Modifications of Vector mesons moving in medium

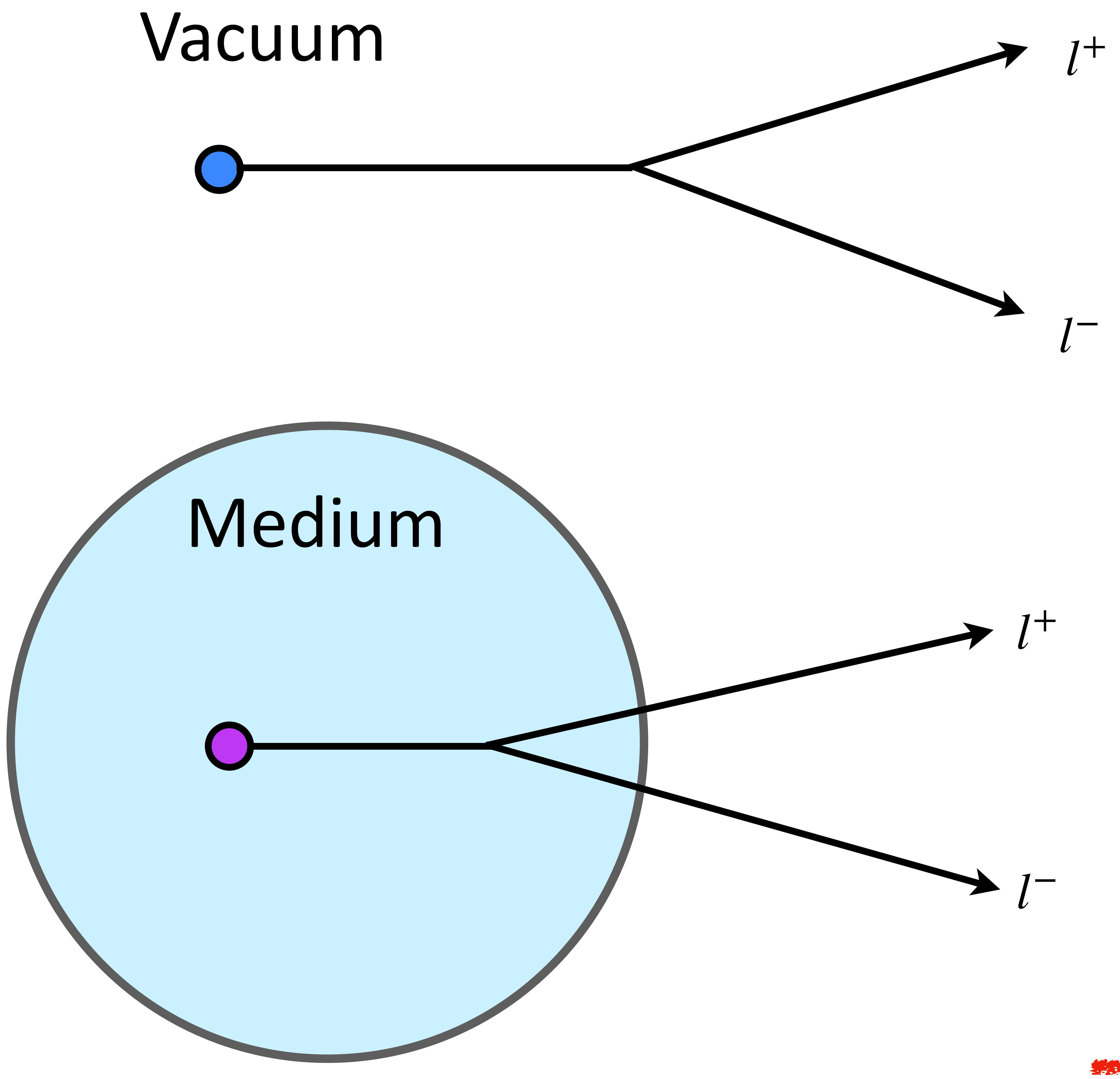
(using QCD sum rules)

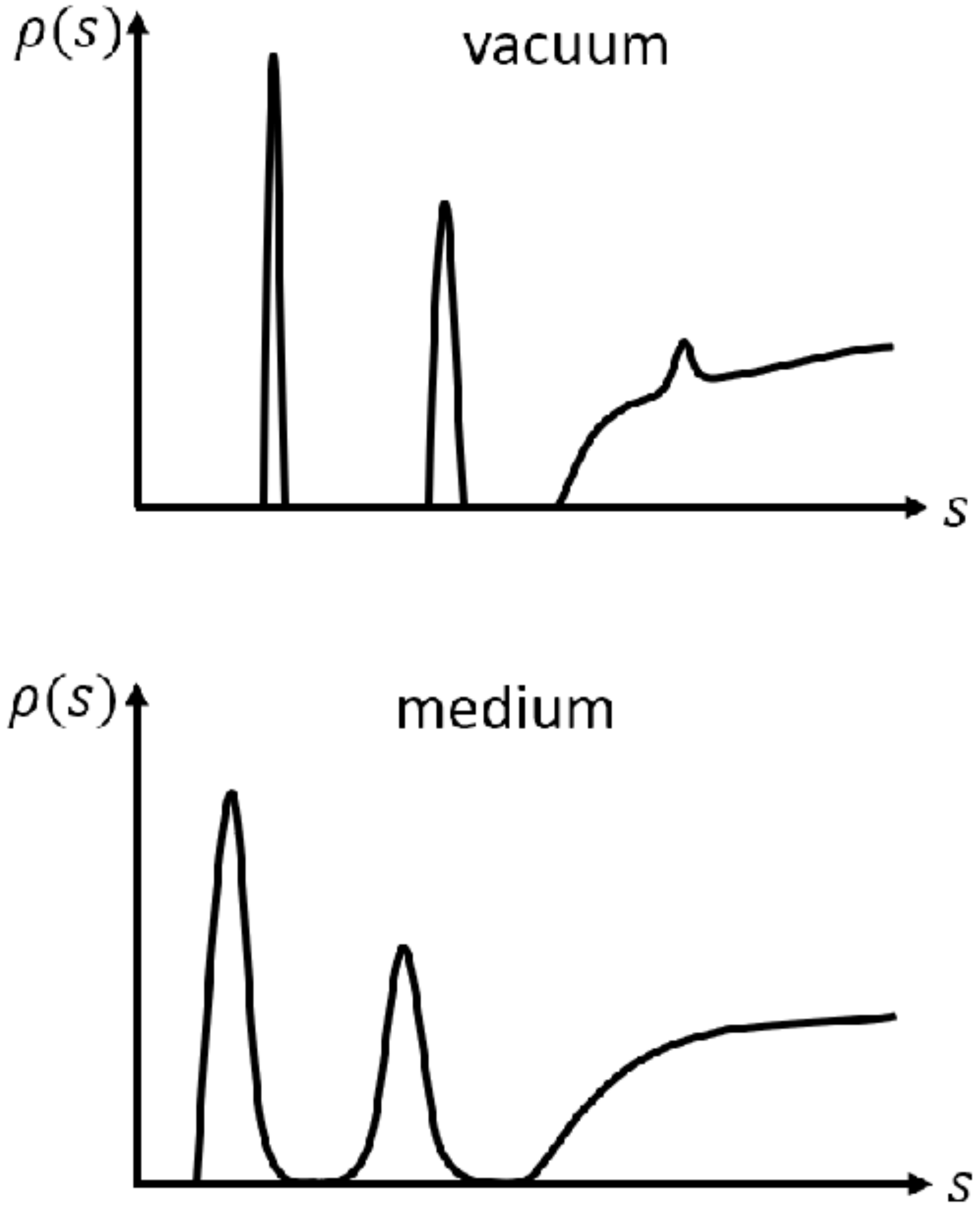
Hyungjoo Kim (Yonsei Univ.)

@ ReiMei Workshop “Hadrons in dense matter at J-PARC”,
22.02.2022

Introduction

Vector mesons are good probes to study in-medium effect.





Mass shift,
Broadening,
Dissociation,…

→ *Modification of QCD vacuum*

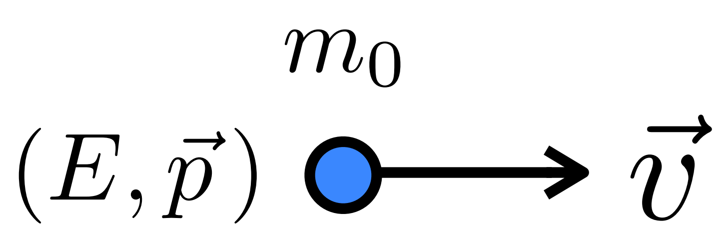
ex) light meson at finite density -> chiral sym.
Quarkonium at finite T -> conf./deconf.

Introduction

In reality, particles produced in experiments have non-zero 3-momentum.

When a massive vector particle is moving with a finite velocity…

In vacuum



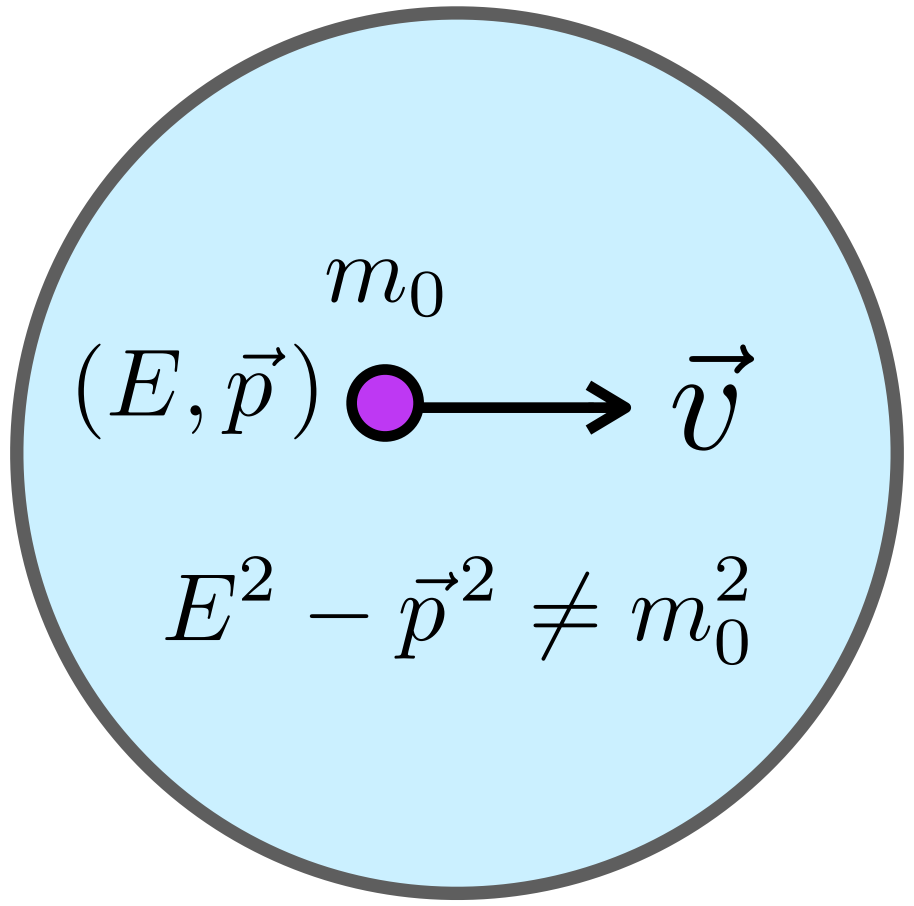
$$E^2 - \vec{p}^2 = m_0^2$$

By Lorentz symmetry,

$$E(v) = \frac{m_0}{\sqrt{1-v^2}} \approx m_0(1 + \frac{1}{2}v^2 + \cdots)$$

Polarization states are not distinguishable

In medium



$$E^2 - \vec{p}^2 \neq m_0^2$$

Lorentz symmetry is broken.

