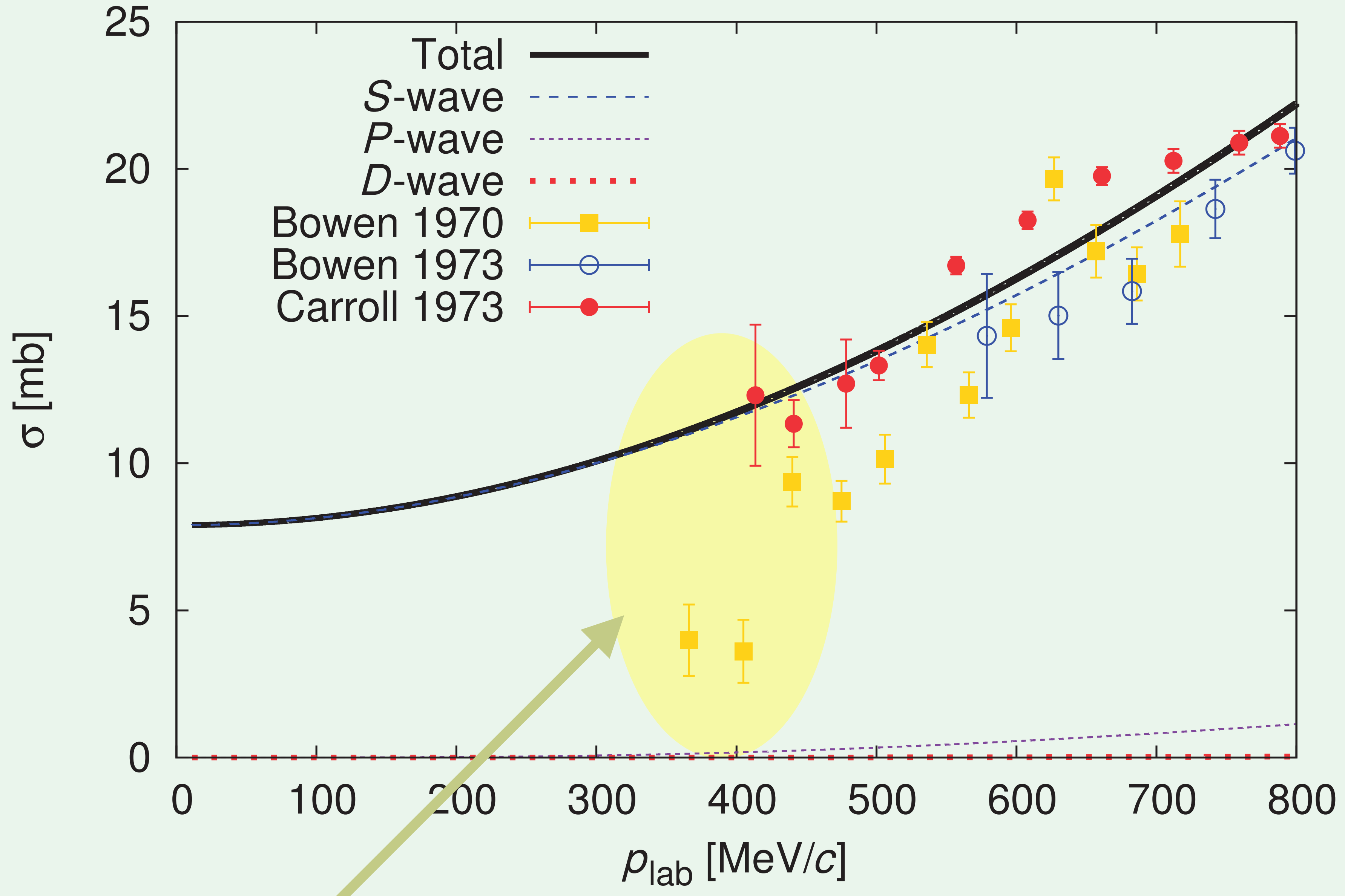


a few % enhancement with $p_{K^+} \sim 500$ MeV

KN scattering in ChPT

Aoki, DJ, PTEP2017,103D01(17)

- I=0 cross sections are not reproduced so well



In particular, increase at $p_{\rm lab} \sim 500$ MeV is not reproduced

Possible wide resonance with S=+1 in unitarized amplitudes

wide resonance in I=0 and S=+1

Aoki, DJ,
PTEP2019,013D01(19)

- **unitarized amplitude T**

$$T = V + VGT$$

V: interaction kernel, given by ChPT

G: KN loop function (I=0, I=1)

one subtraction constant is fixed as a natural value

- **chiral Lagrangian**

most general form up to next-leading-order

L.-S. Geng, Frontiers of Physics 8, 328 (2013)

