Effect of tensor force in nuclear structure

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in collaboration with

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Contents

This talk was based on our recent papers.

1. **Isoscalar pairing correlations by the tensor force in the ground states of ^{12}C, ^{16}O, ^{20}Ne, and ^{32}S nuclei.**

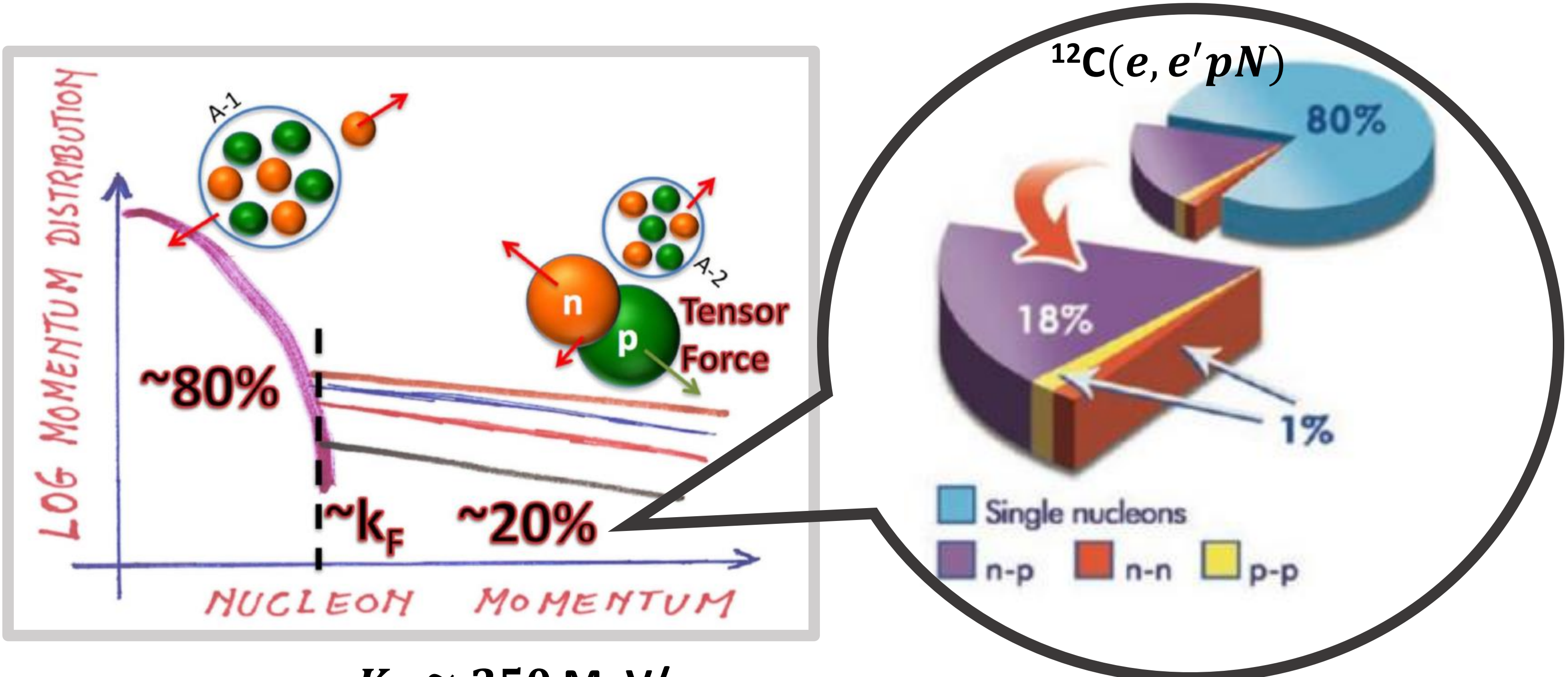
   Ha *et al*. PRC104, 034306(2021)

2. **Tensor Force Effects on the Gamow-Teller Transition for ^{42}Ca, ^{46}Ti and ^{18}O.** Ha *et al*. submitted to EPJA

1. Introduction

❖ Short-range correlation(SRC) pairs in nuclei

Or Hen *et al.*, Rev. Mod. Phys. 89, 045002 (2017). R. Subedi *et al.*, SCIENCE. 320,13 (2008).

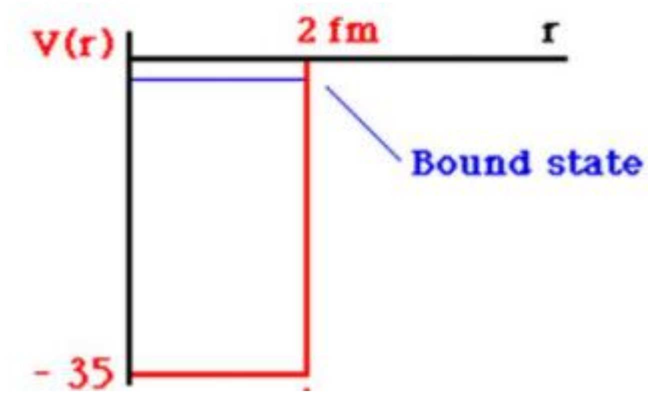


$K_F \cong 250$ MeV/c

- **High-momentum tail from tensor force at k > k_F in N=Z and N > Z nuclei.**
- **Mean field : two-body interaction ➔ one-body interaction**
- **But two-body interaction still remain at the short range region.**

In order to consider the residual interaction, we introduce the pairing correlations.

❖ Tensor force : first evidence from the deuteron



Binding energy	2.225 MeV
Spin, parity	1^+
Isospin	0
Magnetic moment	μ=0.857 μ_N
Electric quadrupole moment	Q=0.282 e fm^2

$$|\psi_d\rangle = 0.98|{}^3S_1\rangle + 0.20|{}^3D_1\rangle$$

$^{2S+1}L_J$ $(L = 0)$ $(L = 2)$

$S = 1$

- ✓ Tensor force(**TF**) mixes two states.
- ✓ Without **TF** deuteron is unbound.

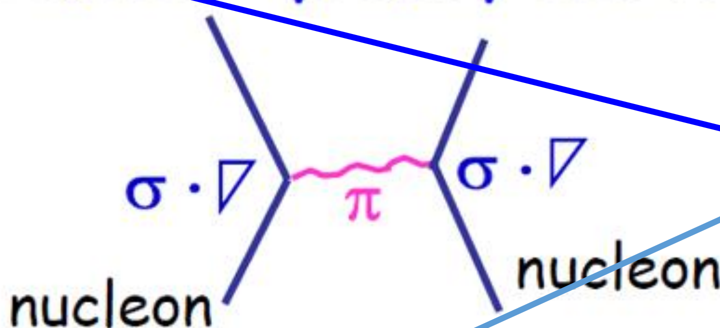
❖ Tensor force (TF) : Non-central force (S=1)

$$\hat{V}_T = V_T(r)\hat{S}_{12}$$

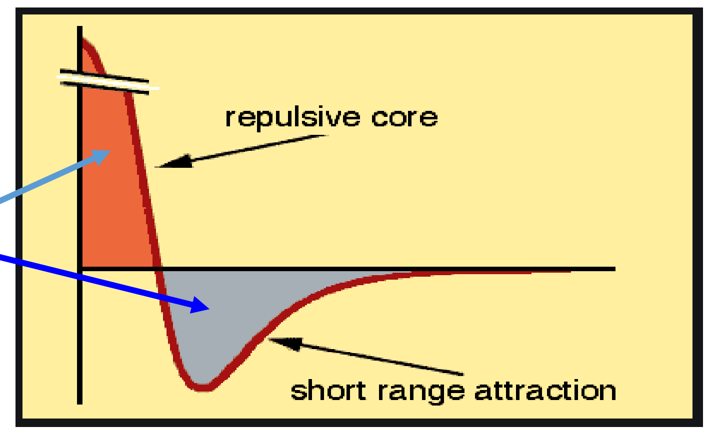
$$\hat{S}_{12} = 3\frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{r})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{r})}{r^2} - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)$$:Tensor operator

Origin of TF :

π meson : primary source



Tensor attraction 80% of entire attraction



ρ meson (~ π+π) : minor (~1/4) cancellation

Ref: Osterfeld, Rev. Mod. Phys. 64, 491 (92)

From Otsuka's talk

❖ How does the tensor force work ?

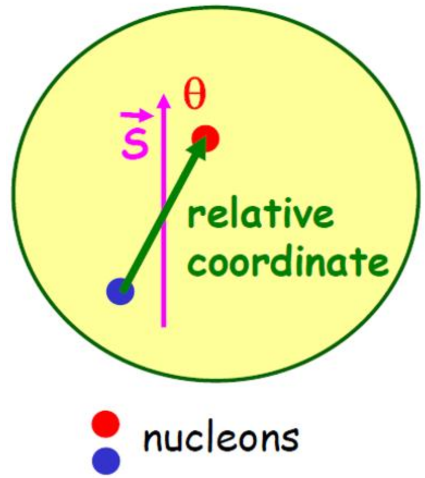
$$\hat{S}_{12} = 3\frac{(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r})}{r^2} - (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) = -2S^2(1-3cos^2\theta) \sim Y_{2,0}$$

$\hat{S}_{12} = 0$ for $\mathrm{S} = 0$

contribute only to S=1 states.

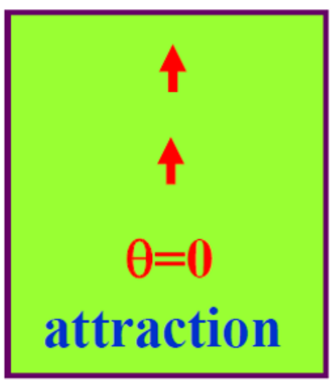
Spin of each nucleon ↑ is parallel, because the total spin must be S=1

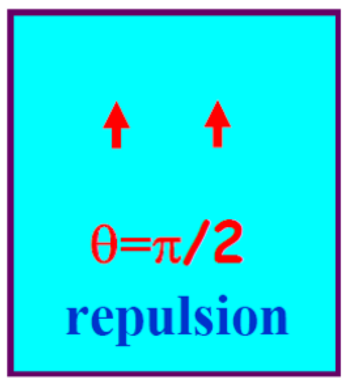
The potential has the following dependence on the angle θ with respect to the total spin $\vec{S}$.

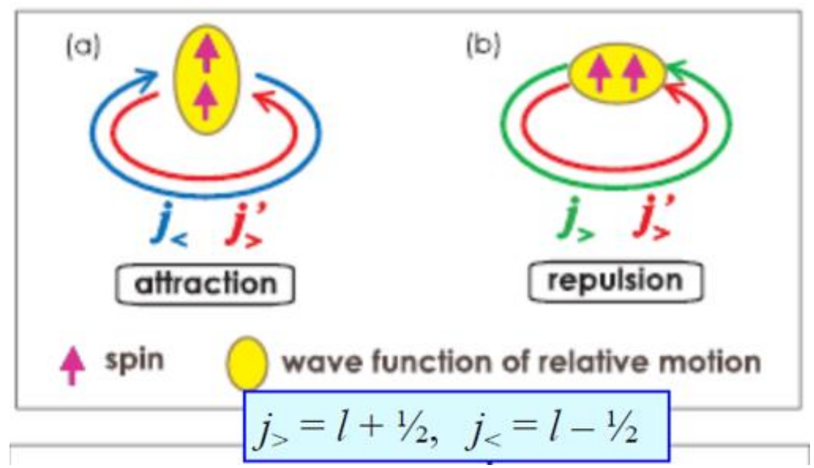


$\hat{V}_T = VT(r)\hat{S}_{12}$ $\quad V_T(r) < 0$: Tensor force

$\hat{V}_T \sim -\hat{S}_{12} \sim 1 - 3cos^2\theta$







❖ New magic number from ^{54}Ca (N=34)!

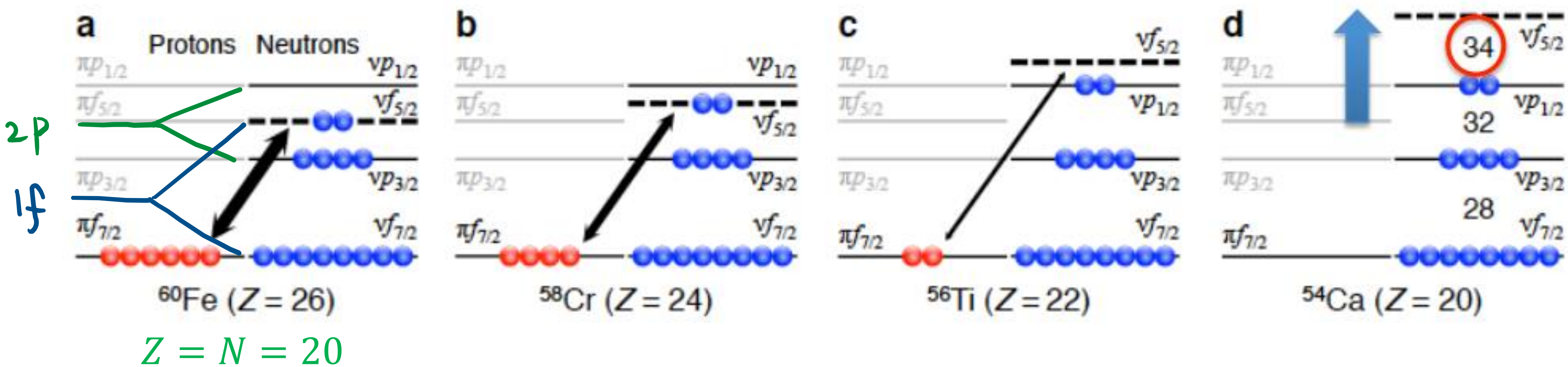
D. Steppenbeck *et al.*
Nature 502,207(2013)

$^{70}Zn^{30+}$ ions at 345MeV by BigRIPS

N=34 magic number
and the shell evolution due to proton-neutron interaction

neutron $f_{5/2}$ - $p_{1/2}$ spacing increases by ~0.5 MeV per one-proton removal from $f_{7/2}$, where tensor and central forces works coherently and almost equally.

note : $f_{5/2} = j_<$ $f_{7/2} = j_>$



From Otsuka's talk

❖ Deuteron structure in nucleus?

PHYSICAL REVIEW LETTERS **121**, 242501 (2018)

Dominance of Tensor Correlations in High-Momentum Nucleon Pairs Studied by (p,pd) Reaction

S. Terashima,[1,2,*] L. Yu,[1] H. J. Ong,[3] I. Tanihata,[1,2,3] S. Adachi,[3] N. Aoi,[3] P. Y. Chan,[3] H. Fujioka,[4] M. Fukuda,[5] H. Geissel,[6,7] G. Gey,[3] J. Golak,[8] E. Haettner,[6,7] C. Iwamoto,[3] T. Kawabata,[4] H. Kamada,[9] X. Y. Le,[1,2] H. Sakaguchi,[3] A. Sakaue,[4] C. Scheidenberger,[6,7] R. Skibiński,[8] B. H. Sun,[1,2,10] A. Tamii,[3] T. L. Tang,[3] D. T. Tran,[3,11] K. Topolnicki,[8] T. F. Wang,[1,2] Y. N. Watanabe,[12] H. Weick,[6] H. Witała,[8] G. X. Zhang,[1,2] and L. H. Zhu[1,2,10]

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The isospin character of p-n pairs at large relative momentum has been observed for the first time in the ^{16}O ground state. A strong population of the $J,T = 1,0$ state and a very weak population of the $J,T = 0,1$ state were observed in the neutron pickup domain of ^{16}O(p,pd) at 392 MeV. This strong isospin dependence at large momentum transfer is not reproduced by the distorted-wave impulse approximation calculations with known spectroscopic amplitudes. The results indicate the presence of high-momentum protons and neutrons induced by the tensor interactions in the ground state of ^{16}O.