$$E(v) \approx m_0(1 + (\frac{1}{2} + \Delta\alpha)v^2 + \cdots)$$

A. Non-trivial velocity dependence

B. Longitudinal $\neq$ Transverse

Motivation

Using **QCD sum rules**,

A. Non-trivial 3-momentum dependence on mass shift

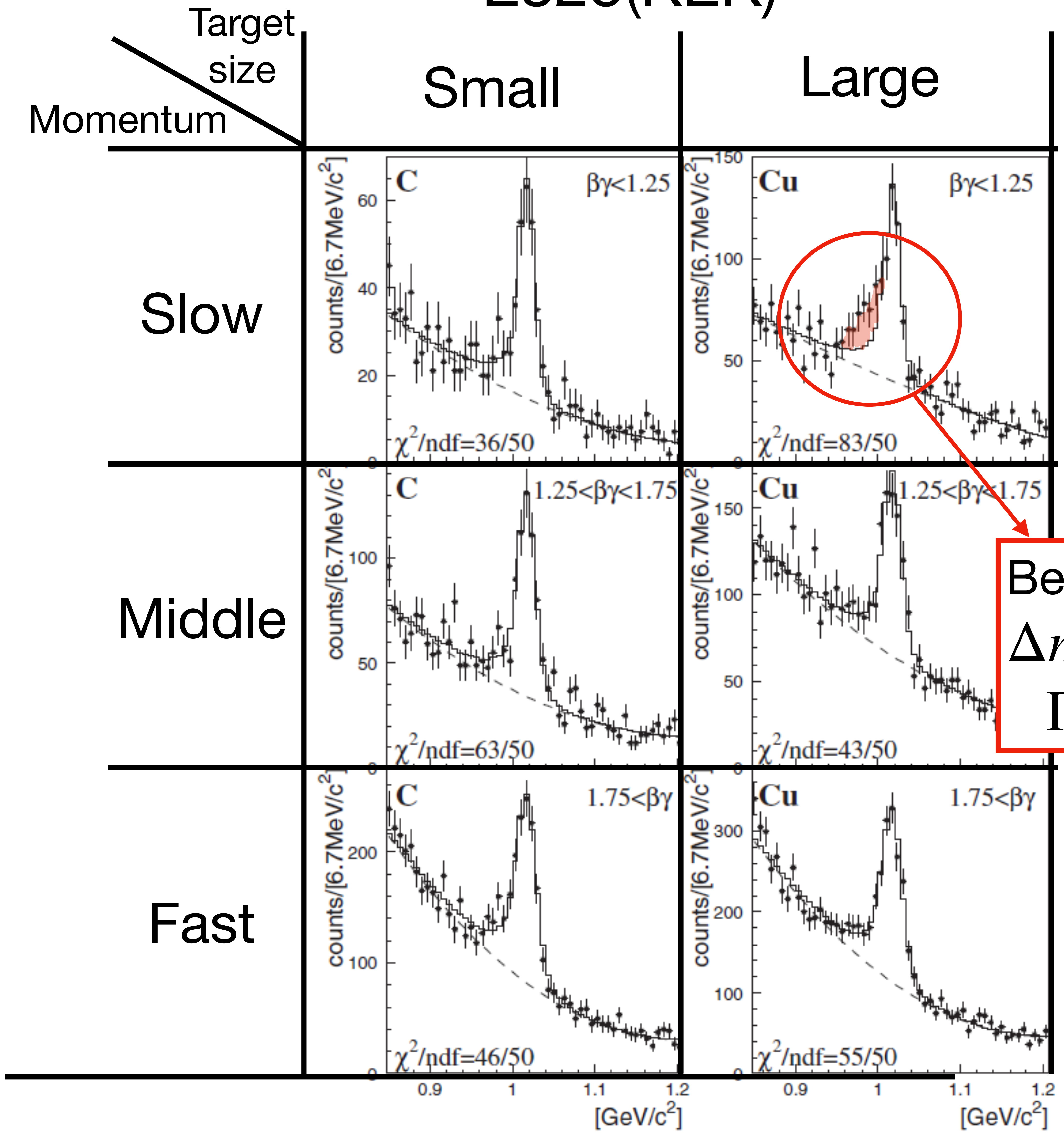
B. Difference between Longitudinal and Transverse polarizations

- **ϕ meson in normal nuclear matter density**
  - PLB 772 (2017) 194-199 H.Kim, P.Gubler, S.H.Lee
  - PLB 805 (2020) 135412 H.Kim, P.Gubler

- **J/ψ and other quarkonium near Tc** (*in progress*)

Some experiments for ϕ

- E325(KEK)



Best fit :
Δm= – 35MeV
Γ = 15MeV

- E16(J-PARC)

Re-confirm E325
Target size dependence
+ momentum dependence

R.Muto et al, PRL 98, 042501 (2007)

Some lattice simulations for Quarkonium

- Momentum-dependence of charmonium spectral functions from lattice QCD
  arXiv:1005.1209 (2010), M. B. Oktay and J. I. Skullerud
  longitudinally polarised J/ψ experiencing smaller medium modifications. Uncertainty is too large.

- Momentum dependences of charmonium properties from lattice QCD
  Nucl.Phys.A 904-905 (2013), H.-T.Ding
  Momentum dependence of charmonium V and P channels at T=0.73Tc,1,46Tc

- S-wave bottomonium states moving in a quark gluon plasma from lattice NRQCD
  JHEP03(2013)084. G. Aarts, C. Allton, S. Kim, et al.
  Momentum dependence of $\Upsilon(1S)$ and η_b. effectively T independent.

- In-medium dispersion relations of charmonia studied by maximum entropy method
  Phys.Rev.D 95 (2017) 1, 014504, A.Ikeda, M.Asakawa, M.Kitazawa
  J/ψ,η_c dispersion relation. But not changed from the vacuum within the error even at T~1.6Tc

- Thermal modifications of charmonia and bottomonia from spatial correlation functions.
  EPJ Web Conf. 175 (2018), 07021, H.-T.Ding, O.Kaczmarek, A.I.Kruse, S.Mukherjee, H.Ohno et al.
  dispersion relation in the P and V channels for both charmonia and bottomonia. Not modified upto 2.25Tc

Basic idea of QCD sum rules in medium

$$\Pi^{\mu\nu}(q_0^2, |\vec{q}\,|) = i \int e^{iq\cdot x} \langle T\{j^\mu(x) j^\nu(0)\}\rangle$$

- $q_0^2 > 0$ with fixed $|\overrightarrow{q}|$,

$$\rho(q_0^2, |\vec{q}\,|) = \frac{1}{\pi} \Delta\Pi(q_0^2, |\vec{q}\,|)$$

$$\Delta\Pi(q_0^2, |\vec{q}\,|) = \frac{\Pi(q_0^2 + i\epsilon, |\vec{q}\,|) - \Pi(q_0^2 - i\epsilon, |\vec{q}\,|)}{2i\epsilon}$$

- $q_0^2 \ll 0$ with fixed $|\overrightarrow{q}|$,

$$\Pi^{\mathrm{OPE}}(q_0^2, |\vec{q}\,|) = \sum_n C_n(q_0^2, |\vec{q}\,|) \langle \mathcal{O}_n \rangle$$

Dispersion relation

$$\underset{\underset{\text{QCD}}{\uparrow}}{\Pi^{\mathrm{OPE}}(q_0^2, |\vec{q}\,|)} = \int_0^\infty d\omega^2 \underset{\underset{\text{Hadron}}{\uparrow}}{\frac{\rho(\omega^2, |\vec{q}\,|)}{\omega^2 - q_0^2}}$$

Strategies for finite 3-momentum in QCDSR

- Separate polarizations w.r.t. 3-momentum

$$\Pi_L = \frac{1}{\vec{q}^2}\Pi^{00}, \quad \Pi_T = -\frac{1}{2}\left(\frac{1}{q^2}\Pi^\mu_\mu + \frac{1}{\vec{q}^2}\Pi^{00}\right)$$

For more details,
PRC57.927 (1998), S.H.Lee
PRC58 (1998) 2939, S.Leupold, U.Mosel
PLB 805 (2020) 135412 H.Kim, P.Gubler

- Change of variables $(q_0^2, |\vec{q}\,|) \rightarrow (q^2, |\vec{q}\,|)$

$$\Pi^{\rm OPE}(q_0^2, |\vec{q}\,|) = \int_0^\infty d\omega^2 \frac{\rho(\omega^2, |\vec{q}\,|)}{\omega^2 - q_0^2}$$

$$\Big\downarrow \; \omega^2 \rightarrow s + \vec{q}^{\,2}$$

$$\Pi^{\rm OPE}(q^2, |\vec{q}\,|) = \int_{-\vec{q}^{\,2}}^\infty ds \frac{\rho(s, |\vec{q}\,|)}{s - q^2}$$

- Absorb trivial momentum dependence inside energy variable
- Remaining momentum dependence stands for pure non-trivial one

- Momentum dependent spectral parameters → (f, m, s_0)

$$\rho(s, |\vec{q}\,|) = f(|\vec{q}\,|)\delta(s - m^2(|\vec{q}\,|)) + \theta(s - s_0(|\vec{q}\,|))\rho_{\rm continuum}(s)$$

: *Simple pole+continuum ansatz*

OPE structure with finite 3-momentum

$$\begin{aligned}\Pi^{\mathrm{OPE}}(q_0,\vec{q}) &= \sum\nolimits_n C_n(q_0,\vec{q})\langle\mathcal{O}_n\rangle \\ &= C_I(q_0,\vec{q}) + C_s(q_0,\vec{q})\langle\mathcal{O}_s\rangle + C_{ns}(q_0,\vec{q})\langle\mathcal{O}_{ns}\rangle \\ &= C_I(q^2) + C_s(q^2)\langle\mathcal{O}_s\rangle + (D(q^2) + E(q^2)\vec{q}^{\,2} + F(q^2)\vec{q}^{\,4} + \cdots)\langle\mathcal{O}_{ns}\rangle\end{aligned}$$

- Non-trivial 3-momentum dependence appears from Wilson coefficients of non-scalar operators, i.e. $E(q^2)$ and $F(q^2)$

- Automatically, $E_L \neq E_T$ and $F_L \neq F_T$ → origins for $L \neq T$

- Not Taylor expansion, determined by operator dimension&spin

OPE structure with finite 3-momentum

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Operators and their V.E.V.s

- For heavy quarkonium at finite T,

Upto dim-4, Scalar : $\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu}$ Non-scalar : $\frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu}$

$$\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \rangle_T = G_0(T) \qquad \langle \frac{\alpha_s}{\pi} G^a_{\mu\alpha} G^{a\alpha}_{\nu} \rangle_T = (u_\mu u_\nu - \frac{1}{4} g_{\mu\nu}) G_2(T)$$

