HaPhy 2021-2: Joint Workshop on Hadron-Nuclear Physics and Astrophysics

Properties of quasi parton-distributions in the large N_c nucleon

with

Maxim V. Polyakov and Asli Tandogan

Hyeon-Dong Son

CENuM, Korea University, South Korea

Hadron Theory Group, Inha University

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Introduction

Parton distribution functions (PDFs)

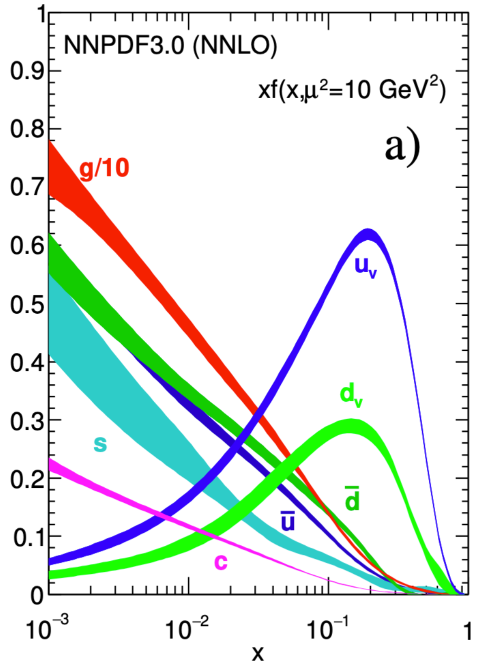
How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

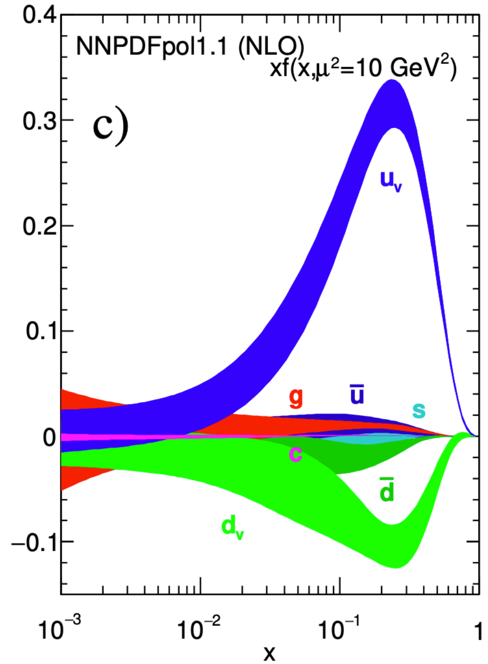
Non-perturbative QCD matrix elements

Factorizations & universality

Plots from PDG 2021



R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)



E. R. Nocera et al. (NNPDF), Nucl. Phys. B887, 276 (2014)

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

Probability density on the light-cone

Non-perturbative QCD matrix elements

Factorizations & universality

Theoretical understanding of PDFs

Effective models (at low renormalization scale)

- providing initial conditions of the QCD evolution
- to understand the detailed mechanism
- Chiral quark-soliton model [D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

Parton distribution functions (PDFs)

How partons (quarks and gluons) are distributed inside a hadron

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Factorizations & universality

Theoretical understanding of PDFs

Lattice QCD

- fundamental difficulties being Euclidean: no direct computation is possible

- Mostly studied using the Mellin moments of the PDFs (large noise at higher moments)

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale

P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **Γ=γ³** or **Γ=γ⁰**

Approaches to PDFs in the limit Pz →∞, or v → 1.

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

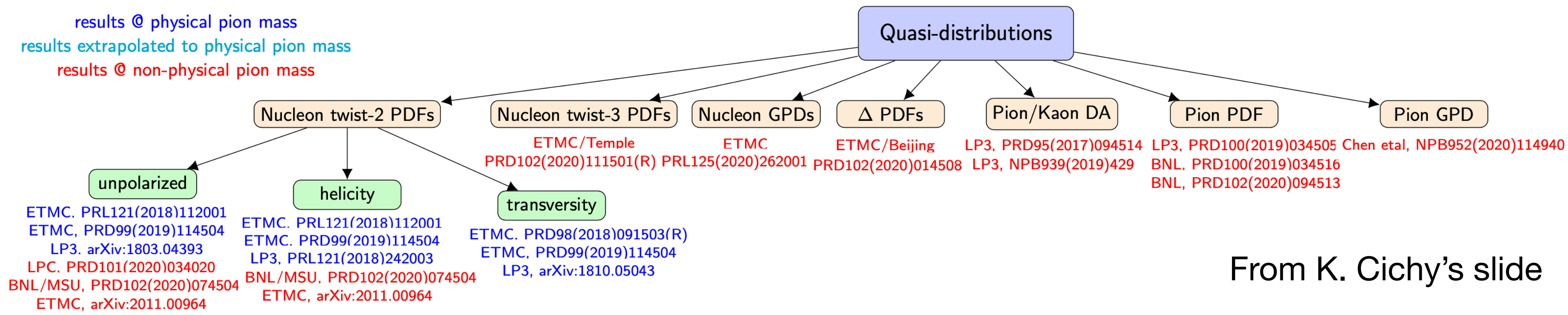
$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} \underline{C(\frac{x}{y},\frac{\mu_R}{\mu},\frac{\mu}{p^z})} q(y,\mu) + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2},\frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

Market results P_z ~ 2-3 GeV

N, π, K / PDFs, DAs, GPDs



From K. Cichy's slide

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N, π, K / PDFs, DAs, GPDs

Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs is needed

Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904
Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..

