Case for quarkyoniclike matter from a constituent quark model

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Phys. Rev. D 104, 094024
arXiv : 2105.12948

Contents

Introduction

Massive Neutron Star

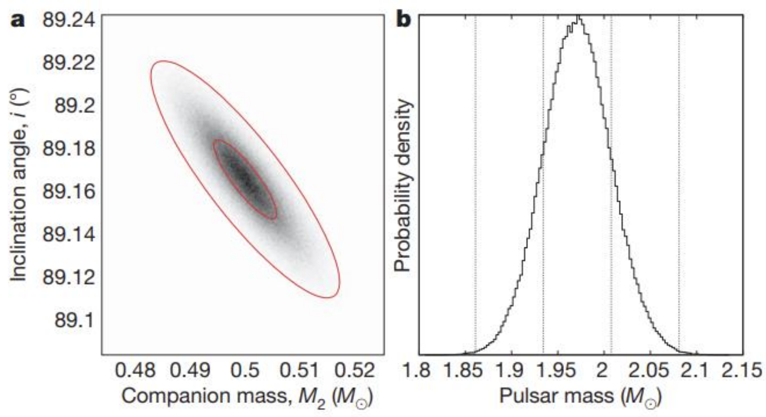


Figure 2 | Results of the MCMC error analysis. a, Grey-scale image shows the

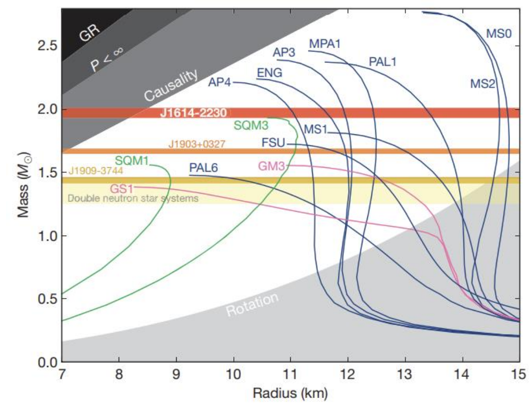
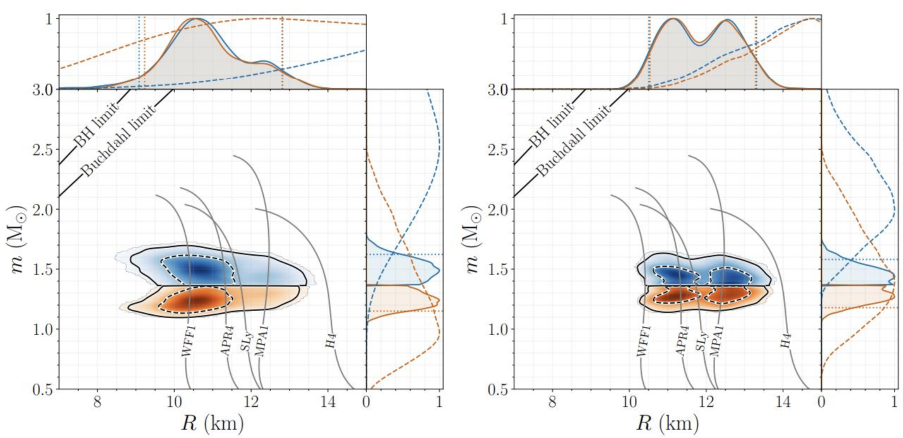


Figure 3 | Neutron star mass–radius diagram. The plot shows non-rotating

P. Demorest, et al. Nature 467, 1081 (2010)

To support the massive neutron stars whose masses are larger than two times the solar mass, it was found that the equation of states for dense matter had to be sufficiently hard.

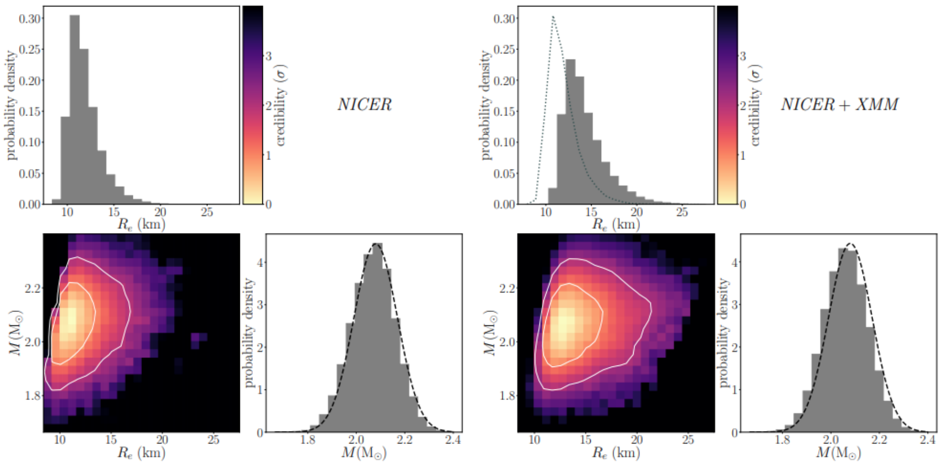


B. P. Abbott, et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 121, no. 16, 161101 (2018)

The tidal deformability constrained via the GW170817 observation requires a relatively soft equation of states.

Speed of Sound

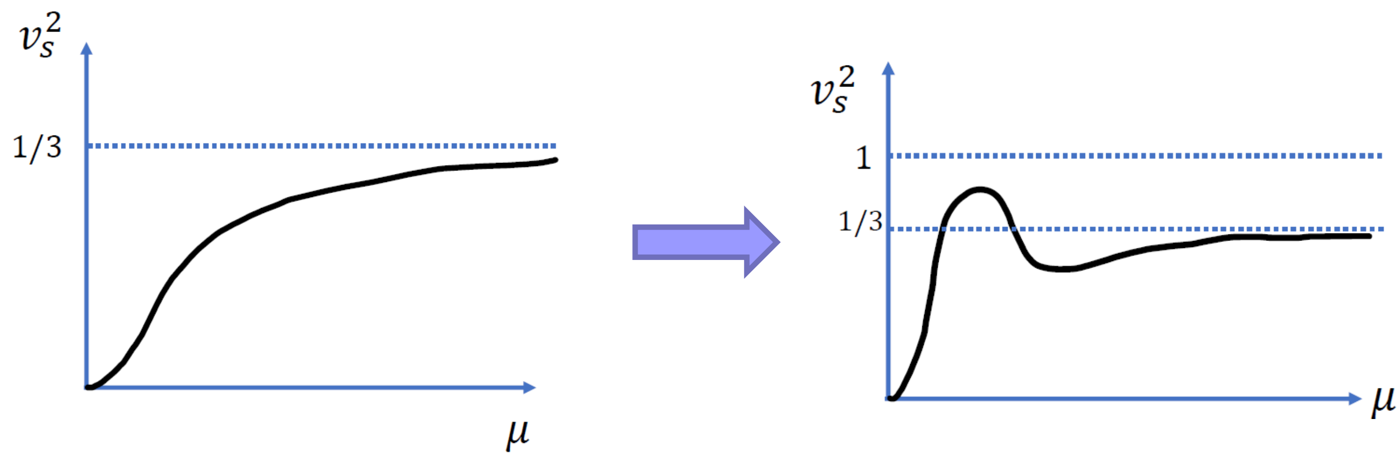
4.2. *The Mass and Radius of PSR J0740+6620*



M. C. Miller, et al. Astrophys. J. Lett. 918, 2 [arXiv:2105.06979 [astro-ph.HE]]

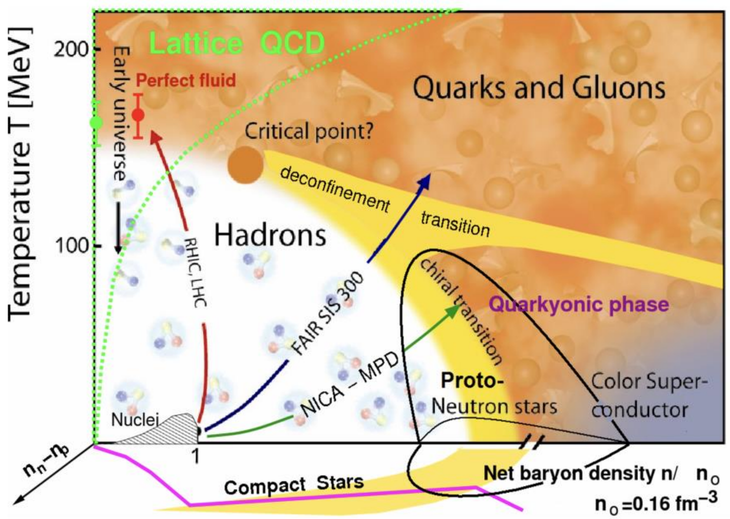
Speed of sound

$$v_s^2 = \frac{dP}{d\epsilon}$$



Quarkyonic Matter

The quarkyonic matter is a hypothetical new phase matter occurring at high baryon density and temperature $T < \Lambda_{QCD}$.