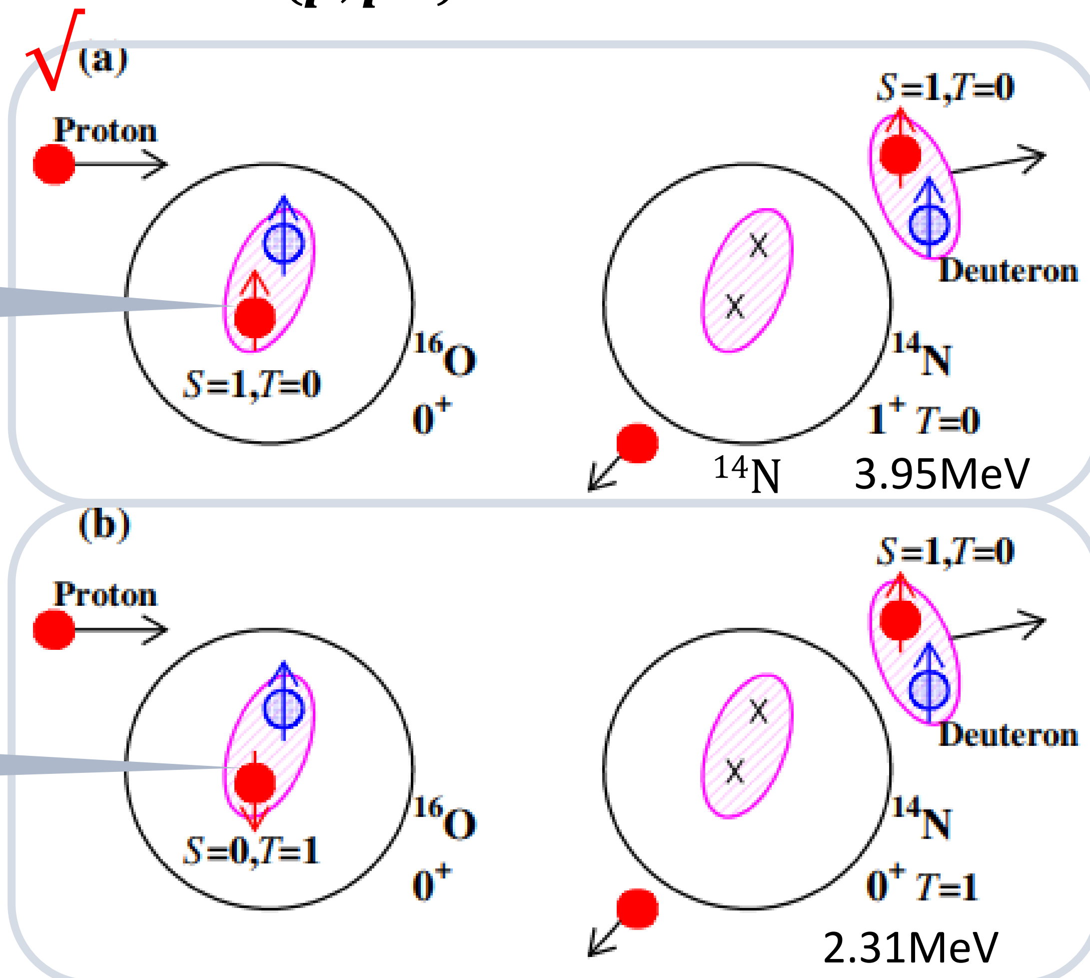
DOI: 10.1103/PhysRevLett.121.242501

❖ Dominance of tensor correlations in high-momentum *n-p* pairs

$^{16}O(p,pd)$ *at* $392MeV$ S.Terashima, *et al.* PRL 121 (2018)

Tensor force (non-central)

Central force



$(-1)^{J+T}$,
$J+T=odd$

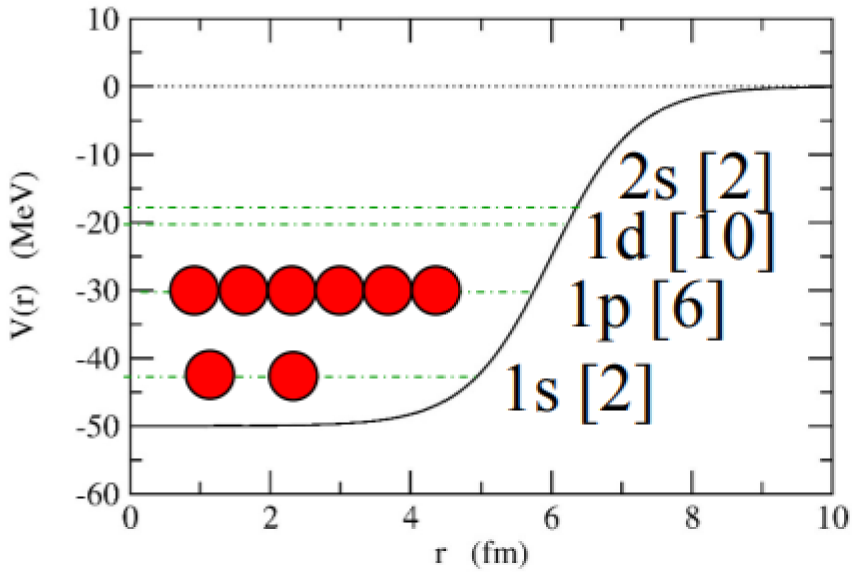
- S=1,T=0 channel in the cross section is dominant.
- The high-momentum *n-p* pairs(S=1,T=0) due to **tensor force** has been observed in the ^{16}O ground state.

2. Formalism

- Deformation
- Pairing correlations

❖ Many-body system

Mean-field approximation



$$H \sim \sum_i \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\rm MF}(\boldsymbol{r}_i) \right)$$

Slater determinant

$$\Psi_{\rm MF}(1,2,\cdots,A)$$
$$= \mathcal{A}[\psi_1(1)\psi_2(2)\cdots\psi_A(A)]$$

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm MF}(\boldsymbol{r}) \right)\psi_k(\boldsymbol{r}) = \epsilon_k\psi_k(\boldsymbol{r})$$

Interacting many-fermion sys.→ non-interacting fermions

the original many-body *H*:

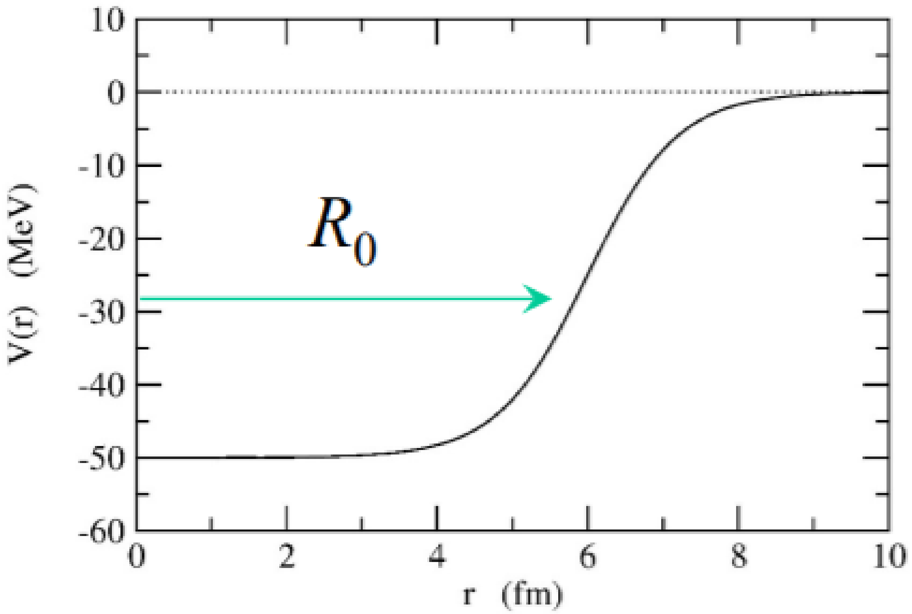
$$H = -\sum_{i=1}^{A} \frac{\hbar^2}{2m}\nabla_i^2 + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j)$$

$$= \sum_{i=1}^{A} \left(-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\rm MF}(\boldsymbol{r}_i) \right) + \frac{1}{2}\sum_{i,j}^{A} v(\boldsymbol{r}_i, \boldsymbol{r}_j) - \sum_i V_{\rm MF}(\boldsymbol{r}_i)$$

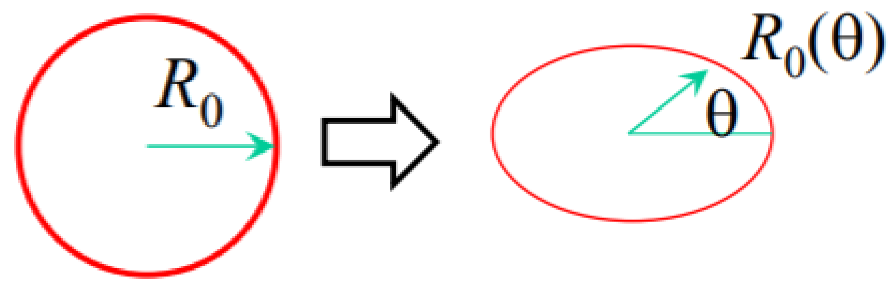
H_{res} = residual

❖ One-particle motion in a deformed potential

a spherical WS potential:



$$V(r) = -\frac{V_0}{1 + \exp\left(\frac{r - R_0}{a}\right)}$$



deformed WS potential

$$V(r) \to V(r,\theta) = -\frac{V_0}{1 + \exp\left(\frac{r - R_0(\theta)}{a}\right)}$$

$$R_0 \to \underbrace{R_0(1 + \beta_2 Y_{20}(\theta))}_{R_0(\theta)}$$

$$V(r,\theta) \sim V_0(r) - \beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta) + \cdots$$

■the effect of Y_{20} term

Eigen-functions for β_2=0 (spherical pot.) :

$$\psi_{nll_z}(\boldsymbol{r}) = R_{nl}(r) Y_{ll_z}(\hat{\boldsymbol{r}})$$

eigen-values: E_{nl} (no dependence on l_z)

The change of energy due to the Y_{20} term (1st order perturbation theory):

$$E_{nl} \rightarrow E_{nl} + \langle \psi_{nll_z} | \Delta V | \psi_{nll_z} \rangle$$

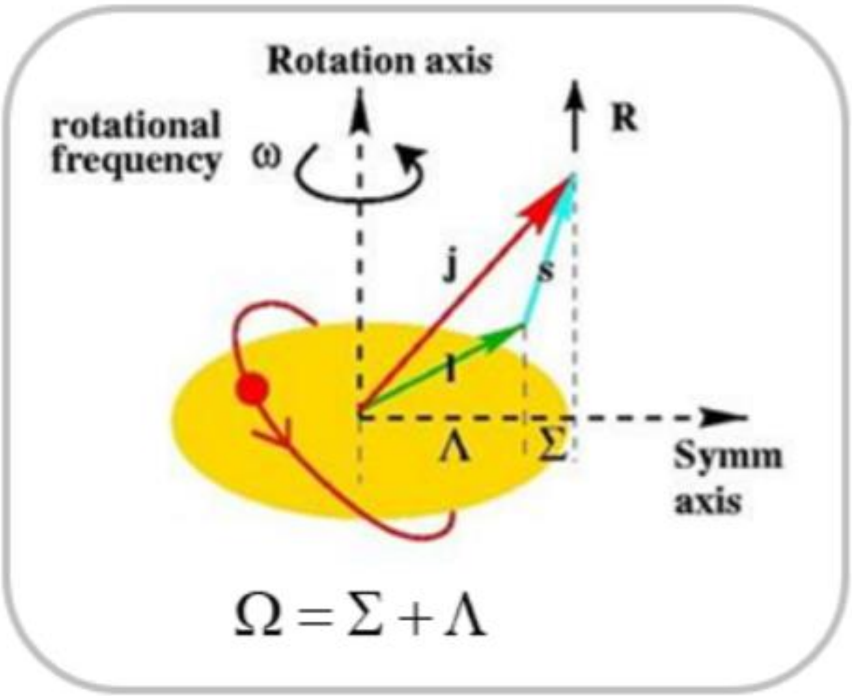
$$\Delta V(r) = -\beta_2 R_0 \frac{dV_0}{dr} Y_{20}(\theta)$$

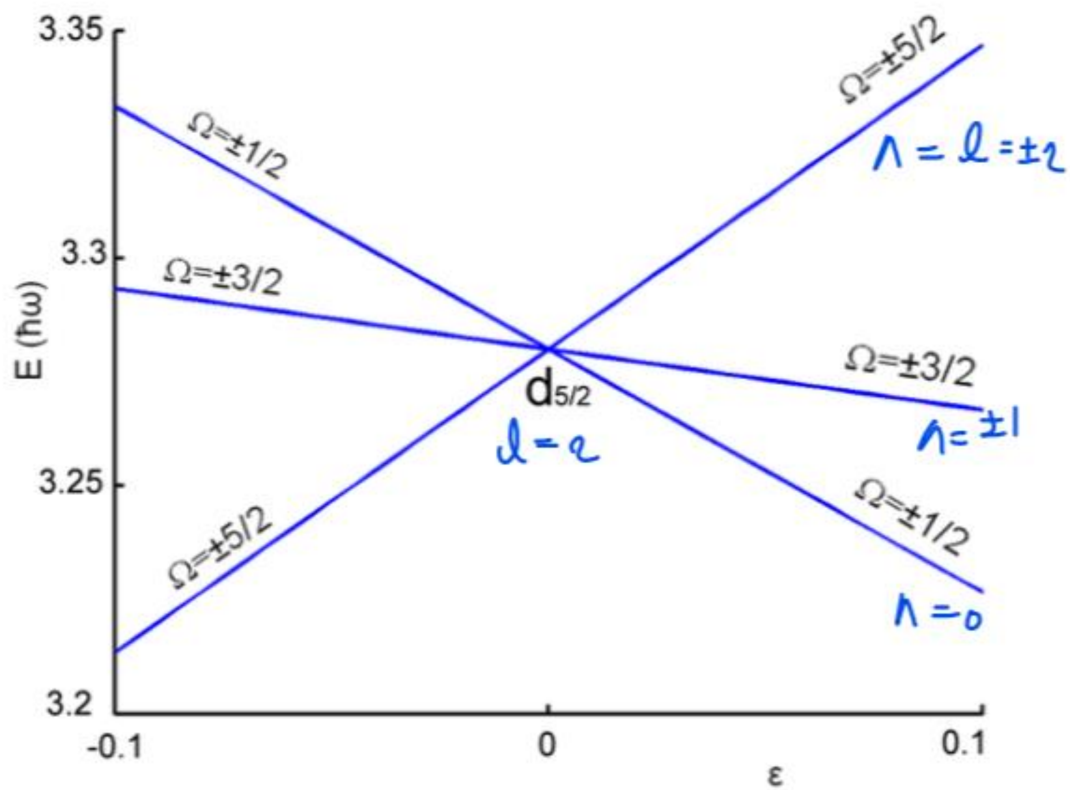
K is good quantum number !!

j projection on z → Ω
l → Λ
J → K
$\Omega = \Lambda \pm \frac{1}{2}$

In spherical basis, **j** is a good quantum number.
But in deformed basis, a projection of J on the nuclear symmetric axis z, **Ω**, is a good quantum number.