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(Quasi-)PDFs in the chiral quark-soliton model

A successful description of the nucleon PDFs at low renormalization scale

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

Nucleon matrix element in Euclidean separation

Lorentz boost → PDFs ~ quasi-PDF

Properties of qPDFs for quarks and antiquarks in the nucleon:

Sum-rules, positivity, evolution in P_z

Gravitational form factor $\bar{c}^q$ is related to the momentum sum-rule:

$\bar{c}^q$ ~ non-convergence of the separate quark EMT operator

Mass decomposition of the nucleon

Interaction strength between the quark- and gluon- subsystems

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[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

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In this talk,

Description of the chiral quark-soliton model

quasi-PDFs within the chiral quark-soliton model

Properties: closed sum-rules for the quasi-PDFs as their Mellin moments

Numerical results for the unpolarized and longitudinally polarized quasi-quark distributions

Comparison of the Dirac structures defining the quasi-PDFs: γ_0 vs γ_3

The polarized antiquark asymmetry

Method

Chiral quark-soliton model

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \; \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

From QCD to the low energy effective theory via the instantons

Parameters: $\bar{\rho} \sim 1/3$ fm & $\bar{R} \sim 1$ fm

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600$ MeV

Dynamically generated quark mass $M = 350$ MeV

Interplays the quark-model and (topological) soliton picture of the baryons

Fully field theoretic: successively describes a wide class of baryon properties

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

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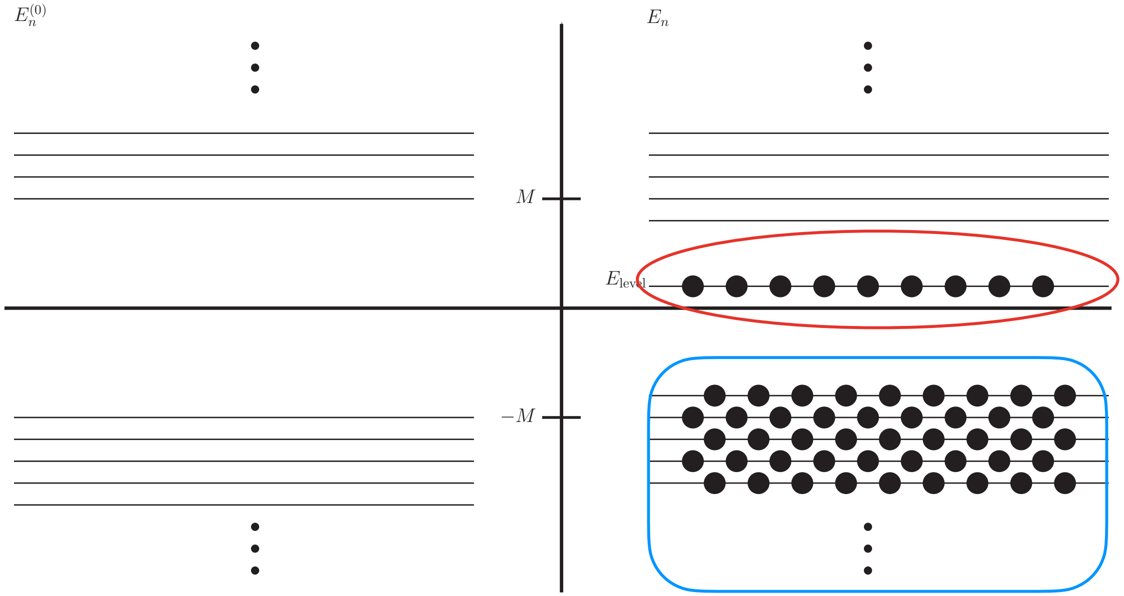
Nucleon as a chiral soliton in the large N_c limit

Quarks are bound by a common pion meanfield, self-consistently generated by their interactions



Hedgehog Ansatz

$$U = \exp[i\gamma_5 \hat{n}^a \tau^a P(r)]$$

Dirac spectra: (n): Grandspin K= J + T and Parity P

$$H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$$

Classical soliton energy

$$\frac{\delta}{\delta U}(N_c E_{\rm level} + E_{\rm cont.})|_{U=U_c} = 0 \quad \longrightarrow \quad M_{\rm cl} = N_c E_{\rm level}(U_c) + E_{\rm cont.}(U_c)$$

Quantum numbers: rotational quantization around the zero-modes

quasi-PDFs in the χQSM

Quark distribution functions: large components

In general, in the large Nc limit:

Isosinglet unpolarised	$u(x) + d(x)$	$\sim\ N_c^2\ \rho(N_c x)$
Isovector polarised	$\Delta u(x) - \Delta d(x)$	

Isovector unpolarised	$u(x) - d(x)$	$\sim\ N_c\ \rho(N_c x)$
Isosinglet polarised	$\Delta u(x) + \Delta d(x)$	

quasi-PDFs acquire overall factor of v

→ follow the same Nc ordering

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Quasi-PDFs in the χQSM

Quasi- quark and antiquark number densities

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar{\psi}_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar{\psi}_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

Isoscalar unpolarized $x\in(-\infty,\infty)$

$$\sum_f q_f(x,v) = N_c M_N v\sum_{n,occ}\int\frac{d^3k}{(2\pi)^3}\delta(k^3+vE_n-vM_Nx)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\Phi_n(\vec{k})\right]$$

Isovector polarized (helicity)

$$\Delta u(x,v)-\Delta d(x,v) = -\frac{1}{3}(2T^3)\frac{N_cM_Nv}{2\pi}\sum_{n,occ}\int\frac{d^2k_\perp}{(2\pi)^2}\delta(k^3+vE_n-vM_Nx)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\tau^3\gamma^5\Phi_n(\vec{k})\right]$$

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\; q(x,v) = \begin{cases} \boxed{N_c B, \qquad \Gamma = \gamma^0} \\ v N_c B, \qquad \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\; xq(x,v) = \begin{cases} \boxed{1, \qquad \Gamma = \gamma^0} \\ v, \qquad \Gamma = \gamma^3 \end{cases}$$

Same as LC PDFs

Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, \qquad \Gamma = \gamma^0 \\ \boxed{g_A^{(3)}. \qquad \Gamma = \gamma^3} \end{cases}$$

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$$\int_{-\infty}^{\infty} dx\; xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Aquire factor of v

Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Sum-rules

Baryon number

$$\int_{-\infty}^{\infty} dx\, q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\, x q(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\, (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

→ better Dirac structure for P_N convergence ?

