Three-nucleon systems, Poincaré covariance and the EMC Effect

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Based on:

- R. Alessandro, A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta,
  Light-Front Transverse Momentum Distributions for $\mathcal{J} = 1/2$ Hadronic Systems in Valence Approximation
  Phys.Rev.C 104 (2021) 6, 065204
- A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta,
  Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body System, **Phys. Rev. C 95, 014001 (2017)**
- E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
  EMC effect, few-nucleon systems and Poincaré covariance,
  Phys. Scr. 95, 064008 (2020)
- MARATHON Coll.
  Measurement of the Nucleon Fn2/Fp2 Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment,
  arXiv:2104.05850, Phys. Rev. Lett 2022, in press
- E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
  The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics
  arXiv:22... In preparation

Outline

1. Motivations
2. The Poincaré covariant framework
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4. Application: "TMDs" of ^{3}He
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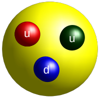
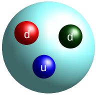
Motivations

- **Phenomenological**: a reliable flavor decomposition needs sound information on the neutron parton structure (PDFs, GPDs, TMDs, etc.).





Accurate and long-lasting experimental efforts in developing effective neutron targets to carefully investigate its electromagnetic responses. $^3\vec{He}$ is special:

$\Rightarrow$ the polarized ^{3}He target, 90% neutron target

(e.g. H. Gao et al, PR12-09-014; J.P. Chen et al, PR12-11-007, @JLAB12)

- Due to the high experimental energies, the accurate theoretical description of a (polarized ^{3}He) has to be *relativistic*

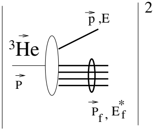
- **Theoretical**: a LF description of three body interacting systems! Bonus: Transverse-Momentum Distributions (**TMDs**) for addressing in a novel way the nuclear dynamics

On the theory side, we need

- A description of dynamics which retains as many general properties as possible,
- to validate sound procedures to *extract* the Nucleon (neutron) structure

In our approach, the key quantity is the *Spectral Function*
($\Rightarrow$ nucleon Green's function in the medium)

$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m \left\{ \langle \Psi_{gr} | a^{\dagger}_{\mathbf{k},\sigma'} \; \frac{1}{E - H + i\epsilon} \; a_{\mathbf{k},\sigma} | \Psi_{gr} \rangle \right\}$$



with

$$H = \sum_{\alpha,\beta} \langle \alpha | H_1 | \beta \rangle \; a^{\dagger}(\alpha) \; a(\beta) + \frac{1}{2} \sum_{\alpha,\beta,\gamma,\eta} \langle \alpha\gamma | H_2 | \beta\eta \rangle \; a^{\dagger}(\alpha) \; a^{\dagger}(\gamma) a(\beta) \; a(\eta) + \ldots \;\; \ldots$$

Diagonal terms: probability density to find a constituent with σ, k with an energy E of the remaining system in the ground state of the bound system.

Quite familiar in nuclear Physics;in hadron physics one introduces the LC correlator:

$$\Phi^{\tau}(x,y) = \langle \Psi_{gr} | \bar{\psi}^{\tau}(x) \mathcal{W}(\hat{n} \cdot A) \psi^{\tau}(y) | \Psi_{gr} \rangle$$

Our point: in valence approximation, one can relate $P_{\sigma'\sigma}(k,E)$ (given in a Poincaré covariant framework) and $\Phi^{\tau}(x,y)$

[Alessandro, Del Dotto, Pace, Perna, Salmè and SS, Phys.Rev.C 104 (2021) 6, 065204]

The Relativistic Hamiltonian Dynamics framework

Why a relativistic treatment ?

General answer: to develop an advanced scheme, appropriate for the kinematics of JLAB12 and of EIC

- The Standard Model of Few-Nucleon Systems, with nucleon and meson degrees of freedom within a non relativistic (NR) framework, has achieved high sophistication **[e.g. the NR $^3\mathrm{He}$ and $^3\mathrm{H}$ Spectral Functions in Kievsky, Pace, Salmè, Viviani PRC 56, 64 (1997)].**
- Covariance wrt the Poincaré Group, $\mathcal{G}_P$, needed for nucleons at large 4-momenta and pointing to high precision measurements.
  Necessary if one studies, e.g., i) nucleon structure functions; ii) nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in ^{2}H), etc
- At least, one should carefully treat the boosts of the nuclear states, $|\Psi_i\rangle$ and $|\Psi_f\rangle$!

Our definitely preferred **framework for embedding** the successful **NR** phenomenology:

Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) + Bakamjian-Thomas (BT) construction of the Poincaré generators for an interacting theory.

In RHD+BT, one can address both Poincaré covariance and locality, general principles to be implemented in presence of interaction:

- Poincaré covariance $\rightarrow$ The 10 generators, $P^\mu \rightarrow$ 4D displacements and $M^{\nu\mu} \rightarrow$ *Lorentz* transformations, have to fulfill

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -\imath(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

  Also $\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

- Macroscopic locality ($\equiv$ cluster separability (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of large spacelike separation (i.e. causally disconnected), rather than for arbitrary (microscopic-locality) spacelike separations. In this way, when a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

  Keister, Polyzou, Adv. Nucl. Phys. 20, 225 (1991) .

  This requires a careful choice of the intrinsic relativistic coordinates.