Larry MaLerran, "FAQS ABOUT QUARKYONIC MATTER"

If the quark Fermi momentum is large enough($k_F \gg \Lambda_{QCD}$), the quark distributed around Fermi surface will be confined into the baryon-like state.

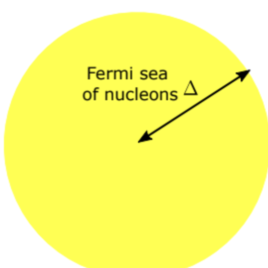
Then, the baryon momentum phase space distribution has a shell-like distribution due to the Pauli blocking effect from the quasi-free quarks occupying the lower distributions.

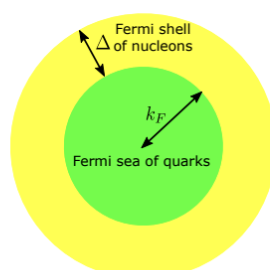
Quarkyonic Matter

At low baryon density quarkyonic matter resembles nuclear matter.

At high density the fermi distribution function of quarkyonic matter is different from purely hadronic or quark matter.

Within the quarkyonic matter framework, nucleon and quark degrees of freedom are described with a single fermi distribution function.



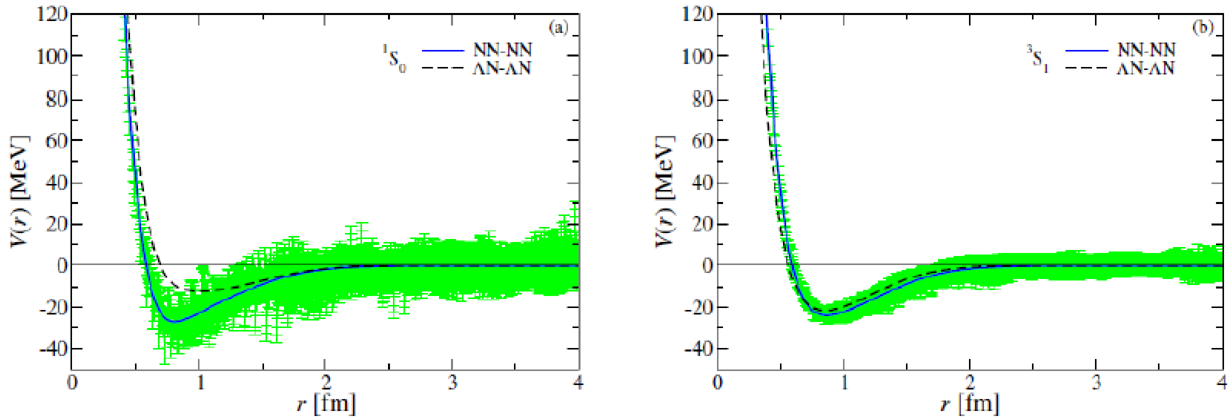


S. Sen and L. Sivertsen, arXiv:2011.04681

Excluded volume model

- To be more realistic

Lattice QCD study for hadron interaction (HALQCD T. Hatsuda et al.)



- Hard core nature can be embodied by semi-classical size v_0

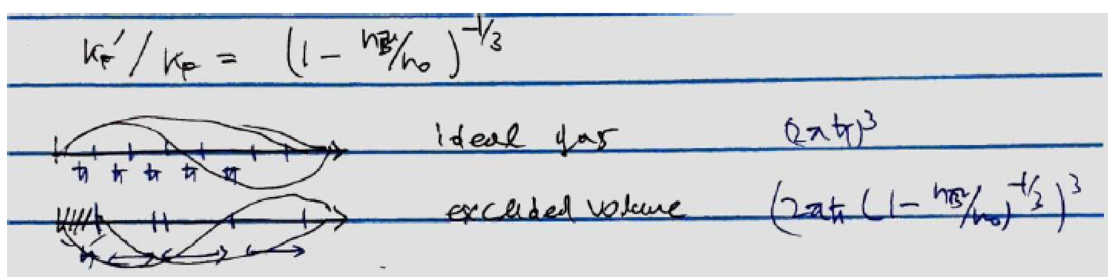
$$V_{ex} = V\ (1 - n/n_0)$$

$$n_0 = 1/v_0$$

$$n_b^{ex} = \frac{n_b}{1 - n_b/n_0}$$

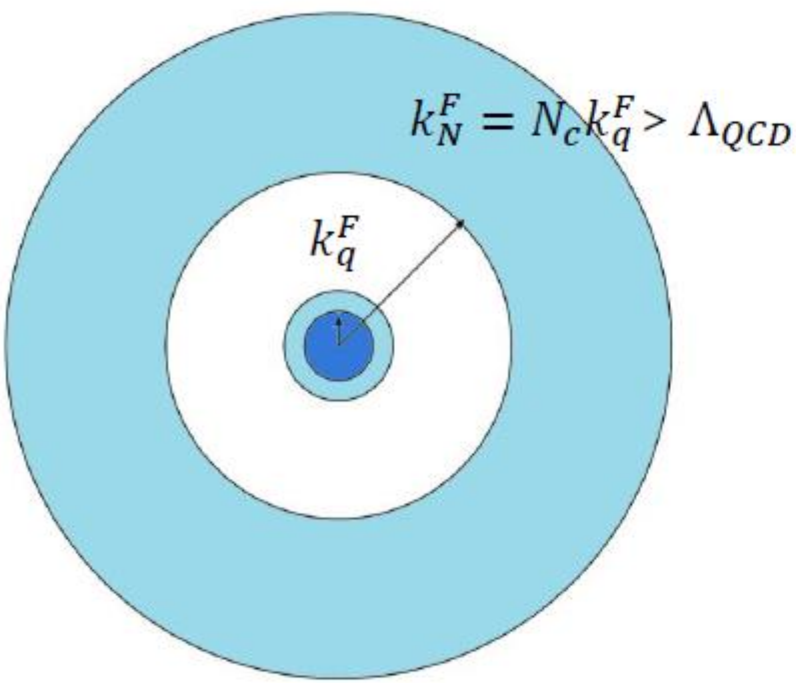
$$= \frac{2}{(2\pi)^3} \int_0^{K_F^b} d^3k$$

Hardcore effective size and excluded volume
→ reduced available space (fast nucleons)