→ Interpretation of the QCD symmetry currents

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Sum-rules

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U(1): charge density (γ^0) vs flux (γ^3)

Sum-rules

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Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v\left(A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0)\right)$

[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

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Sum-rules

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Axial current: $\gamma^3 \sim S^3 g_A^{(3)}$ vs $\gamma^0 \sim \vec{S} \cdot \vec{v}\, g_A^{(3)}$

Numerical results

Numerical calculation

Ansatz for the pion meanfield

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$P(r) = 2\,\mathrm{Arctan}\left(\frac{r_0^2}{r^2}\right) \quad r_0 \approx 1/M$$

within ~10% from the self-consistent solution

Interpolation formula

$$\frac{pM}{p^2+M^2}(U-1) \ll 1$$

Quasi-PDFs have the same order of divergence as the PDFs (v=1)

with smooth convergence in v →1

Logarithmic divergence: Pauli-Villars regularization

$$q(x,v)^{PV} = q(x,v)^{\mathrm{level}} + q(x,v)_{occ} - \frac{M^2}{M_{PV}^2} q(x,v)_{occ}(M \to M_{PV})$$

$$F_\pi^2 = \frac{N_c M^2}{4\pi^2}\log(M_{PV}^2/M^2)$$

$$M = 350\ \mathrm{MeV}$$
$$M_{PV} = 557\ \mathrm{MeV}$$

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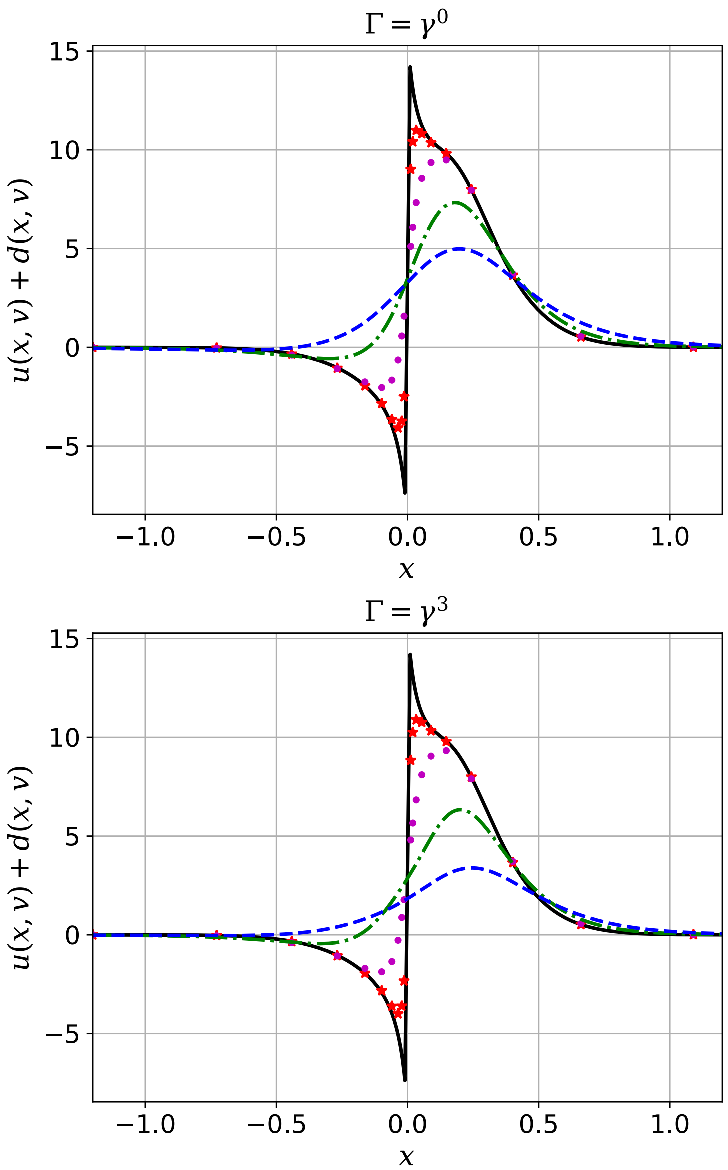
Isoscalar unpolarized

v=1	v=0.999	v=0.99	v=0.9	v=0.7
$P_N/M_N=\infty$	22.3	7.0	2.1	1.0



$\bar{q}(x) = -q(-x)$ (LC PDF)

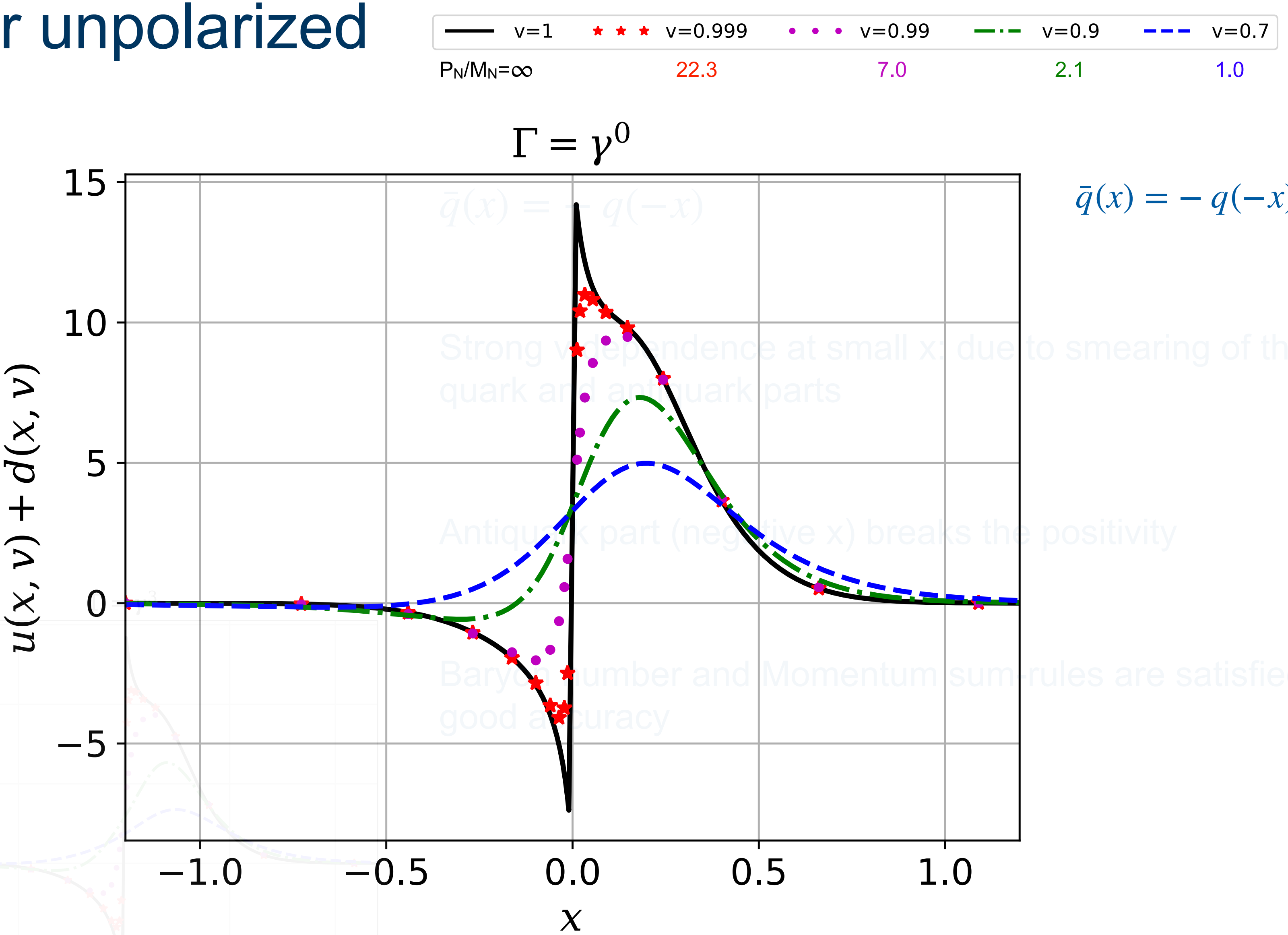
Strong v dependence at small x: due to smearing of the quark and antiquark parts