$$u^\mu = (1,0,0,0)$$

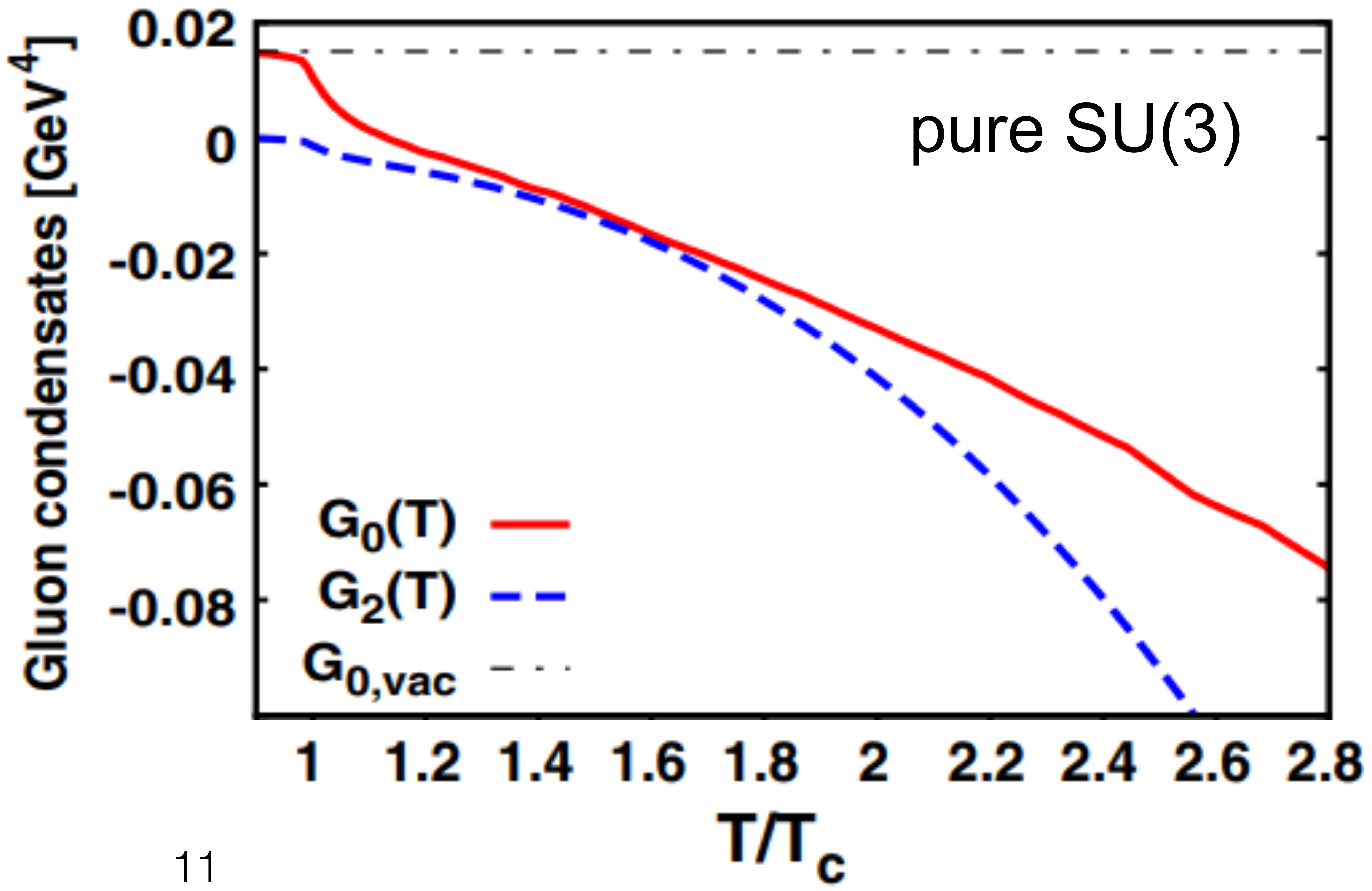
In vacuum

$$G_0(0) = 0.012\,\mathrm{GeV}^4$$

$$G_2(0) = 0\,\mathrm{GeV}^4$$

NPB147, 448 (1979),
NPB147, 385 (1979)

At finite T



PRD.82.054008,
K. Morita, S.H.Lee

Operators and their V.E.V.s

- For ϕ meson in nuclear matter,
  Complete OPE upto dim-6

Scalar : $\bar{s}s,\ G^a_{\mu\nu}G^{a\mu\nu},\ \bar{s}\gamma^\mu(D^\nu G_{\mu\nu})s,\ D^\mu G^a_{\alpha\mu}D_\nu G^{a\alpha\nu},\ \bar{s}t^a\gamma^5\gamma^\mu s\bar{s}t^a\gamma^5\gamma_\mu s,$

Quark non-scalar : $\bar{s}(D^\mu G_{\alpha\mu})\gamma_\beta s,\ \bar{s}\{iD_\alpha, \tilde{G}_{\beta\mu}\}\gamma^5\gamma^\mu s,\ m_q\bar{s}D_\alpha D_\beta s,\ \bar{s}\gamma_\alpha iD_\beta s,\ \bar{s}t^a\gamma^5\gamma_\alpha s\bar{s}t^a\gamma^5\gamma_\beta s,\ \bar{s}\gamma_\alpha D_\beta D_\gamma D_\delta s,$

Gluon non-scalar : $G^a_{\alpha\mu}G^{a\mu}_\beta,\ G^a_{\mu\nu}D_\beta D_\alpha G^{a\mu\nu},\ G^a_{\alpha\mu}D^\mu D^\nu G^a_{\beta\nu},\ G^a_{\alpha\mu}D_\beta D_\nu D^{a\mu\nu},\ G^a_{\alpha\mu}D_\delta D_\gamma G^{a\mu}_\beta$

linear density approximation at normal nuclear matter density $\rho_0 = 0.17\text{fm}^{-3}$

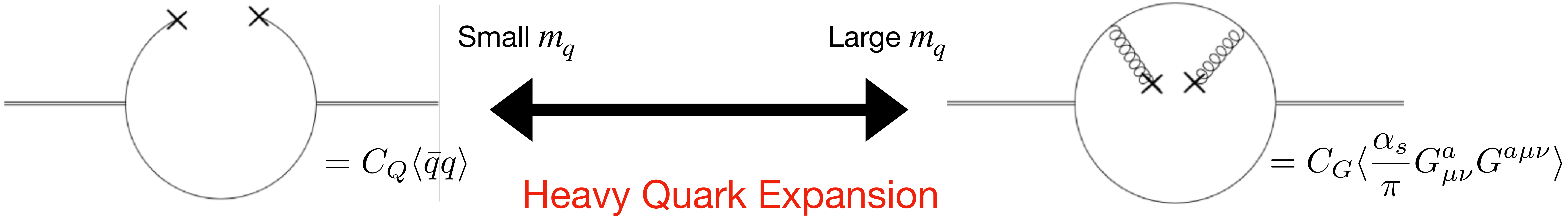
$$\langle \mathcal{O} \rangle_\rho = \langle 0 | \mathcal{O} | 0 \rangle + \rho_0 \langle N | \mathcal{O} | N \rangle$$

For details about V.E.V.s,
- PLB 805 (2020) 135412, H.Kim, P.Gubler
- Prog.Part.Nucl.Phys.106,1 (2019), P.Gubler, D.Satow

Light quark v.s. Heavy quark

Wilson coefficient of gluon operator for light quark is subtle.

$$C_G^{\mathrm{lightquark}}\langle\mathcal{O}_G\rangle \neq \lim_{m_q\to 0} C_G^{\mathrm{heavyquark}}\langle\mathcal{O}_G\rangle \quad \sim m_q^{-1},\ \log(m_q)$$



Small m_q

Large m_q

$= C_Q\langle\bar{q}q\rangle$

$= C_G\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\rangle$

Heavy Quark Expansion

$$\langle\bar{q}q\rangle = -\frac{1}{12m_q}\langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\rangle + \mathcal{O}(\frac{1}{m_q^2})$$

1. Calculate heavy quark expansion of quark operators
2. $C_G^{\mathrm{lightquark}}\langle\mathcal{O}_G\rangle = \lim_{m_q\to 0}\left[C_G^{\mathrm{heavyquark}}\langle\mathcal{O}_G\rangle - C_Q\langle\mathcal{O}_Q\rangle\Big|_{\langle\mathcal{O}_Q\rangle\overset{\mathrm{HQE}}{\to}\sum_i k_i\langle\mathcal{O}_G^i\rangle}\right]$

PLB772,194(2017)
H.J.Kim, P.Gubler, S.H.Lee

Results

- Quarkonium moving at finite temperature
- ϕ meson moving in nuclear matter

Charmonium at rest

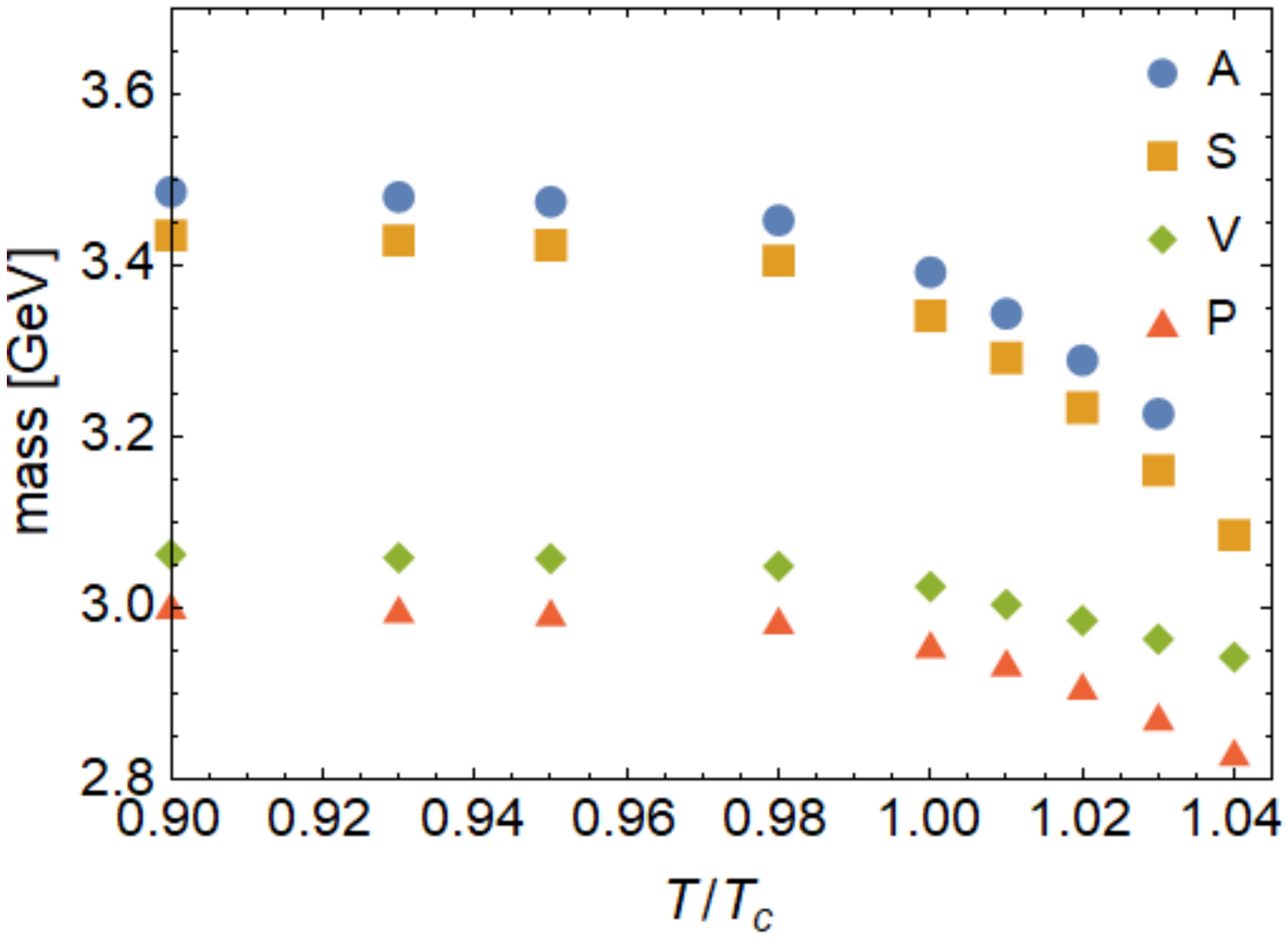
In vacuum

	$A(\chi_{c1})$	$S(\chi_{c0})$	$V(J/\psi)$	$P(\eta_c)$
QCDSR [GeV]	3.49	3.43	3.06	3.00
Exp. [GeV]	3.51	3.41	3.096	2.98

