Light-front holography
with chiral symmetry breaking:

From semiclassical first approximation to ab initio light-front QCD

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ILCAC Seminar
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Outline

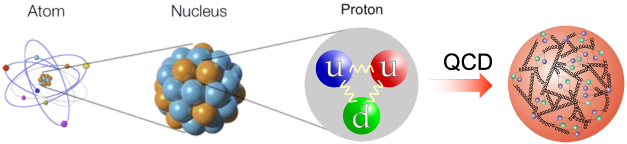
- The role of longitudinal dynamics in light-front Schrödinger wave equation
- Application to light mesons
- Prospects to *ab initio* light-front QCD

Based on:
YL, J.P. Vary, Phys. Lett. B 825,136860 (2022); [arXiv: 2103.09993 [hep-ph]]
YL, J.P. Vary, arXiv:2202.05581 [hep-ph]

Related work:
G. de Téramond, S. Brodsky, Phys. Rev. D (2021);
M. Ahmady et al. Phys. Lett. B (2021); Phys. Rev. D (2021);
C. Weller, G.A. Miller, arXiv:2111.03194 [hep-ph].

QCD in the Hamiltonian formalism

One of the central tasks of HEP is to unravel the fundamental structure of matter:

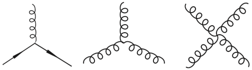


$$i\frac{\partial}{\partial\tau}|\psi_h(\tau)\rangle = H_{\mathrm{QCD}}|\psi_h(\tau)\rangle$$

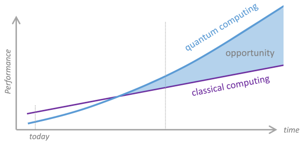
Maximal possible of information: masses, distributions, correlations, reactions, ... (non-perturbative)

Relativistic quantum many-body problem:

$$H_{\mathrm{QCD}} = \sum_i T_i + \sum_{ij} V_{ij}$$

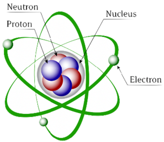
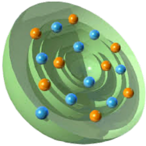
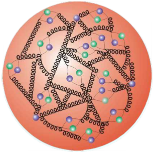


- 😂 Exponential wall: $\dim \mathcal{H} = N^{dN}$, d the spatial dimension
- 😁 Quantum advantage?



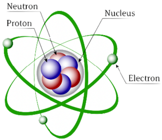
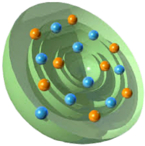
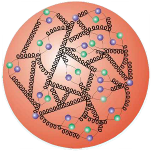
Quantum many-body problems

$$H = \sum_i T_i + \sum_{ij} V_{ij}$$

Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
Non-relativistic, weakly coupling	Non-relativistic, strongly coupling	Relativistic, strongly coupling

Semi-classical first approximation

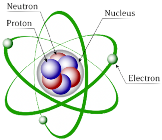
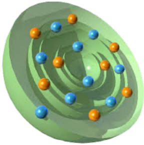
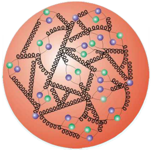
$$H = \sum_i T_i + U_i + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

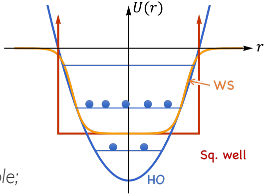
Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
↓	↓	↓
Bohr Model $\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	Shell Model $\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	?

Wilson, Walhout, Harindranath, Zhang, Perry, & Głazek, PRD 1994

Semi-classical first approximation

$$H = \sum_i T_i + U_i + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
↓	↓	↓
Bohr Model 	Shell Model 	?
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2}r^2\right)\psi = E\psi$	

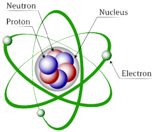


U_i should reproduce the basic physics of the system and ideally is analytically solvable;
e.g., harmonic oscillator potential vs Wood-Saxon potential

Semi-classical first approximation: heavy flavors

$$H = \textcolor{red}{\sum_i T_i + U_i} + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

Atoms



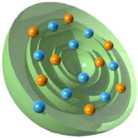
Coulomb interaction

↓

Bohr Model 

$$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$$

Nuclei



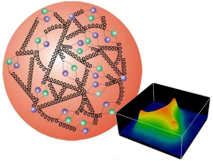
NN, NNN interactions

↓

Shell Model 

$$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$$

Hadrons



QCD interactions

↓

Quark Model

$$\left(\frac{\vec{p}^2}{2m} + \sigma r - \frac{\alpha}{r}\right)\psi = E\psi$$

Relativized quark model: Godfrey, Isgur PRD 1985;
Hamiltonian QCD in Coulomb gauge: Szczepaniak, Swanson, Ji & Cotanch, PRL 1996;
cf. NRQCD: Caswell & Lepage 1986

How heavy is heavy?

[YL, Li, Vary, arXiv:2111.14178 [hep-ph]]

PRL **115,** 222001 (2015) PHYSICAL REVIEW LETTERS week ending 27 NOVEMBER 2015

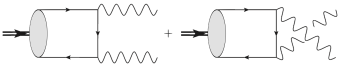
Can Nonrelativistic QCD Explain the $\gamma\gamma^* \to \eta_c$ Transition Form Factor Data?

PRL **119,** 252001 (2017) PHYSICAL REVIEW LETTERS week ending 22 DECEMBER 2017

Next-to-Next-to-Leading-Order QCD Corrections to the Hadronic Width of Pseudoscalar Quarkonium

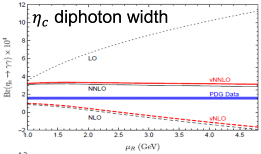
Feng Feng,[1,2] Yu Jia,[1,3,4] and Wen-Long Sang[5,*]

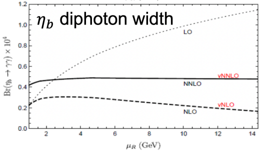
widths and the branching fractions for $\eta_{c,b} \to \gamma\gamma$. We find that severe tension arises between our state-of-the-art NRQCD predictions and the measured η_c hadronic width, and the tension in $\mathrm{Br}(\eta_c \to \gamma\gamma)$ is particularly disquieting. In our opinion, this may signal a profound crisis for the influential NRQCD factorization approach—whether it can be adequately applicable to charmonium decay or not. Our

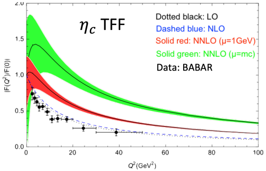


Roughly 1700 3-loop forward-scattering diagrams, divided into 4 distinct cut topologies; Cutkosky rule is imposed

For η_c more than 10σ discrepancy !