Antiquark part (negative x) breaks the positivity

Baryon number and Momentum sum-rules are satisfied in good accuracy

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

Isoscalar unpolarized



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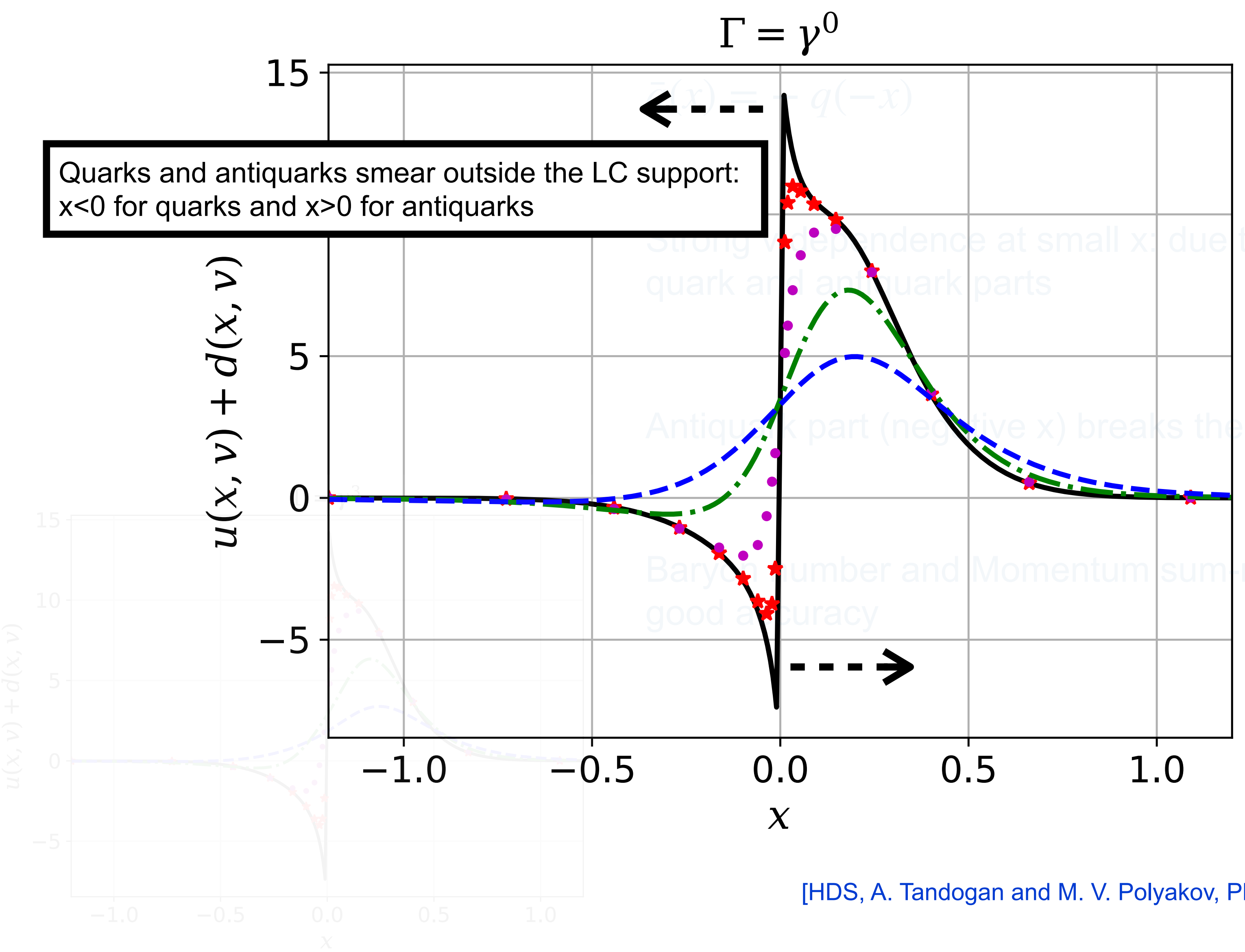

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[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