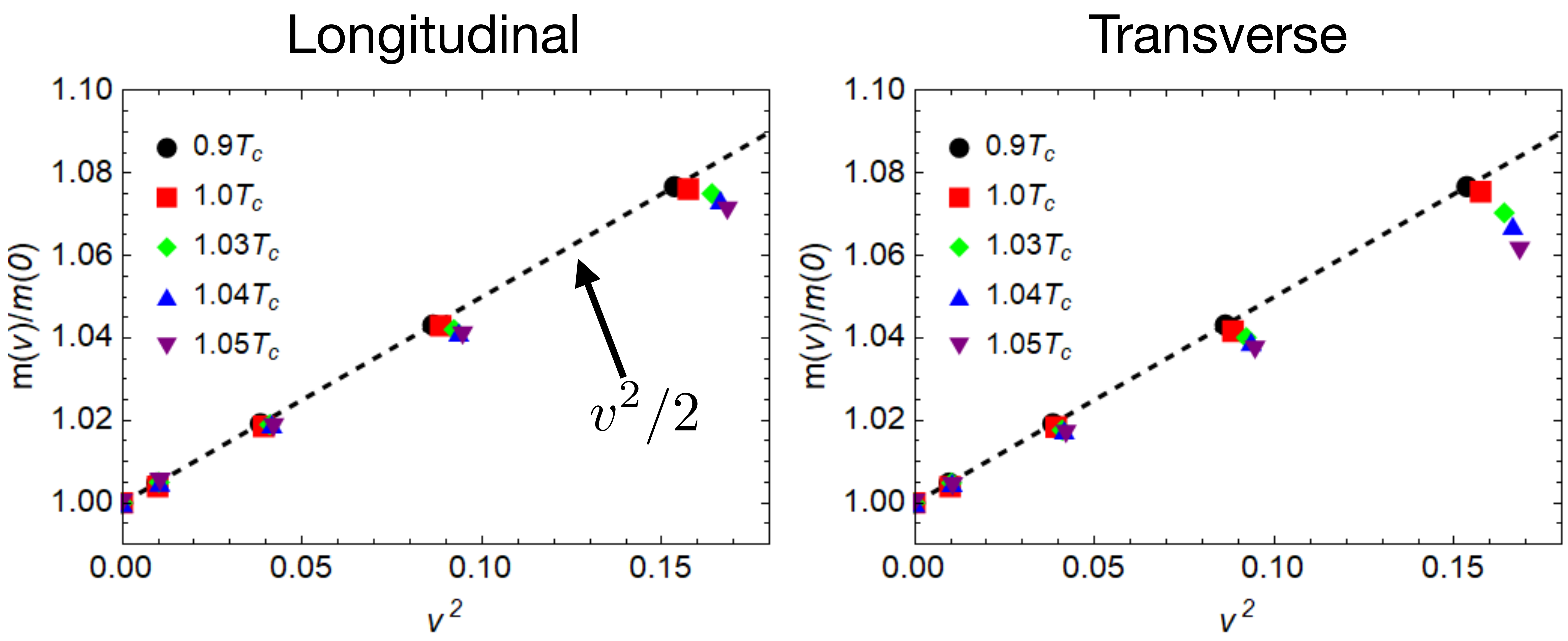
$m_c = 1.26\mathrm{GeV}$

$\alpha_s(8m_c^2) = 0.21$

Near Tc



J/ψ longitudinal v.s. transverse

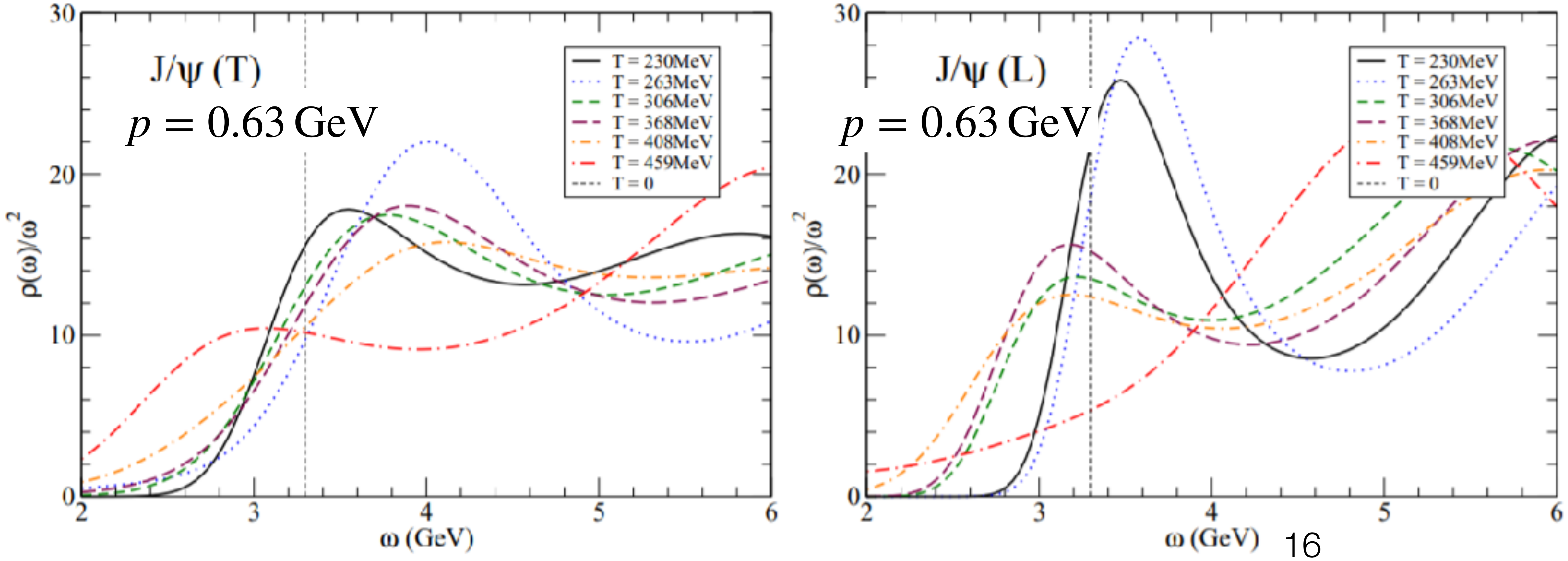


$$v \approx |\vec{q}|/m(0,T)$$

$$\frac{m(v,T)}{m(0,T)} \approx 1 + \left(\frac{1}{2} + \alpha\right)v^2 + \cdots$$

$$\alpha^V_{L,T} < 0,\ |\alpha^V_L| < |\alpha^V_T|$$

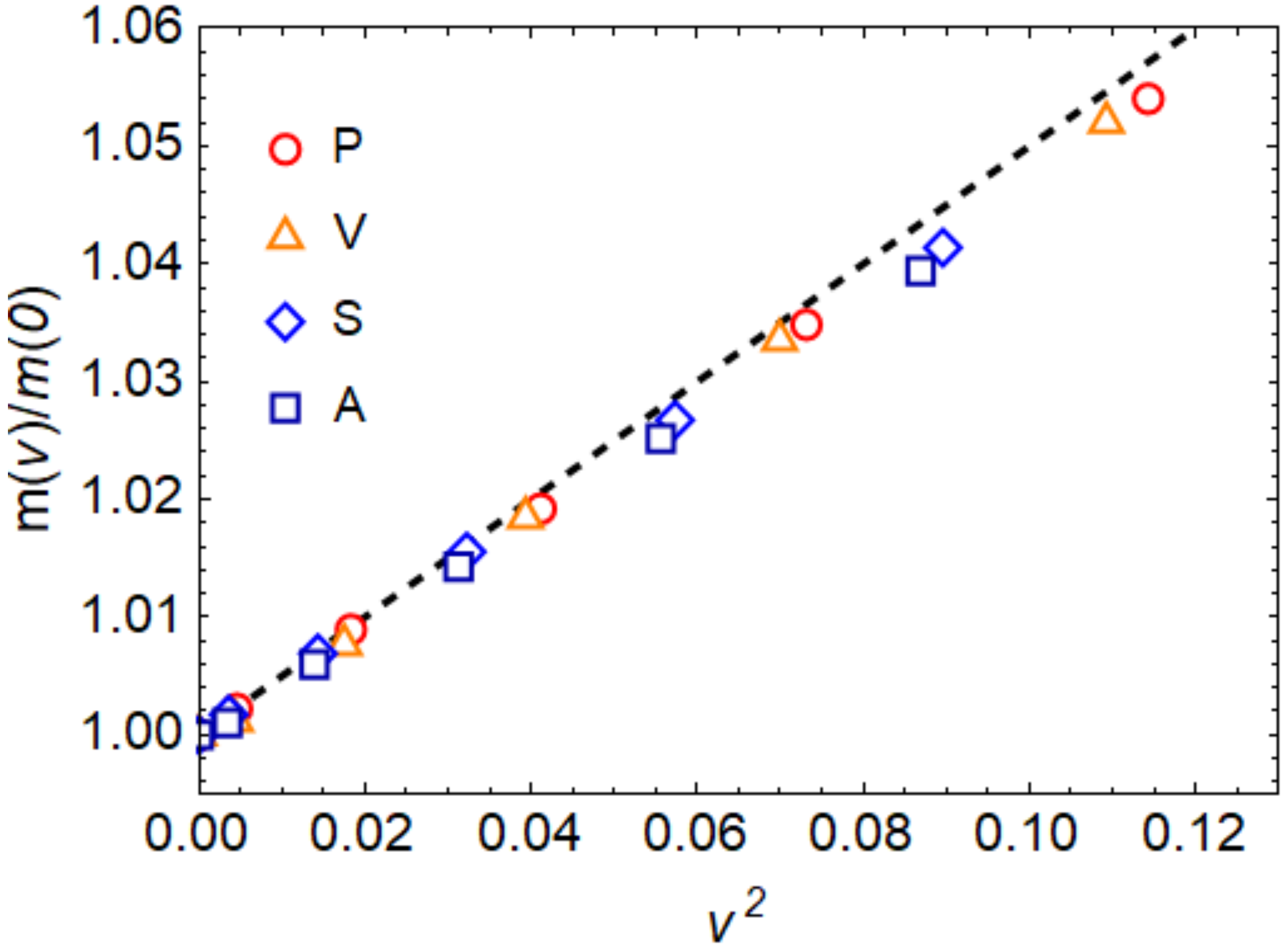
Momentum-dependence of charmonium spectral functions from lattice QCD

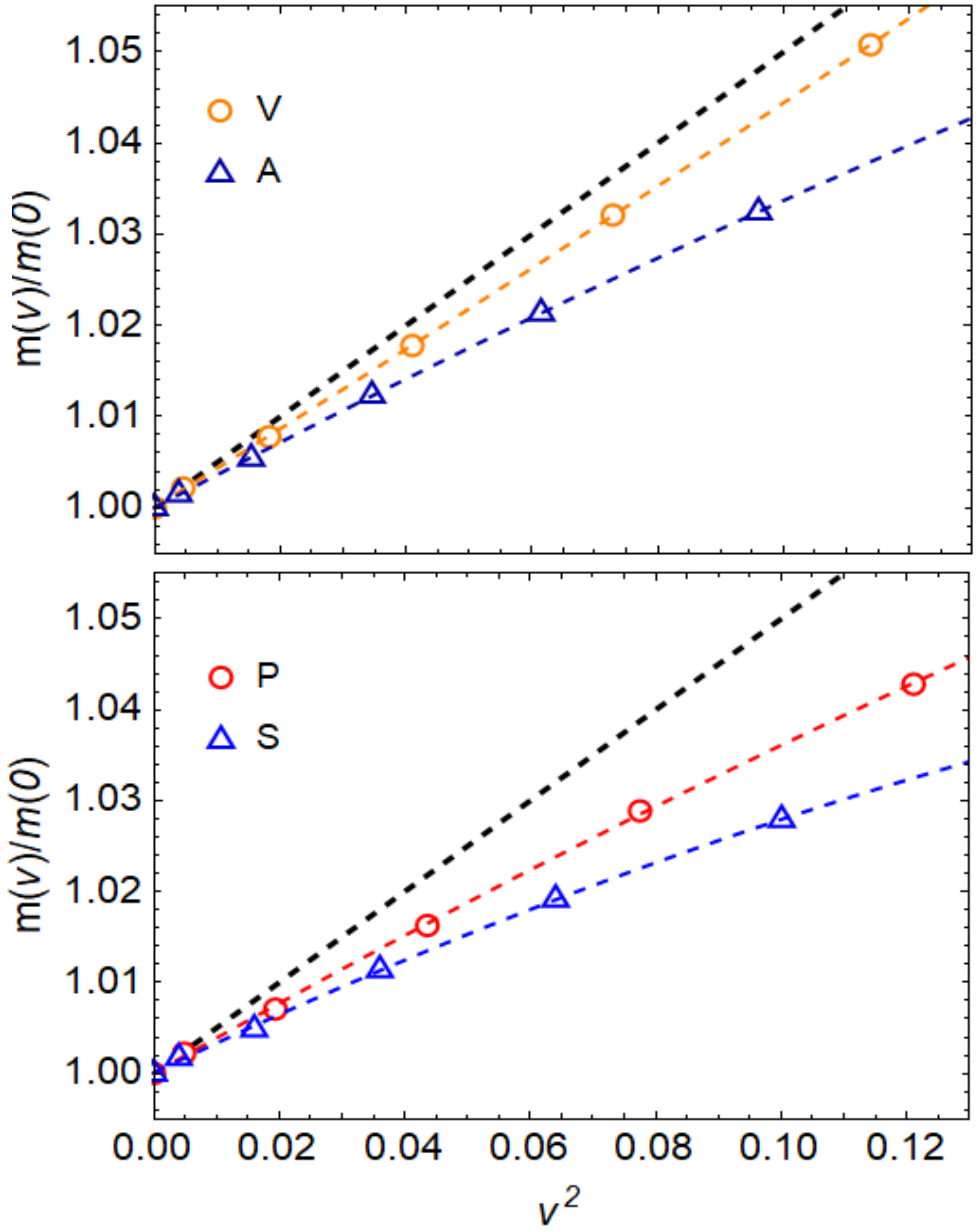


arXiv:1005.1209 [hep-lat]
M.B.Oktay and J.I.Skullerud

“longitudinally polarized J/ψ experiences smaller medium modifications.”