The call for a relativistic formulation is strong!

Light-front dynamics

[Dirac, RMP 1949]

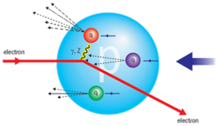
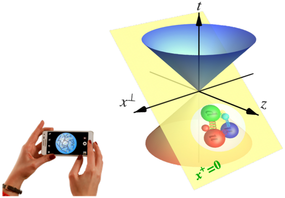


Image credit: CERN Courier

Deep inelastic scattering



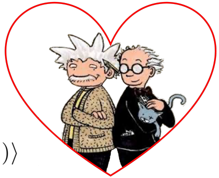
In modern high-energy experiments, the structure of hadron is ``seen'' at a fixed light-front time $x^+ = t + z/c$

- Light-front wave function is frame independent
- Simplification of relativistic dynamics

$$H_{\rm IF} = \sum_i \sqrt{\vec{p}_i^2 + m_i^2} + V \quad \text{vs} \quad H_{\rm LF} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{p_i^+} + V$$

- Direct access to hadron structures, e.g. parton distributions, form factors, OPE
- Schrödinger and Einstein equations are equivalent

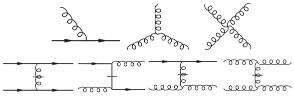
$$\underline{P}^- |\psi_h(P, J, m_J)\rangle = \frac{\vec{P}_\perp^2 + M_h^2}{P^+} |\psi_h(P, J, m_J)\rangle \Leftrightarrow (P^+ \underline{P}^- - \vec{P}_\perp^2)|\psi_h(P, J, m_J)\rangle = M_h^2 |\psi_h(P, J, m_J)\rangle$$



Light-front Schrödinger wave equation (LFSWE)

Light-front QCD in light cone gauge $A^+ = 0$:

$$H_{\text{LFQCD}} = \sum_i \frac{\vec{p}_{i\perp}^2 + m_i^2}{x_i} + U + \sum_{ij} V_{ij}^{(\text{QCD})} - \delta_{ij} U_i$$



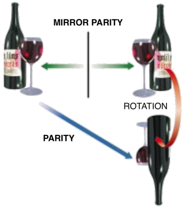
$$\Downarrow$$

$$\Big(\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + U\Big)\psi(x, \vec{k}_\perp) = M^2 \psi(x, \vec{k}_\perp)$$

The effective potential U is expected to implement key physics of QCD:

- ☐ Confinement
- ☐ Chiral symmetry breaking → Gell-Mann Oakes Renner relation: $M_\pi^2 \propto m_q$
- ☐ Regge trajectories: $M^2 \propto n, L, J$
- ☐ Kinematical symmetries, e.g. $m_{\mathsf{P}} = (-1)^J \mathsf{P}, \mathsf{C}$
- ☐ Endpoint asymptotics: $\sim x^a (1-x)^b$
- ☐ Rotational invariance is non-relativistic limit
- ☐ Analytical solutions

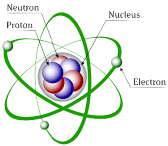
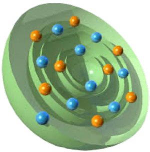
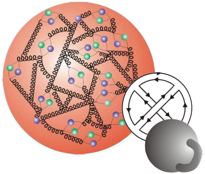
$(t, x, y, z) \overset{m_{\mathsf{P}}}{\to} (t, -x, y, z)$



How to find an effective interaction U for LFQCD?

Semi-classical first approximation: 't Hooft model for LFQCD_{1+1}

$$H = \sum_i \frac{\cancel{p_{i\perp}^2} + m_i^2}{x_i} + U_i + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

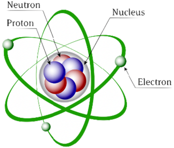
Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD_{1+1} interactions
↓	↓	↓
Bohr Model 	Shell Model 	't Hooft Model
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	$\frac{m_q^2}{x(1-x)}\psi + \frac{g^2}{\pi}\int_{\mathrm{P}} \frac{\psi(x)-\psi(y)}{(x-y)^2} = M^2\psi$

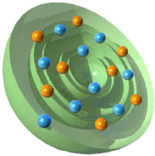
't Hooft, NPB 1976;
LFQCD_{1+1}: Hornbostel, Brodsky, Pauli, PRD 1990

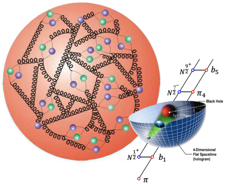
Semi-classical first approximation: light-front holography

$$H = \sum_i \frac{p_{i\perp}^2 + \cancel{m_i^2}}{x_i} + \kappa^4 \zeta_\perp^2 + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

Atoms	Nuclei	Hadrons







Coulomb interaction → Bohr Model 

$$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$$

NN, NNN interactions → Shell Model 

$$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$$

QCD Lite interactions → Light Front Holography

$$\left(\frac{\vec{k}_\perp^2}{x(1-x)} + \kappa^4 \zeta_\perp^2\right)\psi = M^2\psi$$

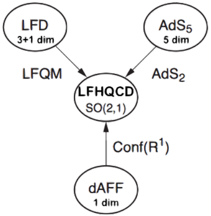
See, Brodsky et al., Phys. Rep. 2015, for a review of LFHQCD

Holographic light-front QCD_{3+1}

[Review: Brodsky, Phys. Rep. '15]

Light front holography is a unique mapping between LFQCD_{3+1} in the chiral limit and string motion in soft-wall AdS/QCD, as consistent with superconformal quantum mechanics

$$\Big[\frac{\vec{k}_\perp^2}{x(1-x)} + \kappa^4 \vec{\zeta}_\perp^2 + 2\kappa^2(J-1)\Big]\psi(\vec{\zeta}_\perp) = M^2\psi(\vec{\zeta}_\perp)$$



where $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$ is mapped to the fifth dimension z.

