

Static properties of light and heavy baryons

Yang, Ghil-Seok
Hoseo University



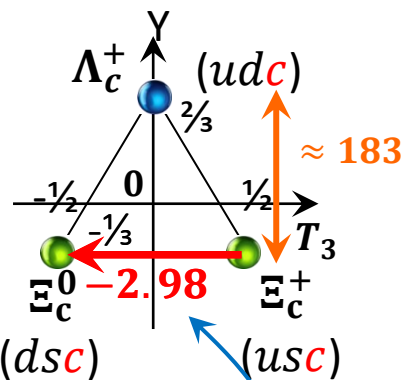
in collaboration with
H.-Ch. Kim(Inha U.), H.-D. Son(CENuM & Ruhr-Uni. Bochum),
M. Polyakov (Ruhr-Uni. Bochum), and M. Praszalowicz(Jagiellonian U)

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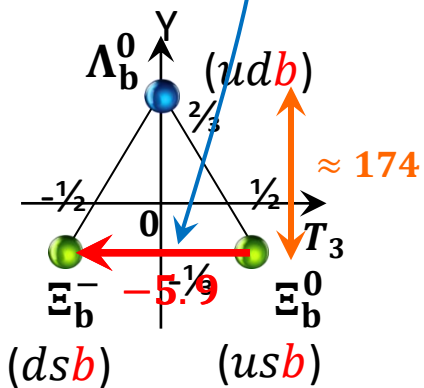
Motivation



$$[\bar{3}] = D(0,1),$$

$$J = 1/2$$

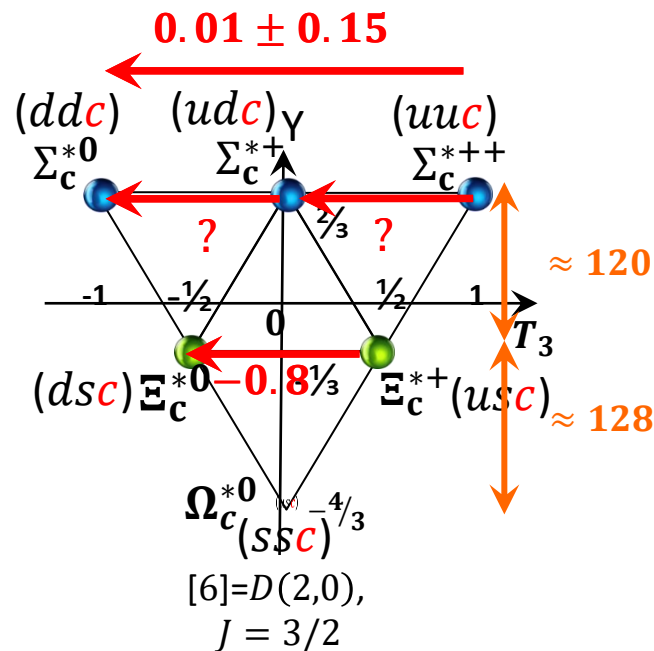
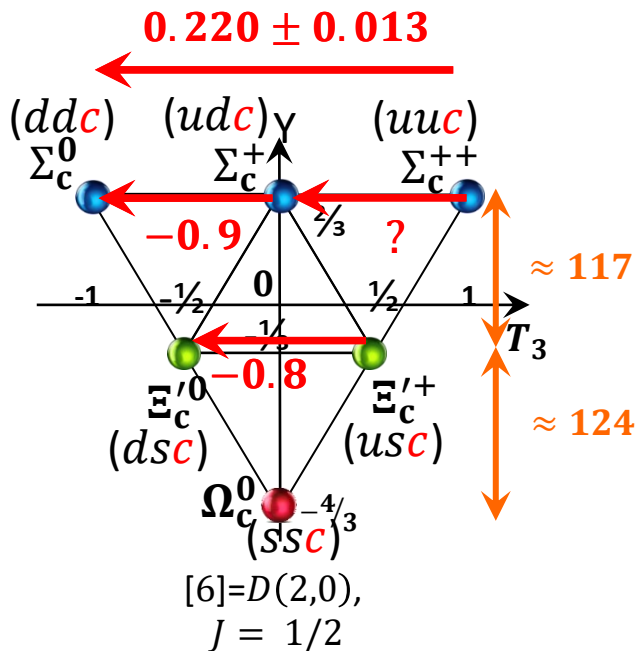
twice ?