8 LECs (4 LECs for I=1, 4 LECs for I=0)

- **data**

up to 800 MeV, where inelastic contributions start to be significant

$K^+p \to K^+p$, total and differential cross sections, p_{lab} =145 to 726 MeV,

which determine I=1 amplitudes very well

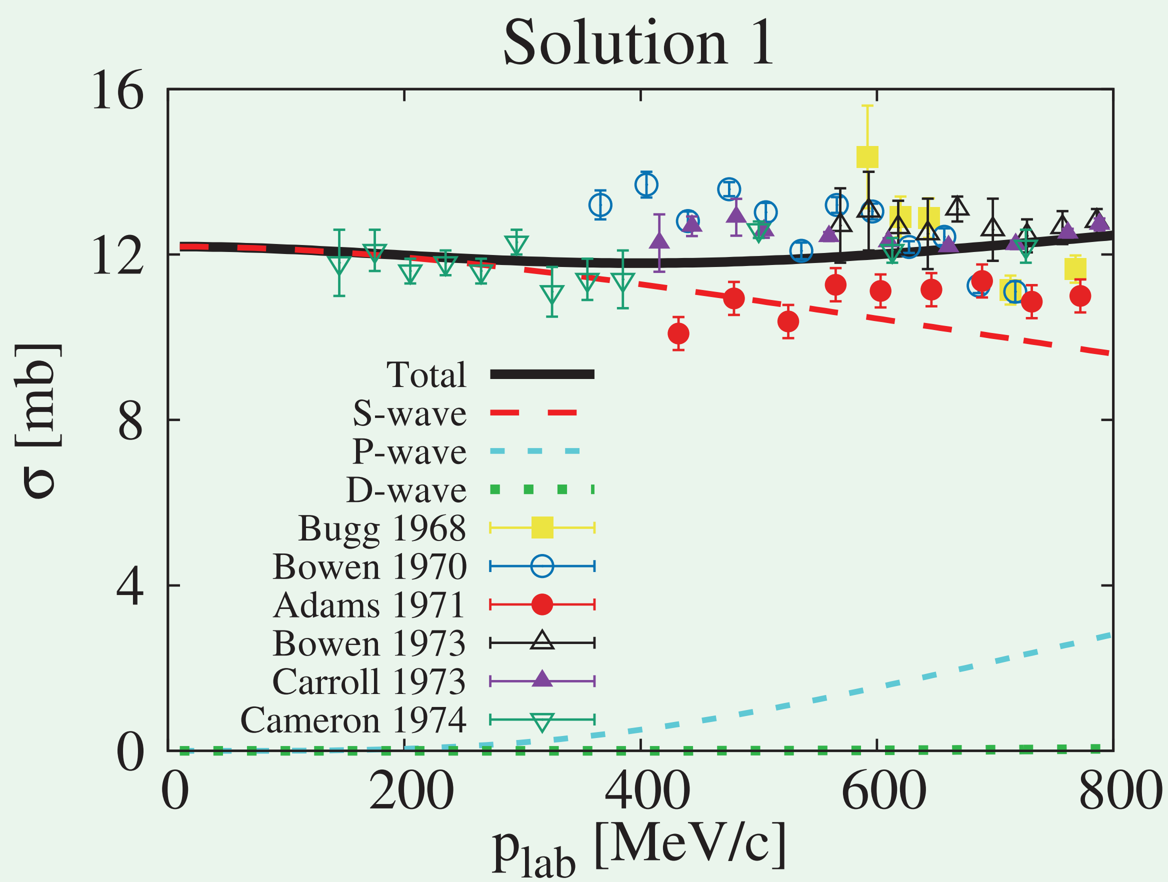
$K^+n \to K^+n$, K^0p, differential cross sections, p_{lab} =526, 604, 640 MeV,

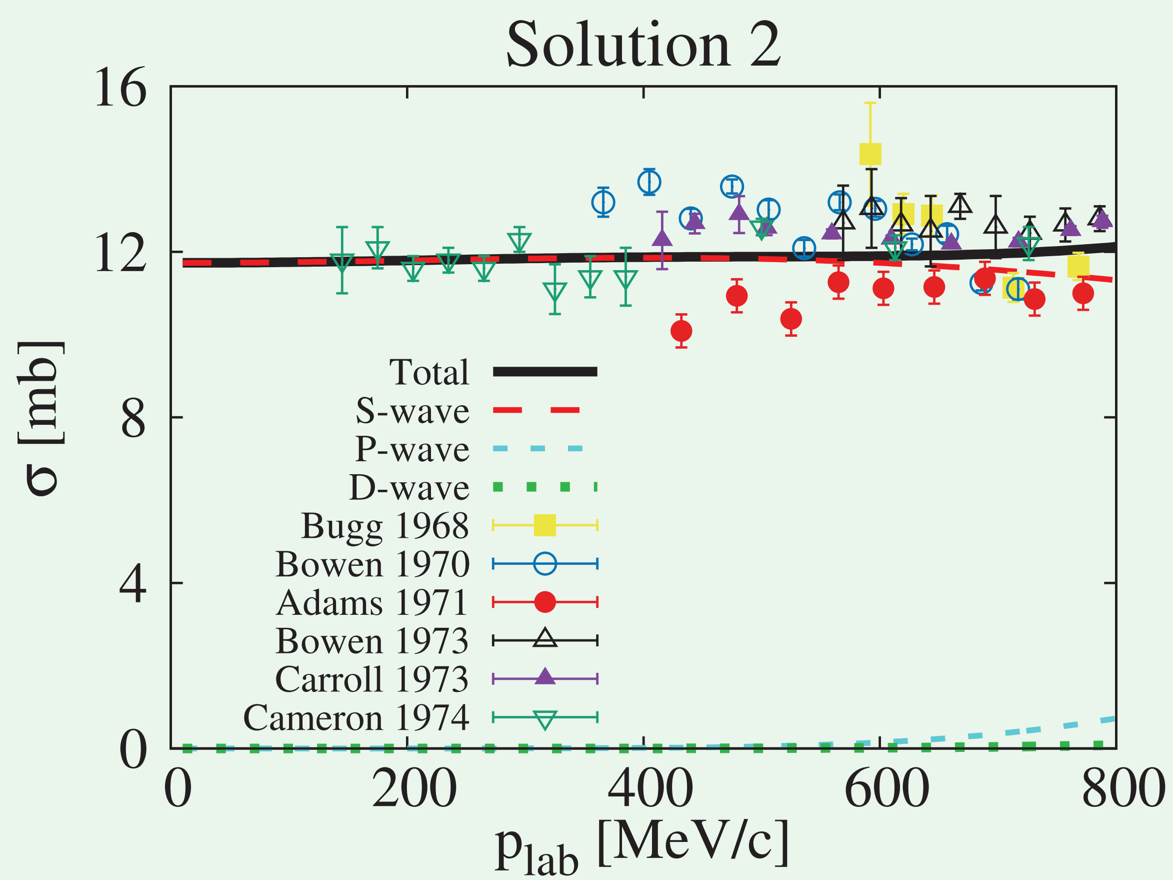
total cross section I=0

I=1 total cross sections

Aoki, DJ, PTEP2019,013D01(19)