Forms of relativistic Dynamics

P.A.M. Dirac, 1949

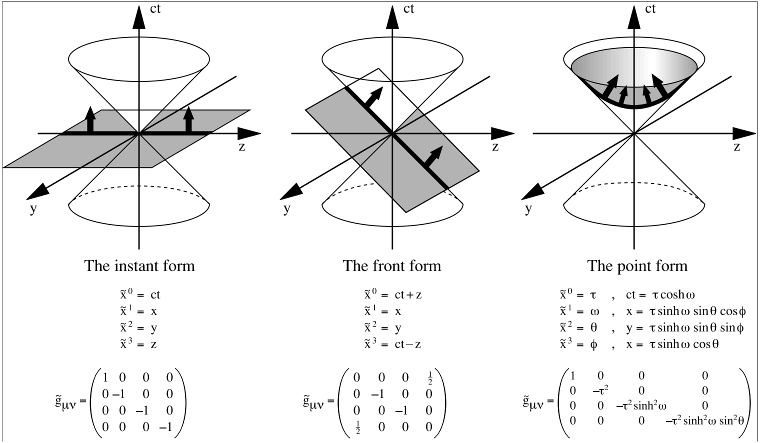


Fig. from **Brodsky, Pauli, Pinsky Phys.Rept. 301 (1998) 299-486**

We choose the Front Form!

The **Light-Front framework** has several advantages:

- 7 Kinematical generators: i) three LF boosts (In instant form they are dynamical!), ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$, iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure : trivial Separation of intrinsic and global motion, as in the NR case. important to correctly treat the boost between initial and final states !
- $P^+ \geq 0 \longrightarrow$ meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
- The infinite-momentum frame (IMF) description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

But within the Bakamjian-Thomas (BT) construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical!

★ The Mass Operator, developed within a *non relativistic framework*, is fully acceptable for a BT construction of the Poincaré generators★

Bakamjian-Thomas construction and LFHD

- An explicit construction of the 10 Poincaré generators, in presence of interactions, was given by Bakamjian and Thomas (PR 92 (1953) 1300).

  The key ingredient is the mass operator :
  i) only the mass operator M contains the interaction;
  ii) it generates the dependence upon the interaction of the three dynamical generators in LFHD, namely P^- and the LF transverse rotations $\vec{F}_\perp$;

- The mass operator is the free mass, M_0, plus an interaction V, or $M_0^2 + U$. The interaction, U or V, must commute with all the kinematical generators, and with the non-interacting angular momentum, as in the non-relativistic case.

- For the two-body case, it allows one to easily embed the NR phenomenology:
  i) the mass equation for the bound state, e.g. the deuteron,

$$[M_0^2(12) + U] \, |\psi_D\rangle = \left[4m^2 + 4k^2 + U\right] \, |\psi_D\rangle = M_D^2 \, |\psi_D\rangle = [2m - B_D]^2 \, |\psi_D\rangle$$

  becomes the Schr. eq. $\left[4m^2 + 4k^2 + 4m \, V^{NR}\right] \, |\psi_D\rangle = \left[4m^2 - 4mB_D\right] |\psi_D\rangle$

  with the identification of U and $4mV^{NR}$ and disregarding $(B_D/2m)^2$.

  ii) The eigensolutions of the mass equation for the continuum are identical to the solutions of the Lippmann-Schwinger equation.

The BT Mass operator for an A=3 system

- For the three-body case the mass operator is

$$M_{BT}(123) = M_0(123) + V^{BT}_{12,3} + V^{BT}_{23,1} + V^{BT}_{31,2} + V^{BT}_{123}$$

where

$M_0(123) = \sqrt{m^2 + k_1^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2}$ is the free mass operator,

$\mathbf{k}_i$ ($i = 1 - 3$) are momenta in the intrinsic reference frame, i.e. the rest frame for a system of free particles: $\mathbf{k}_i = L_f^{-1}(P/M_0)\ \mathbf{p}_i \qquad \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

V^{BT}_{123} is a three-body force

The commutation rules impose to V^{BT} invariance for translations and rotations as well as independence on the total momentum, as it occurs for V^{NR}.

One can assume $M_{BT}(123) \sim M^{NR}$

- Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

  The eigenfuntions of M^{NR} do not fulfill the cluster separability, but we take care of macroscopic locality in the spectral function.

To complete the matter: the spin

- Coupling spins and orbital angular momenta accomplished in the Instant Form of RHD (kinematical hyperplane t=0) through Clebsch-Gordan coefficients, since in this form the three rotation generators are independent of interaction.
- To embed the CG machinery in the LFHD one needs unitary operators, the so-called *Melosh* rotations that relate the LF spin wave function and the canonical one. For a particle of spin (1/2) with LF momentum $\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))\; |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

where

$D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))$ is the standard Wigner function for the $\mathcal{J} = 1/2$ case ,
$R_M(\tilde{\mathbf{k}})$ is the Melosh rotation relating the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving.

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi^\dagger_{\sigma'} \frac{m + k^+ - \imath \boldsymbol{\sigma} \cdot (\hat{z} \times \mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle \tilde{\mathbf{k}}; s\sigma' | \mathbf{k}; s\sigma\rangle_c$$

χ_σ is a two-dimensional spinor.

N.B. If $|\mathbf{k}_\perp| << k^+, m \longrightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

The spin-dependent Light-Front spectral function

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

The Spectral Function: probability distribution to find inside a bound system a particle with a given $\tilde{\boldsymbol{\kappa}}$ when the rest of the system has energy ϵ, with a polarization vector $\boldsymbol{S}$:

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{Tt} {}_{LF}\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_{\mathcal{M}}; ST_z \rangle \times \langle ST_z; \Psi_{\mathcal{M}} | \tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt \rangle_{LF}$$

- $|\Psi_{\mathcal{M}}; ST_z\rangle = \sum_m |\Psi_m; S_z T_z\rangle \; D^{\mathcal{J}}_{m,\mathcal{M}}(\alpha, \beta, \gamma)$ α, β and γ Euler angles of the rotation from the z-axis to the polarization vector $\boldsymbol{S}$
  $|\Psi_m; S_z T_z\rangle = |j, j_z; \epsilon^3; \frac{1}{2} T_z\rangle$ three-body bound eigenstate of $M_{BT}(123) \sim M^{NR}$
- $|\tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; T\tau\rangle_{LF}$ tensor product of a plane wave for particle 1 with LF momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the $[1 + (23)]$ cluster times the fully interacting state of the (23) pair of energy eigenvalue ϵ. It has eigenvalue

  $\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$

  and fulfills the macroscopic locality **(Keister, Polyzou, Adv. N. P. 20, 225 (1991))**.