Deformed states, ±5/2, ±3/2, and ±1/2, are separated from the spherical state $d_{5/2}$.



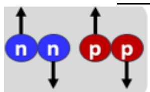
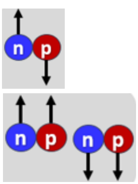


Spherical state occupation (**2j+1**)→ Deformed state ±Ω (**2**)

➢ Single particle states in deformed nucleus become more complex.

❖ Pairing scheme

T=1 (Isovector(IV), isospin- triplet), **T=0 (Isoscalar(IS), isospin- singlet)**

Types	T	S	L	J (K = 0)	
 Like	$T = 1$	$(\alpha\bar{\alpha})$ $S = 0$	$L = 0,2,4...$ (E)	$J = 0,2,4...$ (E)	
		$(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ $S = 1$	$L = 1,3,5...$ (O)	$J = E$	Not allowed
Unlike	$T = 1$	$(\alpha\bar{\alpha})$ $S = 0$	$L = 0,2,4...$ (E)	$J = 0,2,4$ ... (E)	
		TF $(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ $S = 1$	$L = 1,3,5...$ (O)	$J = E$	$^3P_1$ No empirical evidence
	$T = 0$	$(\alpha\bar{\alpha})$ $S = 0$	$L = 1,3,5...$ (O)	$J = 1,3,5$ ... (O)	$^{2S+1}L_J$
		TF $(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ $S = 1$	$L = 0,2,4...$ (E)	$J = O$	$^3S_1$, $^3D_1$

In deformed formalism !

blue: repulsive,
red: attractive

❖ BCS & DBCS

BCS

J=0
T=1

j m   j −m

deformed BCS

K=0

Ω   −Ω

(J=0,1, 2, 3 ···)

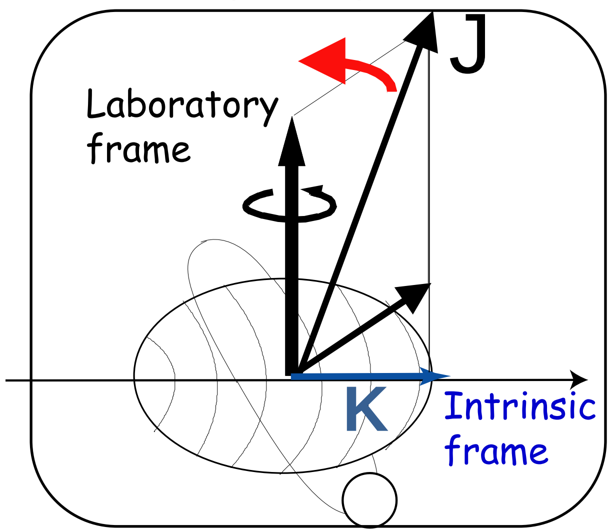
T=0 J=1, 3, 5, ···

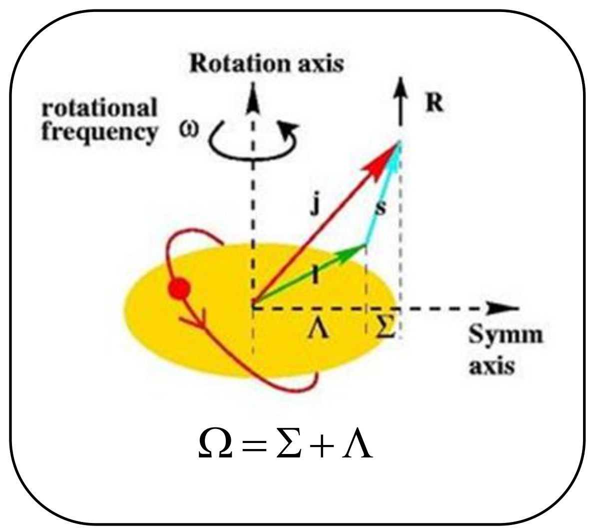
T=1 J=0, 2, 4, ···

Ω= ½
j ≥ Ω

j	Ω
$\frac{1}{2}$	$\frac{1}{2}$
$\frac{3}{2}$	$\frac{1}{2}$
$\frac{5}{2}$	$\frac{1}{2}$
$\frac{7}{2}$	$\frac{1}{2}$
…	

(Coupled system)





➢ The different total angular momenta of the SP basis states would be mixed because the deformed SPS are expanded in terms of the spherical SP bases.

3. Effect of tensor force in the ground states

❖ ^{12}C

Types	T	S	L	J (K = 0)	
Like	T = 1	$(\alpha\bar{\alpha})$ S = 0	L = 0,2,4... (E)	J = 0,2,4... (E)	
		$(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ S = 1	L = 1,3,5... (O)	J = E	
Unlike	T = 1	$(\alpha\bar{\alpha})$ S = 0	L = 0,2,4... (E)	J = 0,2,4 ... (E)	
		TF $(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ S = 1	L = 1,3,5... (O)	J = E	$^3P_1$
	T = 0	$(\alpha\bar{\alpha})$ S = 0	L = 1,3,5... (O)	J = 1,3,5 ... (O)	
		TF $(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$ S = 1	L = 0,2,4... (E)	J = O	$^3S_1, {}^3D_2, {}^3D_3$

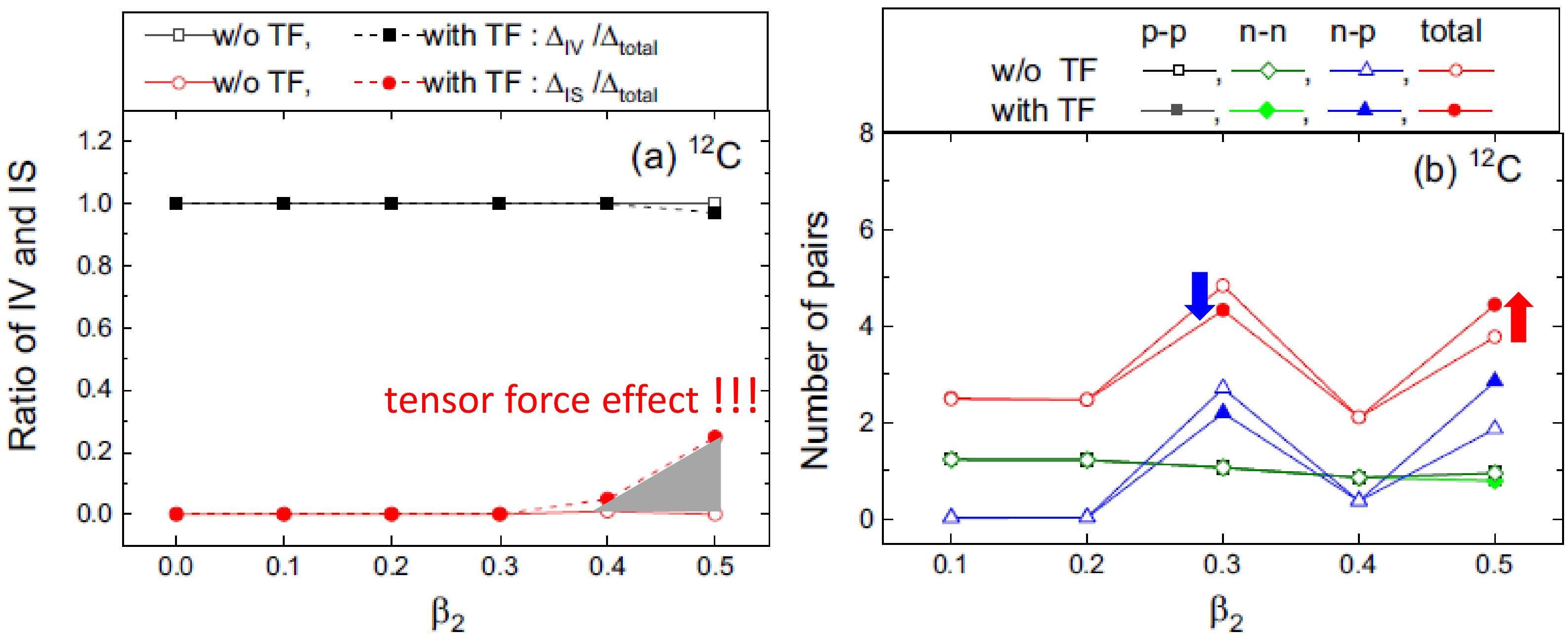
$$< N >_{pp} \sim \frac{\Delta_{pp}^2}{(G_{pp}^{T=1})^2}, < N >_{nn} \sim \frac{\Delta_{nn}^2}{(G_{nn}^{T=1})^2},$$

$$< N >_{np}^{T=0} \sim \frac{(\Delta_{np}^{T=0})^2}{(G_{np}^{T=0})^2}, < N >_{np}^{T=1} \sim \frac{(\Delta_{np}^{T=1})^2}{(G_{np}^{T=1})^2}$$

$^{2S+1}L_J$

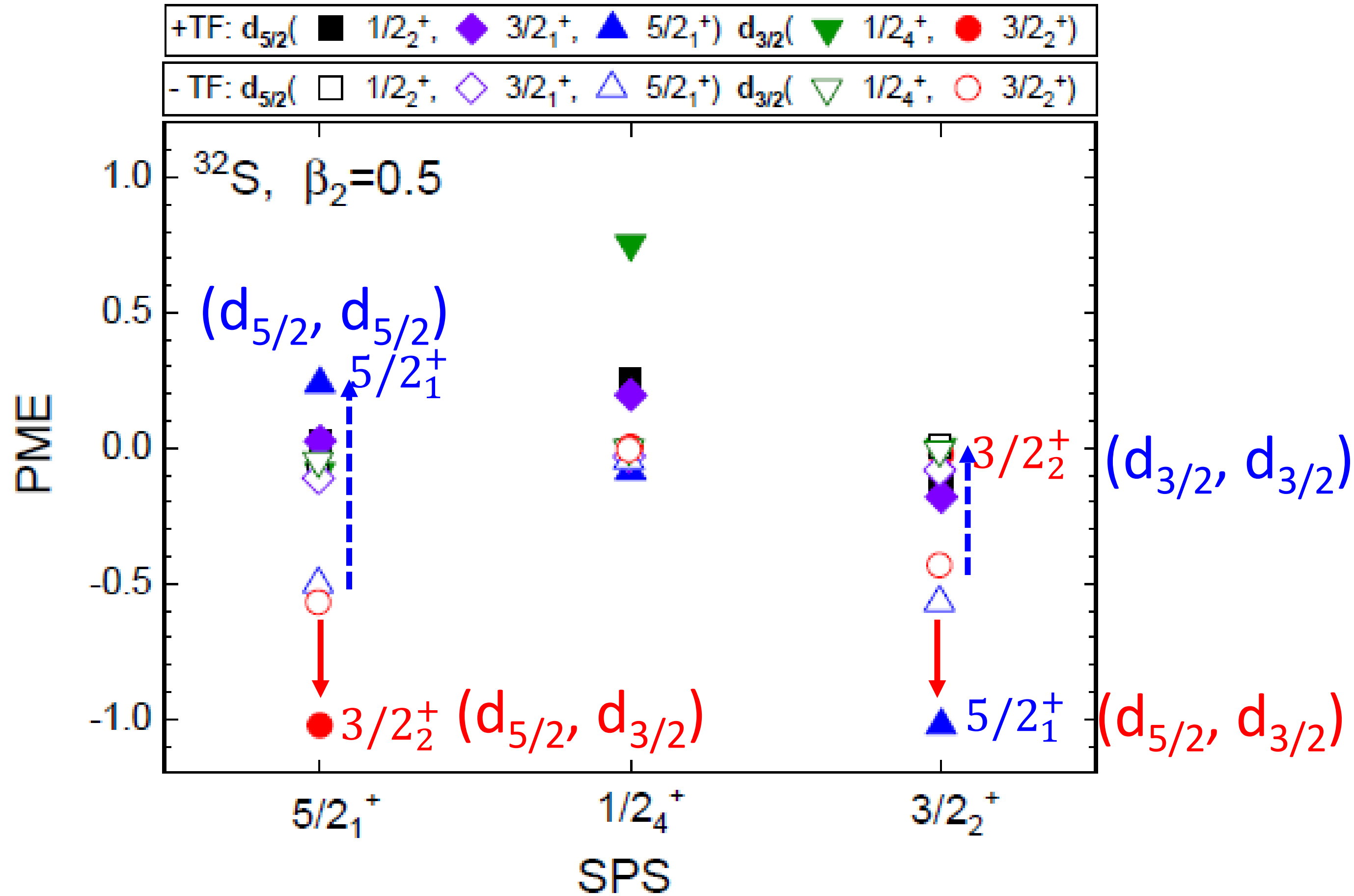
blue: repulsive,

red: attractive



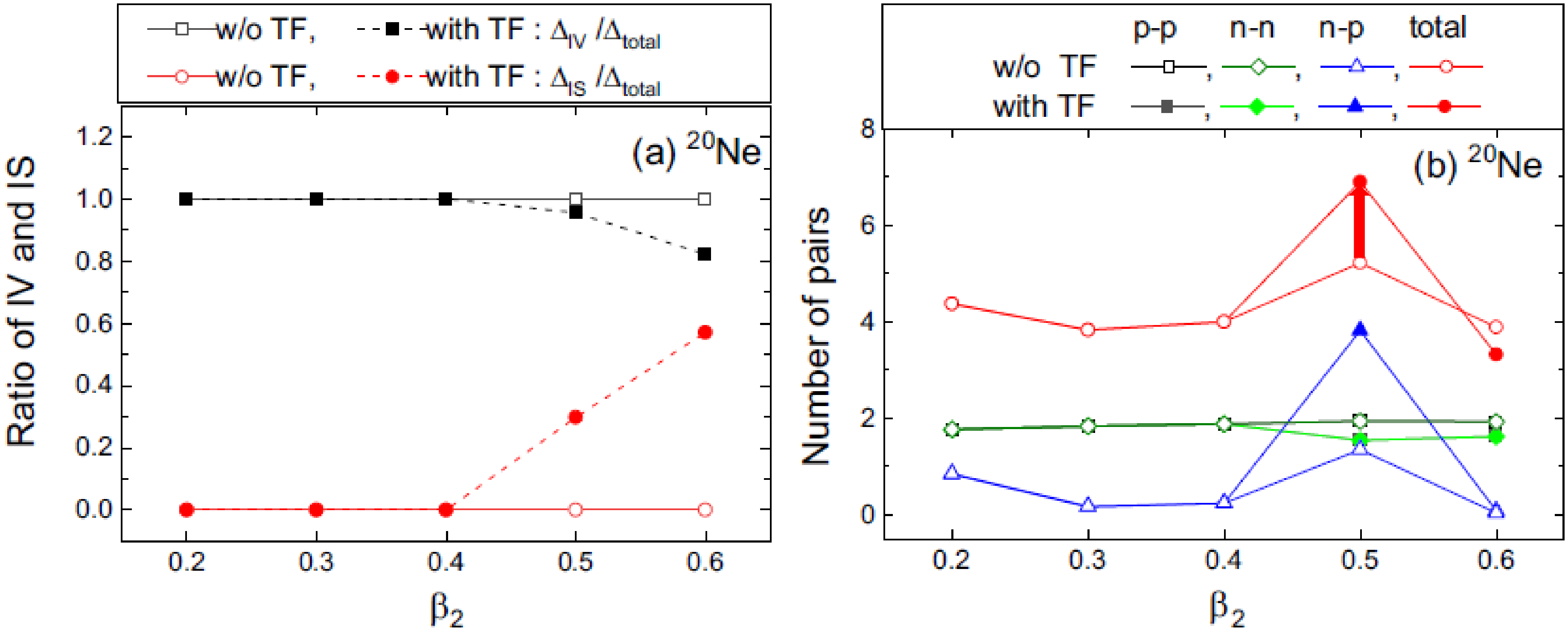
- # of *nn* & *pp* pairs are rarely by the **TF**, while that of *np* pairs changes at certain deformation.
- **TF** increases (decreases) the **PME**s of the T=0 *np* channel by its attractive (repulsive) property around $\beta_2 \sim 0.5$ (0.3)
- **TF is sensitive on the deformation** and may break the IV dominance of the *np* pairing.