- we have two solutions



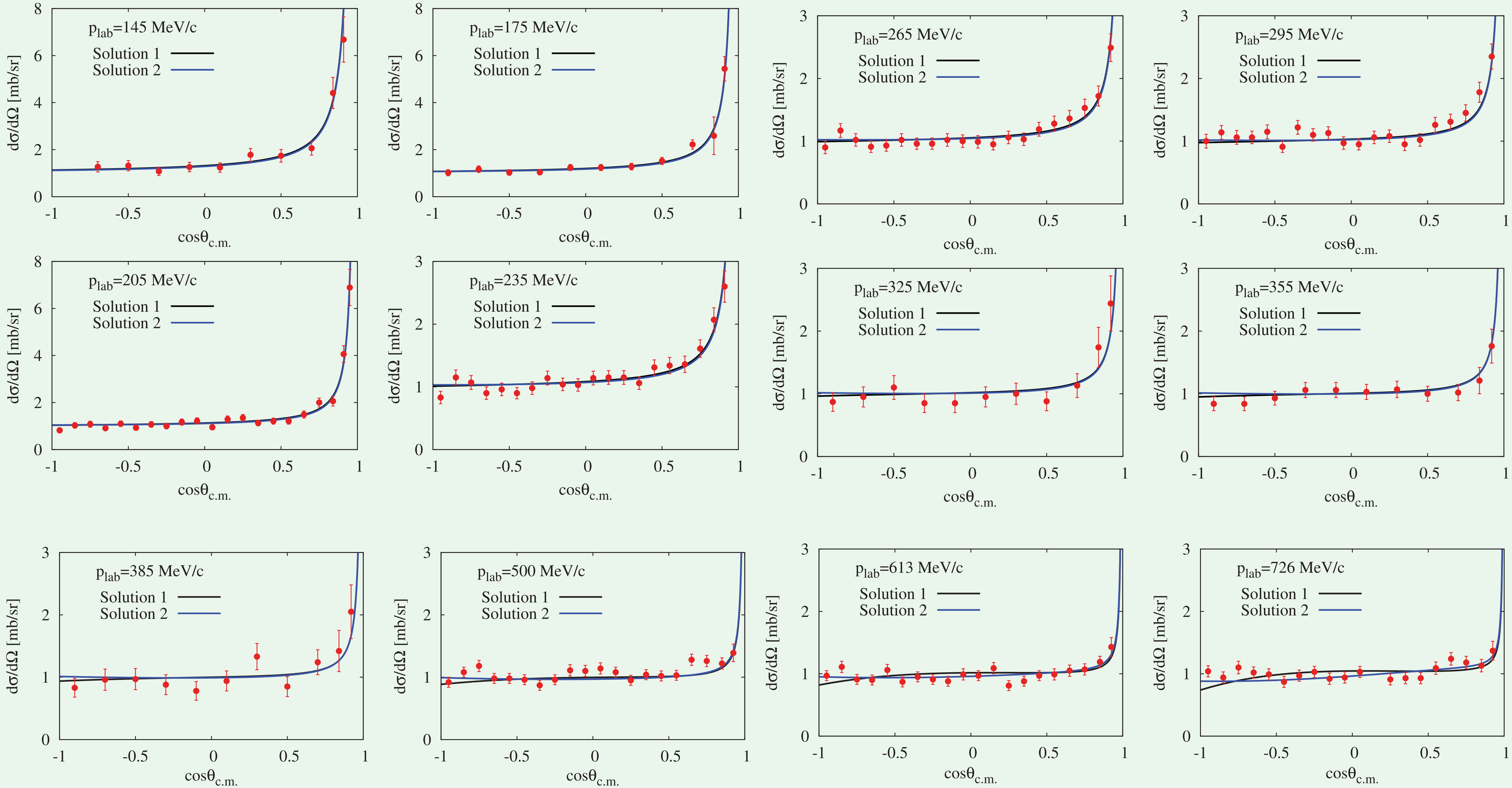


good agreements

solution 1 is consistent with Martin' amplitude and SAID

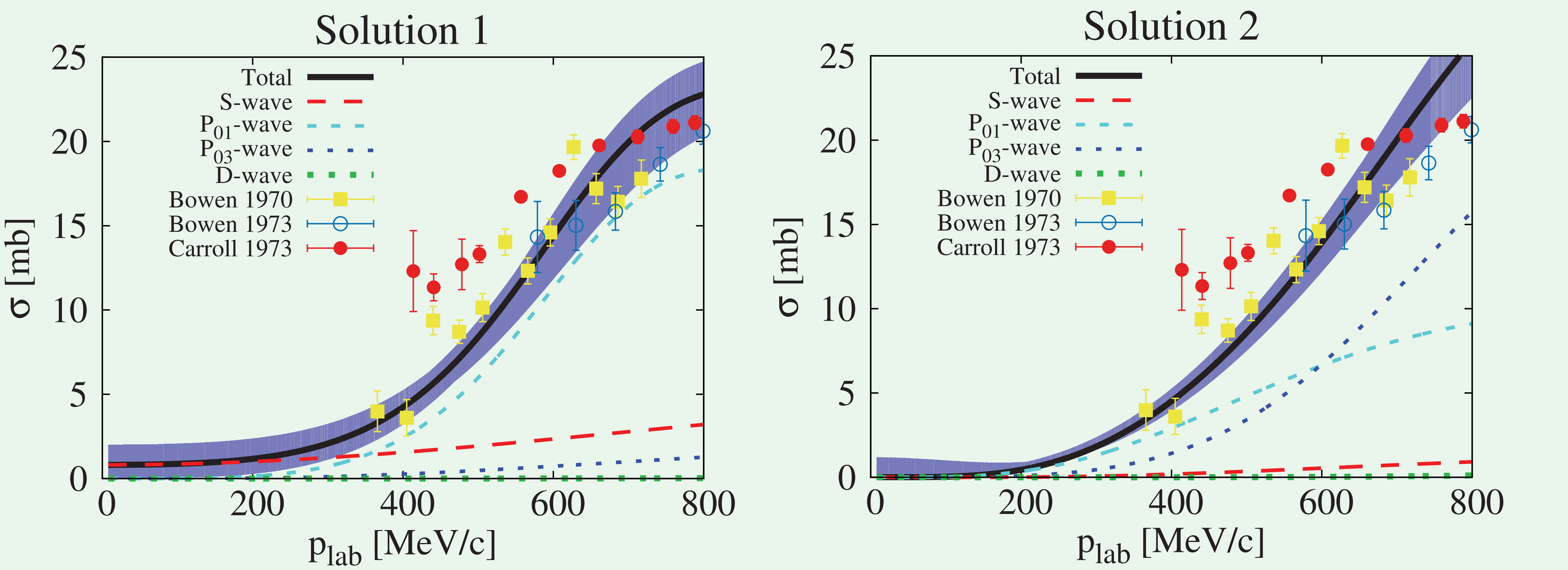
K+p differential cross sections

Aoki, DJ,
PTEP2019,013D01(19)