  $\tilde{\boldsymbol{\kappa}} = (\kappa^+ = \xi \; \mathcal{M}_0(1, 23), \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp)$

The LF overlaps for ^{3}He SF in terms of the IF ones are

$$\overbrace{\langle T\tau;\alpha,\epsilon;JJ_z;\tau_1\sigma,\tilde{\boldsymbol{\kappa}}}^{<\tilde{\boldsymbol{\kappa}}|\times 2N\ state}|\overbrace{j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle}^{3N\ bound\ state}{}_{LF} = \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1'}$$

$$\sqrt{(2\pi)^3\ 2E(\mathbf{k})}\sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}}\sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'}\mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\ \mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

$${}_{IF}\langle T,\tau;\alpha,\epsilon;JJ_z|\mathbf{k}_{23},\sigma_2'',\sigma_3'';\tau_2,\tau_3\rangle\ \langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF}$$

where the effect of boosts is manifest in the Jacobians and in the transformations:

$$\mathcal{D}_{\sigma_i'',\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M^\dagger(\pm\tilde{\mathbf{k}}_{23})]_{\sigma_i''\sigma_i}\ D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma_i'}$$

★ Through the Bakamjian-Thomas construction, one is allowed to approximate the momentum space wave functions for the 2- and 3-body systems

$$\langle\quad\dots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF} = \langle\quad\dots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{NR}$$

preserving the Poincaré covariance but using the successful NR phenomenology, in full

A. Del Dotto et al , PRC 95, 014001 (2017).

Many frames...

★ We stress that, for a correct description of the SF, so that the Macro-locality is implemented, it is crucial to distinguish between different frames, moving with respect to each other:

- The Lab frame, where $P = (M, \vec{0})$
- The intrinsic LF frame of the whole system, (123), where $\tilde{P} = (M_0(123), \vec{0}_\perp)$ with $k^+(123) = \xi \; M_0(123)$ and

$$M_0(123) = \sqrt{m^2 + k^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2}$$

- The intrinsic LF frame of the cluster, (1; 23), where $\tilde{P} = (\mathcal{M}_0(1, 23), \vec{0}_\perp)$, with $\kappa^+(1; 23) = \xi \, \mathcal{M}_0(1, 23)$ and

$$\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} \; + \; \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$$

while $\mathbf{p}_\perp(lab) = \mathbf{k}_\perp(123) = \boldsymbol{\kappa}_\perp(1, 23)$

The LF spin-dependent spectral function for a system with polarization **S**, can be *macroscopically* decomposed in terms of the available vectors:

- the unit vector $\hat{n}$, $\perp$ to the hyperplane $n^\mu x_\mu = 0$. Our choice is $n^\mu \equiv \{1, 0, 0, 1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector **S**.
- the transverse (wrt the $\hat{z}$ axis) momentum component of the constituent, i.e. $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$

$$\boldsymbol{\mathcal{P}}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) = \frac{1}{2}\left[\mathcal{B}^{\tau}_{0,\mathcal{M}} + \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S})\right]_{\sigma'\sigma}$$

The scalar $\mathcal{B}^{\tau}_{0,\mathcal{M}} = Tr\left[\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\right]$ yields the unpolarized spectral function ; the pseudovector $\boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, \mathbf{S}) = Tr\left[\hat{\boldsymbol{\mathcal{P}}}^{\tau}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}}, \epsilon, S)\ \boldsymbol{\sigma}\right]$ is a linear combination of the available pseudovectors,

$$\begin{aligned}\boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S}) &= \mathbf{S}\mathcal{B}^{\tau}_{1,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\tau}_{2,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S}\cdot\hat{z})\mathcal{B}^{\tau}_{3,\mathcal{M}}(\dots) \\ &+ \hat{z}\ (\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\tau}_{4,\mathcal{M}}(\dots) + \hat{z}\ (\mathbf{S}\cdot\hat{z})\mathcal{B}^{\tau}_{5,\mathcal{M}}(\dots)\ .\end{aligned}$$

with $x = \kappa^+(1;23)/\mathcal{M}_0(1;23)$. N.B. The scalar functions $\mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$ depend on the scalars at disposal, i.e., for $\mathcal{J} = 1/2$, $|\mathbf{k}_\perp|$, x, ϵ

By integrating the LF SF on κ^-, equivalent to the integration on the $\epsilon \equiv$ internal energy of the spectator system, one straightforwardly gets the LF spin-dependent momentum distribution

$$\mathcal{N}^{\tau}_{\sigma'\sigma}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \frac{1}{2} \left\{ b_{0,\mathcal{M}}(\dots) + \boldsymbol{\sigma} \cdot \boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) \right\}_{\sigma'\sigma}$$

where $\boldsymbol{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})$ is a pseudovector

$$\boldsymbol{f}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S}) = \mathbf{S}\, b^{\tau}_{1,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S} \cdot \hat{\mathbf{k}}_\perp) b^{\tau}_{2,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\ (\mathbf{S} \cdot \hat{z}) b^{\tau}_{3,\mathcal{M}}(\dots)$$
$$+\hat{z}\ (\mathbf{S} \cdot \hat{\mathbf{k}}_\perp) b^{\tau}_{4,\mathcal{M}}(\dots) + \ \hat{z}\ (\mathbf{S} \cdot \hat{z}) b^{\tau}_{5,\mathcal{M}}(\dots)$$