❖ Pairing matrix elements(PMEs) by *G*-matrix



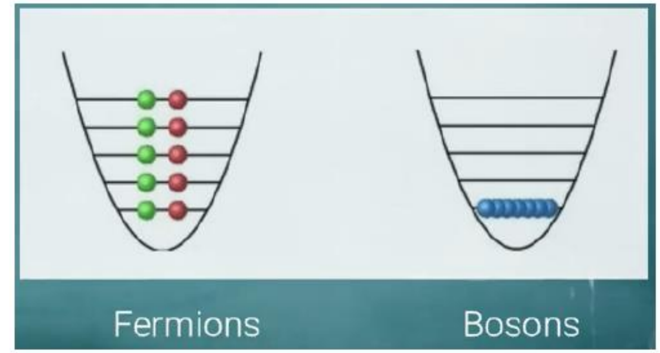
$$\Delta_{p\bar{n}_\alpha} = \Delta_{\alpha p \bar{\alpha} n} = -\sum_{\gamma}\Bigg[\Big[\sum_{J,a,c} g_{\rm np}^{T=1} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} G(aacc, J, T=1)\Big] Re(u^*_{1n_\gamma} v_{1p_\gamma} + u^*_{2n_\gamma} v_{2p_\gamma})$$
$$+\Big[\sum_{J,a,c} g_{\rm np}^{T=0} F^{J0}_{\alpha a \bar{\alpha} a} F^{J0}_{\gamma c \bar{\gamma} c} i\underline{G(aacc, J, T=0)}\Big] Im(u^*_{1n_\gamma} v_{1p_\gamma} + u^*_{2n_\gamma} v_{2p_\gamma})\Bigg],$$

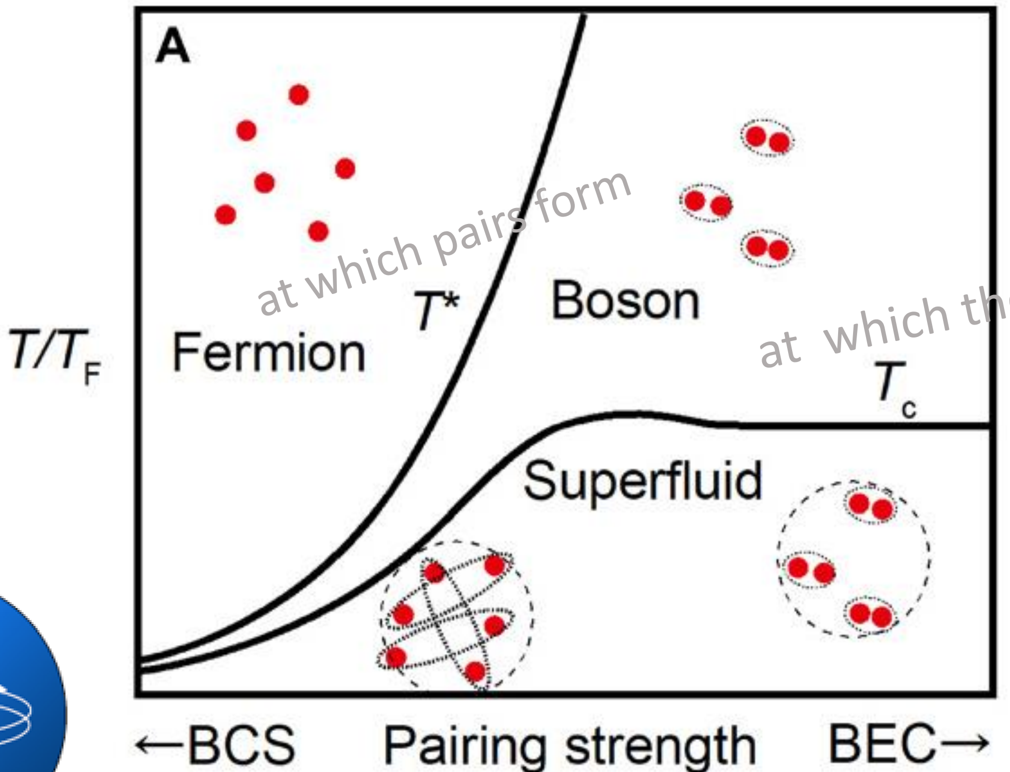
❖ ^{20}Ne

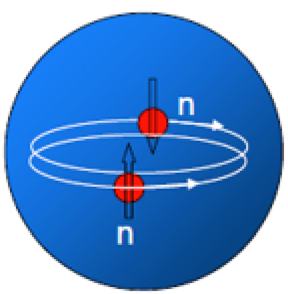


w/o TF, - - with TF : $\Delta_{IV}/\Delta_{total}$
w/o TF, - - with TF : $\Delta_{IS}/\Delta_{total}$
(a) ^{20}Ne
Ratio of IV and IS
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.3
0.4
0.5
0.6
β_2
p-p
n-n
n-p
total
w/o TF
with TF
(b) ^{20}Ne
Number of pairs
8
6
4
2
0
0.2
0.3
0.4
0.5
0.6
β_2

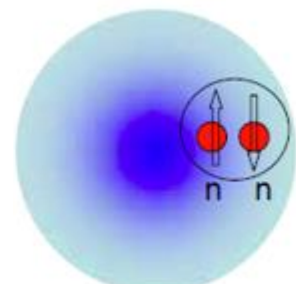
❖ BCS(Bardeen-Cooper-Schrieffer) & BEC(Bose-Einstein condensation)





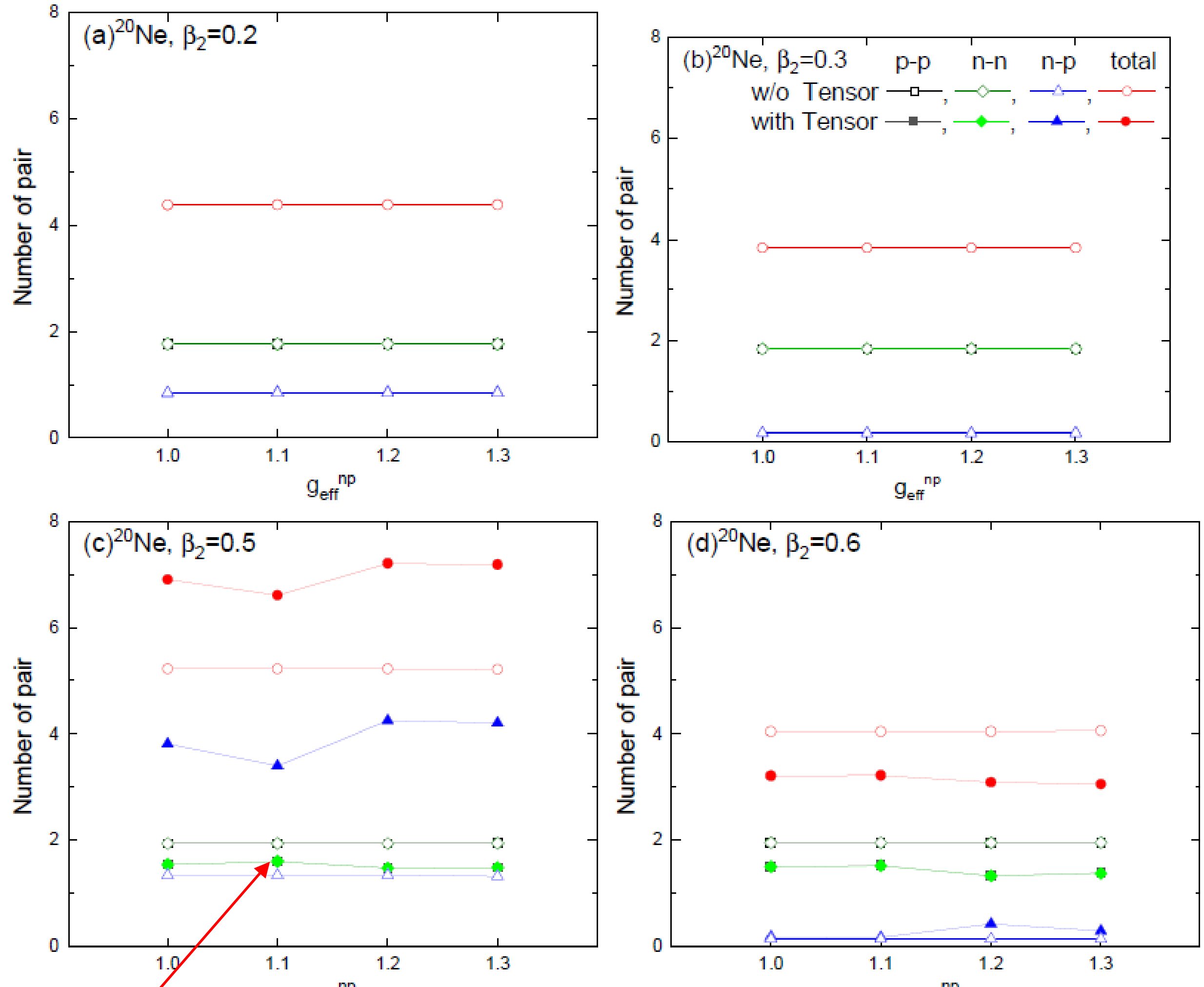


large pair size



small pair size

❖ Effect of IS enhancement



Like $T=1$	$(\alpha\bar{\alpha})$	$S=0$	$L=0,2,4\ldots$ (E)	$J=$ Even
	$(\alpha\bar{\alpha})(\alpha\alpha)(\bar{\alpha}\bar{\alpha})$	$S=1$	$L=1,3,5\ldots$ (O)	$J=$ Even

FIG. 5: (Color online) Number of pairs in terms of the effective np ($T=0$) coupling strength g^{np}_{eff} for ^{16}O. The empirical pairing gaps are fitted for each g^{eff}_{np}. Notations are the same as Fig.1 (b).

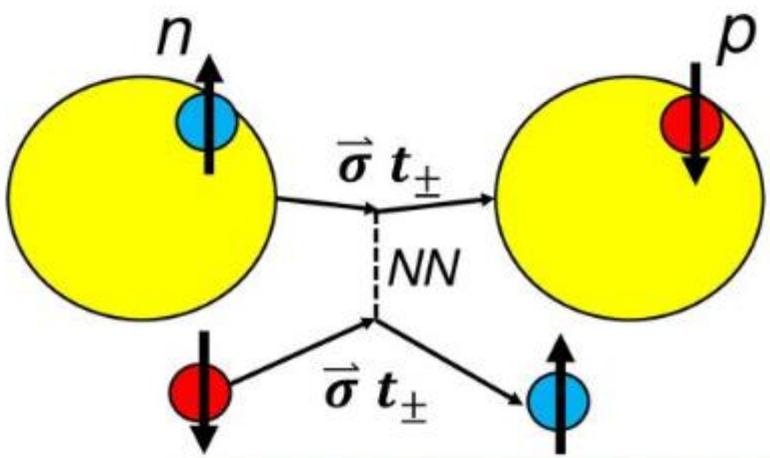
- BEC phase might not occur in ^{20}Ne.

4. Effect of tensor force in the excited state

Gamow-Teller transition and Charge-Exchange (CE) reactions at 100-300 MeV

$\Delta T=1$, $\Delta S=1$, $\Delta L=0$ induced by $\vec{\sigma}\, t_{\pm}$

strength : **B(GT)**

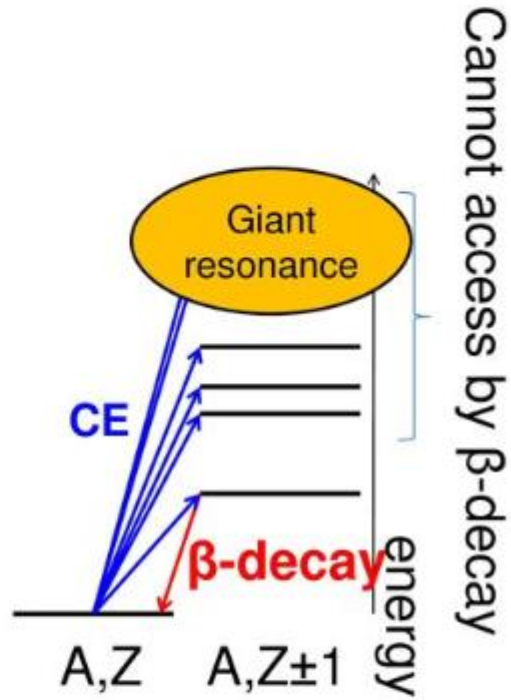


(p,n), (^{3}He,t), (n,p), (t,^{3}He), …

100 – 300 MeV
→ Isovector spin-flip >> Isovector nonspin-flip
& DWIA good

$$\left(\frac{d\sigma}{d\Omega}(q=0)\right)_{(p,n)} = \hat{\sigma}\, B(GT)$$

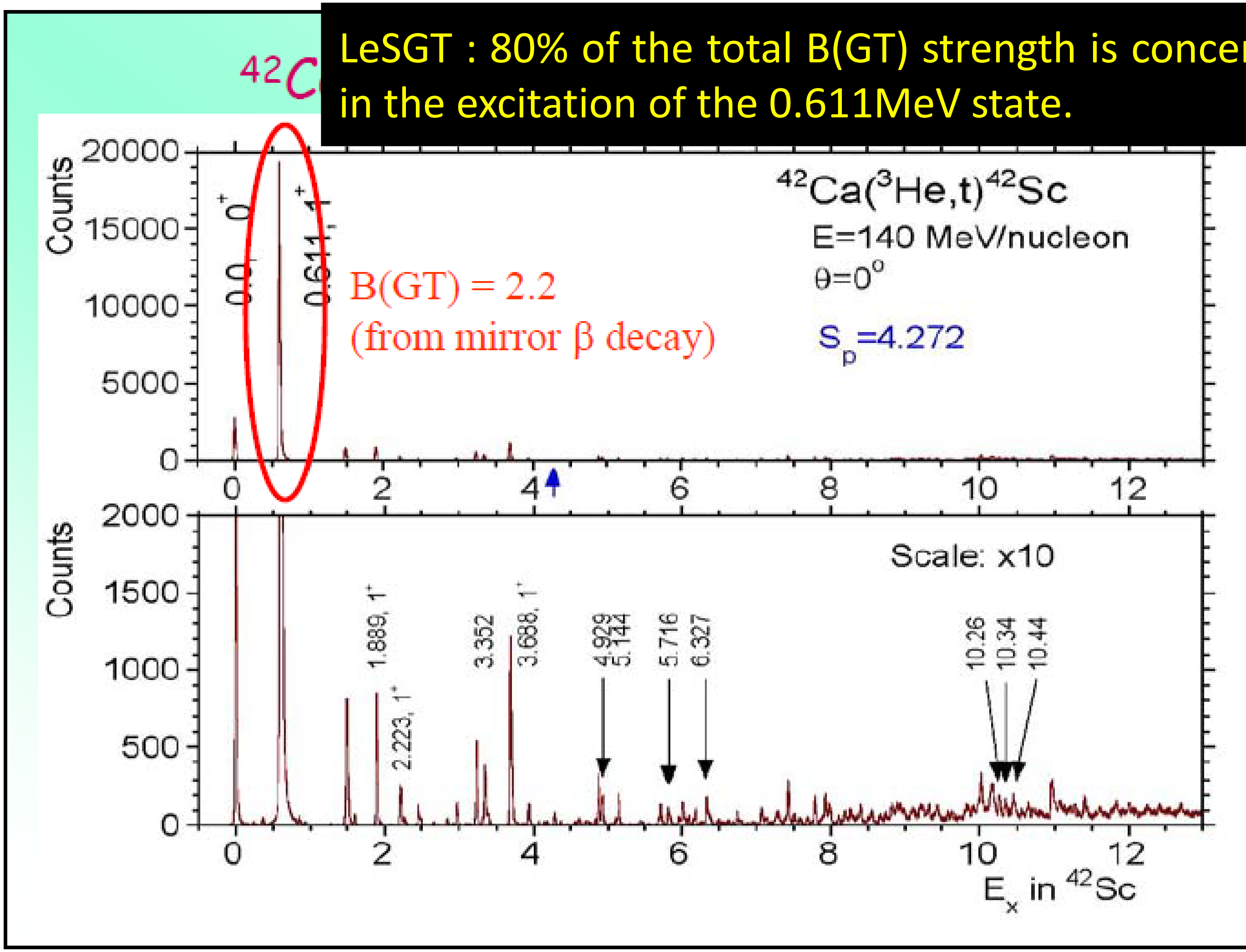
Any (0-50 MeV) Ex!