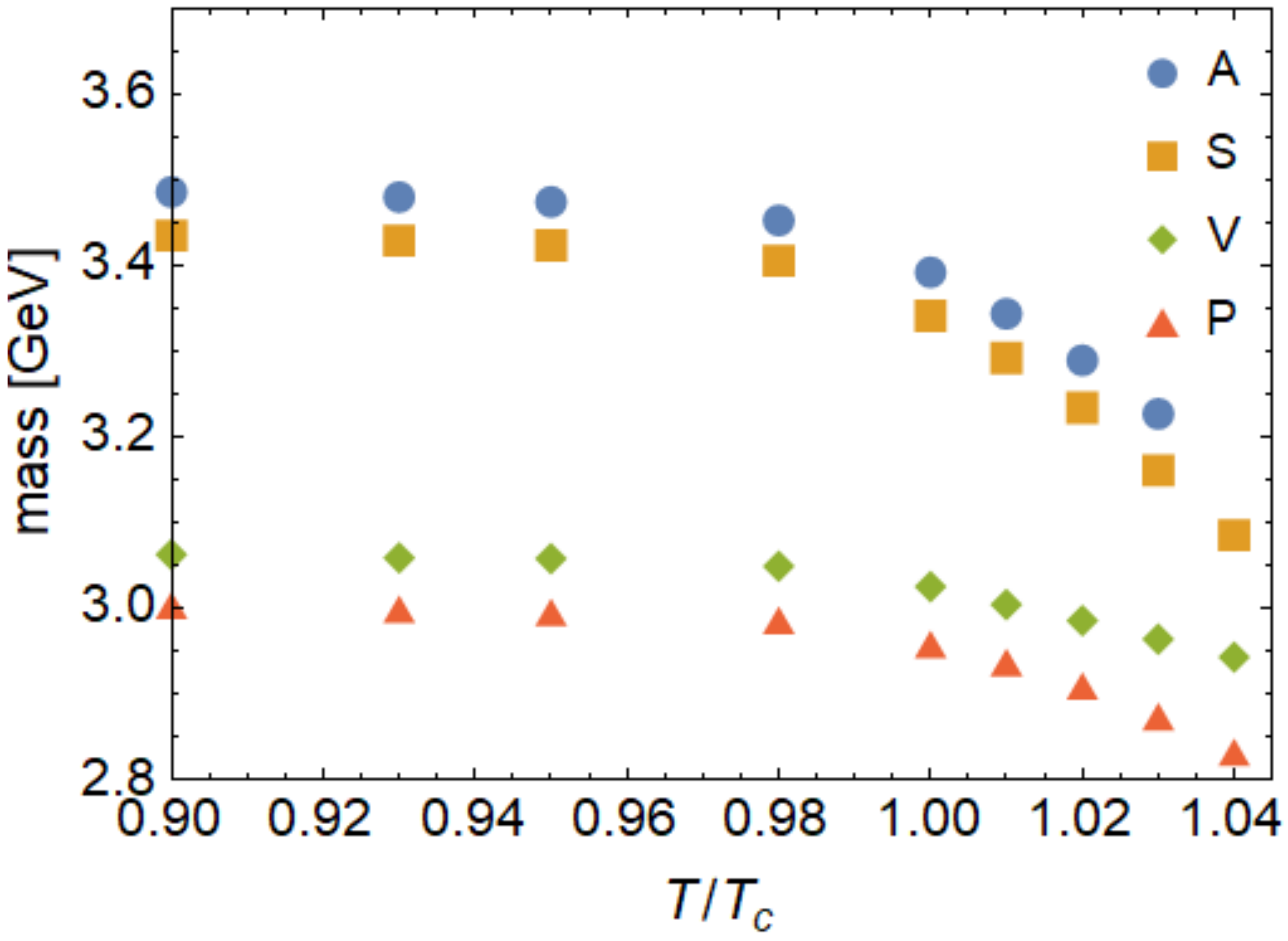
Charmonium S-wave v.s. P-wave


T=1.0 Tc


T=1.03 Tc

$$|\alpha_{\text{S-wave}}| < |\alpha_{\text{P-wave}}|$$

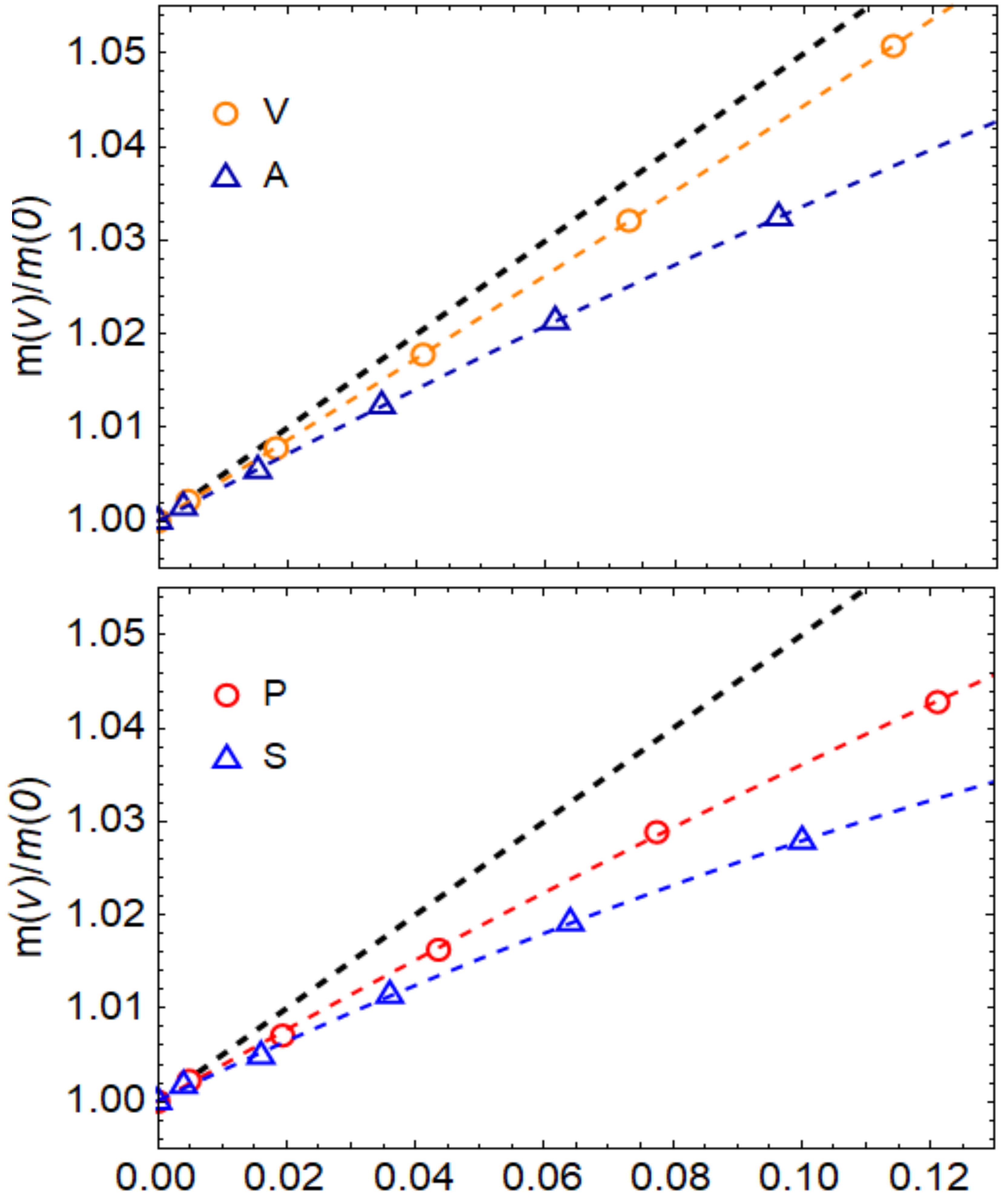
Charmonium S-wave v.s. P-wave



- Sensitivity to T changes

V < P < A < S

T=1.03 Tc

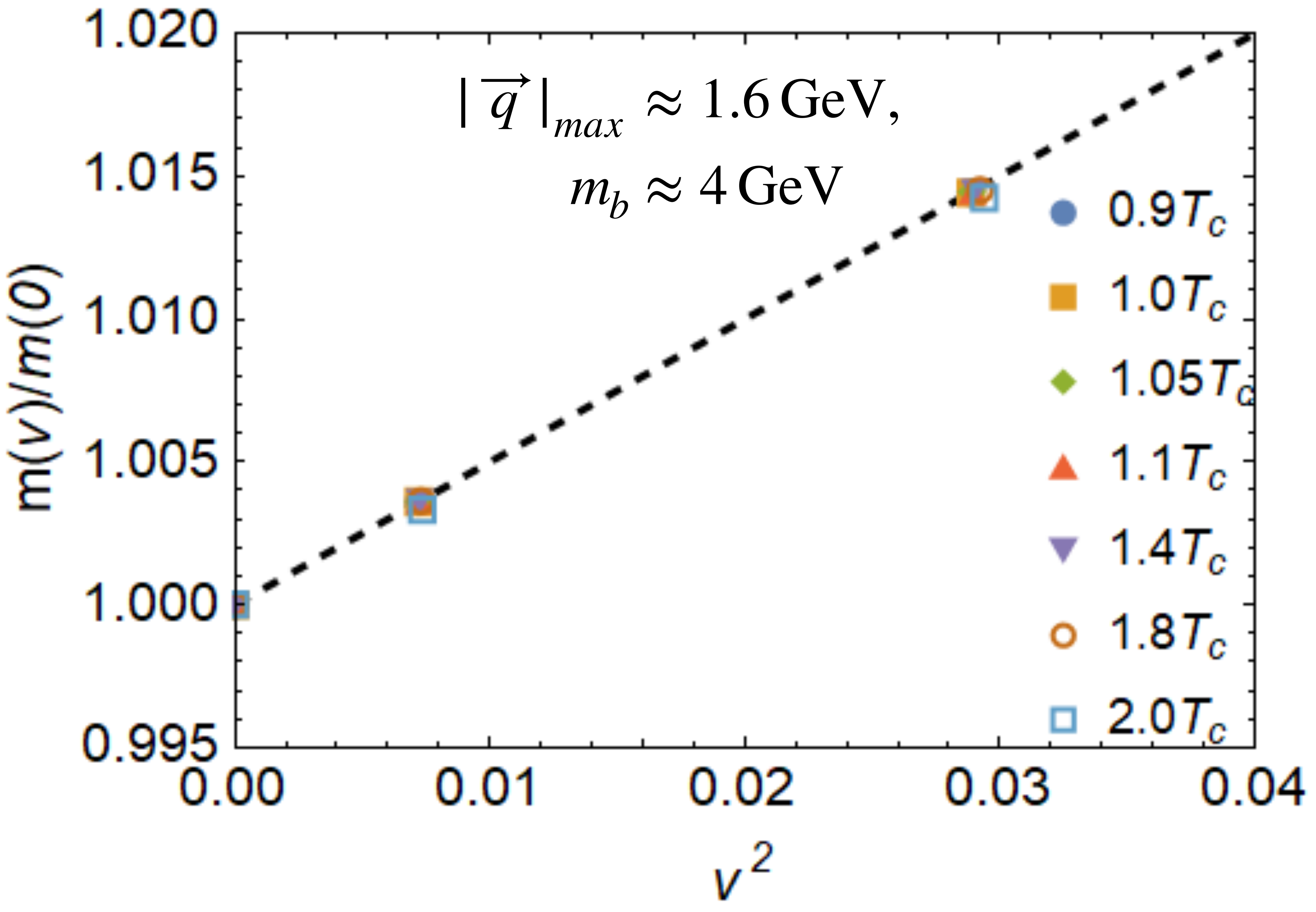


- Deviation from the trivial line

V < P < A < S

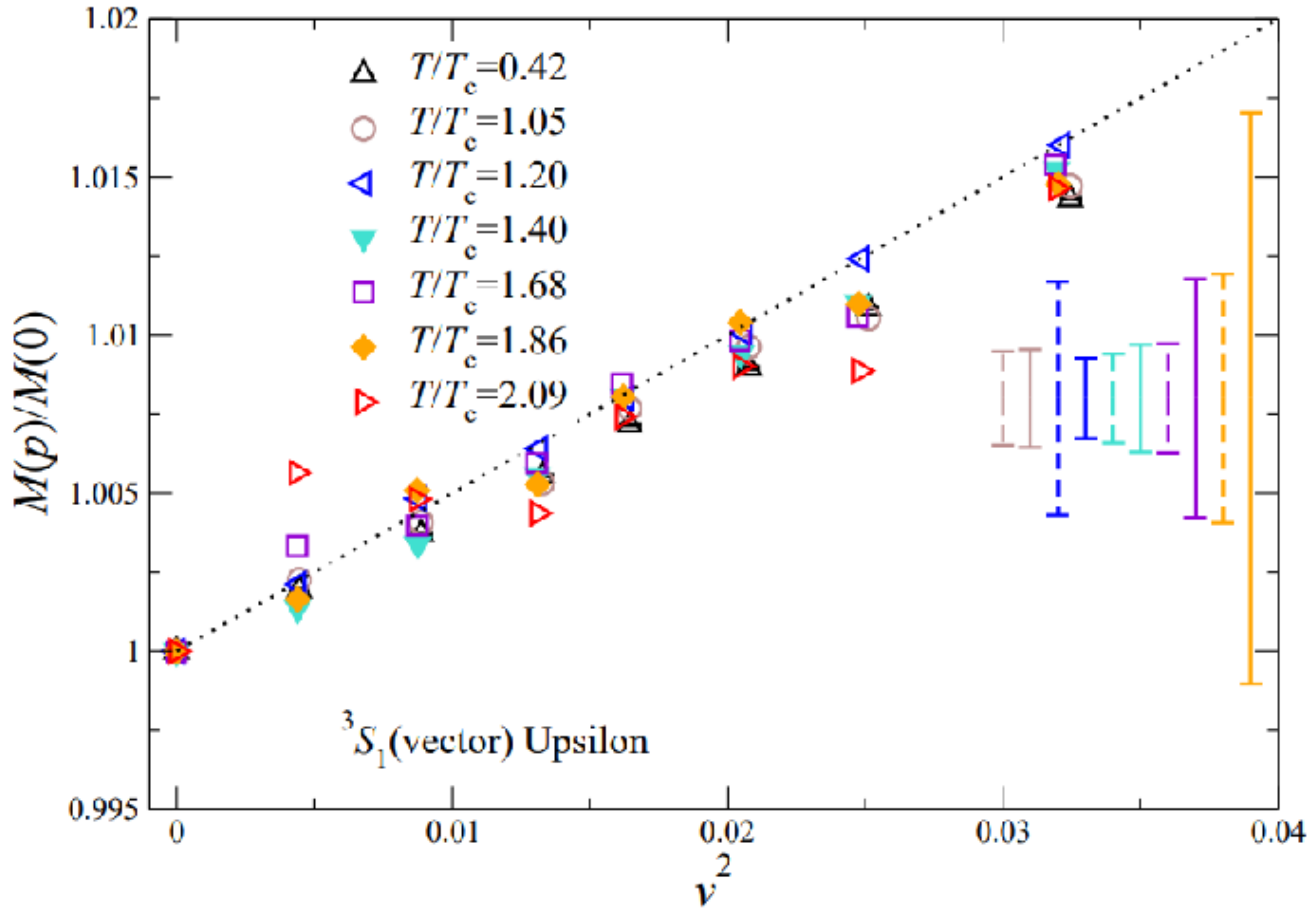
Bottomonium : Upsilon(1S)

- Negligible non-trivial momentum dependence



S wave bottomonium states moving in a quark-gluon plasma from lattice NRQCD

JHEP03(2013)084. G. Aarts, C. Allton, S. Kim, et al.