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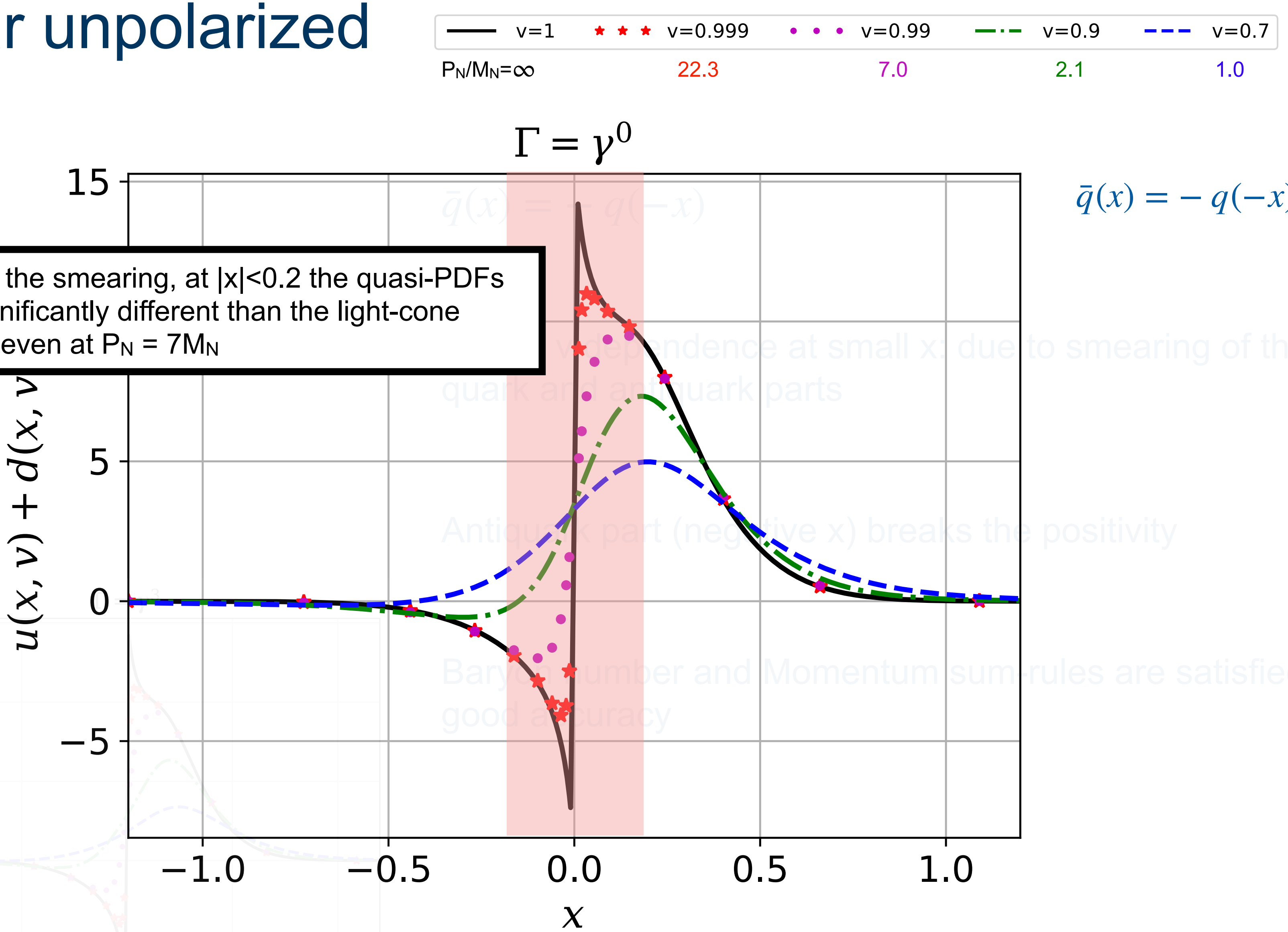
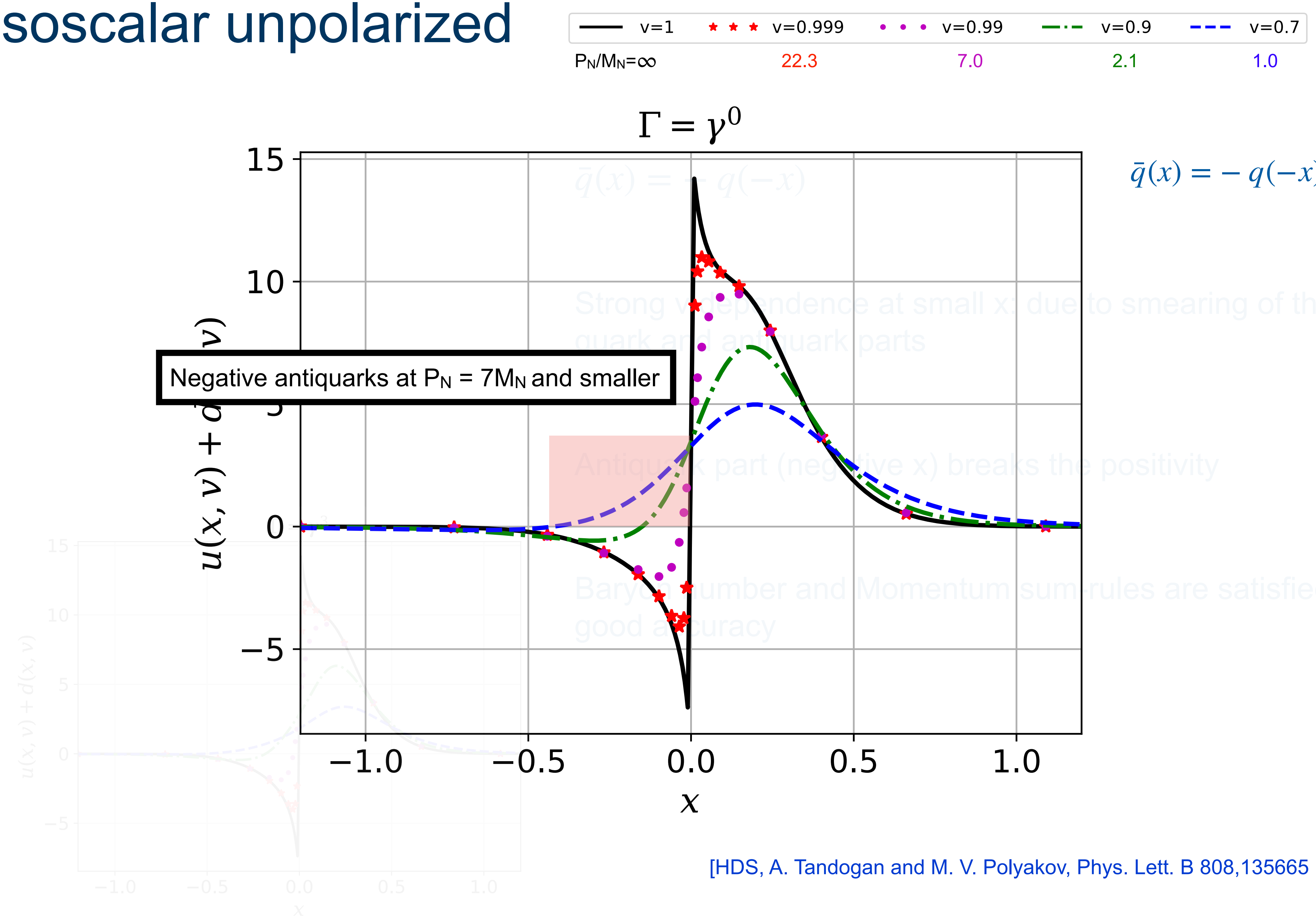