From M. Sasano's talk

❖ Low-energy super GT state(LeSGT) in N=Z+2 nuclei

LeSGT : 80% of the total B(GT) strength is concentrated in the excitation of the 0.611MeV state.



From Y. Fujita's talk

Eur. Phys. J. A (2020) 56:138

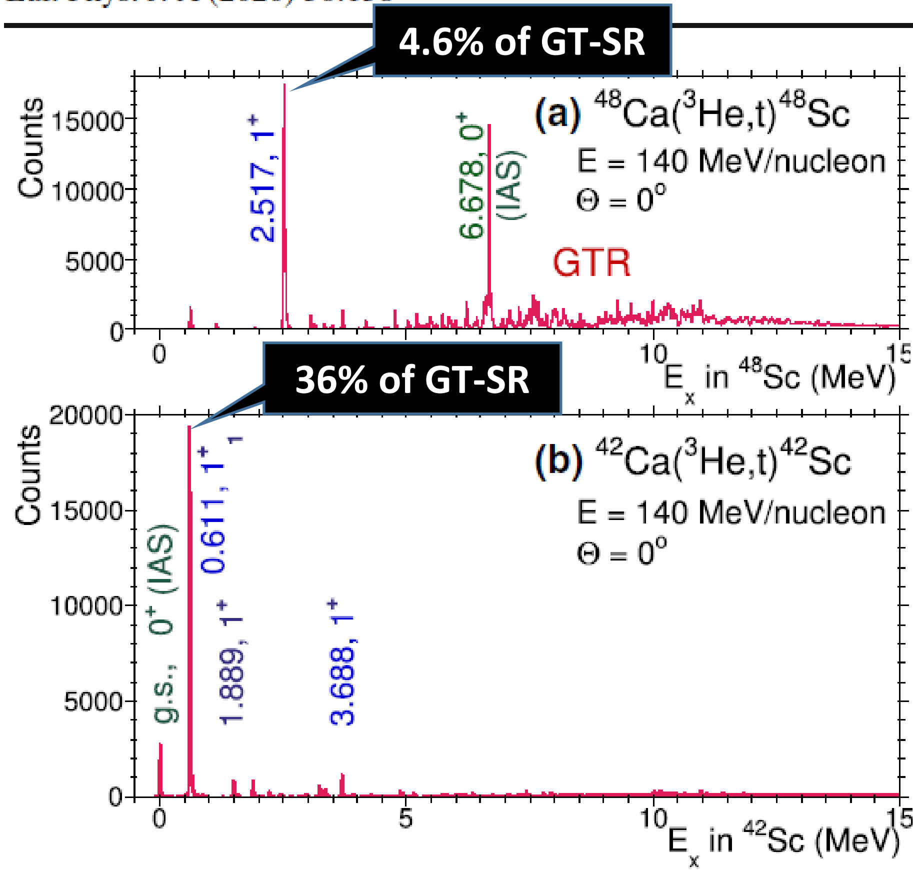
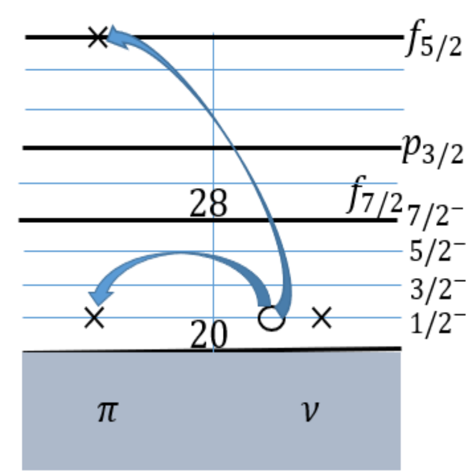


Fig. 1 **a** ^{48}Ca(^{3}He, t)^{48}Sc [32] and **b** ^{42}Ca(^{3}He, t)^{42}Sc [16] spectra taken at an intermediate incident energy of 140 MeV/nucleon and 0°. The vertical scales of **a** and **b** are so normalized that the heights of GT peaks (and IAS peaks) are proportional to their B(GT) [B(F)] values.

❖ **particle-particle interaction**

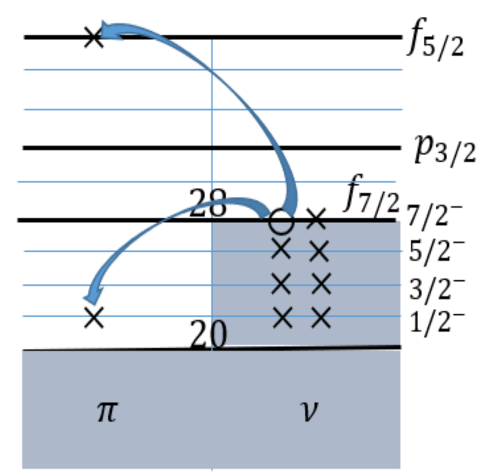
IS-pairing: attractive



$$^{42}_{20}Ca_{22} \rightarrow {}^{42}_{21}Sc_{21}$$

❖ **particle-hole interaction**

IV-pairing : repulsive



$$^{48}_{20}Ca_{28} \rightarrow {}^{48}_{21}Sc_{27}$$

The GT sum-rule (GT-SR) suggests that the total GT strength is four times larger in ^{48}Sc than in ^{42}Sc. In ^{48}Sc, the broad GT-resonance (GTR) structure spreading in the $E_x \approx 5-14$ MeV region carries the main part of the GT-SR strength, while the B(GT) = 1.1 of the sharp 2.517 MeV state is only 4.6% of the GT-SR. In ^{42}Sc, however, 0.611 MeV, $J^\pi = 1^+_1$ LeSGT state carries 36% of the GT-SR and ≈ 80% of the observed GT strength [16]

GT strength Calculations: HFB+QRPA + pairing int.

Bai, Sagawa, Colo et al., PL B 719 (2013) 116

The density dependent contact pairing interactions are adopted for both $T=1$ and $T=0$ channels,

IV $$V_{T=1}(\mathbf{r}_1,\mathbf{r}_2)=V_0\frac{1-P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (1)$$

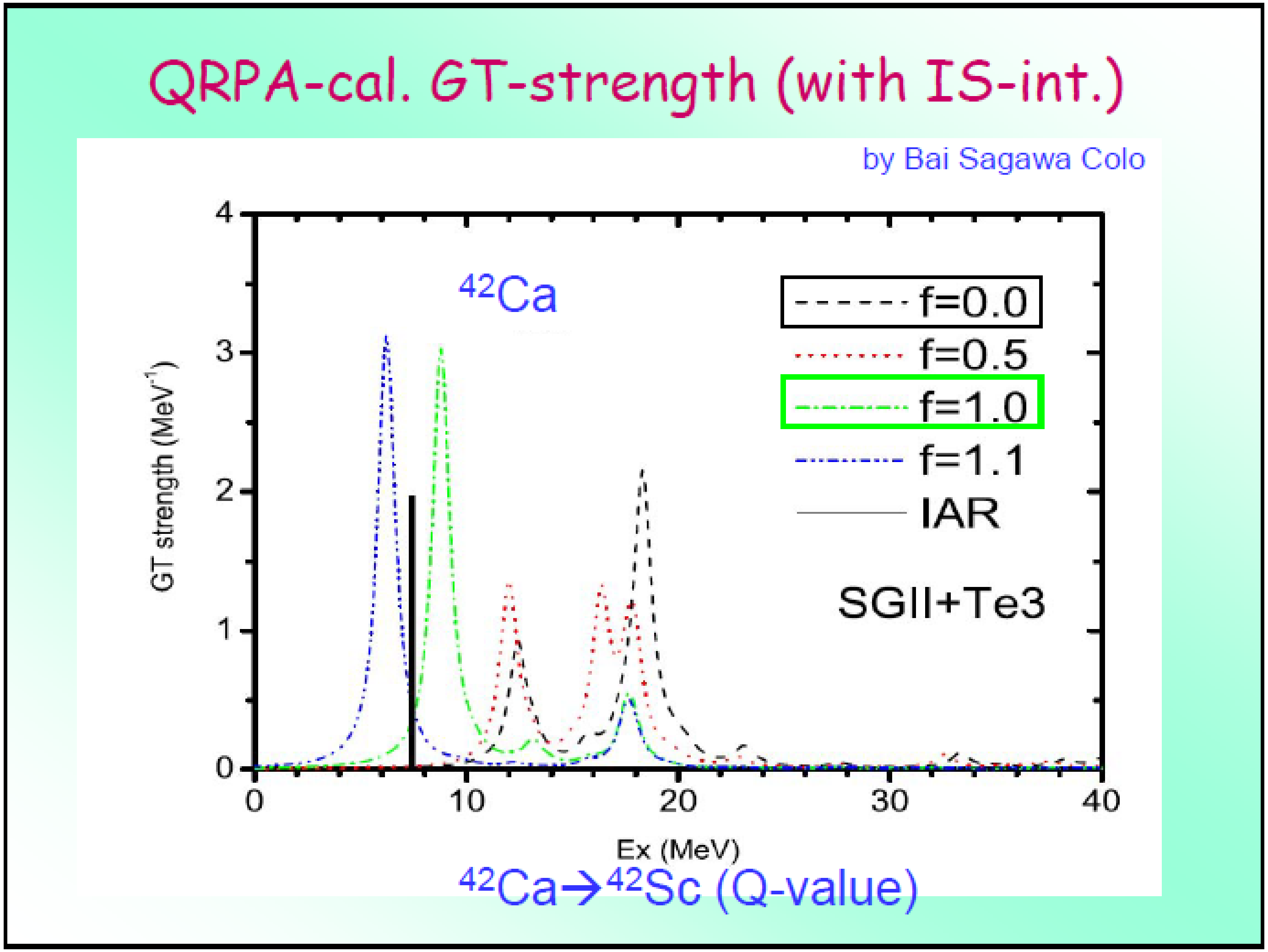
IS $$V_{T=0}(\mathbf{r}_1,\mathbf{r}_2)=fV_0\frac{1+P_\sigma}{2}\left(1-\frac{\rho(\mathbf{r})}{\rho_o}\right)\delta(\mathbf{r}_1-\mathbf{r}_2), \quad (2)$$

Results (using Skyrme int. SGII)

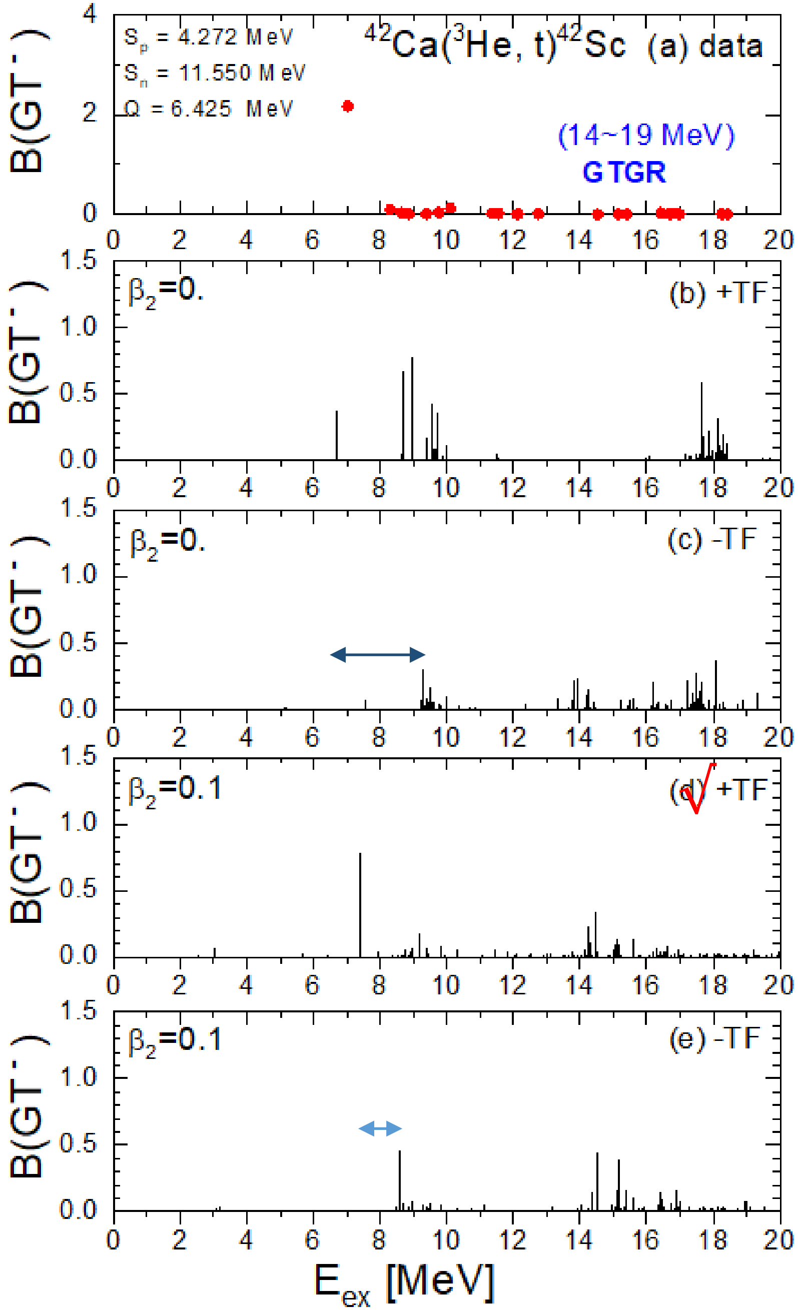
at $f=0$: there is little strength in the lower energy part,

at $f=1.0\sim1.7$: coherent low-energy strength develops!

❖ Low-energy super GT state(LeSGT)



❖ Role of TF in GT states



Tensor force & Deformation ?

$$\frac{g_{pp}}{g_{ph}} = 1.7 \; : {}^{42}\mathrm{Ca}$$

	$\beta_2(E2)$	$\beta_2(FRDM)$	$\beta_2(RMF)$
^{42}Ca	0.245	0.	0.