The decomposition follows from the corresponding one of the SF, and the **scalar functions** $b^{\tau}_{i,\mathcal{M}}(\dots)$ are proper integrals over $\epsilon \equiv$ the spectator energy, present in $\mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$

- The remarkable content of such a decomposition is to make explicit the interplay between transverse momentum component and spin dofs.
- In turn, this can be useful for determining possible *relations* between the so-called Transverse-momentum Distributions (TMDs), in the *valence sector*, i.e. with a minimal number of on-mass-shell constituents inside the *interacting system*.

LC Correlator and LF spin-dependent SF

Let p be the momentum in the laboratory of an off-mass-shell fermion, with isospin τ, in a bound system of A fermions with total momentum P and spin S. The fermion correlator in terms of the LF coordinates is **[e.g., Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]**

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T \; e^{i\ p\xi} \left\langle P,S,A|\bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\right\rangle$$

where $|A,S,P\rangle$ is the A-particle state and $\psi^{\tau}_{\alpha}(\xi)$ the particle field (e.g. a nucleon of isospin τ in a nucleus, or in valence approximation a quark in a nucleon).

The particle contribution to the correlator in valence approximation, i.e. the result obtained if the antifermion contributions are disregarded, is related to the LF SF

$$\Phi^{\tau p}(p,P,S) = \frac{(\not{p}_{on} + m)}{2m}\,\Phi^{\tau}(p,P,S)\,\frac{(\not{p}_{on} + m)}{2m} = \ldots$$

$$= \frac{2\pi\,(P^+)^2}{(p^+)^2\,4m}\,\frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\left\{\,u_{\alpha}(\tilde{\mathbf{p}},\sigma')\,\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\bar{u}_{\beta}(\tilde{\mathbf{p}},\sigma)\,\right\}$$

In deriving this expression it naturally appears the momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is fully interacting.

LC Correlator and TMDs

The correlation function at the leading twist is given by

$$\Phi(p,P,S) = \frac{1}{2}\,\not{P}\,A_1 + \frac{1}{2}\,\gamma_5\,\not{P}\left[A_2\,S_z + \frac{1}{M}\,\widetilde{A}_1\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\right]$$
$$+ \frac{1}{2}\,\not{P}\,\gamma_5\left[A_3\,\not{S}_\perp + \widetilde{A}_2\,\frac{S_z}{M}\,\not{p}_\perp + \frac{1}{M^2}\,\widetilde{A}_3\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\not{p}_\perp\right]$$

where M is the mass of the system.
The functions $A_j,\ \widetilde{A}_j\quad (j=1,2,3)$ can be obtained by proper traces of $\Phi(p,P,S)$ and Γ matrices.
Integrals of $A_j,\ \widetilde{A}_j$ on p^+ and p^- :

$$\mathcal{O}\left[A_j\right] = \frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+\left[A_j\right]$$

give the six time reversal even transverse momentum distributions (TMDs)

$$f(x,\mathbf{p}_\perp^2) = \mathcal{O}\left[A_1\right] \qquad \Delta f(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[A_2\right] \qquad g_{1T}(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_1\right]$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[A_3 + \frac{|\mathbf{p}_\perp|^2}{2M^2}\widetilde{A}_3\right] \qquad h_{1L}^\perp(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_2\right] \qquad h_{1T}^\perp(x,|\mathbf{p}_\perp|^2) = \mathcal{O}\left[\widetilde{A}_3\right]$$

LC Correlator and LF Spectral Function

Alessandro,Del Dotto, Pace, Perna, Salmè, Scopetta, Phys.Rev.C 104 (2021) 6, 065204

The traces of Φ^p can be expressed by traces of the spectral function :

$$\begin{aligned}
\mathrm{Tr}(\gamma^+\Phi^{\mathrm{p}}) &= D\ Tr\left[\hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
\mathrm{Tr}(\gamma^+\gamma_5\ \Phi^{\mathrm{p}}) &= D\ Tr\left[\sigma_z\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
\mathrm{Tr}(\not{\boldsymbol{p}}_\perp\ \gamma^+\ \gamma_5\ \Phi^{\mathrm{p}}) &= D\ Tr\left[\boldsymbol{p}_\perp\cdot\boldsymbol{\sigma}\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right]
\end{aligned}$$

$$D = \frac{(P^+)^2}{p^+}\ \frac{\pi}{m}\ \frac{E_S}{\mathcal{M}_0[1,(23)]}$$

The Integration on p^+ and p^- : $\frac{1}{2}\int \frac{dp^+dp^-}{(2\pi)^4}\ \delta[p^+ - xP^+]\ P^+$ of the traces of Φ^p and of the traces of $\hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}$ gives relations between the TMDs and the functions $b_{i,\mathcal{M}}$

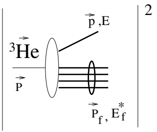
$$f(x,\mathbf{p}_\perp^2) = b_0 \quad \Delta f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \quad g_{1T}(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + \frac{1}{2}\ b_{2,\mathcal{M}}\quad h_{1L}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{3,\mathcal{M}}\quad h_{1T}^\perp(x,|\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2}\ b_{2,\mathcal{M}}$$

There is a one-to-one correspondence between the $\epsilon-$ integral of proper components of the SF (the functions $b_{i,\mathcal{M}}$) and the TMDs of $^3\mathrm{He}$: the latter can be accurately obtained from the wave function!