Due to the smearing, at |x|<0.2 the quasi-PDFs are significantly different than the light-cone PDFs, even at $P_N = 7M_N$

$\bar{q}(x) = -q(-x)$

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808,135665 (2020)]

Isoscalar unpolarized



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Isovector polarized

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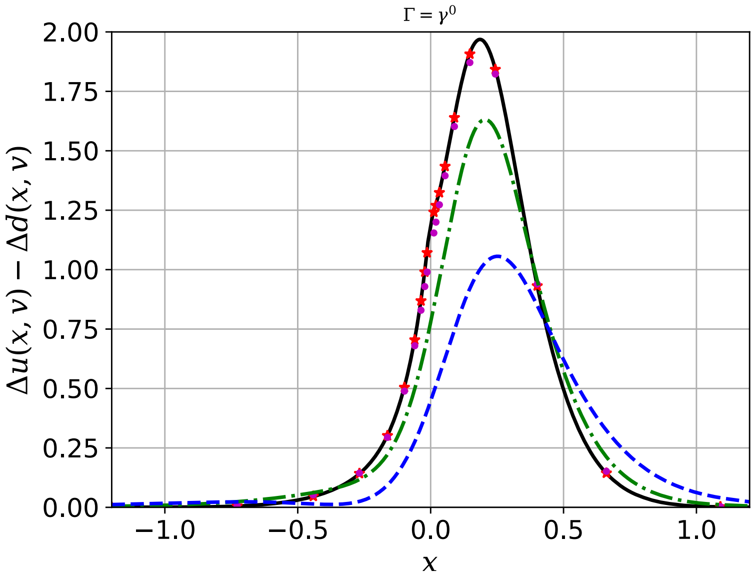

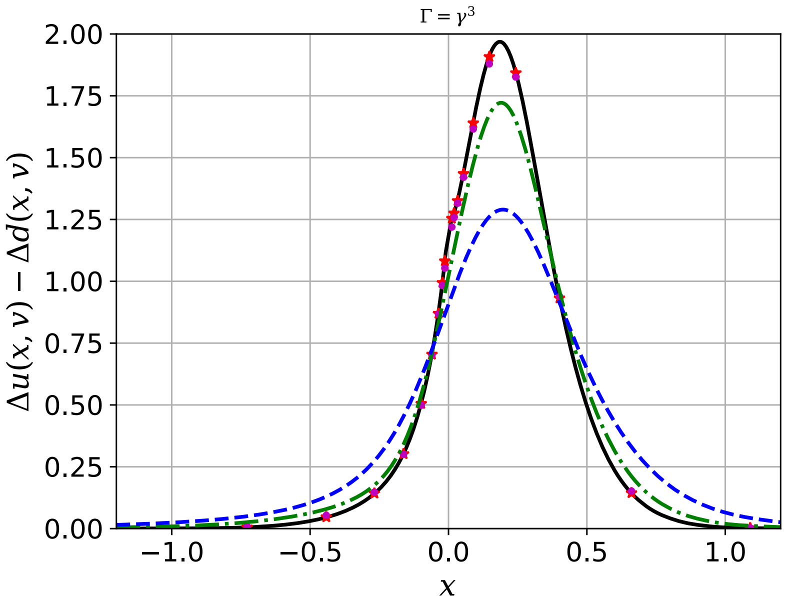

$$\Delta\bar{q}(x) = \Delta q(-x)$$

At v=0.9 (P ~ 2 GeV), qPDF ~ PDF

Sum-rules are satisfied in good accuracy

$\Gamma = \gamma^3$ qPDF converges faster to the lightcone PDF

$$\int_{-\infty}^{\infty} dx\,(\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