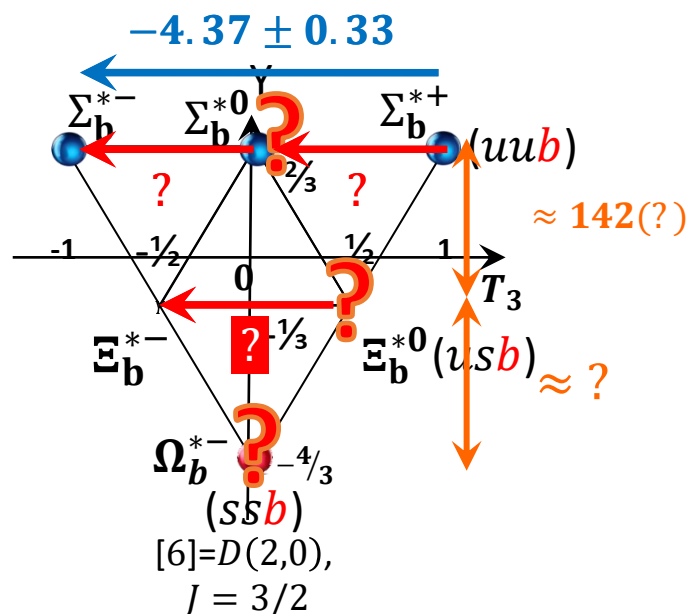
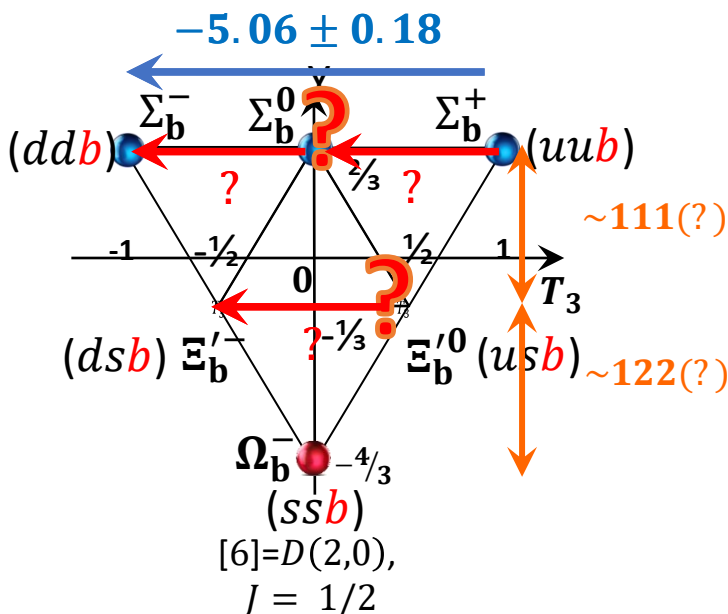
$$[\bar{3}] = D(0,1),$$

$$J = 1/2$$

Why positive ?



Why negative ?



DOUBLY CHARGED BARYONS

($C = +2$)