I=0 total cross sections

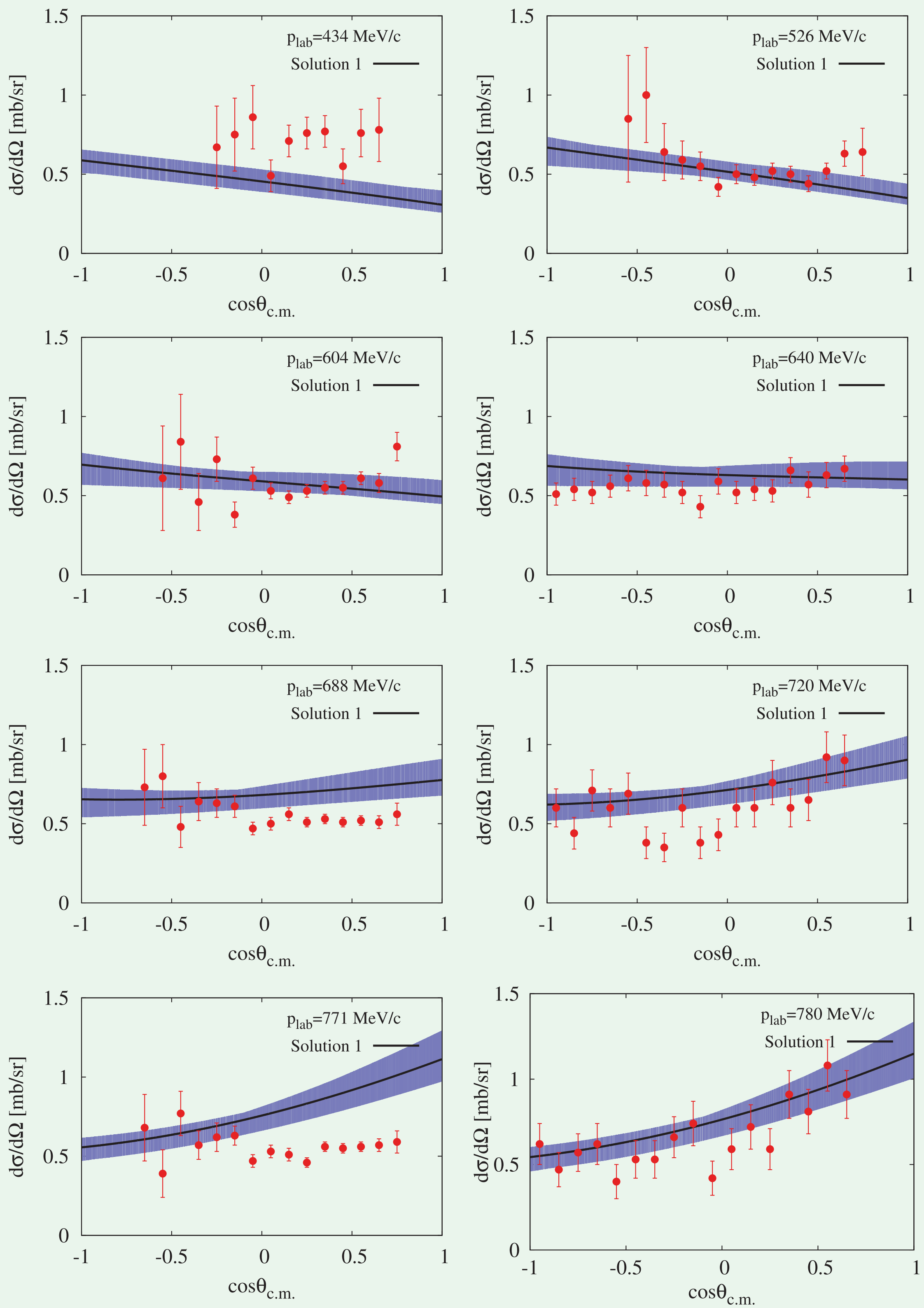
- increase at p_{lab} ~ 500 MeV is reproduced