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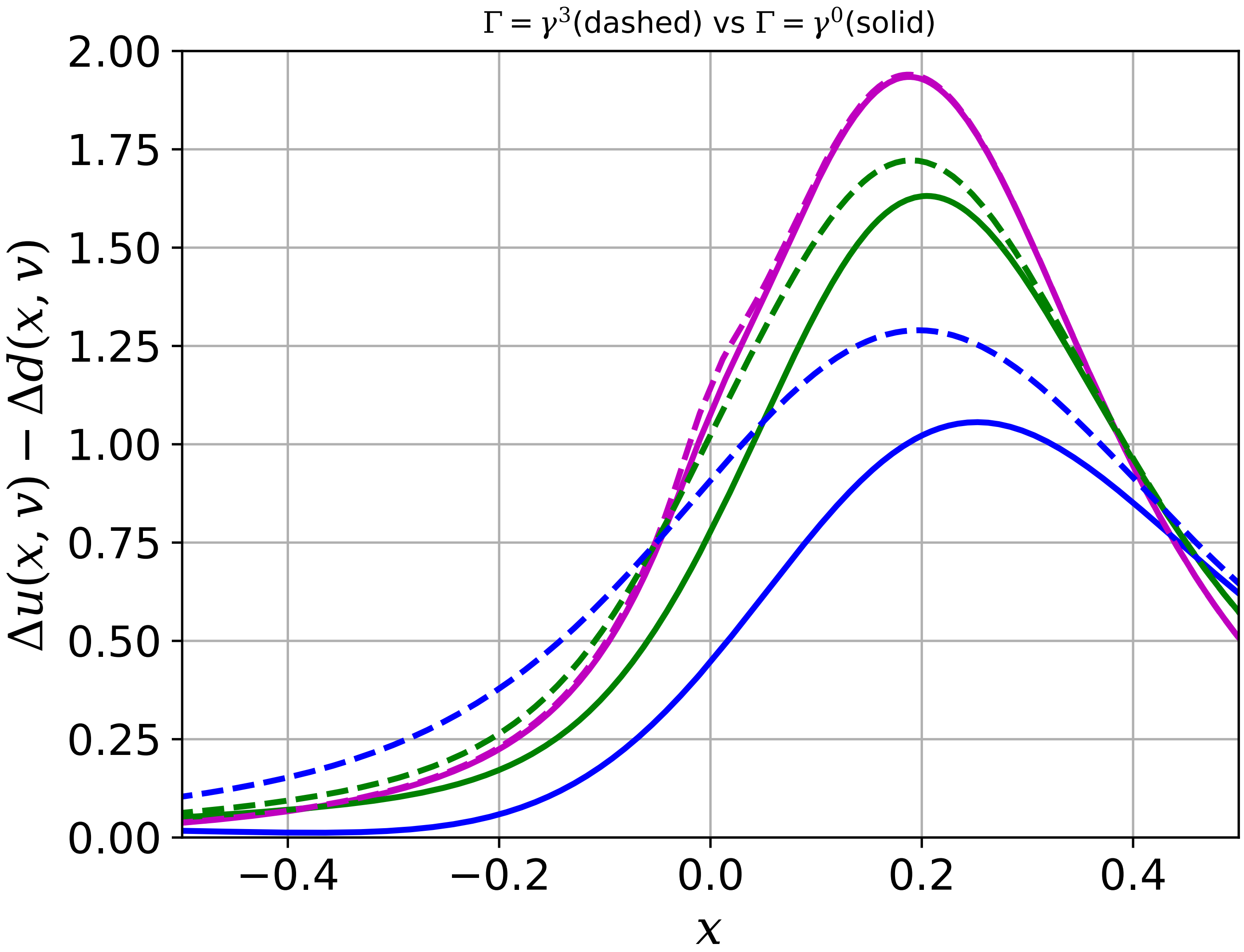
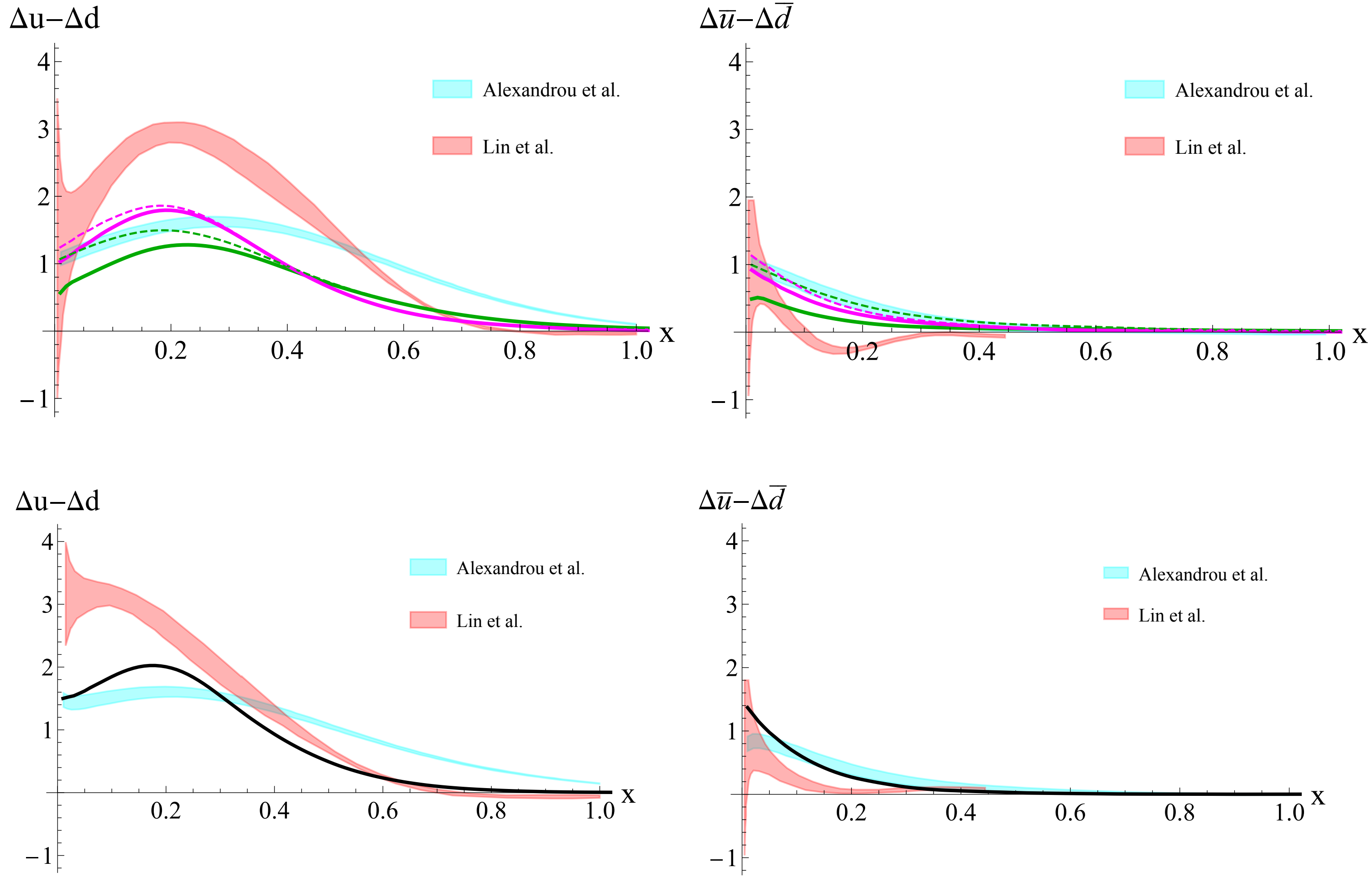


vs. Lattice results

v = 1 — [v = 0.93, Γ = γ^0] - - - [v = 0.93, Γ = γ^3] — [v = 0.77, Γ = γ^0] - - - [v = 0.77, Γ = γ^3]

$P_N/M_N=\infty$ 3.0 GeV 1.4 GeV



$(m_\pi, P_z, \mu) =$ (0.37, 1.4, 2.0) **[Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017],**

(0.135, 3.0, 3.0) **[Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]**

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Antiquark asymmetries in the nucleon