- Unique confining interaction $U_\perp = \kappa^4\zeta_\perp^2 + 2\kappa^2(J-1)$ in the chiral limit
- Meson mass spectra: $M^2_{nmJS} = 2\kappa^2(2n + |m| + J)$
  - Regge trajectory $M^2 \propto n, L, J$
  - Massless pion (chiral limit), $\rho - \pi$ splitting: $M^2_\rho - M^2_\pi = 2\kappa^2$
  - Supersymmetry across hadron sepctrum
- Predicted light-front wave functions (ϕ_{nm} is harmonic oscillator function),

$$\psi_{nmJS}(x, \vec{k}_\perp) = \phi_{nm}\big(\vec{k}_\perp / \sqrt{x(1-x)}\big) \propto e^{-\frac{\vec{k}_\perp^2}{2\kappa^2 x(1-x)}}$$

- Phenomenological successes

Holographic light-front QCD_{3+1}

[Review: Brodsky, Phys. Rep. '15]

Hadron spectrum

- Light mesons: Brodsky, PRL '06
- Heavy-light mesons: Dosch, PRD '15&'17
- Heavy quarkonia & tetraquarks: Nielsen, PRD '18
- Baryons: de Téramond, PRD '15
- Exotica: Zou, PRD '19

Form factors

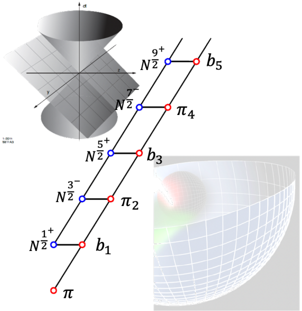
- Gravitational form factors: Brodsky, PRD '08
- Nucleon form factors: Sufian, PRD '17

Parton distributions

- GPDs: de Teramond, PRL '18
- Proton PDFs: Liu, PRL '20

Other applications

- Diffractive ρ productions, Forshaw PRL '12



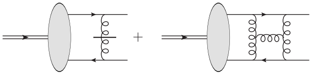
Separation of variables

[Chabysheva, AP 2012]

We saw that there is a natural separation of the transverse and longitudinal d.o.f.'s:

$$\Big\{\underbrace{\frac{\vec{k}_\perp^2}{x(1-x)}}_{\text{chiral limit},T} + \underbrace{\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{x(1-x)}}_{\text{mass term},L} + U\Big\}\psi(x,\vec{k}_\perp) = M^2\psi(x,\vec{k}_\perp)$$

Separation ansatz:



$$U = U_\perp(\zeta_\perp) + U_\parallel(\tilde{z})$$

$$\Rightarrow \quad M^2 = M_\perp^2 + M_\parallel^2, \quad \psi(x,\vec{\zeta}_\perp) = \phi(\vec{\zeta}_\perp)\chi(x)$$

Here, $\tilde{z} = \frac{1}{2}P^+x^- = i\partial/\partial x\big|_{\vec{\zeta}_\perp}$. [Miller & Brodsky, PRC 2020]

The LFSWE can be split into two equations:

$$\Big[\nabla_\zeta^2 + U_\perp(\vec{\zeta}_\perp)\Big]\phi(\vec{\zeta}_\perp) = M_\perp^2\phi(\vec{\zeta}_\perp),$$

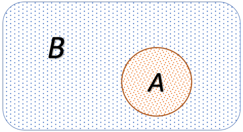
$$\Big[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} + U_\parallel(\tilde{z})\Big]\chi(x) = M_\parallel^2\chi(x)$$

Examples of non-separable interaction based on LFH: Wilson, PRD 1994; Brisdová, PRL 1997, ...

Entanglement entropy

EE measures the quantum entanglement of a subsystem A with the rest part of the system:

$$S_A = -\text{tr}\rho_A \log \rho_A,$$



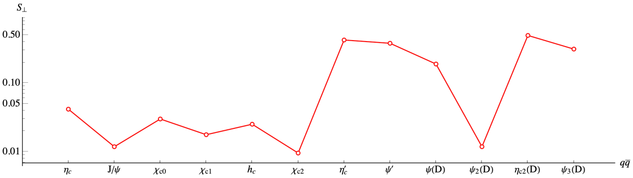
where $\rho_A = \text{tr}_B \rho$. $S_A \nearrow$ if more entanglement.

Examples:

Product state (separable): $|\psi_1\rangle = |0\rangle_A \otimes |1\rangle_B$, $S_A = 0$.

Entangled state: $|\psi_2\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B\big) \neq |\cdot\rangle_A \otimes |\cdot\rangle_B$, $S_A = \log 2 > 0$.

EE of the transverse d.o.f. of charmonia from a non-separable interaction: [YL, Maris, Vary, PRD 2017]



The ground states are approximately separable even though the interaction is non-separable.

[Similar observation in proton: Dumitru, & Kolbusz, arXiv:2202.01803]

The need for the longitudinal dynamics

Invariant mass ansatz (IMA) is based on covariance argument:

$$\frac{\vec{k}_\perp^2}{x(1-x)} \to \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}, \quad \chi(x) = 1 \to \chi(x) = N \exp\Big\{-\frac{(1-x)m_q^2 + x m_{\bar{q}}^2}{2\kappa^2 x(1-x)}\Big\}$$

- ☐ Confinement

  Missing confinement in the longitudinal direction
- ☐ Chiral symmetry breaking → Gell-Mann Oakes Renner relation: $M_\pi^2 \propto m_q$

  $M_\pi^2 = 2m_q^2 \log(\kappa^2/m_q^2 - \gamma_E) + O(m_q^4)$
- ☑ Regge trajectories: $M^2 \propto n, L, J$
- ☐ Kinematical symmetries, e.g. $m_{\mathsf{P}} = (-1)^J \mathsf{P}, \mathsf{C}$