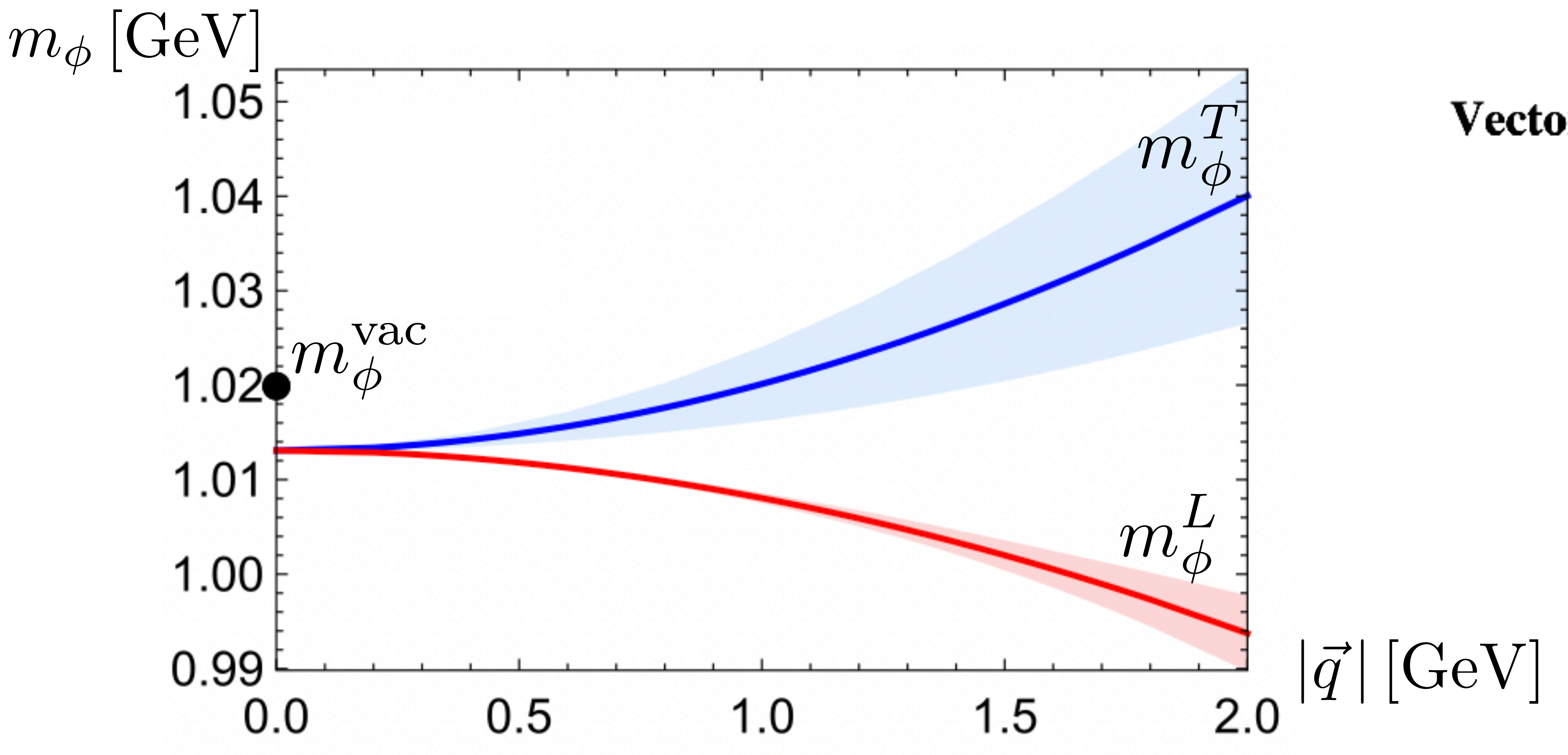
OPE structure,

$$\Pi^{\mathrm{OPE}}(q^2,\vec{q}^2) \approx C_I(q^2) + C_s(q^2)G_0(T) + \left(D(q^2) + E(q^2)\underset{\approx 0.16}{\boxed{\frac{\vec{q}^2}{m_q^2}}}\right)G_2(T)$$

c.f. $\mathcal{O}\left(\frac{|\vec{q}|_{max}}{m_c}\right) \sim 1$

Suppressed by heavy quark mass

ϕ meson result



$$\frac{m_\phi^{L/T}(\rho)}{m_\phi^{\text{vac}}} = 1 + \left(a + b^{L/T}|\vec{q}|^2\right)\frac{\rho}{\rho_0}$$ upto 3 GeV

$$m_\phi^{\text{vac}} = 1.020\,\text{GeV},$$

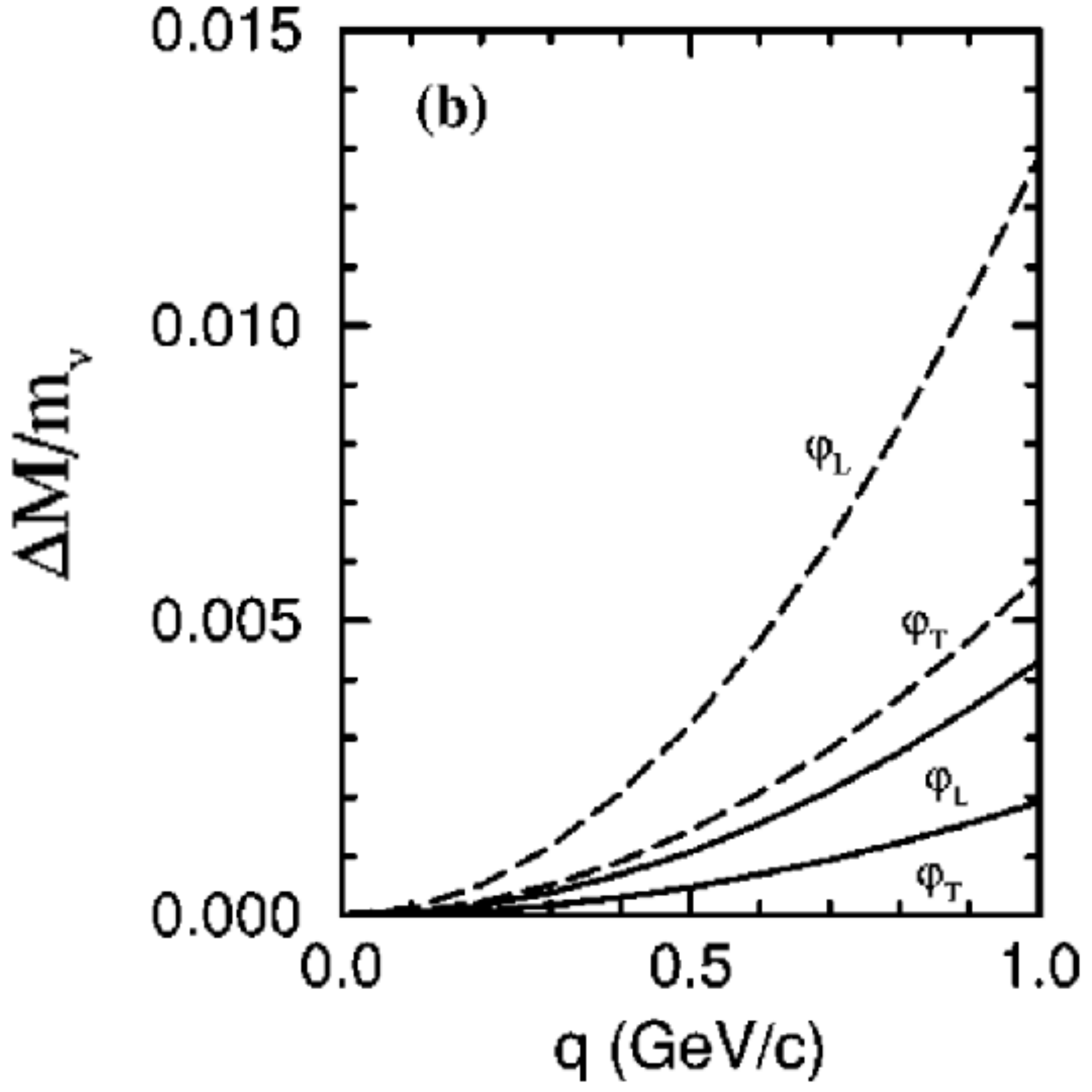
$$a = -0.0067, \qquad m_s\langle\bar{s}s\rangle_N = 52.9\,\text{MeV}$$