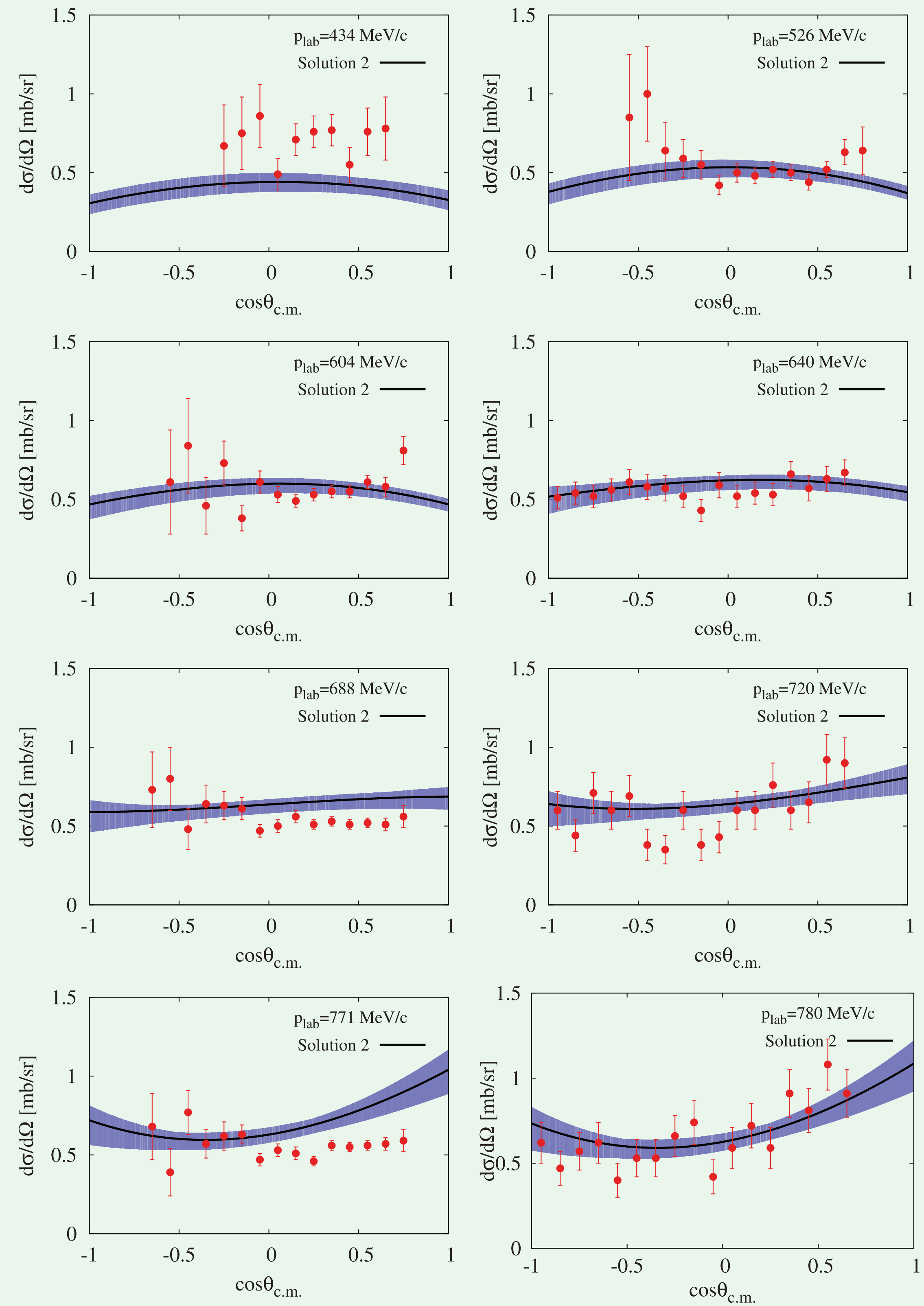
- solution 1: P_{01} amplitude dominate
- solution 2: P_{03} amplitude largely contributed