  $m_{\mathsf{P}} = (-1)^{m+S+1}$, $\mathsf{C} = (-1)^{m+S}$: *requires* $|m| = \max |m_L| = L$, *leading to issues with state id.*
- ☐ Endpoint asymptotics: $\sim x^a (1-x)^b$

  $F_\pi(Q^2) \sim \exp(-cQ^2)$
- ☑ Rotational invariance is non-relativistic limit
- ☑ Analytical solutions

The need for the longitudinal dynamics

Alternative longitudinal wave function: [Gutsche et al. PRD 2014]

$$\chi(x) = N x^{\alpha_1}(1-x)^{\alpha_2}, \quad (\alpha_i = m_i/B)$$

- ☐ Confinement

  Missing confinement in the longitudinal direction
- ☑ Chiral symmetry breaking → Gell-Mann Oakes Renner relation: $M_\pi^2 = Bm_q + 4m_q^4$
- ☑ Regge trajectories: $M^2 \propto n, L, J$
- ☐ Kinematical symmetries, e.g. $m_\mathsf{P} = (-1)^J \mathsf{P}, \mathsf{C}$

  $m_\mathsf{P} = (-1)^{m+S+1}$, $\mathsf{C} = (-1)^{m+S}$: *requires* $|m| = \max|m_L| = L$, *leading to issues with state id.*
- ☑ Endpoint asymptotics: $\sim x^a(1-x)^b$
- ☑ Rotational invariance is non-relativistic limit
- ☑ Analytical solutions

't Hooft model

[Chabyshev AP 2013; de Téramond PRD 2021; Ahmady PLB & PRD 2021]

$$\left[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right]\chi(x) + \frac{g^2}{\pi}\mathrm{P}\int_0^1 \mathrm{d}x' \frac{\chi(x)-\chi(x')}{(x-x')^2} = M_\parallel^2 \chi(x).$$

- Obtained from LFQCD_{1+1} in the 't Hooft limit ($N_c \to \infty, g_s \to 0, g \equiv g_s\sqrt{N_c}$ fixed)

  More general, we can consider the Schwinger model, QED_{1+1} and QCD_{1+1}.

- Confinement from geometry $U_\parallel = g^2|\tilde{z}| = \frac{g^2}{2}P^+|x^-|$

  NR limit: Ahmady, $\propto (2m_q|\tilde{z}| + \zeta_\perp)$ vs $\sqrt{4m_q^2\tilde{z}^2 + \zeta_\perp^2}$ (non-separable); [Wilson PRD 1994; Pirner, PLB 2004; Shuryak 2021]

- Chiral symmetry breaking via Berezinskii-Kosterlitz-Thouless mechanism

  GMOR relation: $M_\pi^2 = 2\sigma m_q + \mathrm{O}(m_q^2)$, where $\sigma = g\sqrt{\pi/3}$. Chiral condensate: $\langle\overline{\psi}\psi\rangle = -gN_c/\sqrt{12\pi} = -f_\pi^2\sigma$

- Regge trajectory $M_\ell^2 = g^2\pi\ell + (m_q^2 + m_{\bar{q}}^2 - 2g^2/\pi)\ln\ell$ for $\ell \gg 1$

- Wave functions are not analytic functions: $\chi(x) \sim x^{\frac{\beta_1}{2}}(1-x)^{\frac{\beta_2}{2}}$, where $\beta_i = 2m_i/\sigma + \mathrm{O}(m_i^2)$ are related to the chiral condensate.

  Numerical solutions can be obtained with basis expansion using Jacobi polynomials. [See, e.g., Mo & Perry 1993]

- Caveat: separability

$$\sim \int \mathrm{d}^2 r'_\perp \delta^2(r_\perp - r'_\perp)\int \mathrm{d}x'\, K(x-x')\chi(x')\phi(\sqrt{x'(1-x')}\vec{r}'_\perp)$$

$\sqrt{x(1-x)}\vec{r}'_\perp$

LMZV model

[YL, Maris, Zhao, Vary PLB 2016; YL & Vary PLB 2022; de Téramond PRD 2021]

$$\Big[\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\Big]\chi(x) - \sigma^2\partial_x\big(x(1-x)\partial_x\chi(x)\big) = M_\parallel^2\chi(x).$$

- Hermitian by construction

  More general, one can consider the Sturm-Liouville operator

- Quadratic confining potential $\langle\tilde{z}|U_\parallel|\tilde{z}'\rangle = \sigma^2\tilde{z}\tilde{z}'\,\frac{j_1(\frac{\tilde{z}'-\tilde{z}}{2})}{2\pi(\tilde{z}'-\tilde{z})}e^{\frac{i}{2}(\tilde{z}'-\tilde{z})} \to \delta(z-z')\frac{1}{4}M^2\sigma^2z^2$

  N.B. there does not exist coordinate operator or coordinate representation in relativistic QM.

- Mass spectra: $M_l^2 = (m_q + m_{\bar{q}})^2 + \sigma(m_q + m_{\bar{q}})(2l+1) + \sigma^2 l(l+1)$

  😂 *For large l, it deviates from the Regge trajectory.*

- Wave functions:

$$\chi_l(x) = Nx^{\frac{\beta_1}{2}}(1-x)^{\frac{\beta_2}{2}}P_l^{(\beta_2,\beta_1)}(2x-1)$$

  where, $\beta_i = 2m_i/\sigma$. WF similar to Gutsche et al., but with a factor of 2 difference.