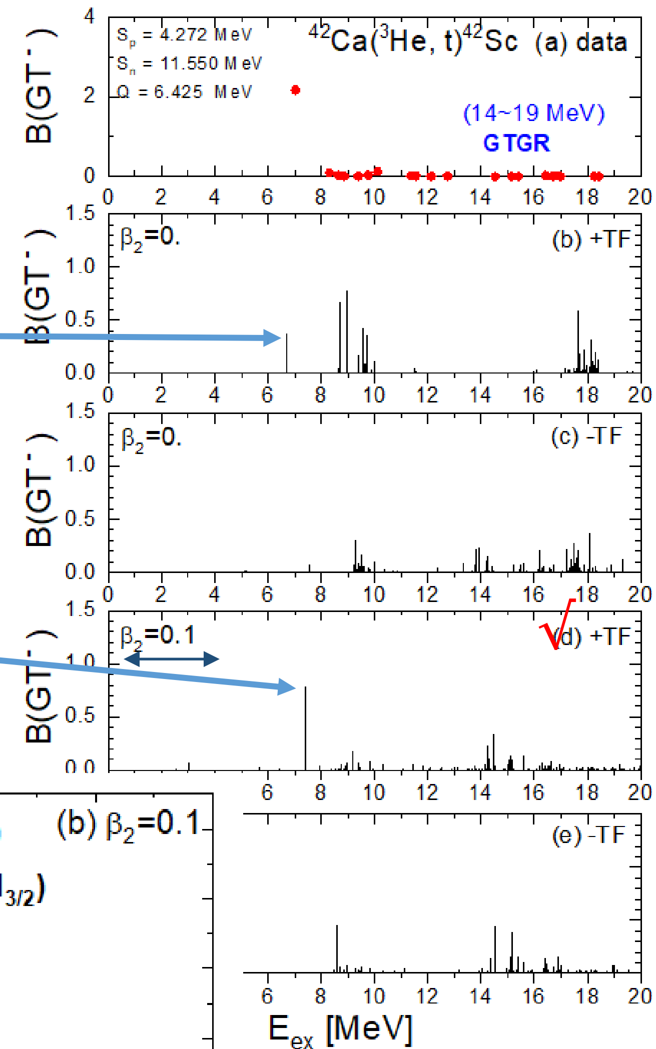
❖ Role of TF in GT states

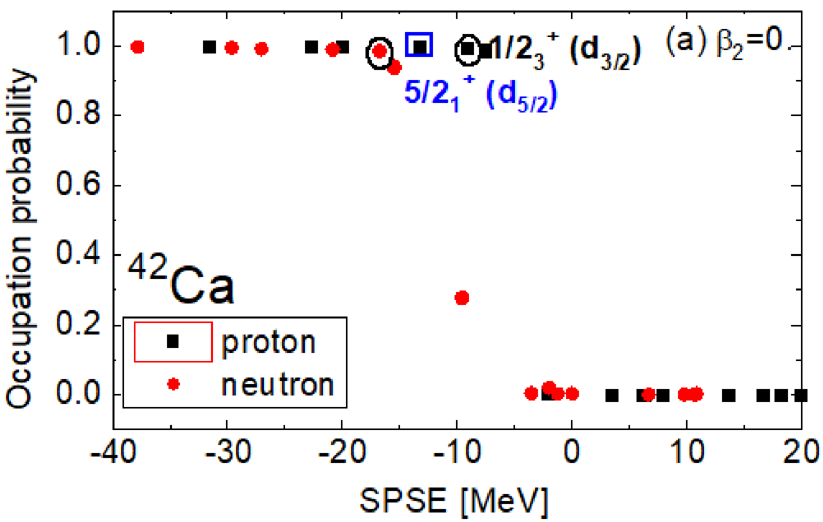
$$\left(\tfrac{7}{2}^-_1, \tfrac{7}{2}^-_1\right), \left(\tfrac{5}{2}^-_1, \tfrac{5}{2}^-_1\right), \left(\tfrac{3}{2}^-_2, \tfrac{3}{2}^-_2\right), \left(\tfrac{1}{2}^-_3, \tfrac{1}{2}^-_3\right) \text{ from } \left(f_{\frac{7}{2}}, f_{\frac{7}{2}}\right)$$

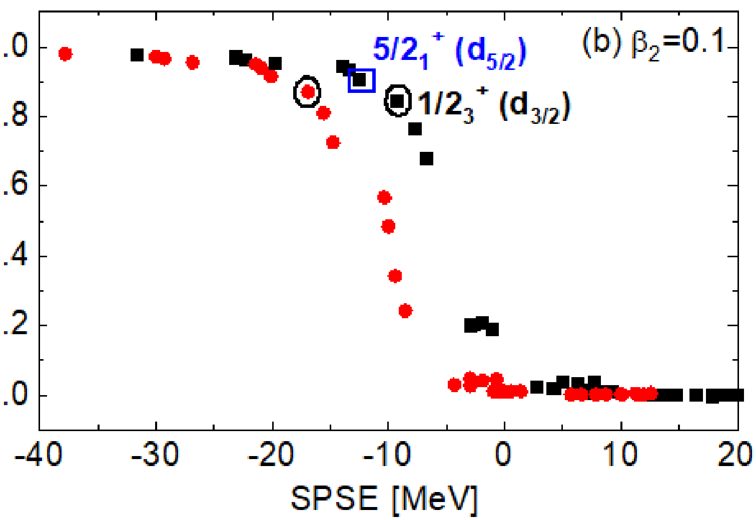
$$\surd \left(\tfrac{1}{2}^+_3, \tfrac{1}{2}^+_3\right) \text{ from } \left(d_{\frac{3}{2}}, d_{\frac{3}{2}}\right)$$

$$\left(\tfrac{1}{2}^-_3, \tfrac{1}{2}^-_3\right) \text{ from } \left(f_{\frac{7}{2}}, f_{\frac{7}{2}}\right)$$

$$\surd \left(\tfrac{3}{2}^+_2, \tfrac{5}{2}^+_1\right) \text{ from } \left(d_{\frac{3}{2}}, d_{\frac{5}{2}}\right)$$







❖ Summary

1. **The noncentral TF effect turns out to be sensitive to the deformation and breaks the IV dominance of the *np* pairing.**

2. **The number of *np* pairs is increased(reduce) by the attractive(repulsive) spin-triplet even(odd) TF.**

3. **The nuclear deformation turns out to be another key factor to determine the number of IS pairs reflecting the TF property.**

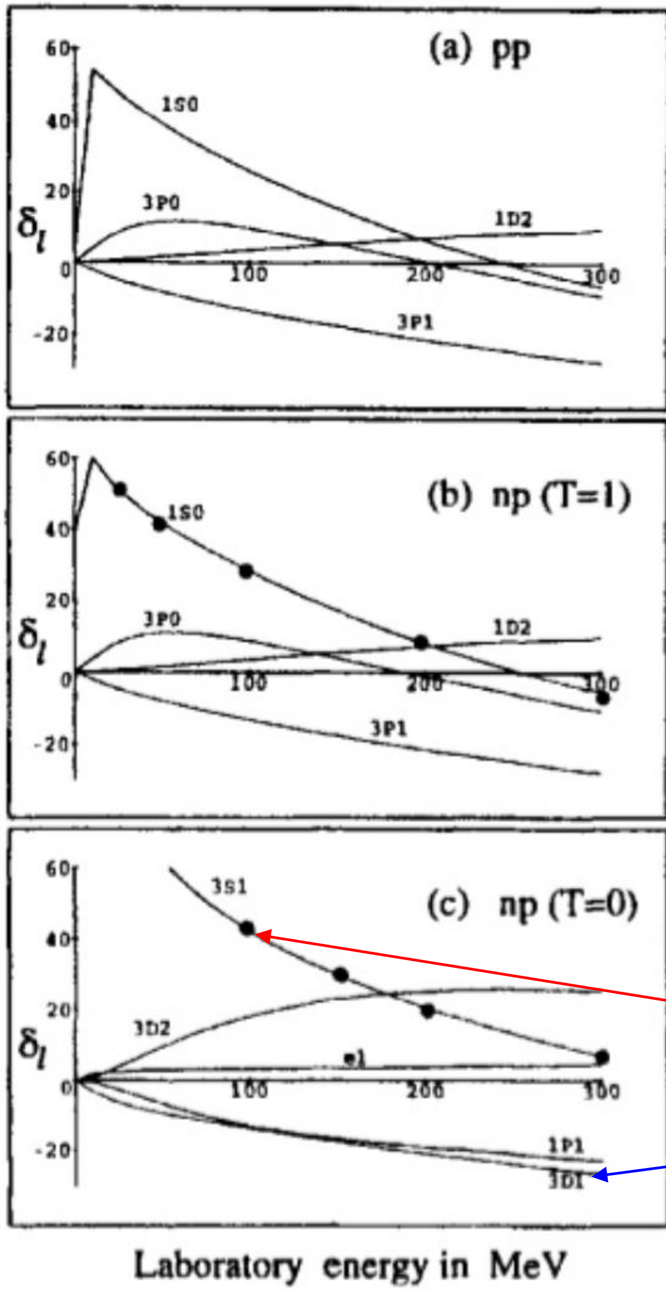
4. **TF plays an important role in producing the low-energy GT peak.**

5. **The attractive TF affects not only the ground state but also plays a crucial role of shifting the main GT peak to the lower excitation energy leading to the LeGST.**

Thank you for your attention!

Back Up

❖ Nucleon-Nucleon Scattering Phase Shifts



IV(isovector), IS(isoscalar)

pp & nn		$^{2S+1}L_J$
IV **T=1** (S)	S =0 (A) L =even(S) J = L $^1S_0$, $^1D_2$	S =1 (S) L =odd(A) J = Even $^3P_0$, $^3P_1$, $^3P_2$

np		
IV **T=1** (S)	S =0 (A) L =even(S) J = L $^1S_0$, $^1D_2$	S =1 (S) L =odd(A) J = Even $^3P_0$, $^3P_1$, $^3P_2$
IS **T=0** (A)	S =0 (A) L =odd(A) J = L $^1P_1$	S =1 (S) L =even(S) J = Odd $^3S_1$, $^3D_1$, $^3D_2$, $^3D_3$

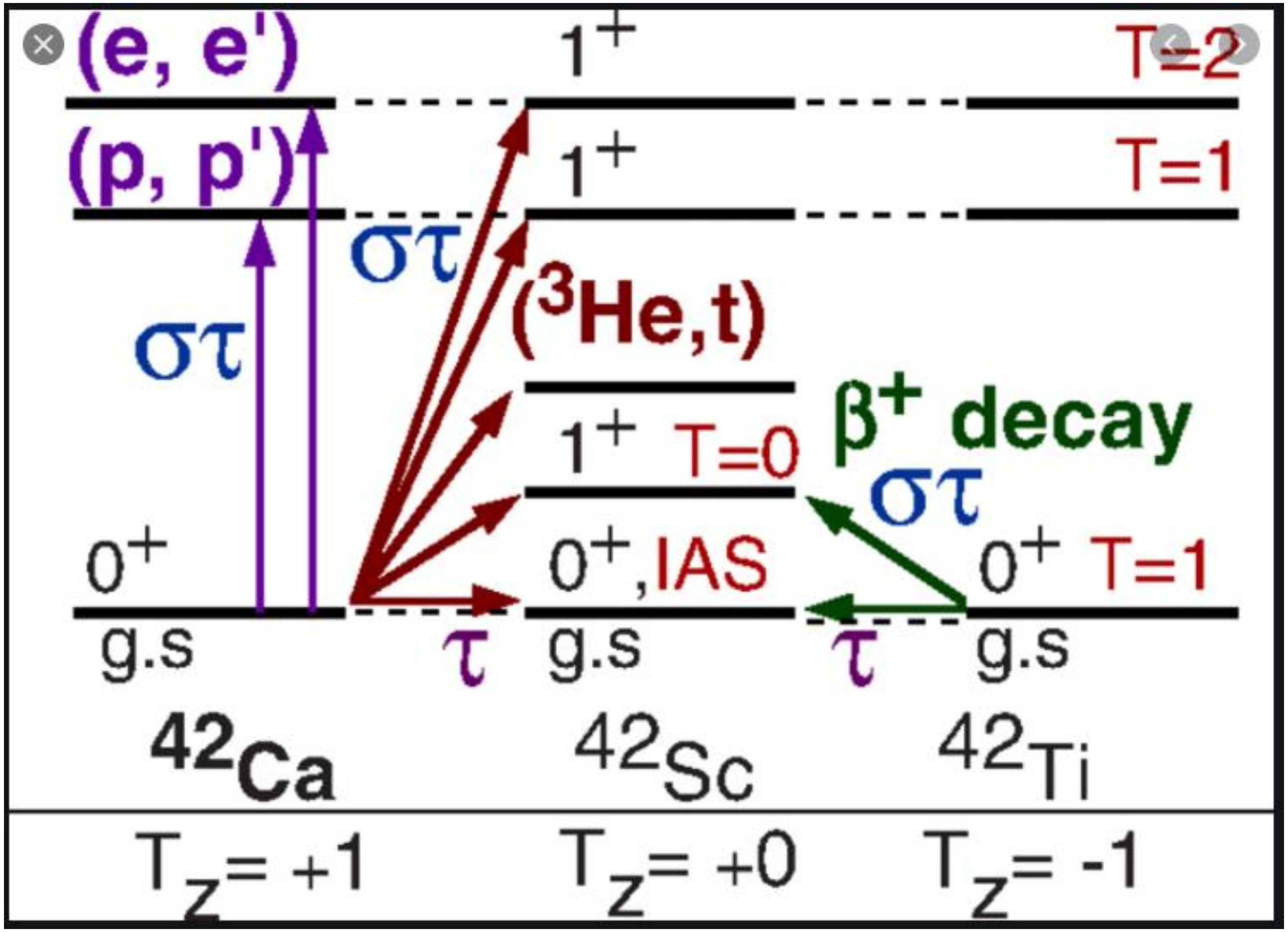
blue: repulsive, red: attractive

❖Six parameters in empirical formula

Table 2.2: Values adopted for the six parameters in Eq. (2.1) for the excitation energies of the first natural parity even multipole states including 2_1^+, 4_1^+, 6_1^+, 8_1^+, and 10_1^+ states. The last two columns are the χ^2 value which fits the parameter set and the total number N_0 of the data points, respectively, for the corresponding multipole state.