Quarkyonic-like baryon shell structure

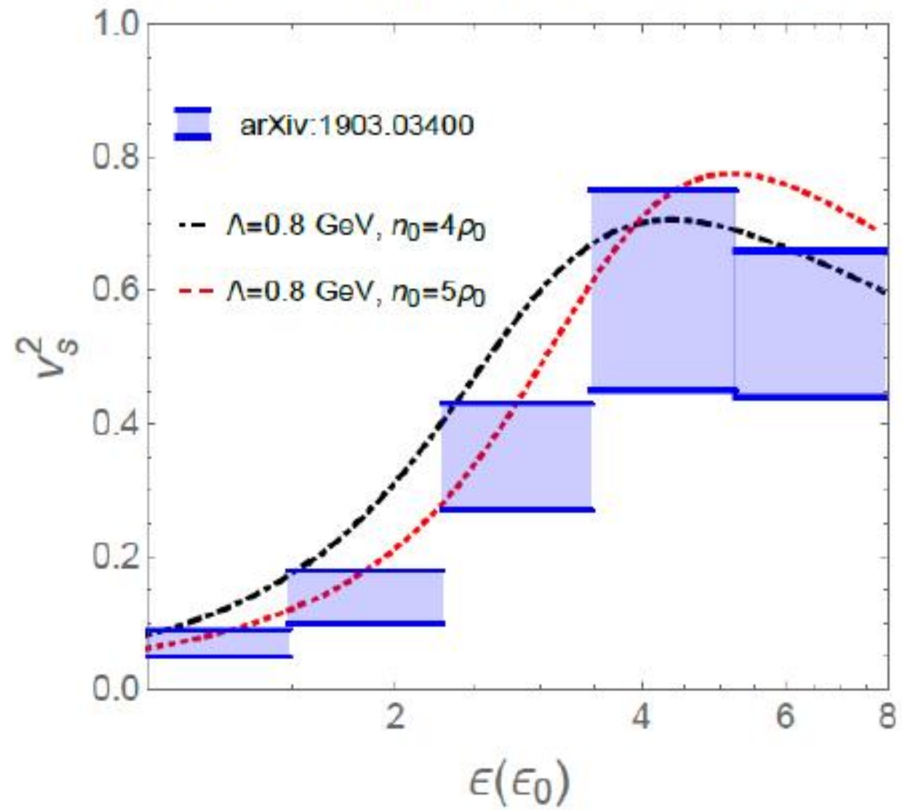
$$\tilde{\epsilon} = 4\left(1 - \frac{n_N^N}{n_0}\right)\int_{k_F}^{k_F+\Delta} \frac{d^3k}{(2\pi)^3}\left((N_c m_Q)^2 + k^2\right)^{\frac{1}{2}} + \frac{2Nc}{\pi^2}\int_0^{k_F/N_c} dkk\left(\Lambda^2 + k^2\right)^{\frac{1}{2}}\left(m_Q^2 + k^2\right)^{\frac{1}{2}}$$



$k_Q < k_F/N_c$

$k_F < k_N < k_F + \Delta$

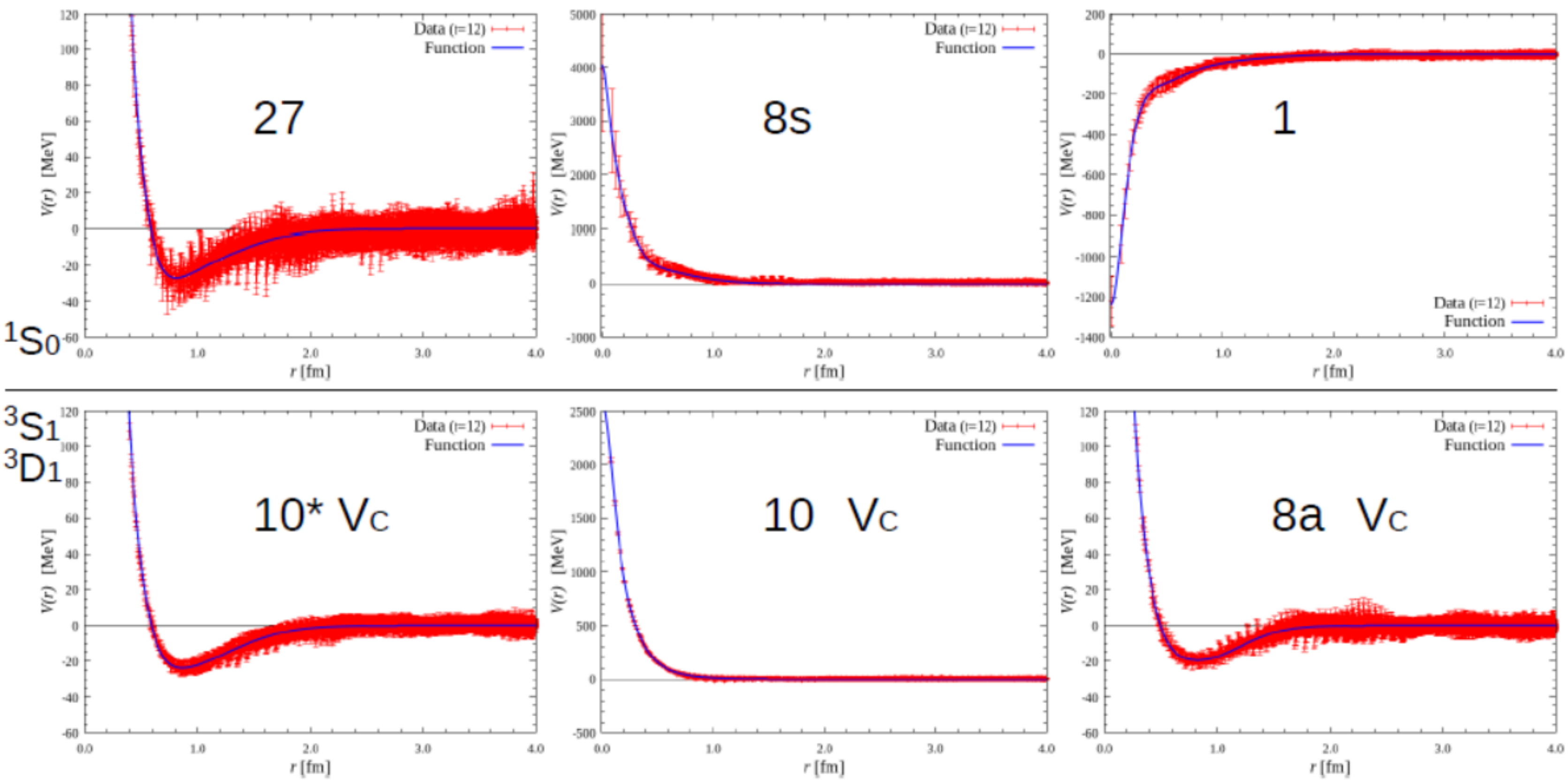
(PRC101 (2020) 035201 K.S.J., L.M., S.S.)



Baryon-baryon interaction in lattice QCD (SU(3) broken)

BB S-wave potentials

(96,96) src
$t - t_0 = 12$



$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1,\ ^3D_1}$$

$M_\pi \simeq 146$, $M_K \simeq 525$ MeV   almost physical point

$M_N \simeq 956$, $M_\Lambda \simeq 1121$, $M_\Sigma \simeq 1201$, $M_\Xi \simeq 1328$ MeV

T. Inoue, QNP2018 conference

Hard cord repulsive interaction – Quark model

$$H=\sum_{i=1}^{N}(m_i+\frac{\mathbf{p}_i^2}{2m_i})-\frac{3}{16}\sum_{i<j}^{N}(V_{ij}^{C}+V_{ij}^{CS}),$$

$$V_{ij}^{C}=-\lambda_i^c\lambda_j^c\left(-\frac{\kappa}{r_{ij}}+\frac{r_{ij}}{a_0}-D\right),$$

Confinement potential

$$V_{ij}^{CS}=\frac{\kappa'}{m_i m_j r_{0ij}^2}\frac{1}{r_{ij}}e^{-(r_{ij}/r_{0ij})^2}\lambda_i^c\lambda_j^c\ \sigma_i\cdot\sigma_j$$