- Chiral symmetry breaking: ground state identical to 't Hooft model in the chiral limit with $\sigma = g\sqrt{\pi/3}$
  - *Mass obeys GMOR relation: $M_\pi^2 = 2\sigma m_q + \mathrm{O}(m_q^2)$*
  - *WF is power law like: $\chi(x) \sim x^{\frac{\beta_1}{2}}(1-x)^{\frac{\beta_2}{2}}$, where $\beta_i = 2m_i/\sigma$*

- Masses & wave functions are analytically known. 😁

Głazek-Trawiński model

[Głazek, APPB 2011, Głazek, Trawiński, PRD 2013]

Introduce a 3rd momentum: $\kappa_\parallel = \frac{m_{\bar q} x - m_q(1-x)}{\sqrt{x(1-x)}}$ and third holographic coordinate $\zeta_\parallel = i\partial_{\kappa_\parallel}$. The confining interaction is a 3D harmonic oscillator $\kappa^4\zeta_\parallel^2$,

$$\Big[\frac{m_q^2}{x} + \frac{m_{\bar q}^2}{1-x} + \kappa^4\zeta_\parallel^2\Big]\chi(x) = M_\parallel^2\chi(x).$$

- Convert light-front kinematics to 3D vector kinematics: $T = \vec\kappa^2 + (m_q + m_{\bar q})^2$, where $\vec\kappa = (\vec\kappa_\perp, \kappa_\parallel)$

  [See, e.g., Heinzl, Lect. Notes Phys. 2001]

- Quadratic confining potential, in terms of $\tilde z$, $U_\parallel \sim -\partial_x\big(x^3(1-x)^3\partial_x\big)$
- Mass spectra obey Regge trajectory
- Wave function generalizes the IMA wave function,

$$\chi(x) = N\exp\Big\{-\frac{m_q^2}{2\kappa^2 x} - \frac{m_{\bar q}^2}{2\kappa^2(1-x)}\Big\}$$

- Rotational symmetry $\kappa^4\vec\zeta^2$.

Even more longitudinal confining potentials

- Potential in terms of Miller-Brodsky longitudinal coordinate $\tilde{z}$, e.g. $\tilde{z}^p$ [Weller & Miller, 2021]
- Collinear effective model and QFT_{1+1} [Burkardt, PRD 1997]

  Related to the ``coordinate rep'n" by Fourier transform: $K(x-x') = \int \mathrm{d}\tilde{z}\, e^{i\tilde{z}(x-x')} U_{\parallel}(\tilde{z})$.
- Wave equation in Sturm-Liouville form:

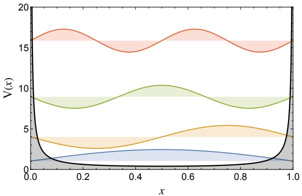
$$-\partial_x p(x) \partial_x \chi + q(x)\chi = \lambda \chi$$

where $U_{\parallel} = -\partial_x p(x) \partial_x + s(x)$ (→ 0), $q(x) = m_q^2/x + m_{\bar{q}}^2/(1-x)$

  - *Sturm-Liouville theorem guarantees the existence of well-behaved solutions*
  - *Non-trivial due to the singularities in LF kinetic energy* $q(x)$ *and in* $1/\sqrt{p(x)}$ *at* $x = 0, 1$
  - *Mathematically, there are only a few well established SL problems directly connected to LFSWE, the LMZV confining potential* $p = x(1-x)$ *is one of them.*

$$\Big[\frac{m_q^2}{x(1-x)} - \sigma^2 \frac{\mathrm{d}^2}{\mathrm{d}x^2}\Big]\chi(x) = M_{\parallel}^2 \chi(x)$$

Another examples with $p(x) = 1$,



Comparison

	't Hooft	LMZV	Głazek-Trawiński	HO
Confinement	$\|\tilde{z}\|$	$\sim \tilde{z}^2$	$\sim \tilde{z}^2$	$\tilde{z}^2$
GMOR	✓	✓	--	--
Regge	✓	linear+quadratic	✓	quadratic
Endpoint	power-law	power-law	Gaussian	sine-like
Analytic	--	✓	✓	--

Other effective $q\bar{q}$ interactions for QCD_{3+1} (a partial list)

Wilson, Walhout, Harindranath, Zhang, Perry, & Głazek, PRD 1994, relativized linear confinement;
Brisudová, Perry & Wilson, PRL 1997, log confinement from $O(\alpha_s)$ similarity renormalization;
Burkardt & Klindworth, PRD 1997, $q\bar{q}$ potential from transverse lattice;
Pauli, EPJC 1999, relativized confining potential based on Krautgärtner-Pauli-Wölz type OGE interaction;
Gubankova, Ji & Cotanch, PRD 2000, linear confinement from truncated flow equation;
Frederico et al., PRD 2002, relativized harmonic oscillator potential;
Pirner & Nurpeissov, PLB 2004, generalizes 't Hooft interaction based on Wilson loop;
Głazek et al., PRD 2004 & 2006, PLB 2017, quadratic confinement from $O(\alpha_s)$ similarity renormalization group;
Shuryak & Zahed, arXiv:2111.01775 [hep-ph], generalizes 't Hooft interaction based on NG model;
⋮

LMZV/BLFQ$_0$ for light mesons

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \vec{\zeta}_\perp^2 + 2\kappa^2(J-1) - \sigma^2 \partial_x (x(1-x)\partial_x).$$

- Mass spectra:

$$M^2_{nmlJS} = 2\kappa^2(2n+|m|+J) + (m_q+m_{\bar{q}})^2 + \sigma(m_q+m_{\bar{q}})(2l+1) + \sigma^2 l(l+1),$$
$$= M_\pi^2 + 2\kappa^2(2n+|m|+J) + \sigma^2 l(l+1) + 2l\sigma(m_q+m_{\bar{q}})$$

Differ from LFH+IMA predictions only for states with longitudinal excitation $l \neq 0$

- Wave functions:

$$\psi_{nmlJS}(x, \vec{k}_\perp) = \phi_{nm}\left(\vec{k}_\perp / \sqrt{x(1-x)}\right)\chi_l(x).$$

where ϕ_{nm} *is harmonic oscillator function,* $\chi_l(x) = N x^{\frac{\beta}{2}} (1-x)^{\frac{\alpha}{2}} P_l^{(\alpha,\beta)}(2x-1)$.