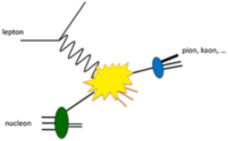
It works for any three-body $J = 1/2$ system in valence approx!
Correspondence:

^{3}He



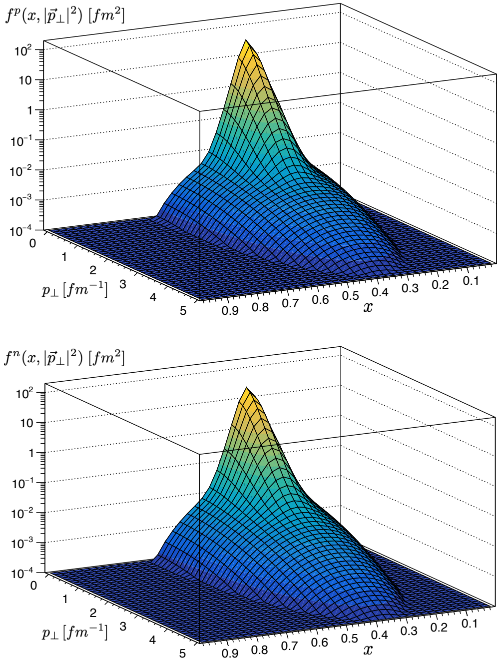
- $p,\ p,\ n$
- $(e, e'p)$ reactions
- p detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations

proton



- $u_v,\ u_v,\ d_v$
- SIDIS
- no q_v detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)

- In the case of $^3\mathrm{He}$ the TMDs could be obtained through measurements of appropriate spin asymmetries in $^3\vec{He}(\vec{e}, e'p)$ experiments: in progress!
- We show in the following our calculation for the TMDs of $^3\mathrm{He}$ **(Alessandro, Del Dotto, Pace, Perna, Salmè, Scopetta, Phys.Rev.C 104 (2021) 6, 065204)** , performed using Av18 + UIX wfs **(Pisa group, A. Kievsky, M. Viviani et al.)**
- Impossible to infer proton properties from $^3\mathrm{He}$, too different dynamics; but a fresh test of LFRHD and of the importance of Relativity in nuclear structure is at hand.



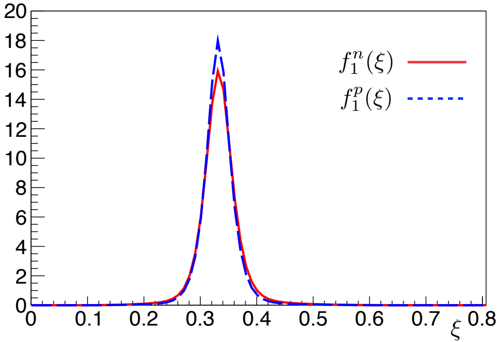
$f^\tau(x, |\mathbf{p}_\perp|^2)$, unpolarized TMD in an unpolarized ^{3}He.

Upper panel: Proton.
Lower panel: Neutron.

Notice the peak @ $x = 1/3$.

The integral over $\mathbf{p}_\perp$ yields the longitudinal light-cone momentum $f_1^\tau(x)$

^{3}He unpolarized light-cone momentum distributions – Pace, Rinaldi, Salmè, SS arXiv:22...

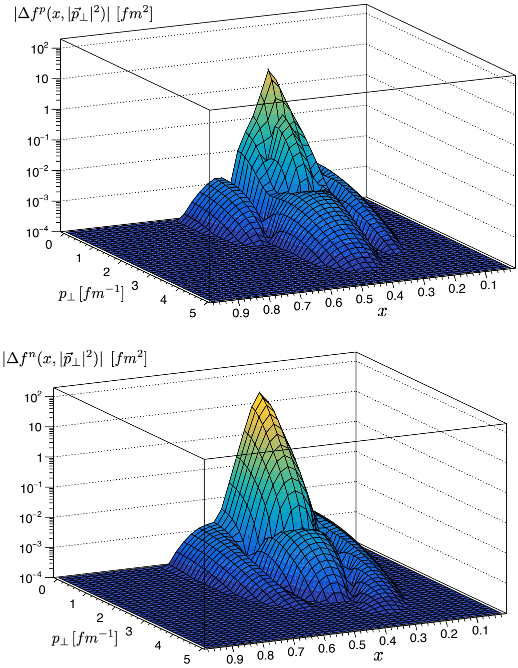


From the normalization of the Spectral Function one has

$$\int_0^1 d\xi \; f_\tau^A(\xi) \;=\; 1 \qquad\qquad f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi, \mathbf{k}_\perp)$$

$$N_A = \int d\xi \; \left[Z f_p^A(\xi) + (A-Z) f_n^A(\xi) \right] = 1 \,, \; MSR = \int d\xi \; \xi \; \left[Z f_p^A(\xi) + (A-Z) f_n^A(\xi) \right] = 1$$

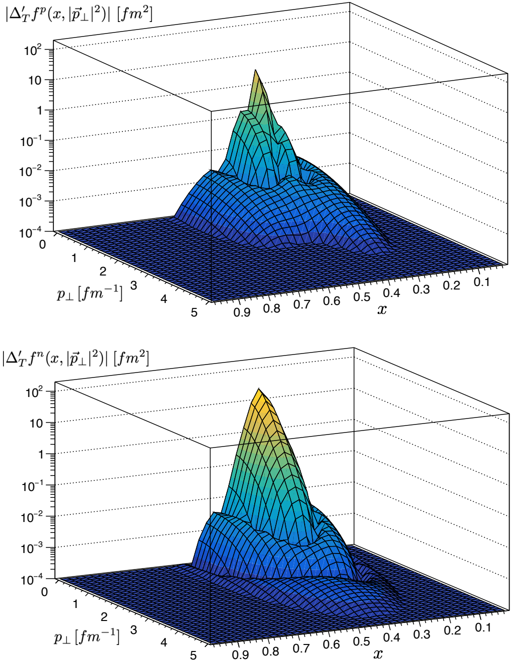
We get $MSR_{calc} = 0.999$ $\longrightarrow$ within LFHD normalization and *MSR* do not conflict!!!
$\longrightarrow$ crucial for the EMC effect (see later).
Well known that in IF they cannot be fulfilled at the same time
(Frankfurt & Strikman; Miller; ...80's)