Hyperfine potential

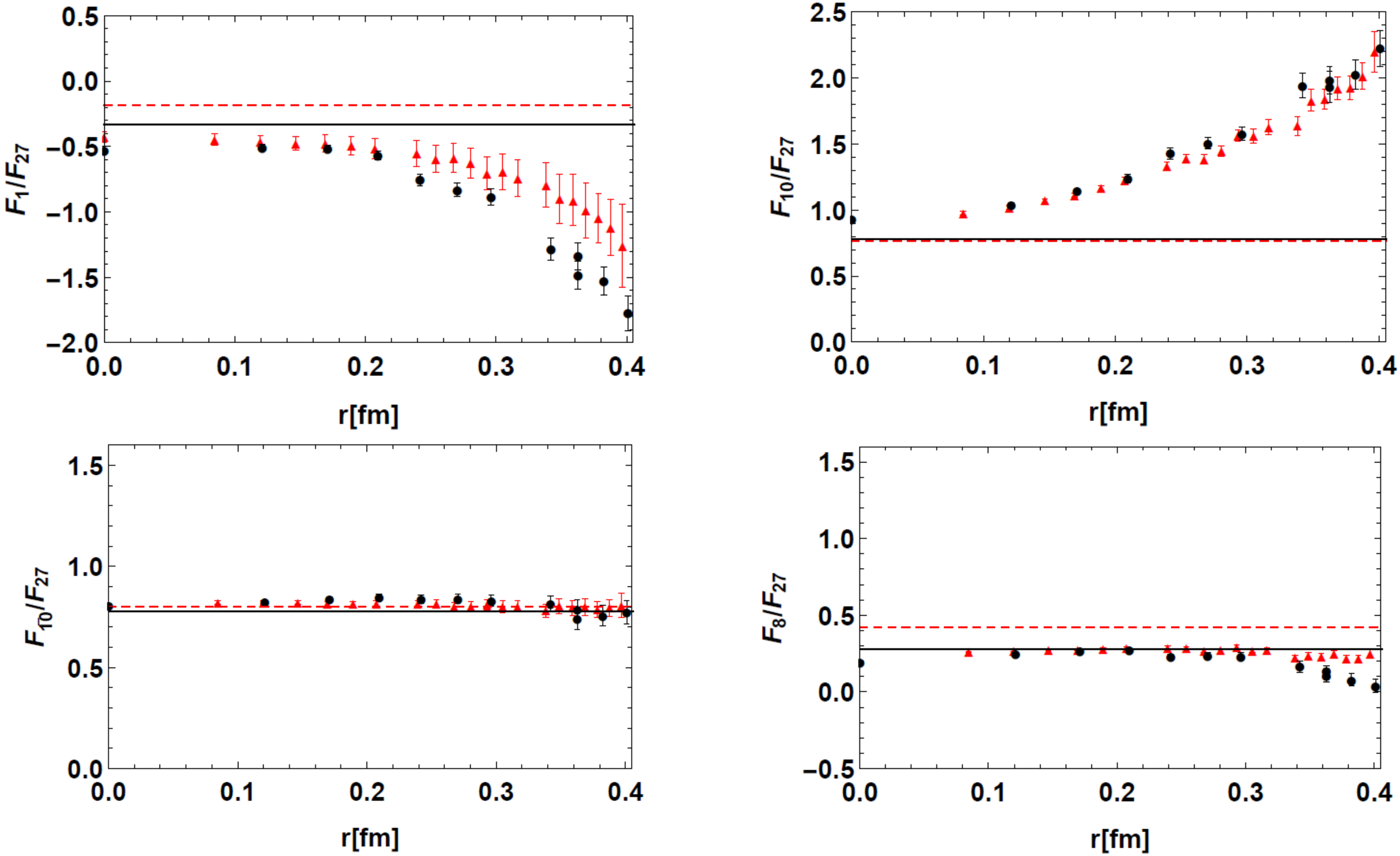
$$V_{\mathrm{CQM}}=\langle H\rangle_{6q}-E_{BB'},$$
$$E_{BB'}=M_B+M_{B'}+K_{\mathrm{rel},BB'}.$$

Comparison with Lattice QCD

SU(3) symmetric case (solid black line)
SU(3) broken case (red dashed line)

(F,S) $SU(3)_F$ symmetric case	(1,0)	(27,0)	(10,1)	$(\overline{10},1)$	(8,1)
$\Delta\mathcal{CS}(F_i)$	-8	24	$\frac{56}{3}$	$\frac{56}{3}$	$\frac{20}{3}$
$V_I(F_i)/V_I(F_{27})$	-0.333	1	0.778	0.778	0.278
LQCD $V_{F_i}/V_{F_{27}}(r=0)$	-0.528	1	0.928	0.807	0.195

(F,S) $SU(3)_F$ breaking case	(1,0)	(27,0)	(10,1)	$(\overline{10},1)$	(8,1)
$V_I(F_i)/V_I(F_{27})$	-0.185	1	0.767	0.801	0.424
LQCD $V_{F_i}/V_{F_{27}}(r=0)$	-0.444	1	0.935	0.785	0.266



A. Park, S.H.Lee, T. Inoue, T. Hatsuda, Eur. Phys. J. A 56, 93(2020)

Quarkyonic Configuration

Quarkyonic configuration

In the low momentum region, quark Fermi sea appears smoothly near the origin of the phase space.

Above the quark modes, due to the presumed Pauli blocking effect, the confined quarks take the higher momentum region of the phase space through a shell-like baryon distribution in the phase space.

$$n_{b_i} \equiv n_s \int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}} \frac{d^3k}{(2\pi)^3},$$

$$n_{\tilde{Q}_l} \equiv 2 \int_0^{k_F^{Q_l}} \frac{d^3k}{(2\pi)^3},$$

$$n_B = \sum_i n_{b_i} + \sum_l n_{\tilde{Q}_l}$$

For a given total baryon number density, the transition to the quarkyoniclike configuration occurs if the nontrivial minimum of the free energy (where $n_{\tilde{Q}} \neq 0$) is obtained.

Quarkyonic configuration

The free energy density at $T \to 0$ can be written as

$$f = \epsilon - Ts$$
$$\to \epsilon = n_s \sum_i \int_{k_F^{b_i}}^{[k_F+\Delta]_{b_i}} \frac{d^3k}{(2\pi)^3} E_{b_i}(k)$$
$$+ 2\sum_l \int_0^{k_F^{Q_l}} \frac{d^3k}{(2\pi)^3} E_{Q_l}(k),$$

where $E_{b_i}(k)$ and $E_{Q_l}(k)$ represent the single baryon and quark energy at each phase cell, respectively.

The extremization condition at a given total baryon density n_B can be found as

$$d\epsilon = \frac{\partial \epsilon}{\partial n_N} dn_N + \frac{\partial \epsilon}{\partial n_{\tilde{Q}}} dn_{n_{\tilde{Q}}},$$
$$= \left(\frac{\partial \epsilon}{\partial n_N} - \frac{\partial \epsilon}{\partial n_{\tilde{Q}}} \right) dn_N = 0.$$

Key Points

1. $N_c = 3$

2. Hard-core interaction from a constituent quark model

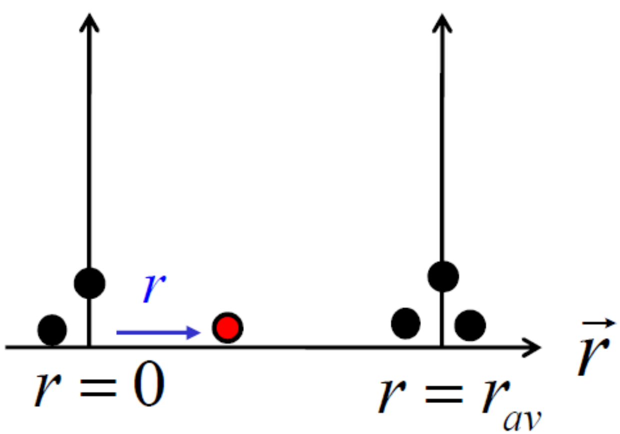
3. $r = \frac{1}{k}$

While the energies will be expressed in terms of the position and momentum coordinates, we can express them into momentum coordinates since the lowest energy mode of the fermions satisfy the minimum uncertainty.