- Pion: $M_\pi^2 = 2\sigma m_{\{u,d\}} + 4m^2_{\{u,d\}}$, $\psi_\pi(x, \vec{k}_\perp) = \mathcal{N}\left[x(1-x)\right]^{\frac{\beta}{2}} e^{-\frac{\vec{k}_\perp^2}{2\kappa^2 x(1-x)}}$

Parameters of our model. The holographic confining strength κ is adopted from LFH

$m_{u,d}$	m_s	κ $(S=0)$	κ $(S=1)$	σ
15 MeV	261 MeV	0.59 GeV	0.54 GeV	0.62 GeV

State identification

- Quantum number assignment: $(n, m, l, S, J) \to (J, \mathsf{P}, \mathsf{C})$
- Exact symmetries: mirror parity $m_\mathsf{P} = (-1)^J \mathsf{P}$ and charge conjugation C: [Soper, PRD 1972; Brodsky PRD 2006]

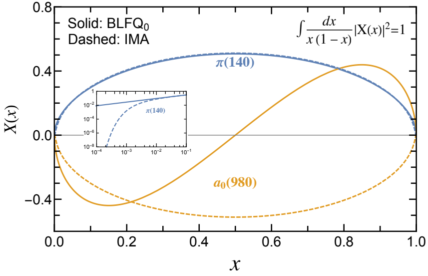
$$m_\mathsf{P} = \int [\mathrm{d}x\,\mathrm{d}^2k_\perp] \psi^*_{s\bar{s}}(x, \vec{k}_\perp)\psi_{-s-\bar{s}}(x, \tilde{k}_\perp) = (-1)^{m+S+1}, \quad (\tilde{k}_\perp = (-k_x, k_y))$$

$$\mathsf{C} = \int [\mathrm{d}x\,\mathrm{d}^2k_\perp] \psi^*_{s\bar{s}}(x, \vec{k}_\perp)\psi_{\bar{s}s}(1-x, -\vec{k}_\perp) = (-1)^{m+l+S}.$$

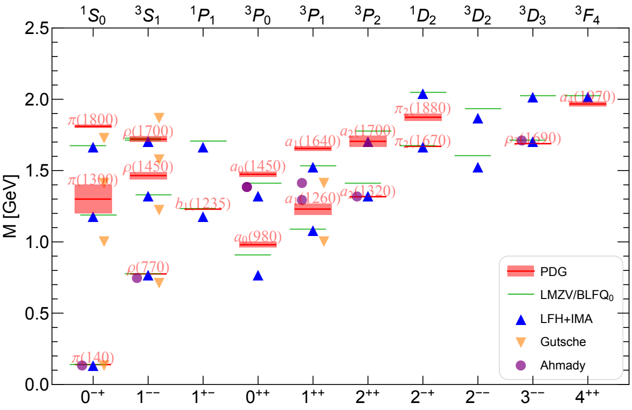
- Approximate symmetries: $\mathsf{P} \doteq (-1)^{L+1}, \mathsf{C} \doteq (-1)^{L+S}$, with $|m| = \max m_L \equiv L$
- Violation of transverse rotational symmetry in LFD leads to the lift of mass degeneracy in m_J.

IMA vs BLFQ$_0$ predictions for the ground-state scalar a_0 (0^{++})

	IMA	LMZV/BLFQ$_0$
quantum numbers	$n=0, m=1, S=1, J=0$	$n=0, m=0, l=1, S=1, J=0$
mass squared	$M_\pi^2 + 2\kappa^2$	$M_\pi^2 + 2\sigma^2 + 4\sigma m_q$
spin alignment	$\downarrow\downarrow$	$\uparrow\downarrow + \downarrow\uparrow$
wave function	$\mathcal{N}\frac{k_\perp}{\sqrt{x(1-x)}} e^{-\frac{\vec{k}_\perp^2 + m_q^2}{2\kappa^2 x(1-x)}}$	$\mathcal{N}(2x-1)\left[x(1-x)\right]^{\frac{\beta}{2}} e^{-\frac{\vec{k}_\perp^2}{2\kappa^2 x(1-x)}}$
LCDA	$\mathcal{N}\sqrt{x(1-x)} e^{-\frac{m_q^2}{2\kappa^2 x(1-x)}}$	$\mathcal{N}(2x-1)\left[x(1-x)\right]^{\frac{\beta+1}{2}}$



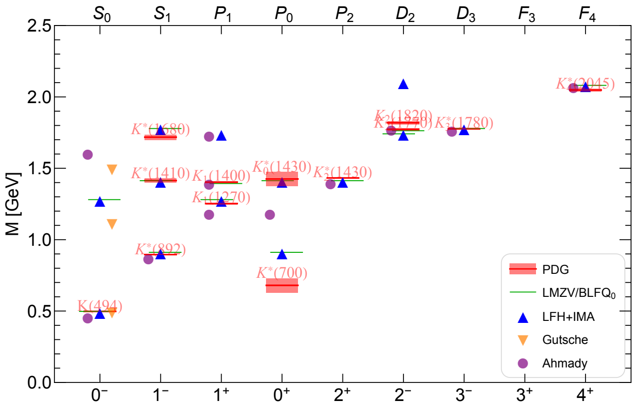
Spectrum of light mesons $q\bar{q}$



- For states without longitudinal excitations, the predictions are identical to those of LFH with IMA
- For states with longitudinal excitations, a_0, $\rho(1700)$, b_1, a_2 and ρ_2, the predictions are improved

[LFHQCD: Brodsky, Phys. Rep. '14; PRC '20; Gutsche, PRD '13; Ahmady '21&'21]

Spectrum of kaon



- For states without longitudinal excitations, the predictions are identical to those of LFH with IMA
- For states with longitudinal excitations, K_1, K_2, the predictions are improved

[LFHQCD: Brodsky, Phys. Rep. '14; PRC '20; Gutsche, PRD '13; Ahmady '21&'21]

Rotation symmetry

Violation of rotation symmetry leads to a lift of the mass degeneracies in different m_J projections.

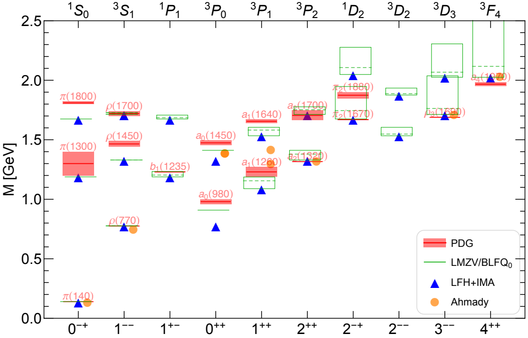
- In model w. longitudinal dynamics, e.g. LMZV/BLFQ$_0$, this can be incorporated in state id.
- Violation of Regge scaling in LMZV model leads to large spreads of mass eigenvalues for high-J states
- In LFH+IMA, only states with $|m| = \max |m_L| = L$ can be described

$$M^2_{[nmlJ,\mathrm{BLFQ}_0]} = M^2_\pi + 2\kappa^2(2n + |m| + J) + 4l\sigma m_q + \sigma^2 l(l+1),$$

$$M^2_{[nmJ,\mathrm{IMA}]} = M^2_\pi + 2\kappa^2(2n + L + J).$$

Comparison of state identification of selected light mesons with longitudinal excitations.