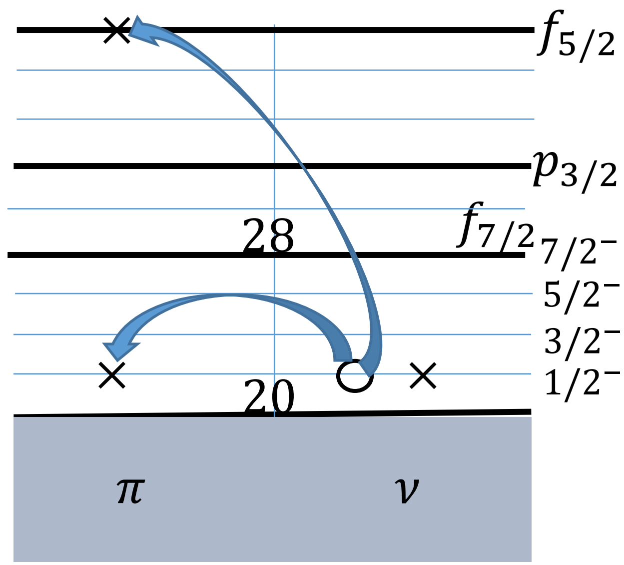
J_1^π	α (MeV)	γ	β_p (MeV)	β_n (MeV)	λ_p	λ_n	χ^2	N_0
2_1^+	68.37	1.34	0.83	1.17	0.42	0.28	0.126	557
4_1^+	268.04	1.38	1.21	1.68	0.33	0.23	0.071	430
6_1^+	598.17	1.38	1.40	1.64	0.31	0.18	0.069	375
8_1^+	1,438.59	1.45	1.34	1.50	0.26	0.15	0.053	309
10_1^+	2316.85	1.47	1.36	1.65	0.21	0.14	0.034	265

❖Isobaric analogue state (IAS)



❖ **particle-particle interaction**

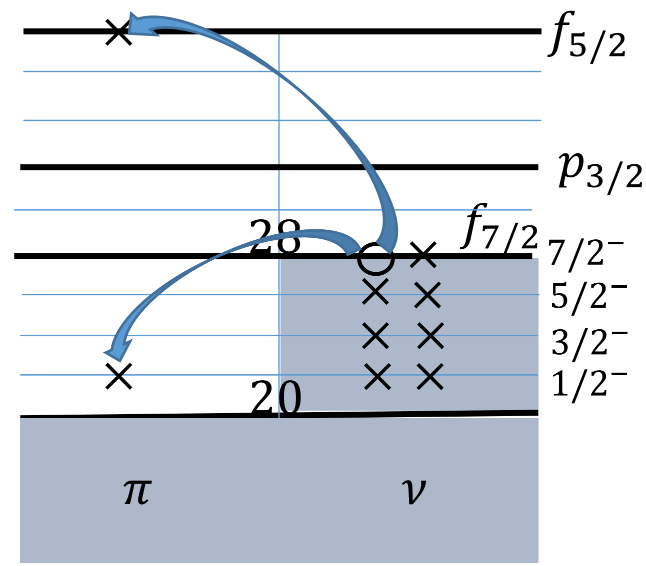
IS-pairing: attractive



$$^{42}_{20}Ca_{22} \rightarrow ^{42}_{21}Sc_{21}$$

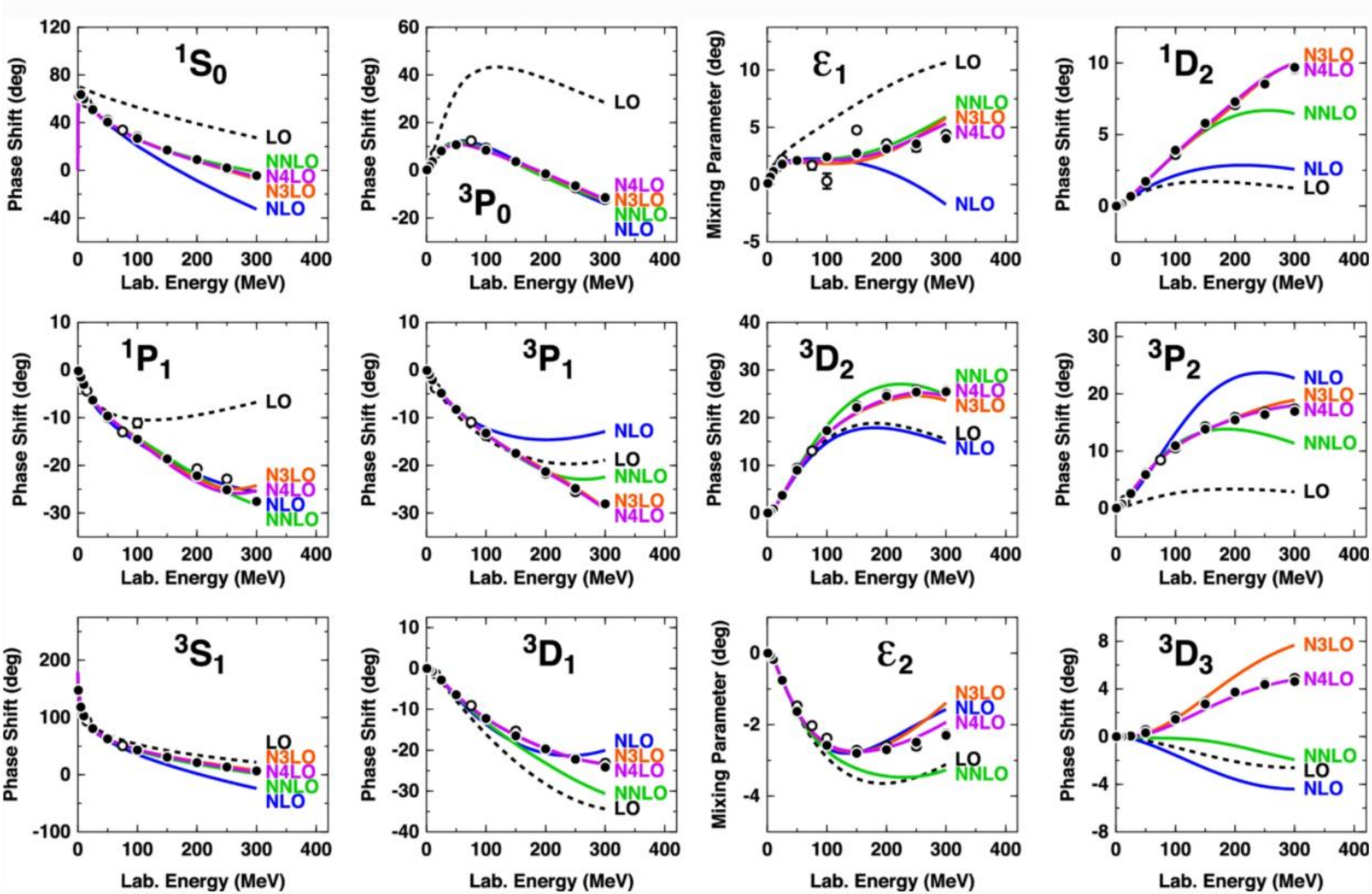
❖ **particle-hole interaction**

IV-pairing : repulsive

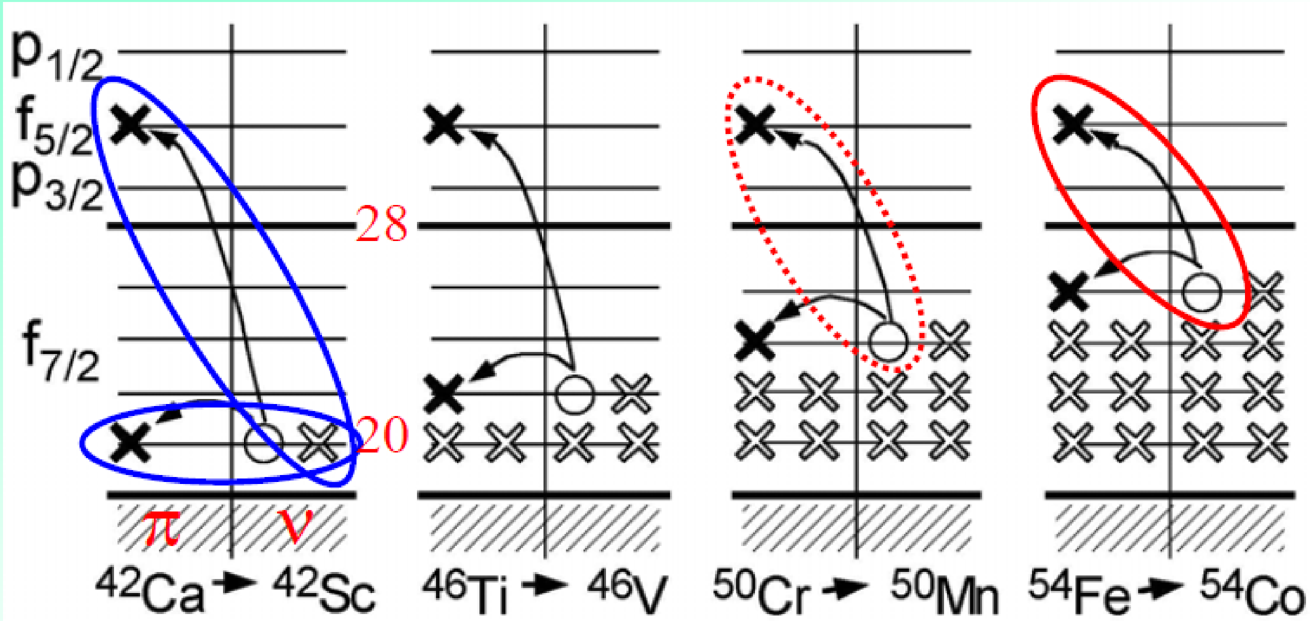


$$^{48}_{20}Ca_{28} \rightarrow ^{48}_{21}Sc_{27}$$

❖ Nucleon-Nucleon Scattering Phase Shifts



SM Configurations of GT transitions



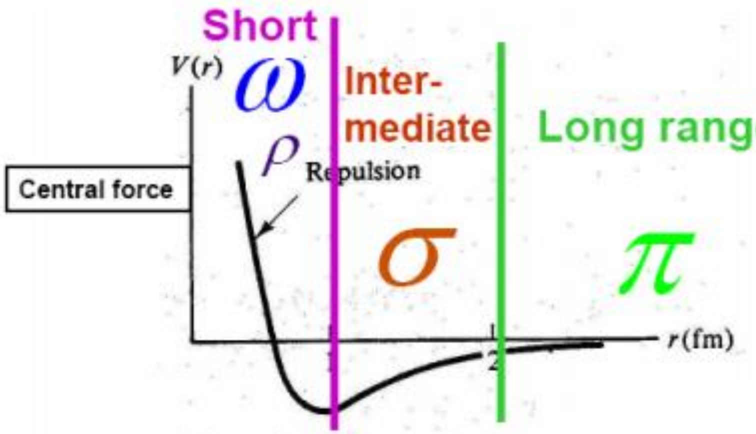
π-p - ν-p configurations
sensitive to IS pairing int.
➔attractive
(spin-triplet, IS int. is stronger than spin-singlet, IV int.)

particle-hole configurations
+ IV-type excitation (στ)
➔repulsive

by Engel, Bertsch, Macchiavelli

✓ ***IS (T=0) pairing can play important roles !***

The short and long range tensor force



Lecture notes of R. Machleidt
at the 2005 RIKEN summer school

Tensor force π ρ

Spin-orbit force ω σ

$\pi(138)$ $$V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ Long-ranged tensor force

$\sigma(600)$ $$V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1 - \frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ intermediate-ranged, attractive central force plus LS force

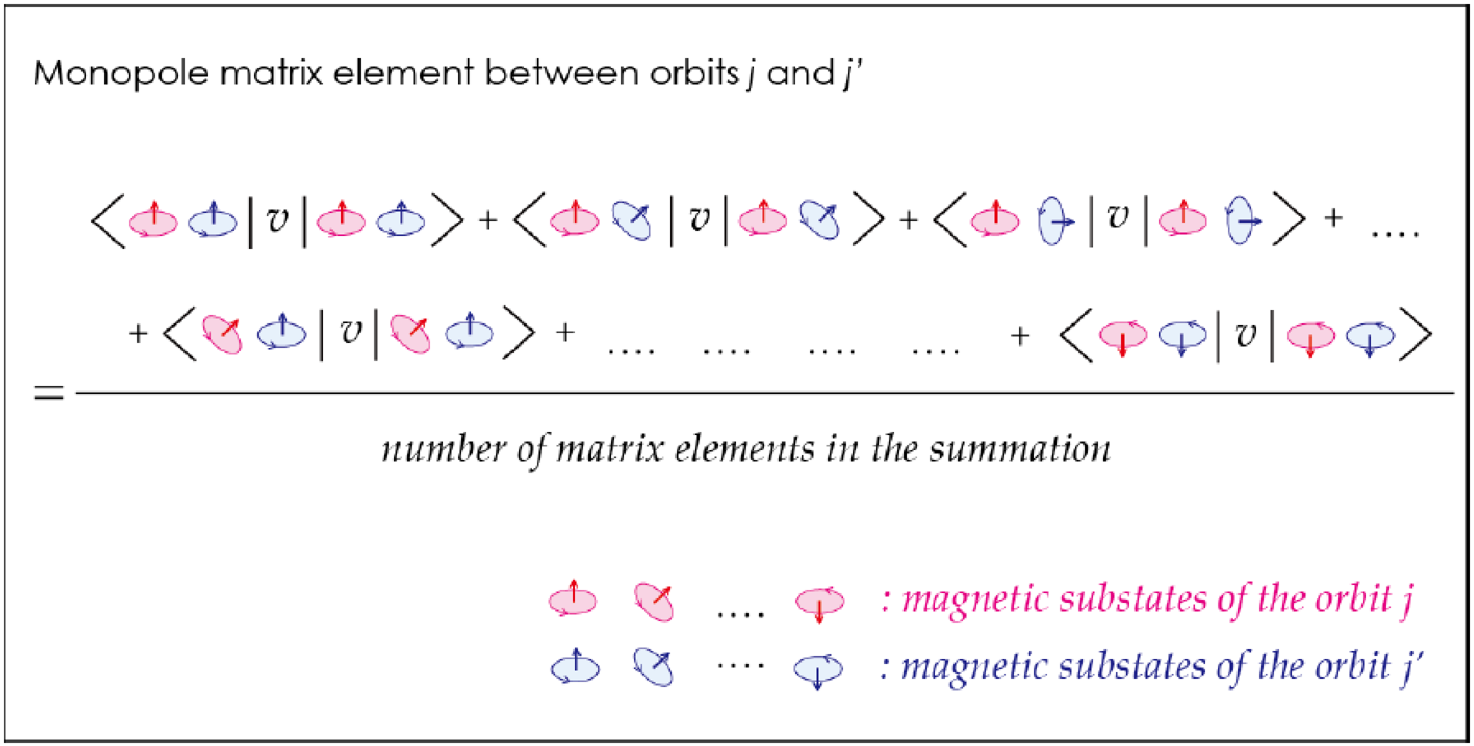
$\omega(782)$ $$V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1 - 3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$$ short-ranged, repulsive central force plus strong LS force

$\rho(770)$ $$V_\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$$ short-ranged tensor force, opposite to pion

Monopole matrix element between orbits *j* and *j'*

$$V^{m}_{nn}(j,j') = \frac{\sum_{(m,m')} \langle j,m\,;\,j',m' | \hat{v}_{nn} | j,m\,;\,j',m' \rangle}{\sum_{(m,m')} 1}$$

v_{nn} is interaction; *m*, *m'* are magnetic substates



< Tensor Force >

Tensor potential $\hat{V}_T = V_T(\vec{r})\left[3\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2\right]$ " $\hat{S}_{12}$