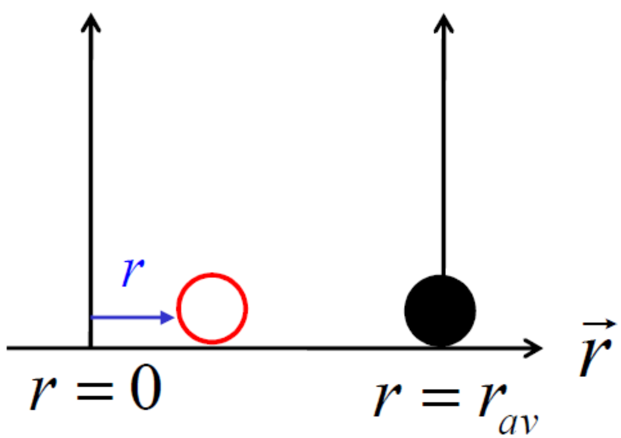
Quark model approach I

Quark model approach

Excitation energy of the quark

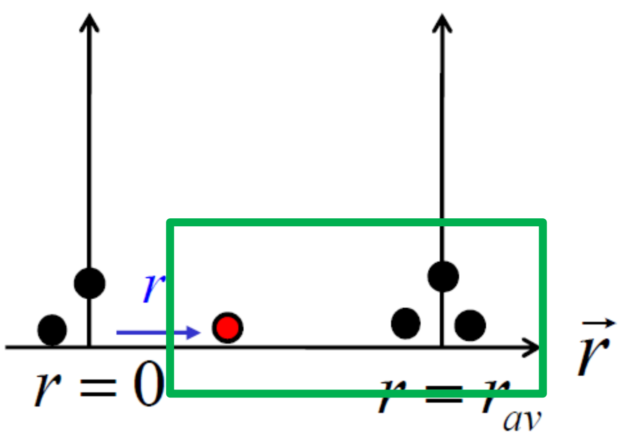


Excitation energy of the Baryon



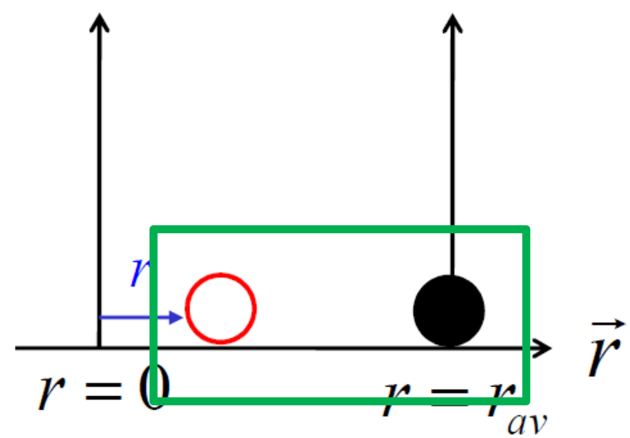
Quark model approach

Excitation energy of the quark



4-quark configuration

Excitation energy of the Baryon



dibaryon configuration

Hamiltonian

$$H = \sum_{i=1}^{N} (m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \frac{3}{16} \sum_{i<j}^{N} (V_{ij}^C + V_{ij}^{CS}),$$

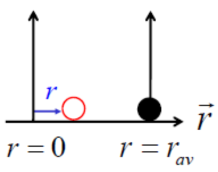
$$V_{ij}^C = -\lambda_i^c \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D \right),$$

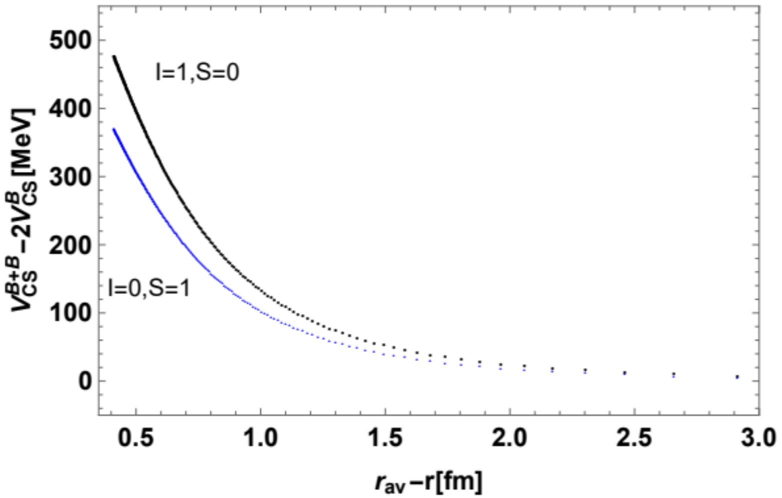
Confinement potential

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j$$

Hyperfine potential

Baryon excitation





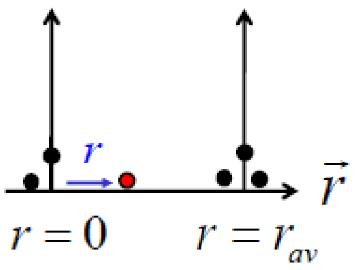
$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j$$

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r - r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

m_B : nucleon mass
$a_B = 0.218$ GeV·fm
$b_B = 1.474$ fm

Quark excitation

Quark excitation



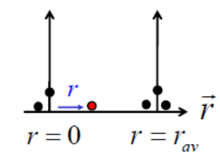
$$\mathcal{X} \equiv -\sum_{i<j}^{N} \langle \lambda_i^c \lambda_j^c \, \sigma_i \cdot \sigma_j \rangle$$
$$= N(N-10) + \frac{4}{3} S(S+1) + 4C_F + 2C_C.$$

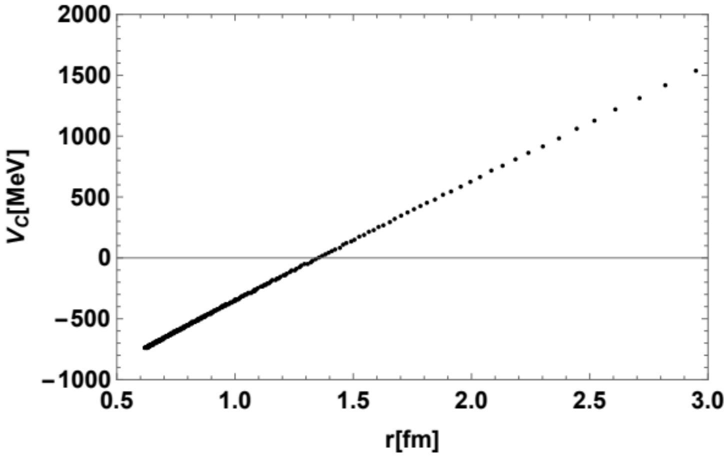
$\chi = -8$ for both nucleon and ud diquark.

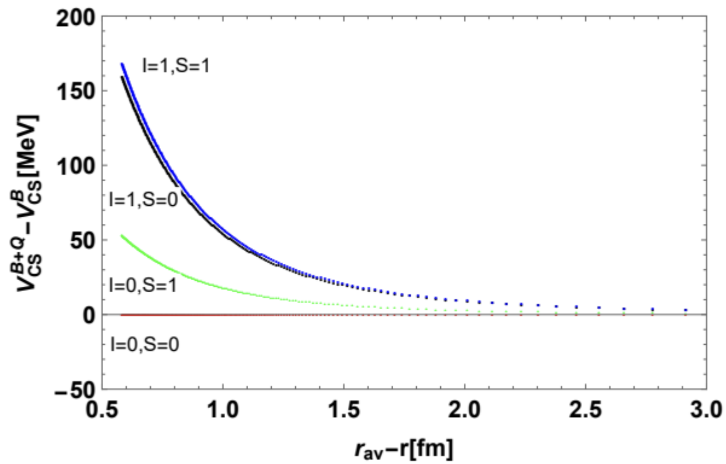
→ The excitation of a d quark will not cost any color-spin energy as the attractive (ud) diquark remains intact.