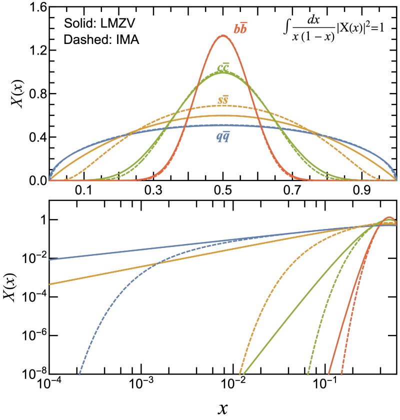
meson	PDG			LMZV/BLFQ$_0$					LFH+IMA		
	J^{PC}	$N^{2S+1}L_J$	(GeV)	n	$\|m\|$	l	$\|m_J\|$	(GeV)	n	$\|m\|$	(GeV)
$a_0(980)$	0^{++}	$1\,^3P_0$	0.98(20)	0	0	1	0	0.91	0	1	0.78
$a_1(1260)$	1^{++}	$1\,^3P_1$	1.230(40)	0	1	0	0	1.1	0	1	1.1
				0	0	1	1	1.2			
$b_1(1235)$	1^{+-}	$1\,^1P_1$	1.229(32)	0	0	1	0	1.23			
				0	1	0	1	1.19	0	1	1.19
$\rho(1700)$	1^{--}	$1\,^3D_1$	1.720(20)	0	0	2	0	1.73	2	0	1.33
				0	0	2	1	1.73			

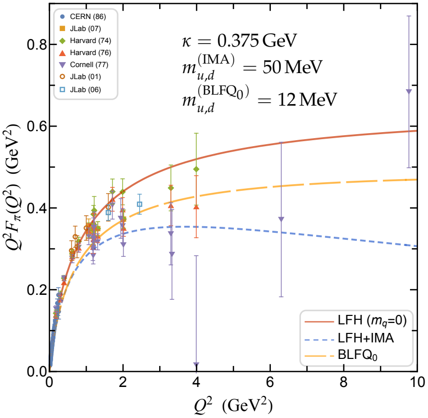


Endpoint behavior

The endpoint behavior has an impact on hadronic observables in high energy collisions as hard kernels T_H are sensitive to the endpoint singularities.

$$F_\pi(Q^2) = \int \frac{\mathrm{d}x}{x(1-x)} \left|X(x)\right|^2 \exp\left[-\frac{1-x}{x}\frac{Q^2}{4\kappa^2}\right]$$



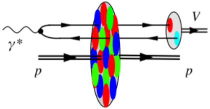


Longitudinal wave function without longitudinal dynamics

- Adopt the transverse holographic wave functions $\phi_{nm}(\vec{z}_\perp)$ and longitudinal wave functions $\chi_l(x)$ as the building block
  A hadron state is the superposition of all basis states allowed by the (kinematical) symmetries: Lorentz boosts, rotational symmetry along z, m_P, C, ...

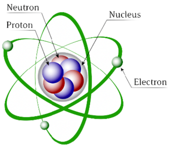
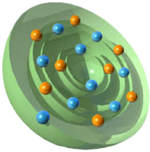
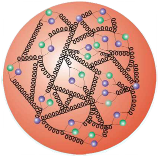
$$\psi^{(\lambda)}_{s\bar{s}/V}(x,\vec{r}_\perp) = \sum_{n,m,l} C^{\lambda}_{nmls\bar{s}}\,\phi_{nm}(\sqrt{x(1-x)}\vec{r}_\perp)\chi_l(x) \propto x^{\frac{\beta}{2}}(1-x)^{\frac{\alpha}{2}} e^{-\frac{\kappa^2}{2}x(1-x)\vec{r}^2_\perp}.$$

- Adopt a set of physical observables, decay constant, radius, leptonic/radiative width etc to fix the basis parameters $\{\alpha,\beta,\kappa\}$ and the basis coefficients $C_{nmls\bar{s}}$
  Additional constraints: orthogonality, angular momentum coupling with Clebshch-Gordan coefficients, ...
- Application to vector meson diffractive production [Li, YL, Chen, Lappi, and Vary, arXiv:2111.07087]
- Similar to the moments reconstruction used in DSE/BSE [Shi, PRL 2019]



Semi-classical first approximation

$$H = \sum_i \frac{p_{i\perp}^2 + m_i^2}{x_i} + \kappa^4 \zeta_{i\perp}^2 + U_{\parallel}(\tilde{z}_i) + \sum_{ij} V_{ij} - \delta_{ij} U_i$$

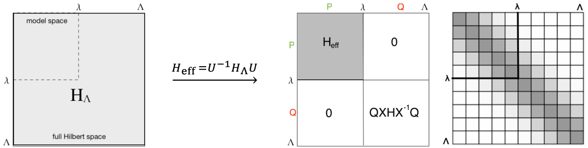
Atoms	Nuclei	Hadrons
		
QED	chiral EFT	QCD
↓	↓	↓
Coulomb interaction	NN, NNN interactions	Effective LFQCD interactions
↓	↓	↓
Bohr Model	Shell Model	Light Front Holography
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2} r^2\right)\psi = E\psi$	$\left(\frac{\vec{k}_\perp^2}{x(1-x)} + \kappa^4 \zeta_\perp^2 + U_{\parallel}\right)\psi = M^2\psi$

From semi-classical first approximation to ab initio calculation

$$H = \sum_i h_i + \sum_{ij} V_{ij}^{(2)}$$

where $h_i = T_i + U_i$*, whose eigenstates are chosen to span the Hilbert space* $\mathcal{H}$ *(ideally,* h_i *is analytically solvable)*