K+n elastic scattering

solution 1

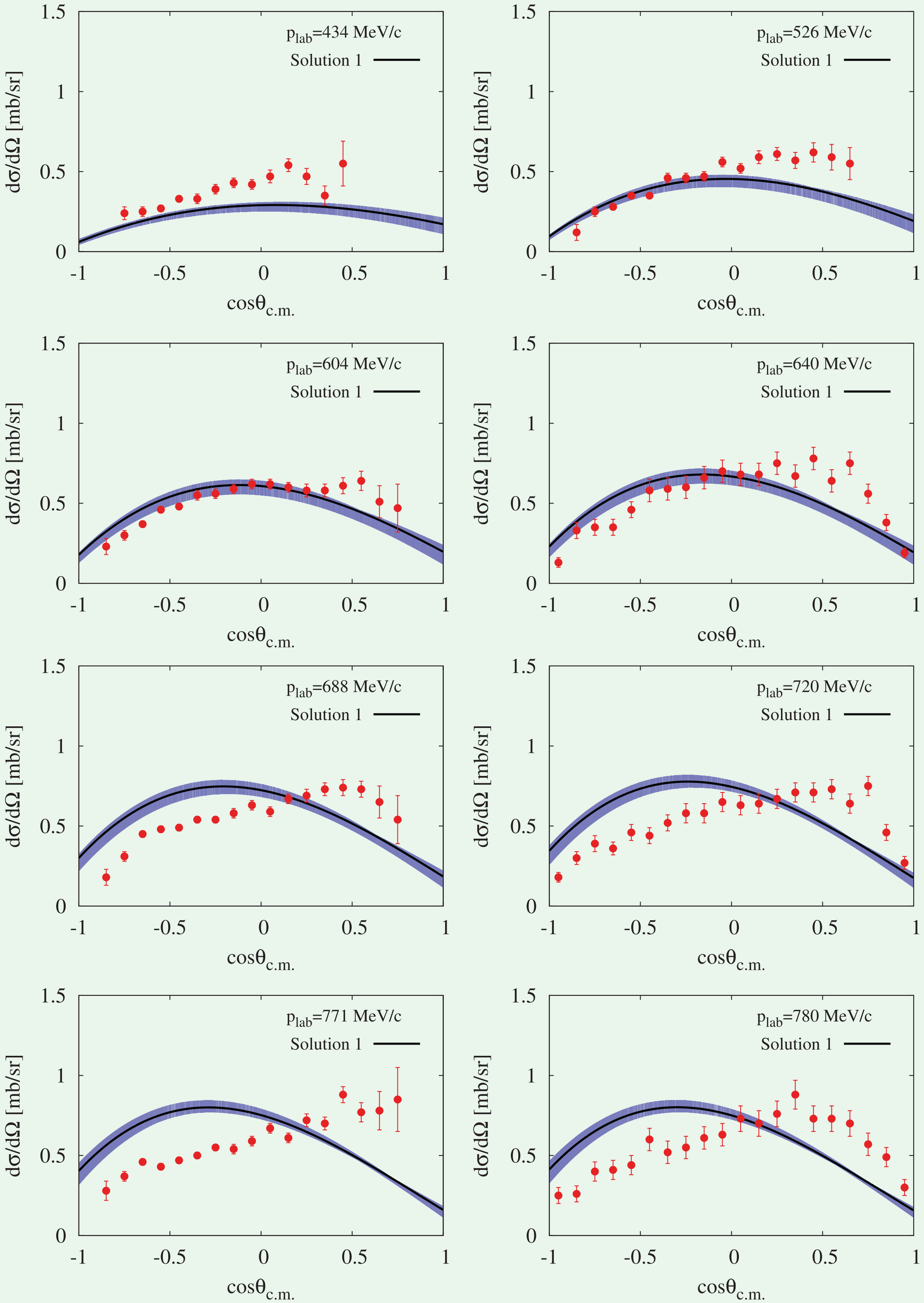


solution 2

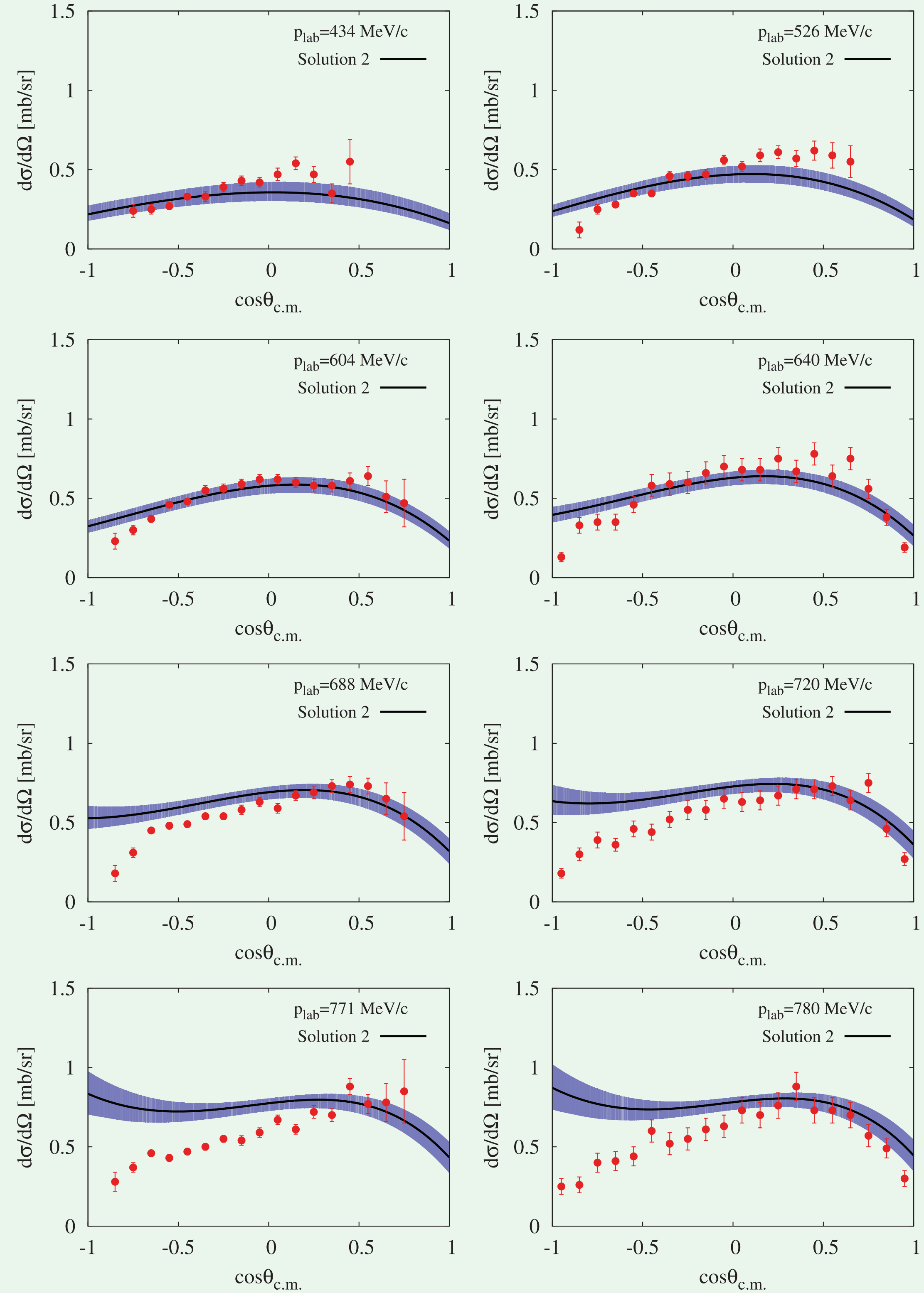


$K^+ n \to K^0 p$ charge exchange scattering

solution 1



solution 2

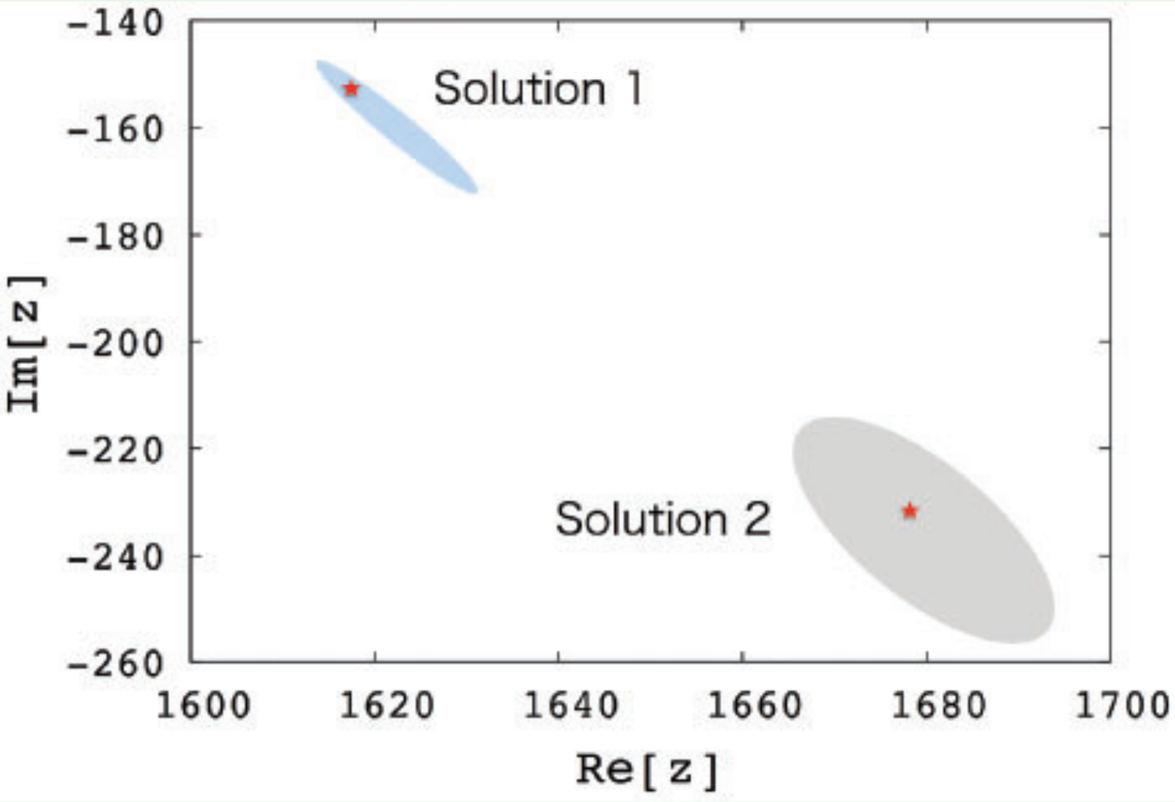


Possible broad resonance with S=+1