Quark excitation

Quark excitation







$$V_{ij}^{C} = -\lambda_i^c \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D \right),$$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma \left(r H(r_{\max} - r) + r_{\max} H(r - r_{\max}) \right) + R_n \left(\frac{a_Q}{|r - r_{av}|} e^{-\frac{|r - r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}} \right)$$

m_D : diquark mass
$m_Q = 0.343$ GeV
$\sigma = 0.962$ GeV/fm
$a_Q = 0.2$ GeV·fm
$b_Q = 0.745$ fm

$H(r)$: Heaviside step function
$R_n = 0.7$

Results

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r - r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma\left(rH(r_{\max} - r) + r_{\max}H(r - r_{\max})\right) + R_n\left(\frac{a_Q}{|r - r_{av}|} e^{-\frac{|r-r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}}\right)$$

$$\Delta E = E_d(k_F^d) + m_D - E_n(k_F^n = k_F^d).$$

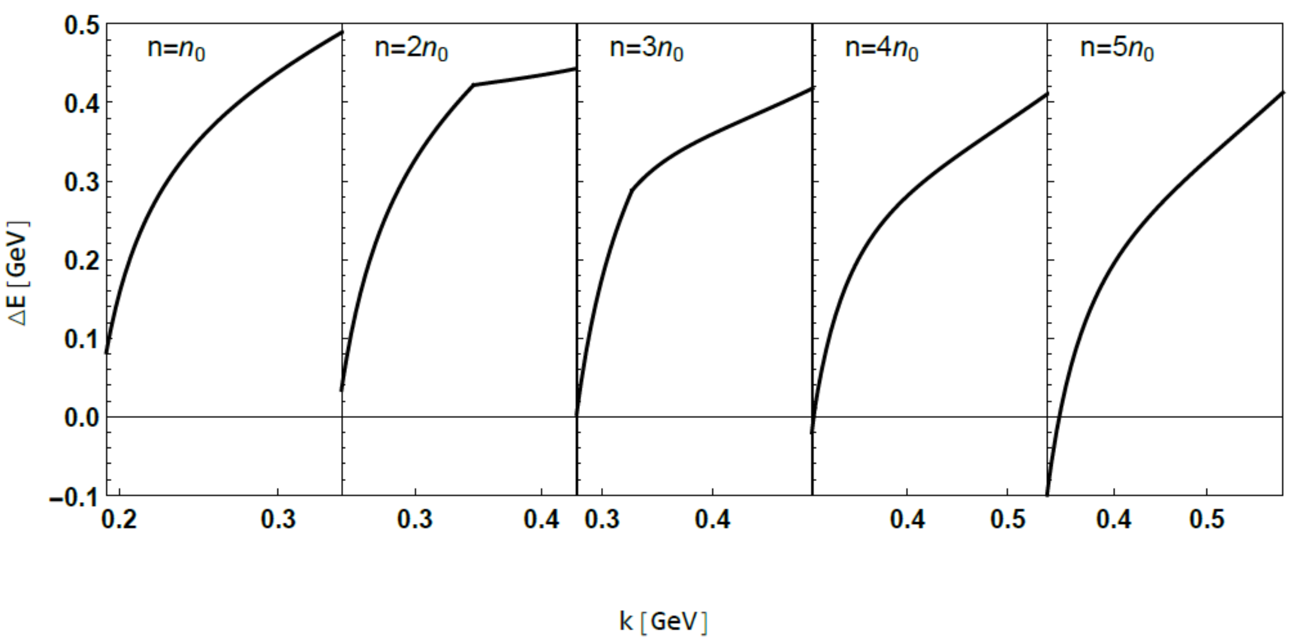


FIG. 5. ΔE where $R_n = 0.7$. Here, n_0 is a normal nuclear density.

Results

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r - r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma\left(rH(r_{\max} - r) + r_{\max}H(r - r_{\max})\right) + R_n\left(\frac{a_Q}{|r - r_{av}|} e^{-\frac{|r-r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}}\right)$$

$$\Delta E = E_d(k_F^d) + m_D - E_n(k_F^n = k_F^d).$$

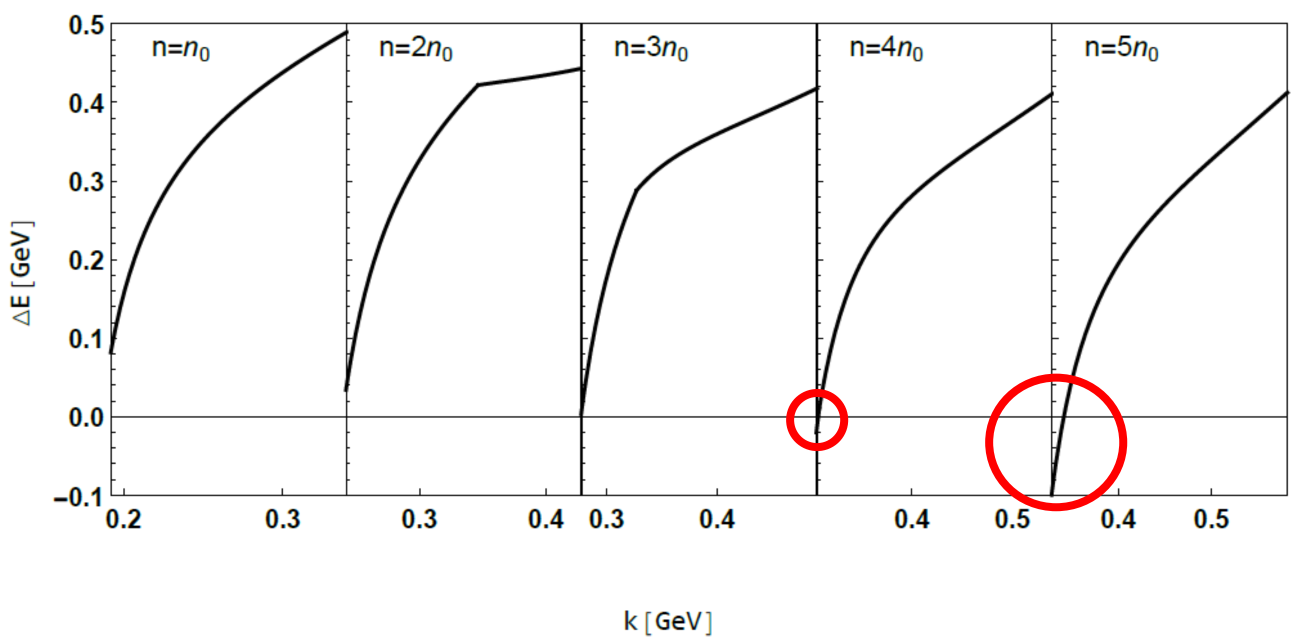
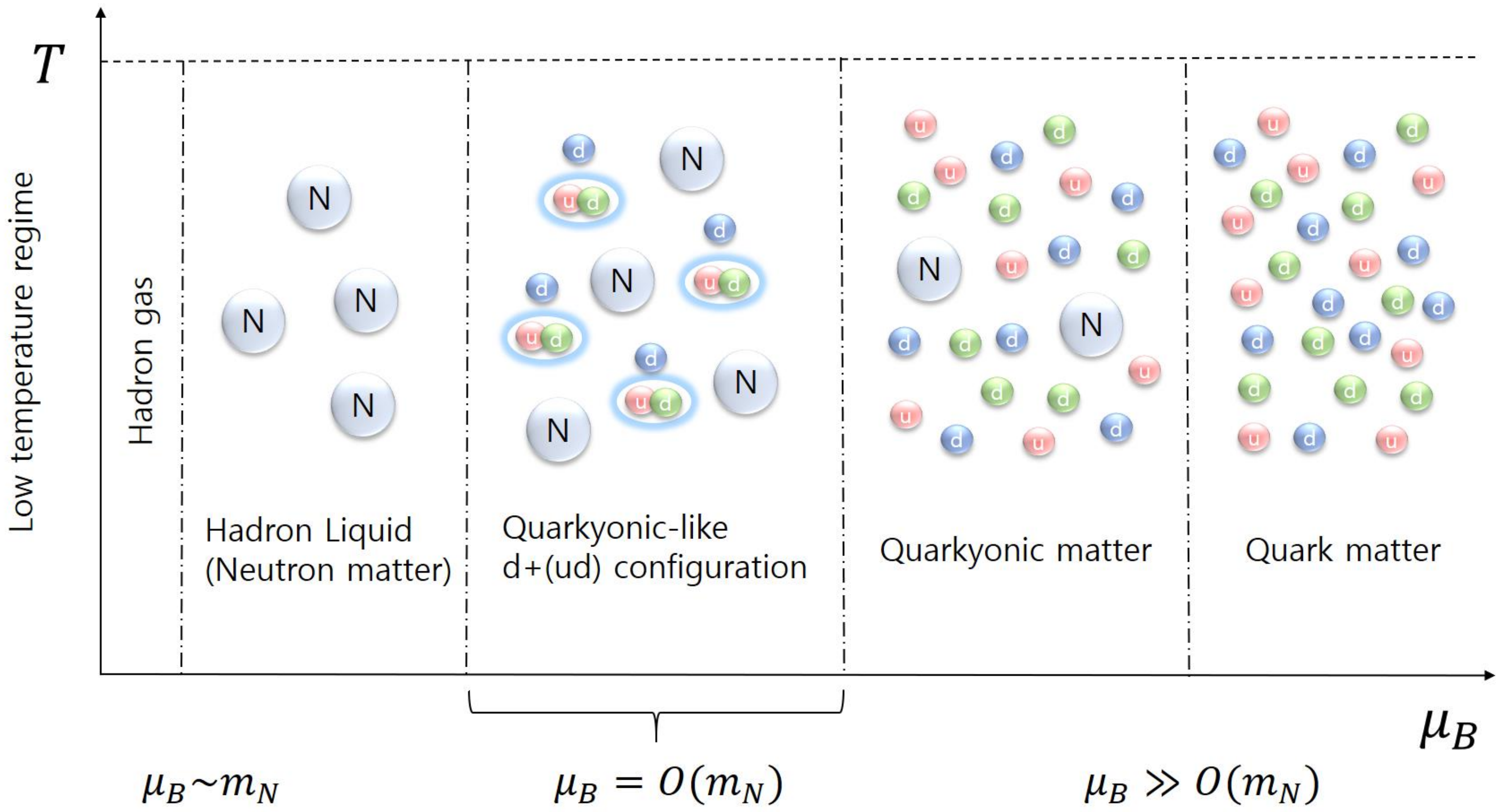