- In practice, the basis space is finite truncated, $\mathcal{H} \to \mathcal{P}_A$ and $H \to H_{\text{eff}}[\mathcal{P}_A]$
- H_{eff} defined in $\mathcal{P}_A$ can be viewed as similarity RG transformation $\to$ non-linear matrix equations



- Perturbative expansion
- Cluster expansion: $S_A = \sum_{i=1}^{A} s_i \approx \sum_{i=1}^{a} s_i$, where $a < A$

[Wilson et al., PRD 1994]
[Barrett, PPNP 201]

$$H_{\text{eff}} = e^{-S_A} H e^{+S_A} = \sum_i h_i + \sum_{ij} \widetilde{V}_{ij}^{(2)} + \sum_{ijk} \widetilde{V}_{ijk}^{(3)} + \cdots + \sum_{i_1, i_2, \cdots, i_A} \widetilde{V}_{i_1 i_2 \cdots i_A}^{(A)}$$

$$\approx \sum_i h_i + \sum_{ij} \widetilde{V}_{ij}^{(2)} + \cdots + \sum_{i_1, i_2, \cdots, i_a} \widetilde{V}_{i_1 i_2 \cdots i_a}^{(a)}$$

Converge to the exact results if $a \to A$ *or* $\mathcal{P}_A \to \mathcal{H}$

Beyond semi-classical first approximation

Extending the effective interaction with one-gluon exchange interaction by Krautgärtner et al. based on perturbative Bloch-Wilson/Okubo-Suzuki-Lee transformation [YL, Maris, Zhao, Vary, PLB 2016]

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \zeta_\perp^2 - \sigma^2 \partial_x \left[x(1-x) \partial_x \right] - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{s'}(k') \gamma_\mu u_s(k) \bar{v}_{\bar{s}}(\bar{k}) \gamma^\mu v_{\bar{s}'}(\bar{k}')$$

- Alternative $q\bar{q}$ interactions [Brisudova PRL 1997 and many others]
- Truncation up to $q\bar{q}g$ [BLFQ Col. (Lan et al.) PLB 2021]
- Lessons from strongly interacting non-relativistic quantum many-body calculations [Hergert, FP 2020]

Mass spectra & wave functions

- $c\bar{c}, b\bar{b}$: YL, Maris, Vary, PRD '17
- $B_c(b\bar{c})$: Tang, PRD '18
- heavy-light mesons B, D, B_s, D_s: Tang, EPJC '20
- light mesons $q\bar{q}, s\bar{q}$: Jia, PRC '19; Qian, PRC '20;
- nucleons: Mondal, PRD '20; Xu, PRD '21
- tetraquark: Kuang, '22

Form factors

- elastic form factors: YL, Maris, Vary, PRD '18
- (semi-)leptonic decay: Li, PRD '18 & '19; Tang, PRD '21
- radiative decay: YL, '21

Parton structures

- π: Lan, PRL '19 & PRD '20
- $c\bar{c}, b\bar{b}$: Adhikari, PRC '18&'21; Lan, PRD '20
- nucleons: Mondal, PRD '20; Xu, PRD '21; Liu '22

Summary

- The role of longitudinal dynamics in a semi-classical first approximation to QCD based on light-front holography
- A survey of recent work on the longitudinal confining interaction, in particular, the implementation of chiral symmetry breaking
- Prospects towards ab initio calculations based on first approximation

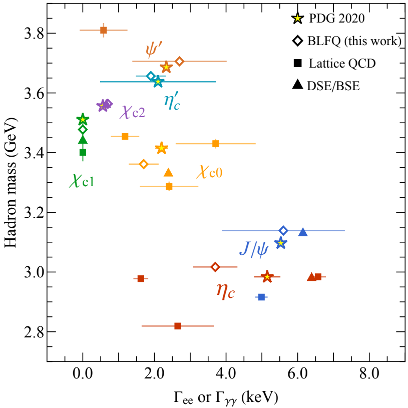
fin

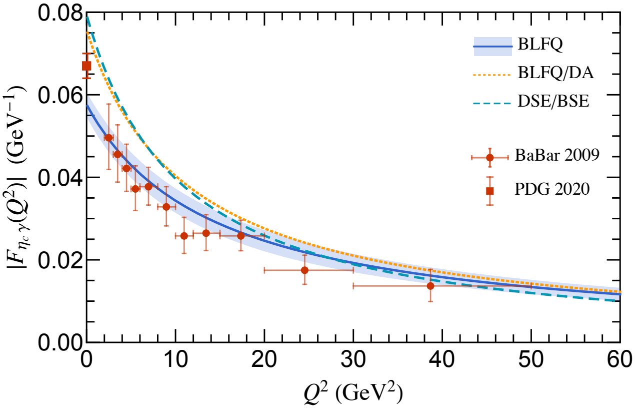
Best Wishes for the Year of Tiger

backup slides

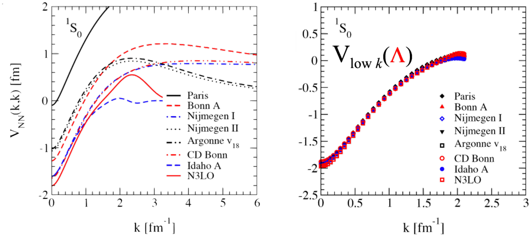
Application to charmonia

$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 \zeta_\perp^2 - \sigma^2 \partial_x [x(1-x)\partial_x] - \frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \bar{u}_{s'}(k')\gamma_\mu u_s(k) \bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}')$$

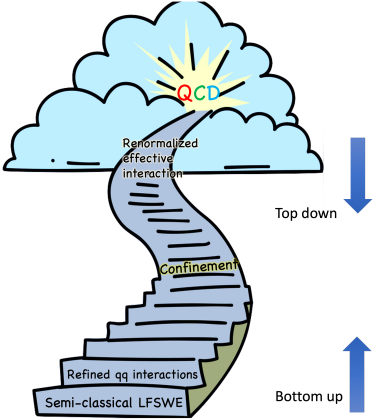




Bottom-up and top-down



Lessons from nuclear physics: universality across the RG evolved NN forces



Jacob's ladder for LFQCD