- a resonance pole around 1650 MeV (p_{lab} = 500 MeV) with a large width

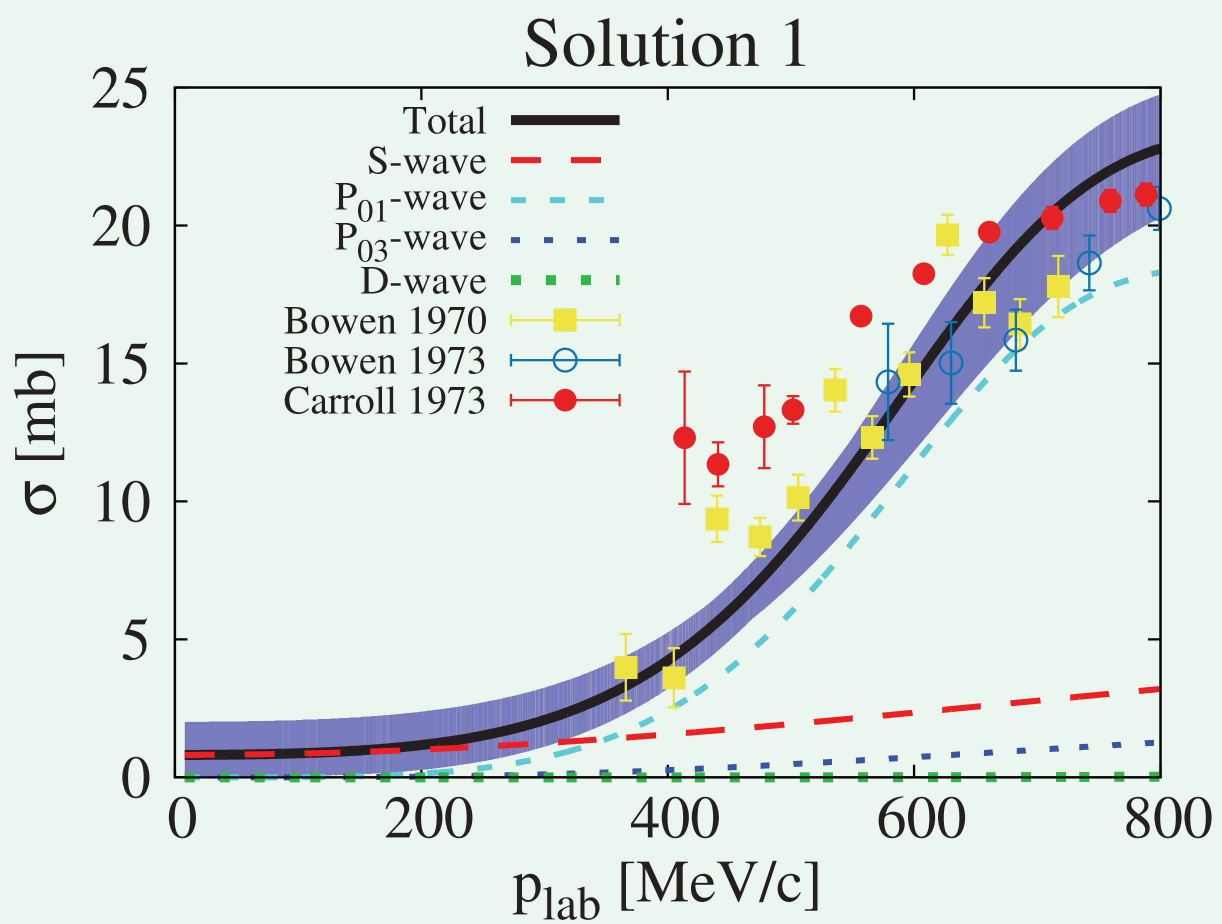
Table 3. The resonance states of Solutions 1 and 2.

amplitude (J^P)		mass [MeV]	width [MeV]
Solution 1	$P_{01}\left(\frac{1}{2}^+\right)$	1617	305
Solution 2	$P_{03}\left(\frac{3}{2}^+\right)$	1678	463