FIG. 5. ΔE where $R_n = 0.7$. Here, n_0 is a normal nuclear density.

Results



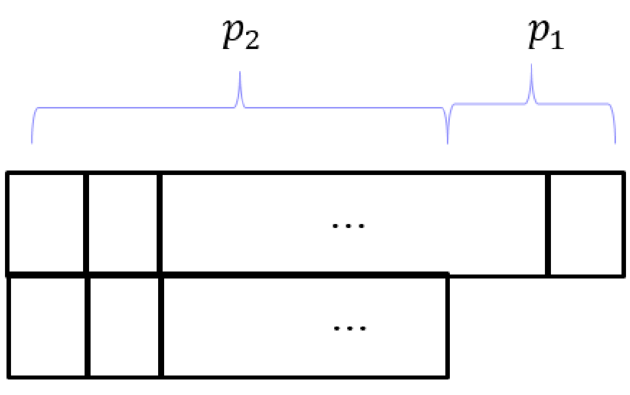
T
Low temperature regime
Hadron gas
Hadron Liquid
(Neutron matter)
Quarkyonic-like
d+(ud) configuration
Quarkyonic matter
Quark matter
μ_B
$\mu_B \sim m_N$
$\mu_B = O(m_N)$
$\mu_B \gg O(m_N)$

Quark model approach II

Color-spin interaction

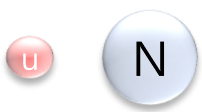
$$V_{CS} = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j$$
$$\equiv \frac{1}{m_u m_u} H_{CS},$$

$$-\sum_{i<j}^{n} \lambda_i^c \lambda_j^c \sigma_i \cdot \sigma_j = n(n-10) + \frac{4}{3} S(S+1) + 4C_F + 2C_C,$$
$$4C_F = \frac{4}{3}(p_1^2 + p_2^2 + 3p_1 + 3p_2 + p_1 p_2),$$



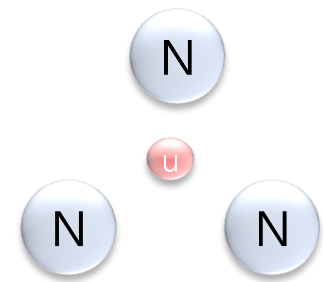
Quark excitation

1 quark + 1 baryon

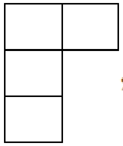
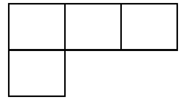


1 quark + 2 baryons

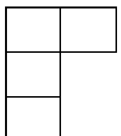
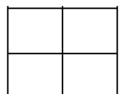
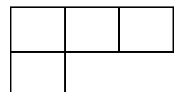
1 quark + 3 baryons



1 Quark + 1 Baryon : 4-quark state

Color :  , Flavor $\otimes$ Spin : 

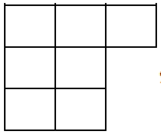
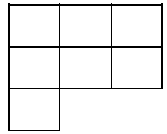
Flavor states of 4 quarks : $\mathbf{8} \times \mathbf{3} = \mathbf{3} + \overline{\mathbf{6}} + \mathbf{15}$

Flavor and spin :  $\mathbf{3}(S=0,1)$,  $\overline{\mathbf{6}}(S=0,1)$,  $\mathbf{15}(S=0,1)$.

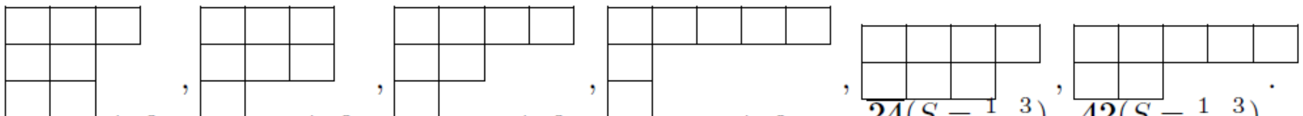
$$\Delta H_{CS}^{\rm 4q} = H_{CS}^{\rm 4q} - H_{CS}^{\rm 1b}$$

$$\Delta H_{CS}^{\rm avg} = \frac{1}{\sum_{F,S} d_F d_S} \sum_{F,S} d_F d_S \Delta H_{CS}^{\rm 4q}.$$

1 Quark + 2 Baryons : 7-quark state

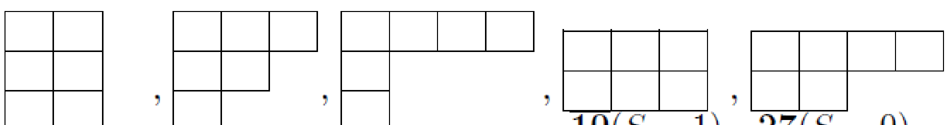
Color :  , Flavor $\otimes$ Spin : 

Flavor states of 7-quark : $\mathbf{8} \times \mathbf{8} \times \mathbf{3} = \mathbf{3}_{(m=3)} + \overline{\mathbf{6}}_{(m=3)} + \mathbf{15}_{(m=4)} + \mathbf{15}' + \overline{\mathbf{24}}_{(m=2)} + \mathbf{42}$

Flavor and spin : 

$\mathbf{3}(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{6}}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{15}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{15}'(S=\frac{1}{2},\frac{3}{2})$, $\overline{\mathbf{24}}(S=\frac{1}{2},\frac{3}{2})$, $\mathbf{42}(S=\frac{1}{2},\frac{3}{2})$.