Absolute value of the nucleon longitudinal-polarization distribution, $\Delta f^\tau(x, |\mathbf{p}_\perp|^2)$, in a longitudinally polarized ^{3}He.

Upper panel: Proton.
Lower panel: Neutron.

N.B. longitudinal wrt the virtual-photon axis.



Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x, |\mathbf{p}_\perp|^2)$, in a ^{3}He transversely polarized in the same direction of the nucleon polarization.

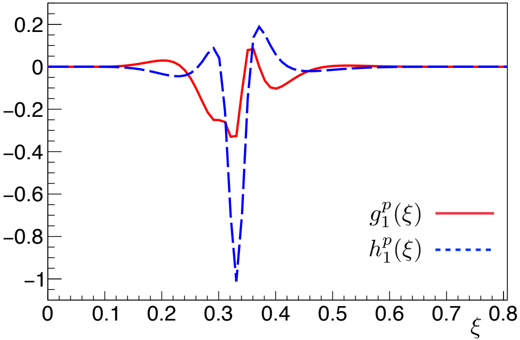
Upper panel: Proton.
Lower panel: Neutron.

N.B. transverse wrt the virtual-photon axis.

(Small) Difference wrt the previous slide due to relativistic effects

The ^{3}He polarized proton light cone momentum distributions

Pace, Rinaldi, Salmè, SS, arXiv:22...



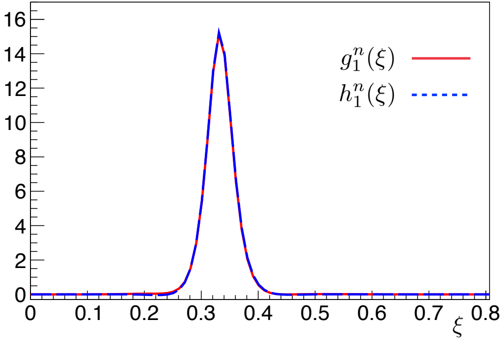
$g_1^p(\xi)$ longitudinal-polarization distribution

$h_1^p(\xi)$ transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off ^{3}He. Work in progress to LF update our NR results $\longrightarrow$ important for JLab12, EIC

The ^{3}He light-cone polarized neutron momentum distributions

Pace, Rinaldi, Salmè, SS, arXiv:22...



$g_1^n(\xi)$ longitudinal-polarization distribution

$h_1^n(\xi)$ transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off ^{3}He. Work in progress to LF update our NR results $\longrightarrow$ important for JLab12, EIC

Effective polarizations

Key role in the extraction of neutron polarized structure functions and neutron Collins and Sivers single spin asymmetries, from the corresponding quantities measured for ^{3}He

Effective longitudinal polarization (axial charge for the nucleon)

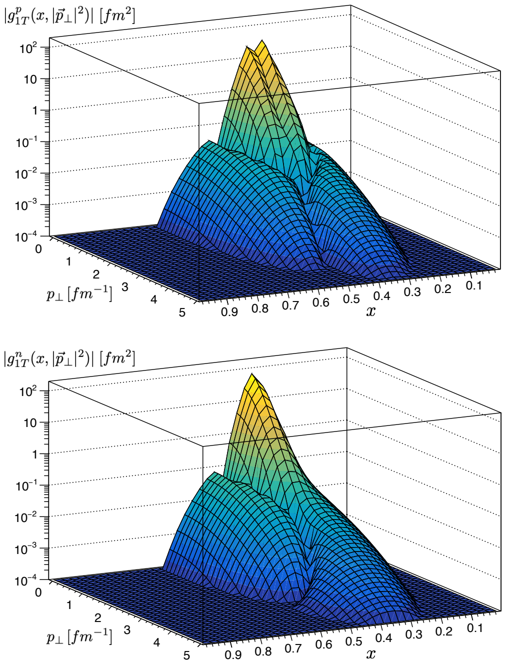
$$p_{||}^{\tau} = \int_0^1 dx \int d\mathbf{p}_{\perp} \; \Delta f^{\tau}(x, |\mathbf{p}_{\perp}|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p_{\perp}^{\tau} = \int_0^1 dx \int d\mathbf{p}_{\perp} \; \Delta'_T f^{\tau}(x, |\mathbf{p}_{\perp}|^2)$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

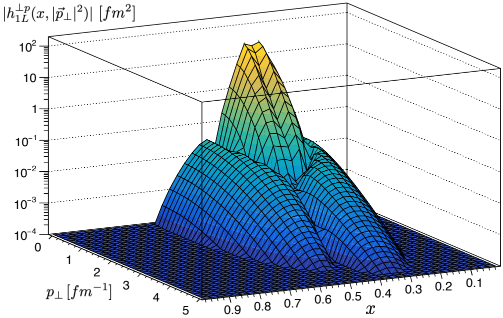
- The difference between the LF polarizations and the non relativistic results are up to 2% in the neutron case (larger for the proton ones, but it has an overall small contribution), and should be ascribed to the intrinsic coordinates, implementing the Macro-locality, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework: $p_{||}^{\tau}(NR) = p_{\perp}^{\tau}(NR)$

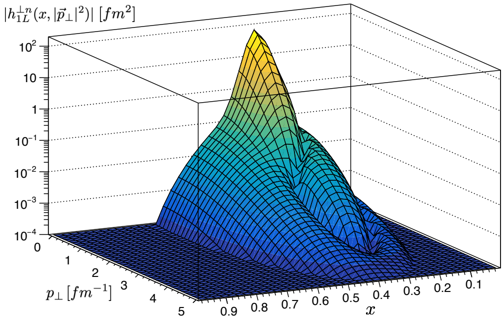


Absolute value of the nucleon longitudinal-polarization distribution, $g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He.