Possible broad resonance with S=+1

- increase around plab = 500 MeV is explained by a resonance



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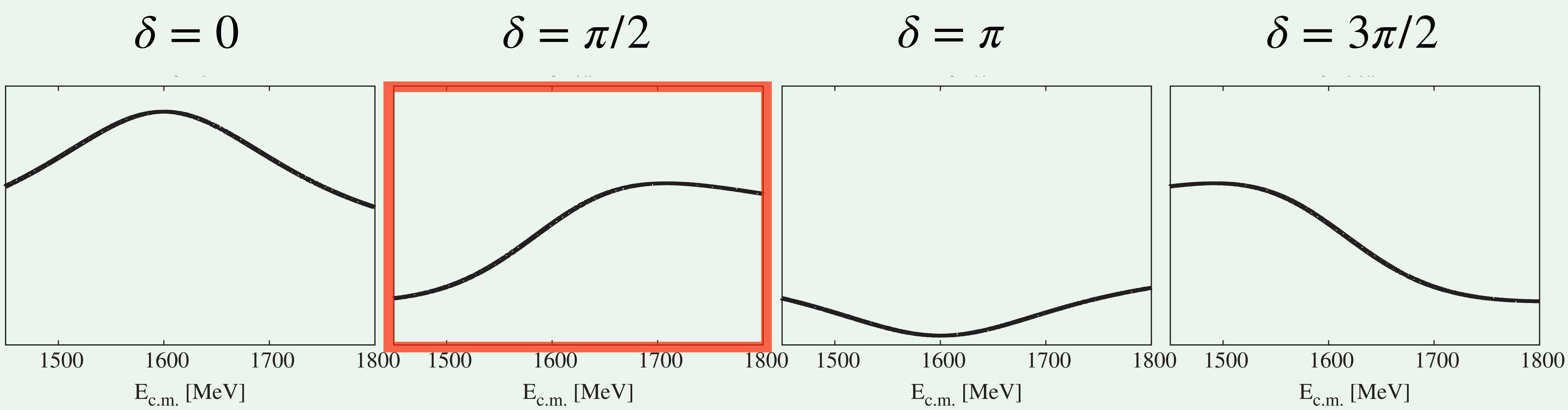
- resonance does not always have a peak structure
  → Fano resonance

Fano resonance

- resonance pole + back ground with a relative phase

$$f(E) = \frac{i}{E - M + i\Gamma/2} + be^{i\delta}$$



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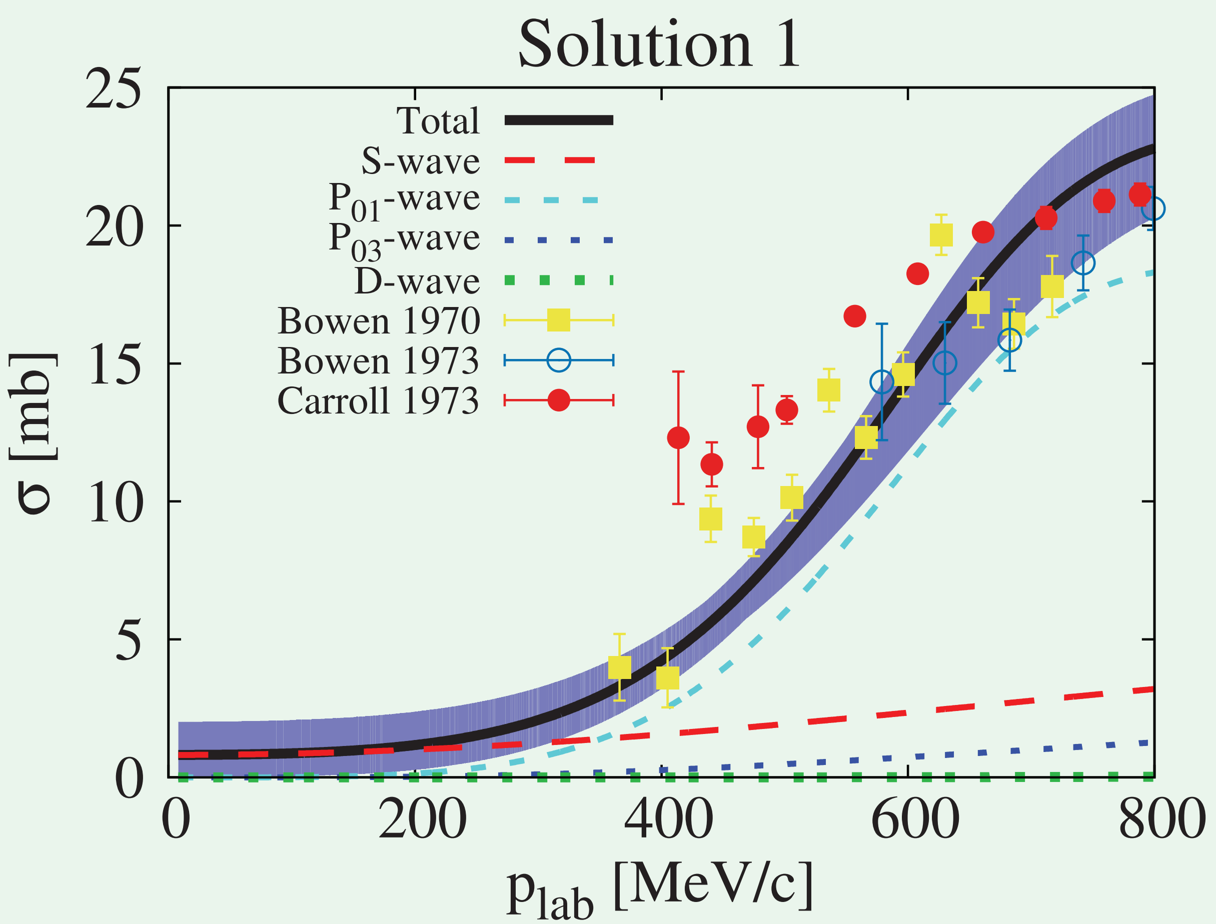

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Summary

- in-medium quark condensate can be evaluated by correlation function in soft limit
  - it connects to quark condensate to the low energy constants or scattering properties
  - S=+1 channel is much better than S=-1, because it is free from baryon resonance.
- NNLO chiral perturbation theory
  - describes I=1 (K^+p) elastic scattering amplitude very well up to 800 MeV
  - poorly reproduces I=0 scattering amplitudes
  - cannot provide increase at p_{lab} = 500 MeV in I=0 total cross section
- unitarized KN amplitude
  - describes K^+p elastic scattering amplitude very well again
  - reproduces well $K^+n \to K^+n$, K^0p amplitudes
  - provides a broad resonance with S=+1 and I=0
- owing to presence of resonance at p_{lab} = 500 MeV, one need $K^+n \to K^+n$, K^0p amplitudes below there for better extrapolation to extract in-medium quark condensate.

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