$$b^T = 0.0067 \pm 0.0034\,\text{GeV}^{-2},$$

$$b^L = -0.0048 \pm 0.0008\,\text{GeV}^{-2}$$

Vector mesons in the nuclear medium with a finite three-momentum

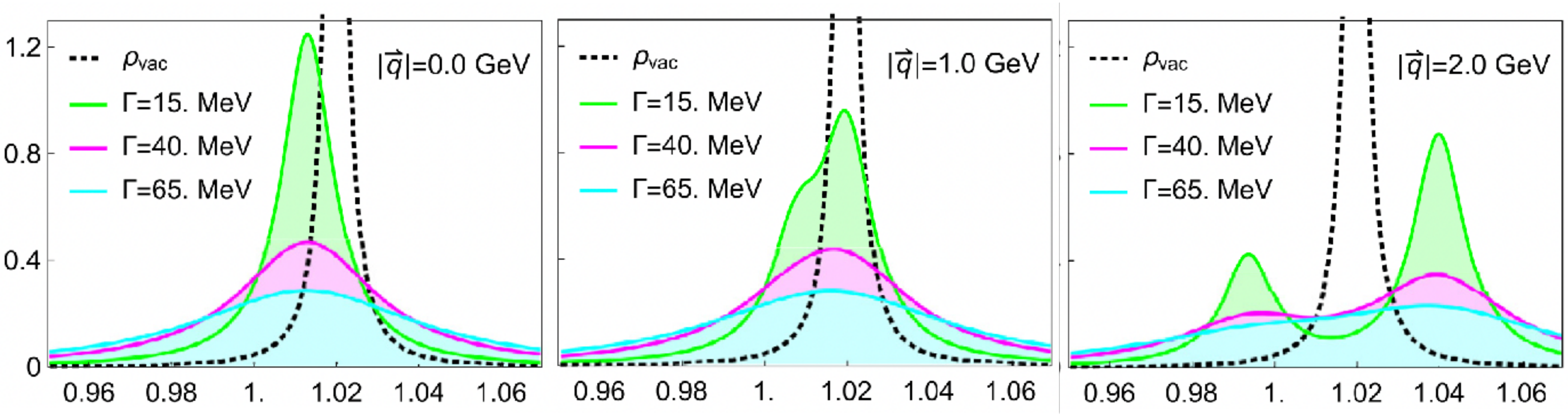
PRC57.927 (1998), S.H.Lee



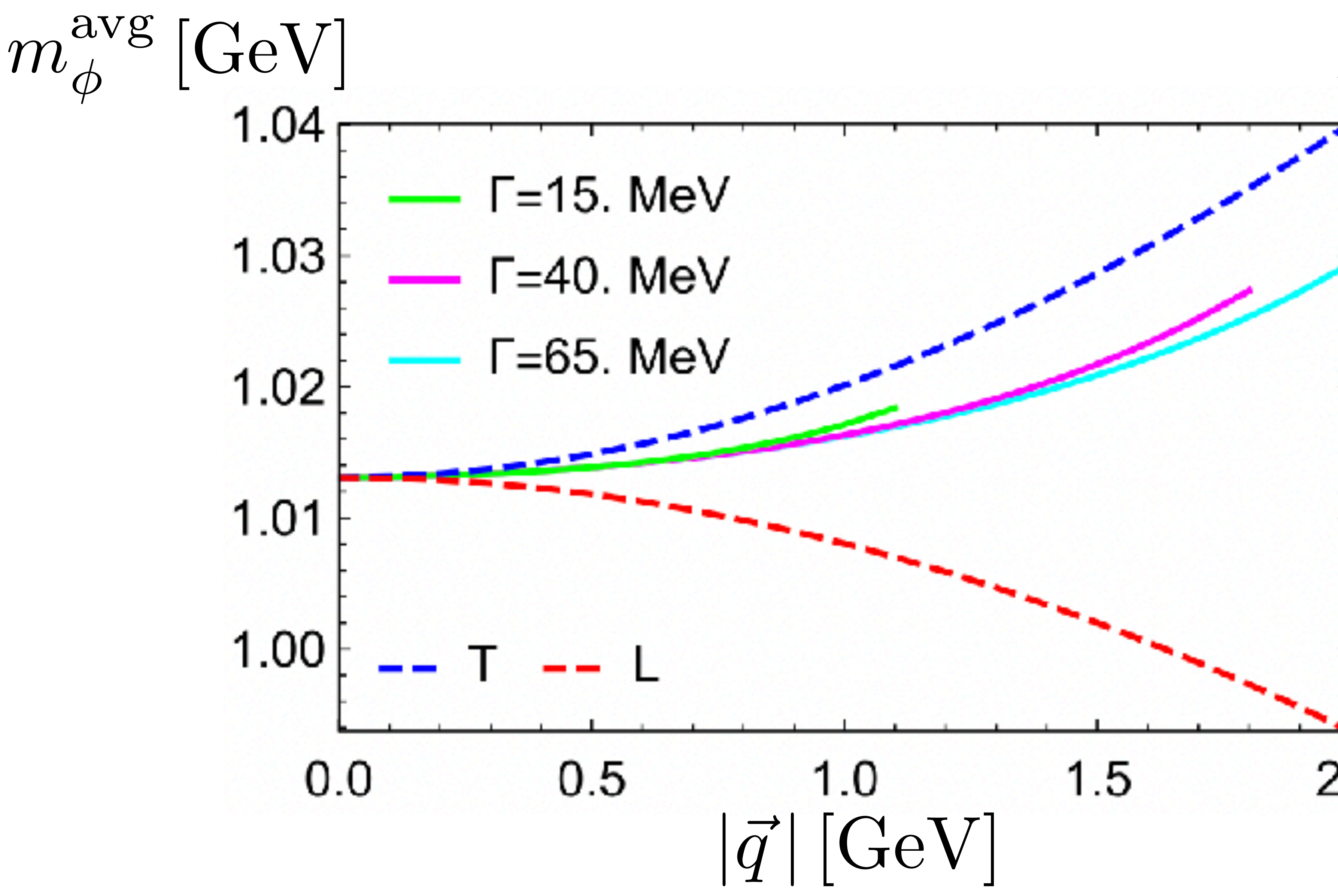
Finite width (Breit-Wigner form)

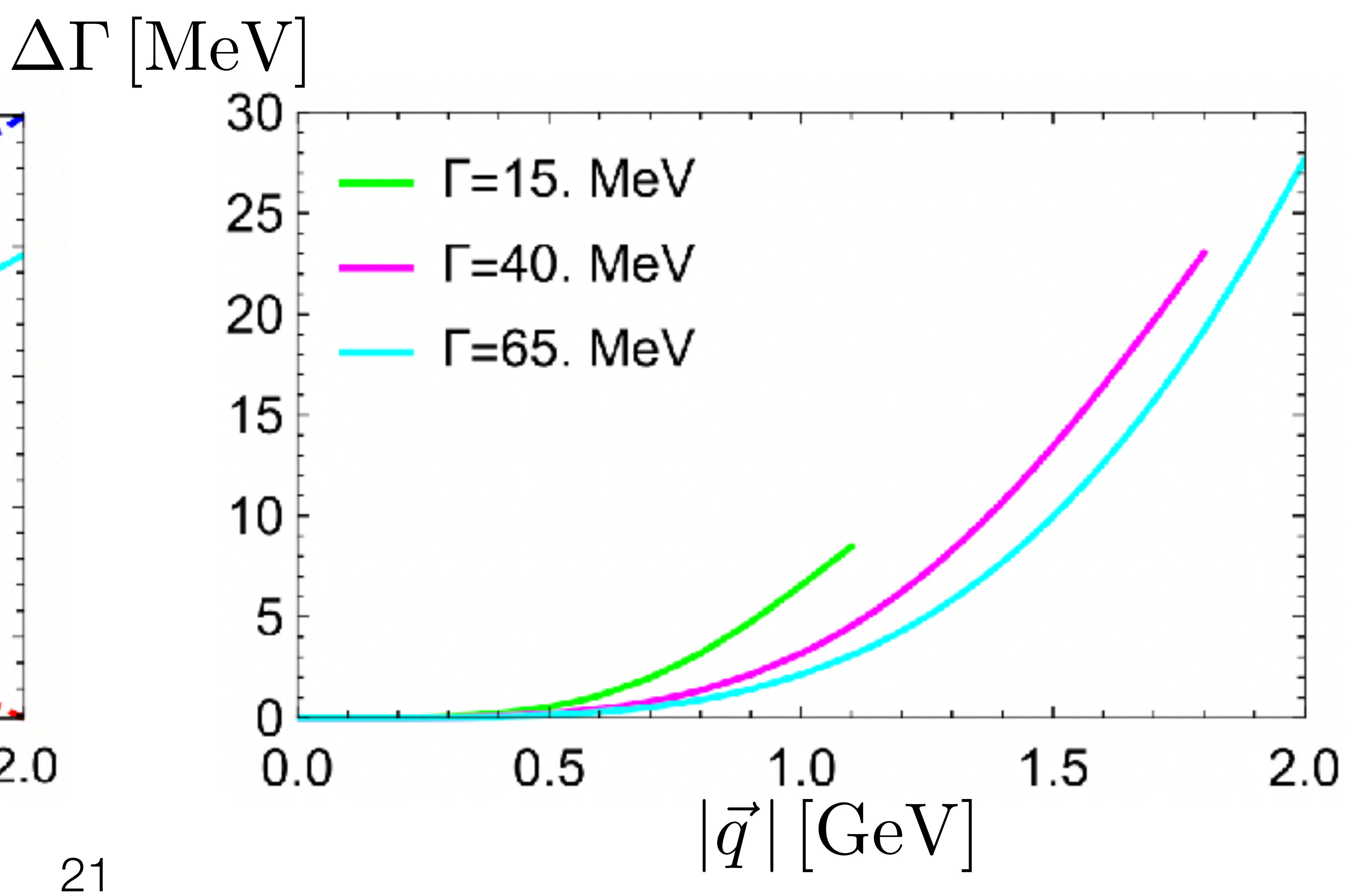
- Averaged spectra

$$\frac{\rho_L + 2\rho_T}{3}$$



- Single Breit-Wigner peak





Summary

We investigated non-trivial effects appear when ϕ meson and quarkonium are moving in hot/dense medium using QCDSR.