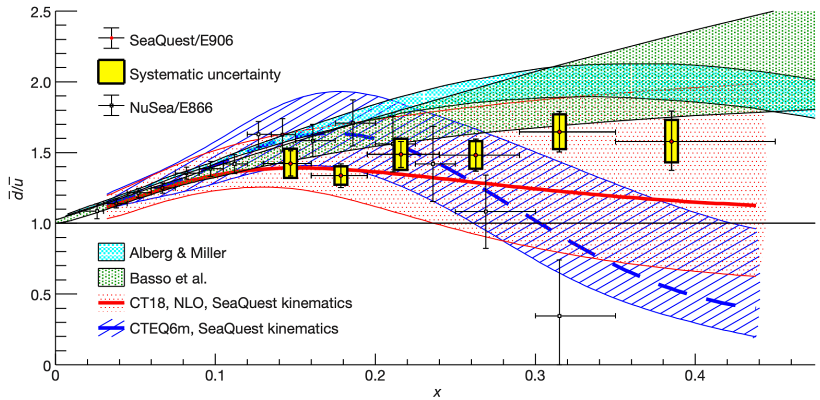


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

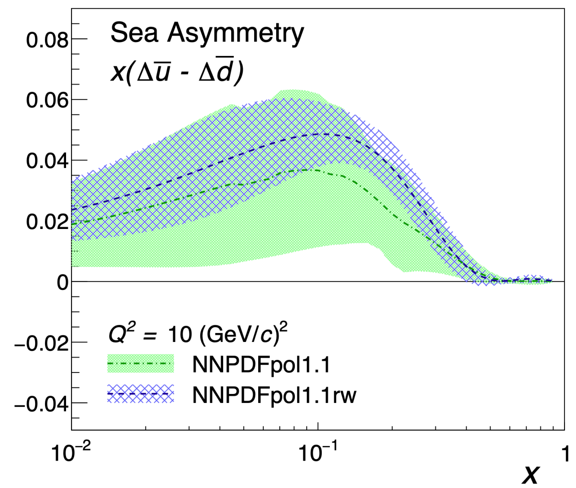
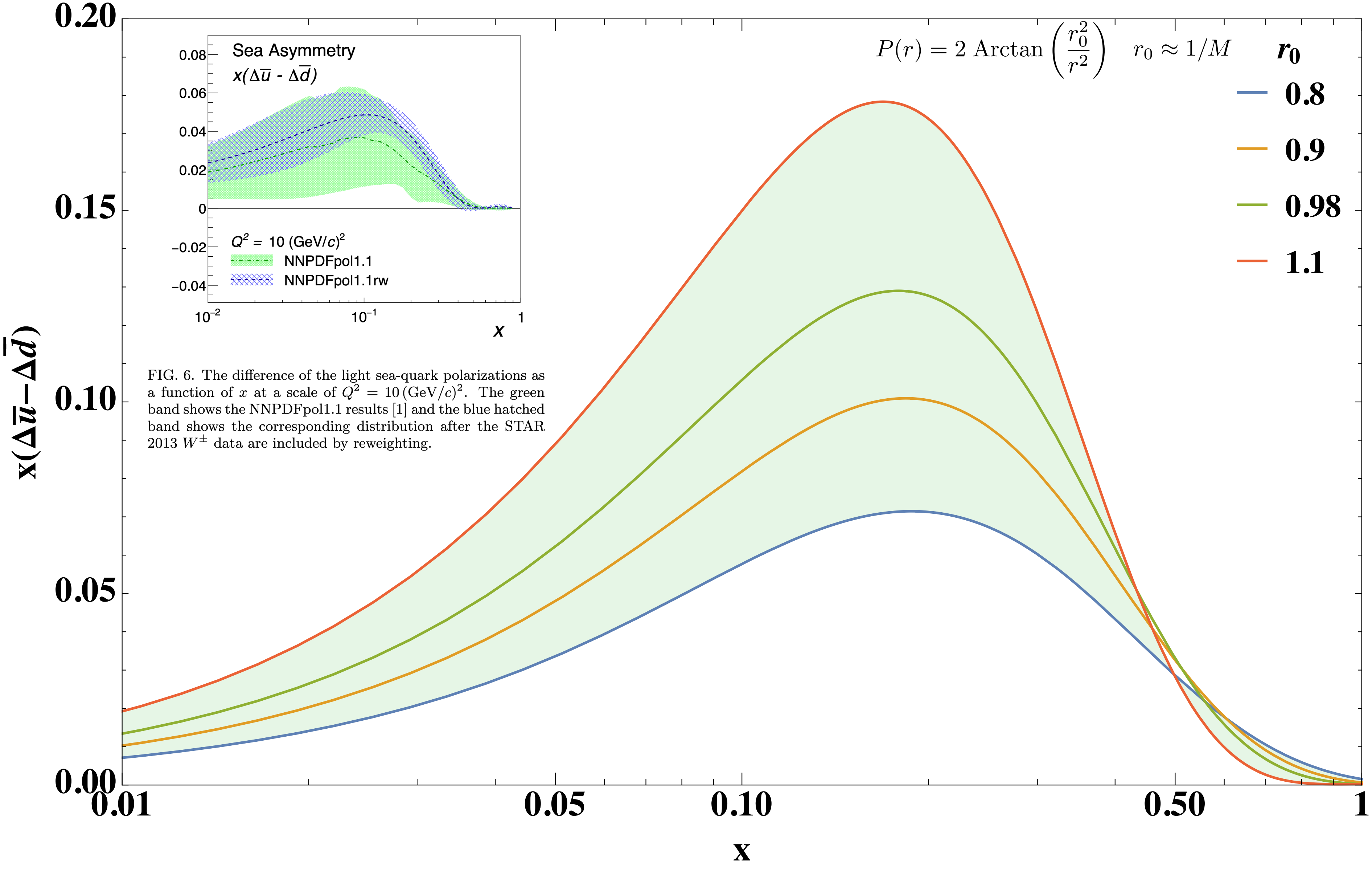


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

Antiquark flavor asymmetry: model case



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Closing remarks

Summary

χQSM is a working framework for understanding the (quasi-)PDFs

Sum-rules: $\bar{c}^q$, 'better' Γ for the convergence to the PDFs

Good convergence of the isovector polarized

vs. poor $\mathbf{P_z} \to \infty$ convergence for the isoscalar unpolarized quasi-PDF

Antiquark flavor asymmetry is predicted

Future tasks

Transversity PDF and other small components in the large Nc

Quantitative comparison vs Lattice: scale evolution, large x

gluon PDFs (targeting the EIC physics)

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Thank you very much!