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = (\vec{A}\cdot\vec{B}) + i\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$

$$(\vec{\sigma}\cdot\vec{r})^2 = (\vec{r}\cdot\vec{r}) + i\vec{\sigma}\cdot(\vec{r}\times\vec{r}) = r^2$$

$$2(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r}) = (\vec{\sigma}_1\cdot\vec{r}+\vec{\sigma}_2\cdot\vec{r})^2 - (\vec{\sigma}_1\cdot\vec{r})^2 - (\vec{\sigma}_2\cdot\vec{r})^2$$

$$= (\vec{\sigma}_1\cdot\vec{r}+\vec{\sigma}_2\cdot\vec{r})^2 - r^2 - r^2$$

$$= \{(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{r}\}^2 - 2r^2 \qquad (\because s = \tfrac{1}{2}\sigma,\ \sigma_1+\sigma_2 = 2S)$$

$$\therefore (\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r}) = \frac{4(\vec{S}\cdot\vec{r})^2 - r^2}{2}$$

$$= 2(\vec{S}\cdot\vec{r})^2 - r^2$$

$$\therefore \hat{S}_{12} = 3\frac{(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2 \ *$$

$$= 3\frac{2(\vec{S}\cdot\vec{r})^2 - r^2}{r^2} - 2S^2 + 3$$

$$\therefore \hat{S}_{12} = 2\left[\frac{3(\vec{S}\cdot\vec{r})^2}{r^2} - S^2\right] = 2\left[3S^2\cos^2\theta - S^2\right]$$

$$\boxed{\hat{S}_{12} = -2S^2(1-3\cos^2\theta)}$$

*

$$(\vec{\sigma}_1+\vec{\sigma}_2)^2 = \sigma_1^2 + \sigma_2^2 + 2(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

$$(\vec{\sigma}_1\cdot\vec{\sigma}_2) = \frac{1}{2}\left[(\sigma_1+\sigma_2)^2 - \sigma_1^2 - \sigma_2^2\right]$$

$$= \frac{1}{2}\left[4S^2 - 4S_1^2 - 4S_2^2\right]$$

$$= 2S^2 - 2\left(\tfrac{1}{2}\right)\left(\tfrac{3}{2}\right) - 2\left(\tfrac{1}{2}\right)\left(\tfrac{3}{2}\right)$$

$$= 2S^2 - \frac{3}{2} - \frac{3}{2}$$

$$= 2S^2 - 3$$

i) $S=0$, $\hat{S}_{12} = 0$.

ii) $S=1$, $\theta = 0°$ $\hat{S}_{12} = -2(-2) = 4$ ← attractive

$\theta = \frac{\pi}{2}$ $\hat{S}_{12} = -2(1-0) = -2$ ← repulsive

$$\hat{V}_T = V_T(r)\hat{S}_{12}, \qquad V_T(r) < 0$$

$$\hat{V}_T \sim -\hat{S}_{12} \sim 1 - 3\cos^2\theta = Y_{2,0}$$

Ground-state correlations

$\beta\beta$ decay : $0^+_{gs} \rightarrow 0^+_{gs}$

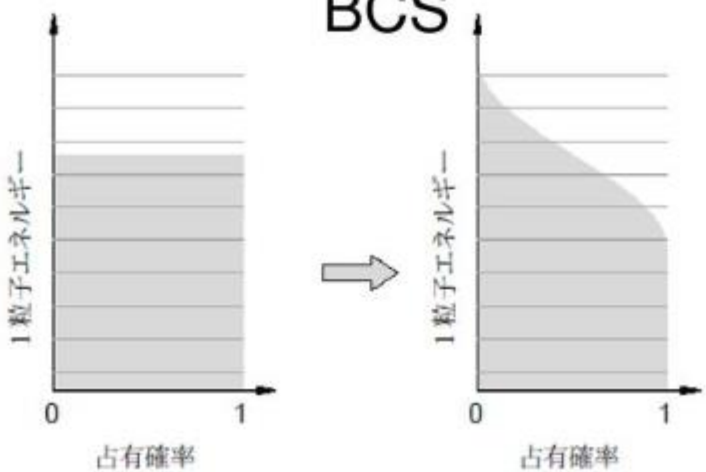
Important components of *NN* interaction:

1) Pairing between like nucleons → BCS

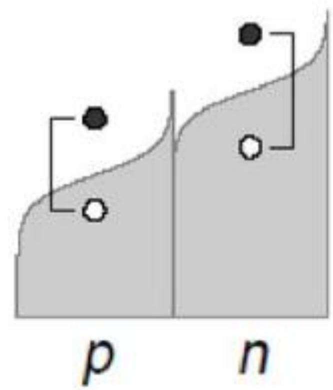
2) Proton-neutron interaction, in particular QQ force

→ RPA **R**andom-**P**hase **A**pproximation

BCS



RPA



proton-neutron QRPA

BCS

- Ground state ansatz

$$|\mathrm{BCS}\rangle = \prod_{j,m>0} (u_j + v_j\, c^\dagger_{jm} c^\dagger_{j-m})|\ \rangle$$

- Variation with constraints

$$\frac{\partial}{\partial u_j}\langle \mathrm{BCS}|H'|\mathrm{BCS}\rangle = 0$$

$$H' = H - \lambda\widehat{N}$$

$$\langle \mathrm{BCS}|\widehat{N}|\mathrm{BCS}\rangle = N$$

$$u_j^2 + v_j^2 = 1$$

- quasiparticle

$$a^\dagger_{jm} = u_j\, c^\dagger_{jm} + v_j\, \tilde{c}_{jm}$$

$$\tilde{a}_{jm} = u_j\, \tilde{c}_{jm} - v_j\, c^\dagger_{jm}$$

$$a_{jm}|\mathrm{BCS}\rangle = 0$$

RPA

- Excitation Operator
  charge-changing modes

$$Q^\dagger_{\omega,J} = \sum_{pn}(X^{pn}_{\omega,J}\,[a^\dagger_p a^\dagger_n]_J - Y^{pn}_{\omega,J}\,[\tilde{a}_p\,\tilde{a}_n]_J)$$

- Equation of motion

$$\langle 0|[\delta Q, [H, Q^\dagger_\omega]]|0\rangle = \omega\langle 0|[\delta Q, Q^\dagger_\omega]|0\rangle$$

- RPA equation

$$\begin{bmatrix} A & B \\ B & A \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = \omega\begin{bmatrix} C & 0 \\ 0 & -C \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix}$$

quasi-boson approximation

$$\Rightarrow \quad C_{p'n',pn} = \delta_{p'p}\,\delta_{n'n}$$

B (GT) derivation

★ β decay :fundamental, but E_x range :limited "Q-window limitation"

★ (p, n) reaction at intermediate energies (E = 100-500 MeV)

"proportionality" : B(GT) and σ(0°)

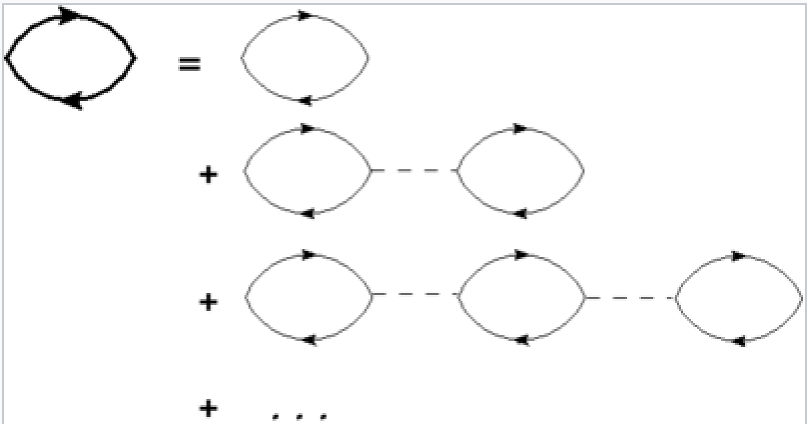
$$\sigma(0^\circ) = K N_{\sigma\tau} \, | J_{\sigma\tau}(0^\circ) |^2 \, B(\mathrm{GT})$$

⇒ Breakthrough against "Q-window limitation"

but resolution : rather poor (ΔE = 200-400 keV)

$J_{\sigma\tau}$: volume integral of the effective interaction $V_{\sigma\tau}$at momentum transfer $q \approx 0$.
$K(\omega)$: kinematic factor.
ω:total energy transfer.
$N_{\sigma\tau}$: distortion factor.
$\sigma(0)$: unit cross section for the GT transition at $q = \omega = 0$.

❖ RPA



Ring diagrams that are summed up in order to obtain the RPA approximation. Above bold lines stand for interacting Green functions, non-bold lines stand for non-interaction Green function, and dashed lines stand for two-body interactions.

In a weak perturbing external filed, the response of system is related to the 2-body Green’s function.

RPA provides an approximation scheme to calculate the 2-body Green’s function.

In dielectric function e(k,w) describes the dielectric response to the plan-wave Electric field E(w,k)exp^i(kr-wt).

The contribution to the dielectric function from the total potential is assumed to average out, so that only the potential at wave vector k contribute.

Coexistence of BCS- and BEC-Like Pair Structures in Halo Nuclei

K. Hagino,[1] H. Sagawa,[2] J. Carbonell,[3] and P. Schuck[4,5]

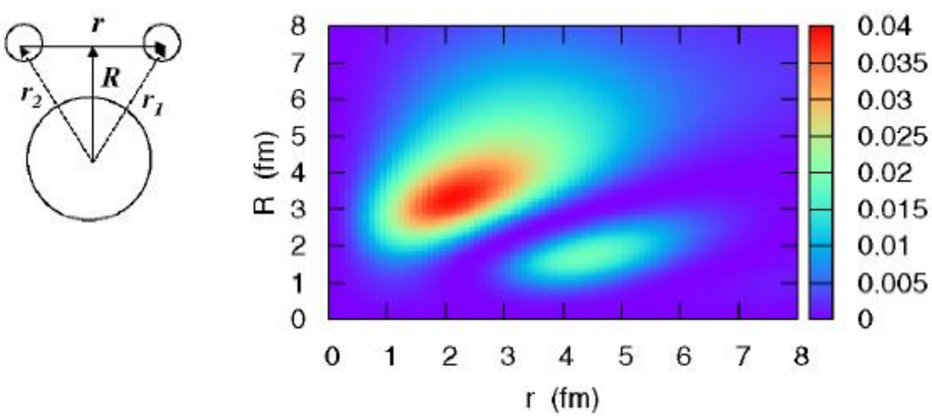


FIG. 1 (color online). A two-dimensional plot for the ground state two-particle wave function $r^2R^2|f_{L=0}(r,R)|^2$, for ^{11}Li. It is plotted as a function of the relative distance between two neutrons r and the distance between the center of mass of the two neutrons and the core nucleus R, as denoted in the inset.

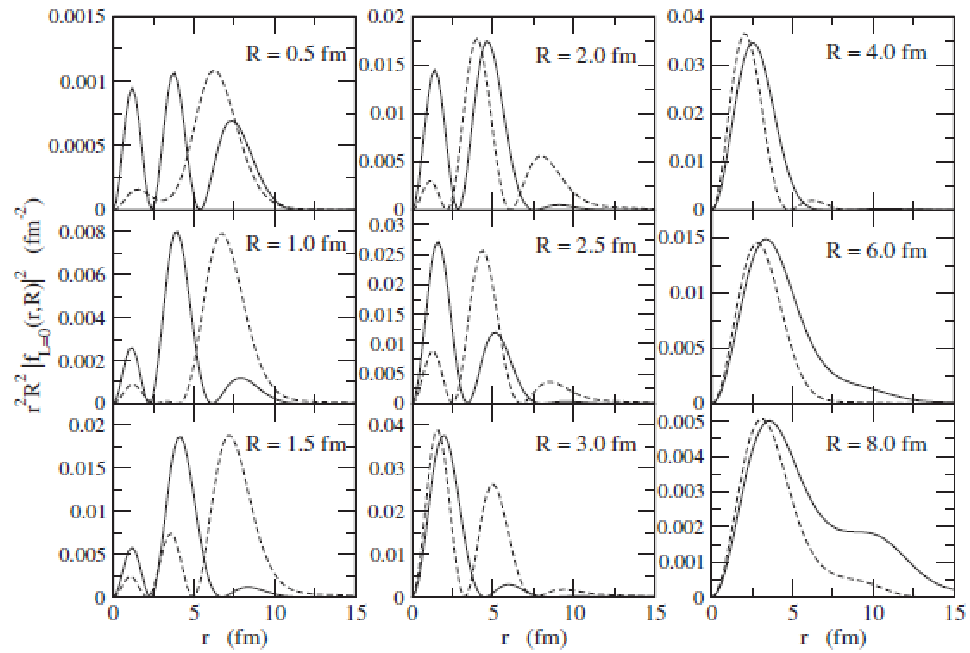
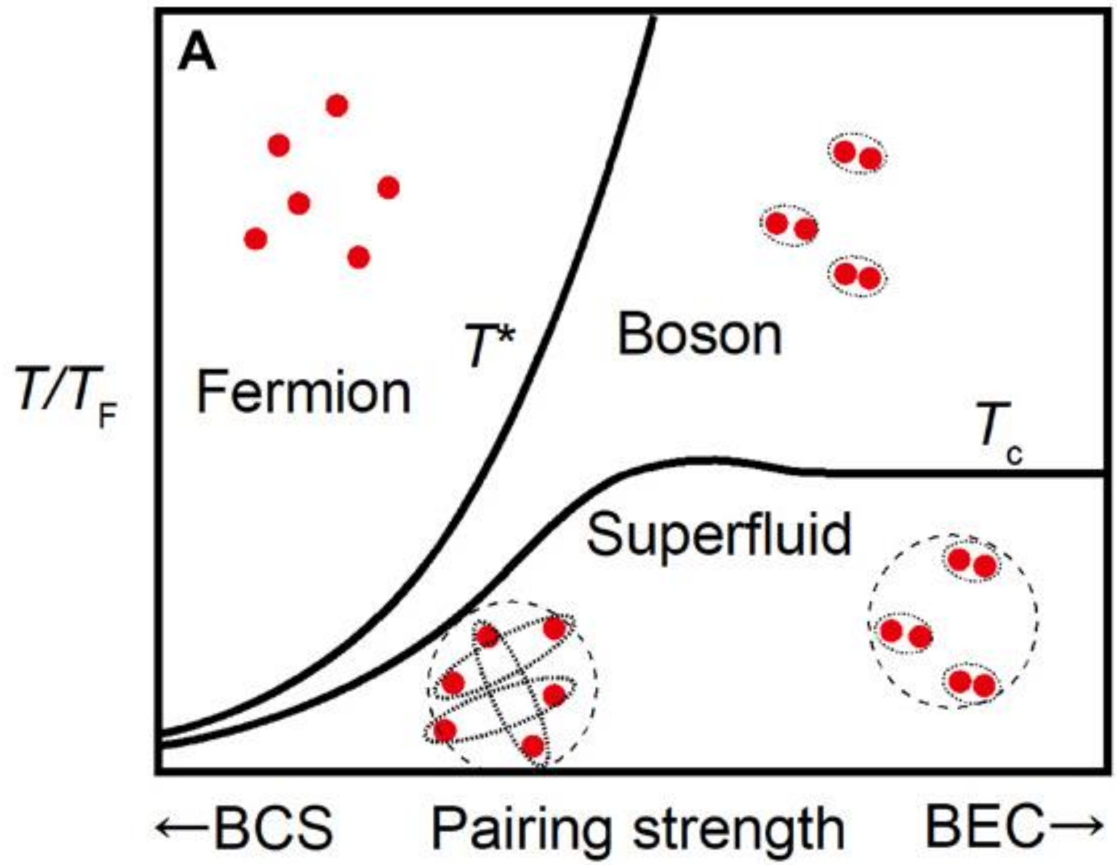


FIG. 2. The ground state two-particle wave functions $r^2R^2|f_{L=0}(r,R)|^2$ as a function of the relative distance between the neutrons r at several distances R from the core. The solid lines correspond to the two-particle wave functions of ^{11}Li, while the dashed lines denote those of ^{16}C. Notice the different scales on the ordinate in the various panels.


A
T/T_F
Fermion
T*
Boson
T_c
Superfluid
←BCS
Pairing strength
BEC→

Tayler series,

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots$$

here $x - x_0 = h$ for convergence

$x = x_0 + h$ if) h is small

$$f(x_0 + h) = f(x_0) + f'(x_0)\,h + \frac{f''(x_0)}{2!}h^2 + \cdots$$