For quarkonium,

- J/ψ : Longitudinal < Transverse
- Strength of the non-trivial momentum dependence : V<P<A<S
- Upsilon has small momentum dependence

For ϕ meson,

- Mass shift of T and L have a opposite sign
- Averaged mass shift is positive. Effective width increases

backup

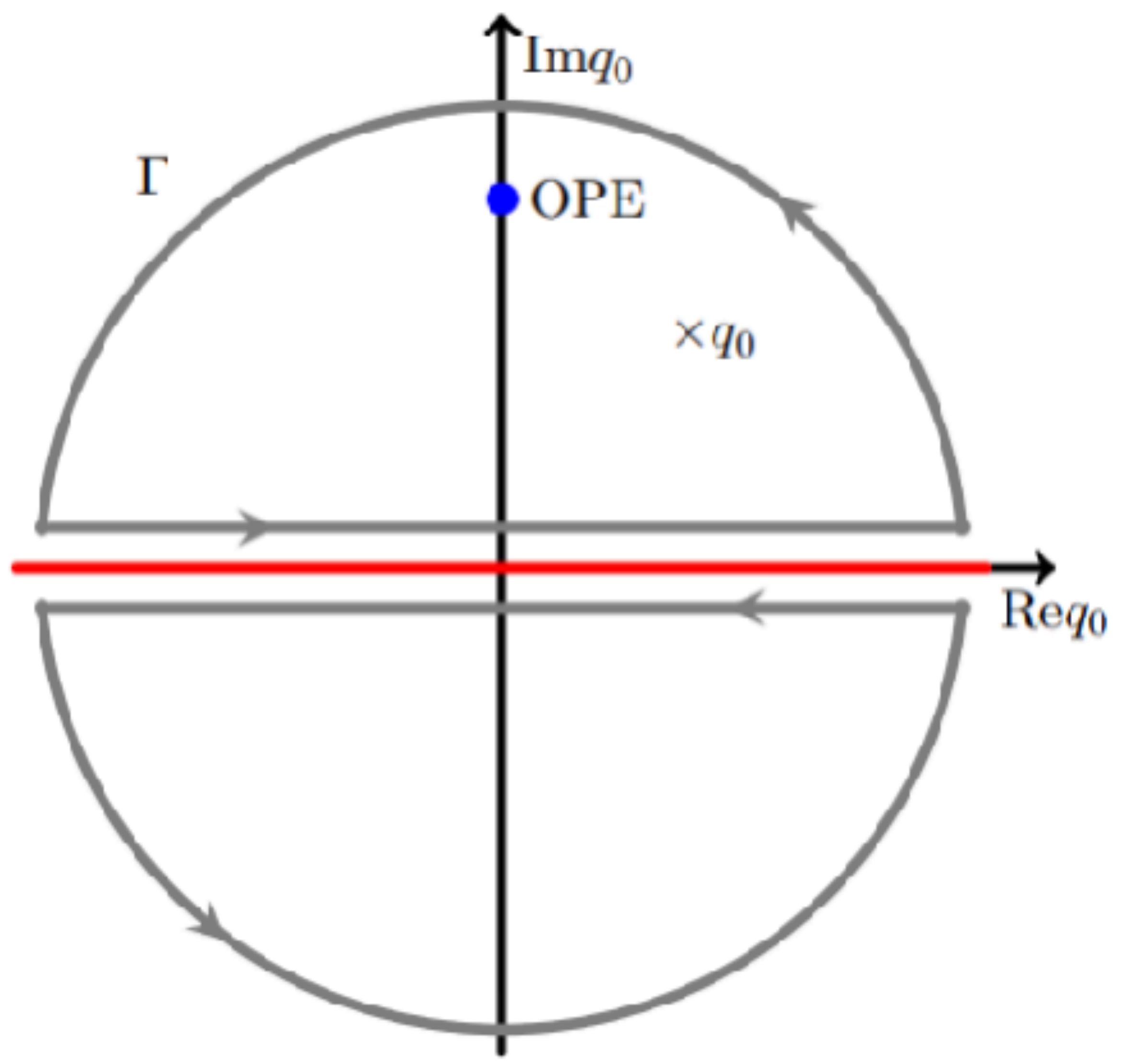
QCD sum rule in medium with finite momentum

$$\Pi_{\mu\nu}(q_0,\vec{q}) = i\int e^{iq\cdot x}\langle T\{j_\mu(x)j_\nu^\dagger(0)\}\rangle$$

dispersion relation

$$\Pi(q_0,|\vec{q}|) = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi(\omega,|\vec{q}|)}{\omega - q_0}$$

$$\Delta\Pi(\omega,|\vec{q}|) = \frac{1}{2i}(\Pi(\omega+i\epsilon,|\vec{q}|) - \Pi(\omega-i\epsilon,|\vec{q}|)$$



A. Self-adjoint

$$j^\dagger = j$$

A. Parity

$$\Pi_{\mu\nu}(q_0,\vec{q}) = (-1)^\mu(-1)^\nu\Pi_{\mu\nu}(q_0,-\vec{q})$$

A. Time reversal

$$\Pi_{\mu\nu}(q_0,\vec{q}) = (-1)^\mu(-1)^\nu\Pi_{\mu\nu}(-q_0,\vec{q})$$

A. Translational

$$\Pi_{\mu\nu}(q_0,\vec{q}) = \Pi_{\nu\mu}(-q_0,-\vec{q})$$

⇨

$$\boxed{\Pi_{\mu\nu}(q) = \Pi_{\nu\mu}(q)}$$

⇨

$$\boxed{\begin{array}{c}\Pi(q_0,|\vec{q}|) = \Pi(-q_0,|\vec{q}|) \\ \Delta\Pi(-q_0,|\vec{q}|) = -\Delta\Pi(-q_0,|\vec{q}|)\end{array}}$$

QCD sum rule in medium with finite momentum

$$\hat{\Pi}^{OPE}(M^2, |\vec{q}|) = \int_{-\vec{q}^2}^{\infty} ds\, e^{-s/M^2} \rho(s, |\vec{q}|)$$

$$\hat{\Pi}^{OPE}(M^2, \vec{q}^2) = e^{-4m_q^2/M^2} A(M^2)\Big[1 + \alpha_s a(M^2) + b(M^2)G_0(T) + \Big(c(M^2) + \underline{d(M^2)}\frac{\vec{q}^2}{m_q^2}\Big)G_2(T)\Big]$$

For vector,

$$d_L^V = -\frac{\omega(G(-\frac{3}{2}, \frac{7}{2}, \omega) - 6G(-\frac{1}{2}, \frac{7}{2}, \omega) + 6G(\frac{1}{2}, \frac{7}{2}, \omega))}{6G(\frac{1}{2}, \frac{5}{2}, \omega)}$$

$$d_T^V = \frac{\omega(-2G(-\frac{3}{2}, \frac{7}{2}, \omega) + 3G(-\frac{1}{2}, \frac{7}{2}, \omega) + 8G(\frac{1}{2}, \frac{5}{2}, \omega) - 6G(\frac{1}{2}, \frac{7}{2}, \omega)))}{12G(\frac{1}{2}, \frac{5}{2}, \omega)}$$

$$\omega = 4m_q^2/M^2$$

$$G(b, c, \omega) = \frac{1}{\Gamma(c)} \int_0^\infty dt\, e^{-t} t^{c-1} (\omega + t)^{-b}$$: Whittaker function

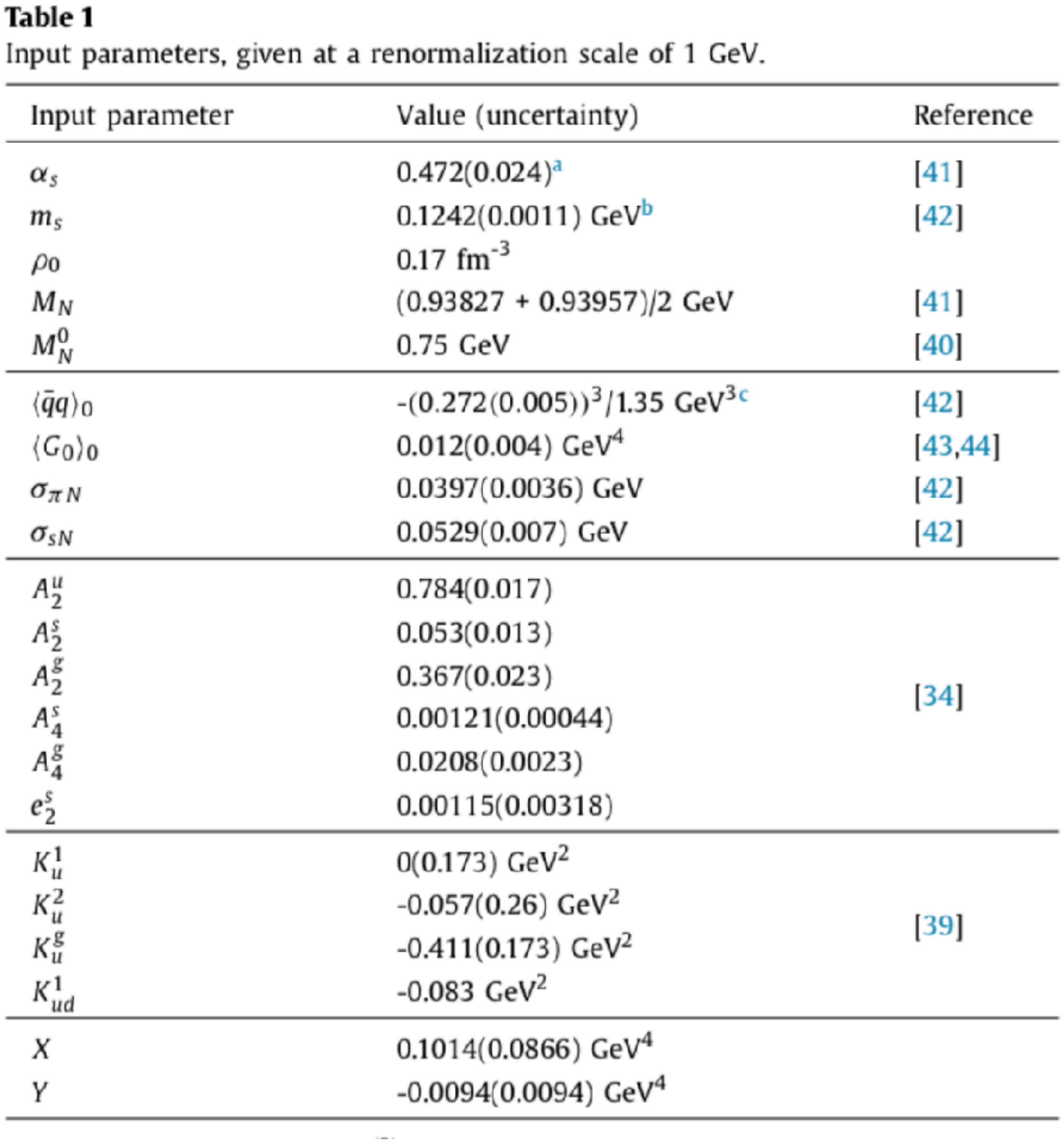


Table 1

Input parameters, given at a renormalization scale of 1 GeV.

Input parameter	Value (uncertainty)	Reference
α_s	0.472(0.024)[a]	[41]
m_s	0.1242(0.0011) GeV[b]	[42]
ρ_0	0.17 fm^{-3}	
M_N	(0.93827 + 0.93957)/2 GeV	[41]
M_N^0	0.75 GeV	[40]
$\langle\bar{q}q\rangle_0$	-(0.272(0.005))^3/1.35 GeV^3[c]	[42]
$\langle G_0\rangle_0$	0.012(0.004) GeV^4	[43,44]
$\sigma_{\pi N}$	0.0397(0.0036) GeV	[42]
σ_{sN}	0.0529(0.007) GeV	[42]
A_2^u	0.784(0.017)	[34]
A_2^s	0.053(0.013)	
A_2^g	0.367(0.023)	
A_4^s	0.00121(0.00044)	
A_4^g	0.0208(0.0023)	
e_2^s	0.00115(0.00318)	
K_u^1	0(0.173) GeV^2	[39]
K_u^2	-0.057(0.26) GeV^2	
K_u^g	-0.411(0.173) GeV^2	
K_{ud}^1	-0.083 GeV^2	
X	0.1014(0.0866) GeV^4	
Y	-0.0094(0.0094) GeV^4