Upper panel: Proton.
Lower panel: Neutron.

Notice $|g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)| \sim |h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)|$, next slide.





Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.

Upper panel: Proton
Lower panel: Neutron

Notice $|h_{1L}^{\perp\tau}(x, |\mathbf{p}_\perp|^2)| \sim |g_{1T}^{\tau}(x, |\mathbf{p}_\perp|^2)|$, previous slide.

From the general principles implemented in the SF, TMDs receive contributions from both $L = 0$ and $L = 2$ orbital angular momenta (in the one-body density matrix). The relative weights depend upon the TMD.

Interestingly, R. Jacob et al. [NPA 626, 937 (1997)] and B. Pasquini et al. [PRD 78, 034025 (2008)] suggested approximate relations between TMDs, viz

$$\Delta f(x, |\mathbf{p}_\perp|^2) = \Delta'_T f(x, |\mathbf{p}_\perp|^2) \; + \; \frac{|\mathbf{p}_\perp|^2}{2M^2} \; h^\perp_{1T}(x, |\mathbf{p}_\perp|^2)$$

$$g_{1T}(x, |\mathbf{p}_\perp|^2) = \; - \; h^\perp_{1L}(x, |\mathbf{p}_\perp|^2)$$

$$(g_{1T})^2 \; + \; 2 \; \Delta'_T f \;\; h^\perp_{1T} = 0$$

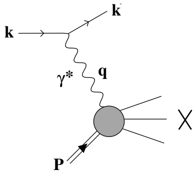
In our approach:

- the first relation is recovered retaining only the $L = 0$ contribution. Taking into account both $L = 0, \; 2$, the difference between the lhs and rhs is small for the neutron, not negligible for the proton;
- the second relation holds in modulus, since if the $L = 0$ component, tiny for those TMDs, is retained the minus sign works, while the dominant $L = 2$ contribution leads to a plus sign.
- The third relation does not hold, even if the $L = 2$ contribution is vanishing. Noteworthy, the integration on k_{23}, imposed by **Macro-locality**, spoils the relation:
  $\Rightarrow$ its effect becomes measurable **!** (Importance of 2-3 interaction!)

The EMC effect: the beginning

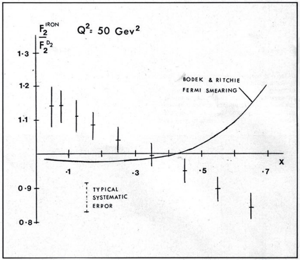
almost 40 years ago, the European Muon Collaboration (EMC) measured

$$R(x) = F_2^{^{56}Fe}(x)/F_2^{^{2}H}(x)$$



Expected result: $R(x) = 1$ up to small corrections due to the nucleon Fermi motion

Result:
Aubert et al. Phys.Lett. B123 (1983) 275
1488 citations (inSPIRE)



Naive parton model interpretation:

"Valence quarks, in the bound nucleon,
are in average slower
that in the free nucleon"

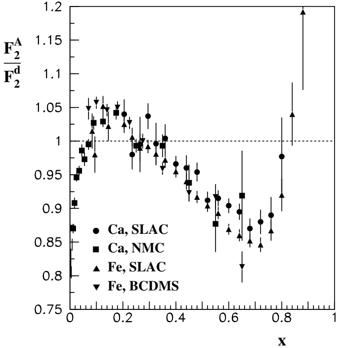
What happens in coordinate space?

Is the bound proton bigger than the free one???

EMC effect: more details

For nuclear DIS, $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$

- $x \leq 0.3$ "Shadowing region"
  coherence effects, γ^* interacts with partons belonging to different nucleons
- $0.2 \leq x \leq 0.8$ "EMC (binding) region": mainly valence quarks involved
- $0.8 \leq x \leq 1$ "Fermi motion region"
- $x \geq 1$ "TERRA INCOGNITA"
  Superfast quarks in superfast nucleons: few ones! Small σ, big errors



main features: universal behaviour independent on Q^2; weakly dependent on A; scales with the density $\rho \rightarrow$ global property? Or due to correlations...Local...

Explanation (exotic) advocated: confinement radius bigger for bound nucleons, quarks in *bags* with 6, 9, 3A quark, pion cloud effects...
Alone or mixed with conventional ones...

EMC effect: explanations?

Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

- the knowledge of nuclear PDFs is crucial for the analysis of heavy ions collisions;
- neutron parton structure measured with nuclear targets; several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models

One has probably to go beyond (not treated here...) **(R. Dupré and S.S., EPJA 52 (2016) 159)**

- **Hard Exclusive Processes (GPDs)**
- **SIDIS (TMDs)**

Status of "Conventional" calculations for light nuclei:

- **IF (NR) Calculations**: qualitative agreement but no fulfillement of both particle and MSMR... Not under control
- **LF Calculations**: in heavy systems, mean field approaches do not find an EMC effect in the valence region (**Miller and Smith, PRC C 65 (2002) 015211**);
  For light nuclei, no realistic calculations available (approximate attempt in **Oelfke, Sauer and Coester NPA 518 (1990) 593**)

Hadronic Tensor and Nuclear Structure Function F_2

The hadronic tensor is found to be **(Pace, Rinaldi, Salmè, SS Phys. Scri. 2020)**

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2 \, \kappa^+} \, \frac{1}{\xi} \, \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \; w_{N,\sigma}^{\mu\nu}(p,q)$$

with $w_{N,\sigma}^{\mu\nu}(p,q)$ that for a bound nucleon. In the Bjorken limit the nuclear structure function F_2^A can be obtained from the hadronic tensor

$$F_2^A(x) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2 \, \kappa^+} \, \frac{1}{\xi} \, \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \, (-x) \, g_{\mu\nu} \; w_{N,\sigma}^{\mu\nu}(p,q) =$$

$$= \sum_N \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \, d\kappa^+}{(2\pi)^3 \, 2 \, \kappa^+} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \, \frac{P_A^+}{p^+} \, \frac{x}{z} \; F_2^N(z)$$

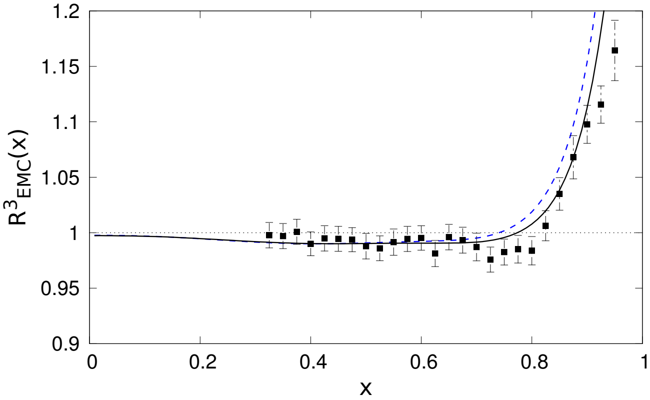
where $x = \frac{Q^2}{2P_A \cdot q}$ is the Bjorken variable, $\xi = \frac{\kappa^+}{\mathcal{M}_0(1,23)} \neq x$, $z = \frac{Q^2}{2p \cdot q}$ and $F_2^N(z) = -z \, g_{\mu\nu} \sum_\sigma w_{N,\sigma}^{\mu\nu}(p,q)$ the nucleon structure function .

In general, one cannot integrate on ϵ to obtain the momentum distribution: $\xi \propto \epsilon$.

Actually, in the Bj limit it can be done!

F_2 and the EMC effect can be calculated through the unpolarized TMD !

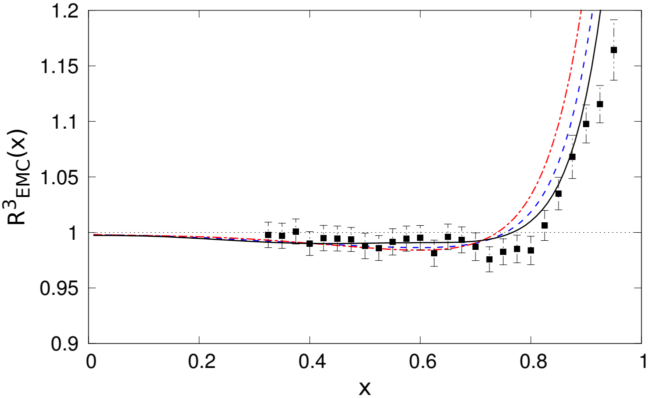
Pace, Rinaldi, Salmè, SS arXiv:22...



$$R^3_{EMC} = R^3_2(x)/R^2_2(x) \qquad R^A_2(x) = \frac{A\ F^A_2(x)}{Z\ F^p_2(x)+(A-Z)\ F^n_2(x)}$$

- **Solid line: with Av18 description of ^{3}He**, Dashed line: including three-body forces (U-IX) with "SMC" nucleon structure functions **(Adeva et al PLB 412, 414 (1997)).**
- **Full squares**: data from J. Seely et al., PRL. 103, 202301 (2009) reanalyzed by S. A. Kulagin and R. Petti, PRC **82**, 054614 (2010)

Pace, Rinaldi, Salmè, SS arXiv:22...



- Av18+UIX LF results with different nucleon F_2. Solid: SMC, Dashed: NMC; Red: GRV (1998): mild but annoying dependence, due to the neutron uncertainty
- Full squares data from J. Seely et al., PRL. 103, 202301 (2009) reanalyzed by S. A. Kulagin and R. Petti, PRC **82**, 054614 (2010)
- Conclusions: small but solid effect; waiting for MARATHON data; essential the extension to ^{4}He (which presents a bigger effect)

Conclusions & Perspectives

- **A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed.** The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework.
  N.B. Normalization and momentum sum rule are both automatically fulfilled.
- ★ Macro-locality can be implemented, as it must be and plays a role in precision experiments (see also TMD's relations).
- ★ The Spectral Function is related to the valence contribution to the correlator introduced for a QFT description of SiDIS reactions involving the nucleon, applied for the first time to ^{3}He.

  ★★ General principles fulfilled by the LF Spectral function entail relations among T-even twist-2 (and also twist-3) valence TMDs, with interesting angular momentum dependence.
- encouraging calculation of $^3\mathrm{He}$ EMC, shedding light on the role of a reliable description of the nucleus. Also the LC spin-depedent momentum distributions are available, for both longitudinal and transverse polarizations of the nucleon.
  Crucial extension to ^{4}He!
- Analyses of exclusive reactions, with polarized initial and final states, for accessing nuclear TMD's in ^{3}He are in progress