Flavor states of 2-baryon : $\mathbf{8} \times \mathbf{8} = \mathbf{1} + \mathbf{8}_{(m=2)} + \mathbf{10} + \overline{\mathbf{10}} + \mathbf{27}$

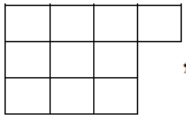
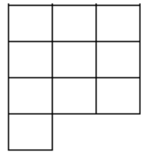
Flavor and spin : 

$\mathbf{1}(S=0)$, $\mathbf{8}(S=1)$, $\mathbf{10}(S=1)$, $\overline{\mathbf{10}}(S=1)$, $\mathbf{27}(S=0)$

$$\Delta H^{7q}_{CS} = H^{7q}_{CS} - \sum_{2b} w_{2b} H^{2b}_{CS}$$

$$\Delta H^{avg}_{CS} = \frac{1}{2\sum_{F,S} d_F d_S m_{FS}} \sum_{F,S} d_F d_S m_{FS} \Delta H^{7q}_{CS}.$$

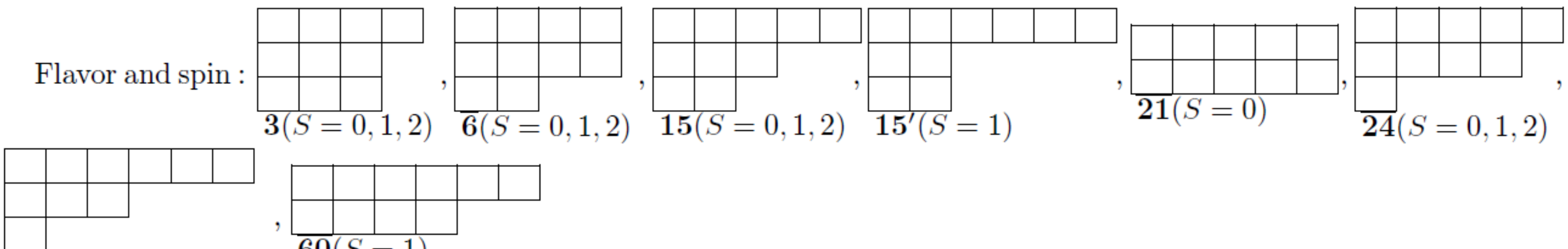
1 Quark + 3 Baryons : 10-quark state

Color :  , Flavor $\otimes$ Spin : 

Flavor states of 10 quarks

: $\mathbf{8}\times\mathbf{8}\times\mathbf{8}\times\mathbf{3} = \mathbf{3}_{(m=10)} + \overline{\mathbf{6}}_{(m=12)} + \mathbf{15}_{(m=18)} + \mathbf{15}'_{(m=6)} + \overline{\mathbf{21}}_{(m=2)} + \overline{\mathbf{24}}_{(m=12)} + \mathbf{42}_{(m=9)} + \mathbf{48}_{(m=2)} + \overline{\mathbf{60}}_{(m=3)} + \mathbf{90}$

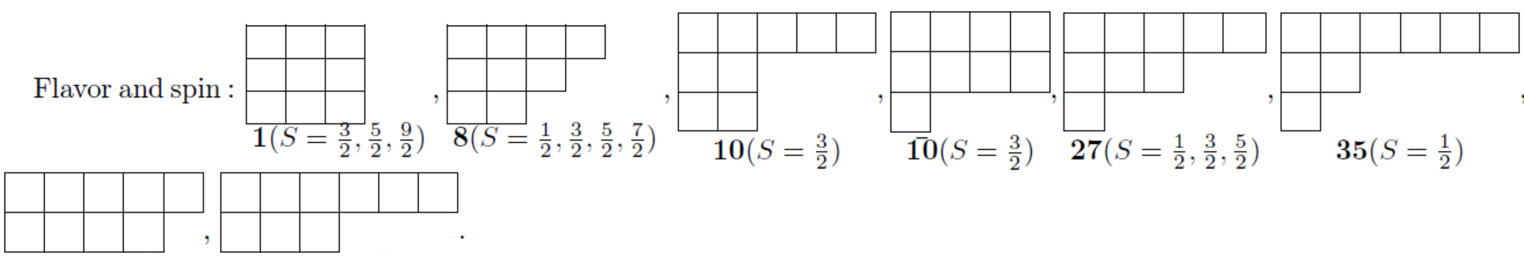
Flavor and spin :



$\mathbf{3}(S=0,1,2)$, $\overline{\mathbf{6}}(S=0,1,2)$, $\mathbf{15}(S=0,1,2)$, $\mathbf{15}'(S=1)$, $\overline{\mathbf{21}}(S=0)$, $\overline{\mathbf{24}}(S=0,1,2)$, $\mathbf{42}(S=0,1,2)$, $\overline{\mathbf{60}}(S=1)$

Flavor states of 3-baryon : $\mathbf{8}\times\mathbf{8}\times\mathbf{8} = \mathbf{1}_{(m=2)} + \mathbf{8}_{(m=8)} + \mathbf{10}_{(m=4)} + \overline{\mathbf{10}}_{(m=4)} + \mathbf{27}_{(m=6)} + \mathbf{35}_{(m=2)} + \overline{\mathbf{35}}_{(m=2)} + \mathbf{64}$

Flavor and spin :



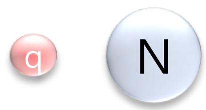
$\mathbf{1}(S=\frac{3}{2},\frac{5}{2},\frac{9}{2})$, $\mathbf{8}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2})$, $\mathbf{10}(S=\frac{3}{2})$, $\overline{\mathbf{10}}(S=\frac{3}{2})$, $\mathbf{27}(S=\frac{1}{2},\frac{3}{2},\frac{5}{2})$, $\mathbf{35}(S=\frac{1}{2})$, $\overline{\mathbf{35}}(S=\frac{1}{2})$, $\mathbf{64}(S=\frac{3}{2})$.

$$\Delta H_{CS}^{10q} = H_{CS}^{10q} - \sum_{3b} w_{3b} H_{CS}^{3b}$$

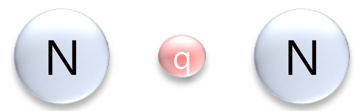
$$\Delta H_{CS} = \frac{1}{3\sum_{F,S} d_F d_S m_{FS}} \sum_{F,S} d_F d_S m_{FS} \Delta H_{CS}^{10q}.$$

Quark excitation

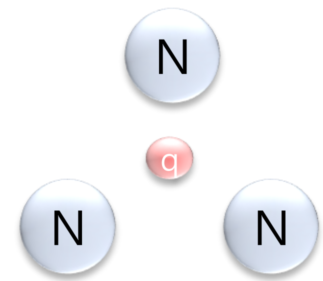
1 quark + 1 baryon



1 quark + 2 baryons

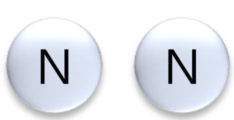


1 quark + 3 baryons



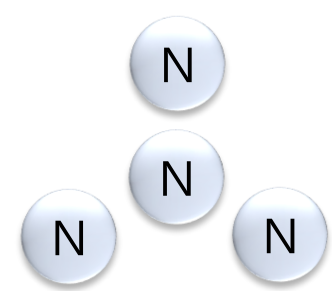
Baryon excitation

1 baryon + 1 baryon



1 baryon + 2 baryons

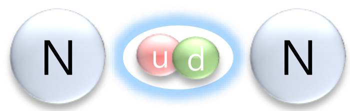
1 baryon + 3 baryons



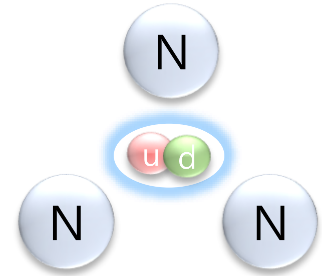
Diquark excitation

1 diquark + 1 baryon

1 diquark + 2 baryons

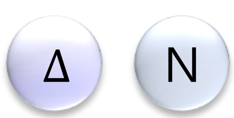


1 diquark + 3 baryons



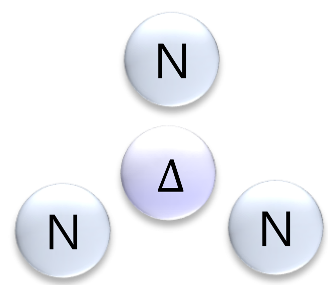
Delta excitation

1 baryon + 1 baryon



1 baryon + 2 baryons

1 baryon + 3 baryons



Summary

1. Based on the fact that the constituent quark model reproduces the recent lattice result on baryon-baryon interaction at short distance, we analyzed to what extent the quarkyonic modes appear as one increases the density.

2. We analyzed the excitation modes of the baryon and quarks in the presence of a neighbouring nucleon.

3. We found that the initial excitation may involve the d-quark from a neutron, which will leave the most attractive (ud) diquark intact.