$\Xi_{cc}^{++} = ucc$, $\Xi_{cc}^+ = dcc$, $\Omega_{cc}^+ = scc$

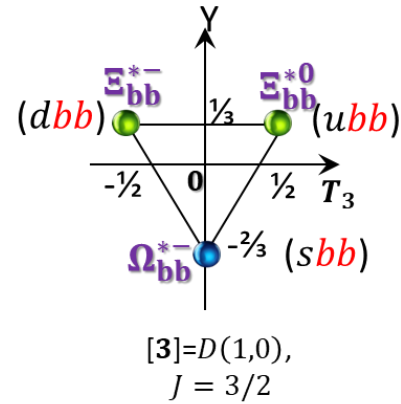
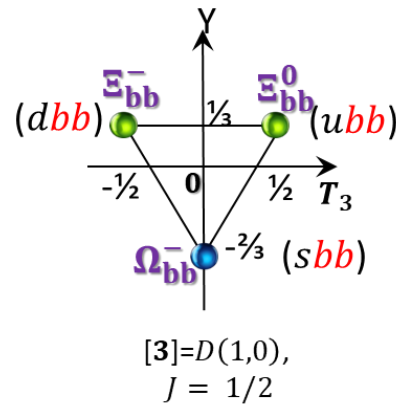
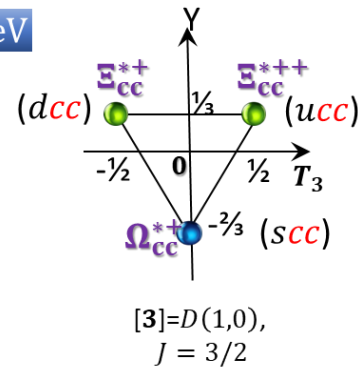
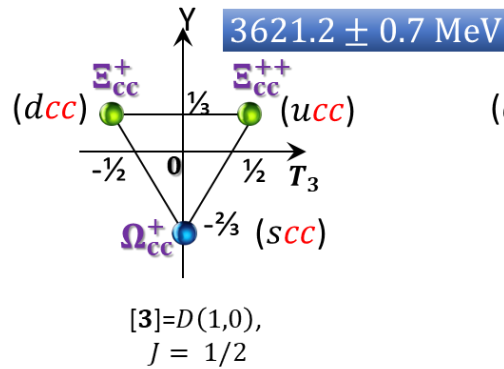
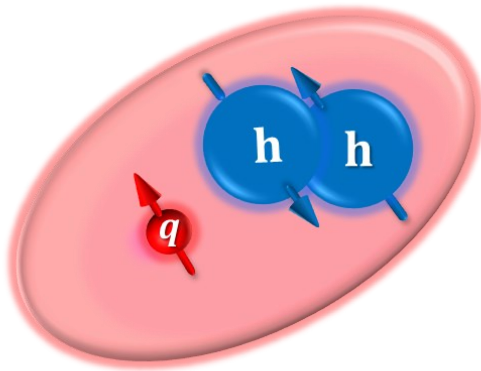
Ξ_{cc}^{++} $I(J^P) = ?(??)$

Ξ_{cc}^{++} MASS

3621.2 ± 0.7 MeV

Ξ_{cc}^{++} MEAN LIFE

$(2.56 \pm 0.27) \times 10^{-13}$ s

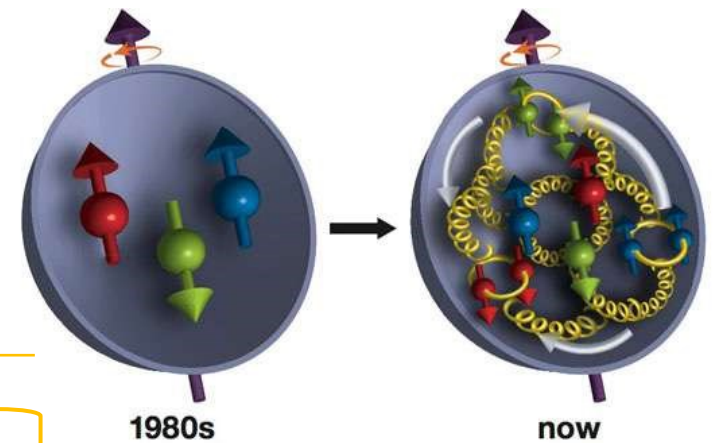


- **Theoretical framework: Chiral soliton model**

Large N_c arguments allows us to consider a classical pion mean field (Witten): Relativistic Mean Field Approximation



Fig: <https://phys.org/news/2017-03-proton.html>



1980s

now

sea quarks
(pion mean fields)

The presence N_c valence quarks creates the pion mean fields and valence quarks are self-consistently bound by it in the large N_c limit. One can put the real-world value 3 into N_c at the end of the calculation.

- **Theoretical framework: Chiral soliton model**



: Effective and relativistic low energy theory

: **Large N_c limit : meson fields**
 → **Soliton (No quark degree of freedom)**

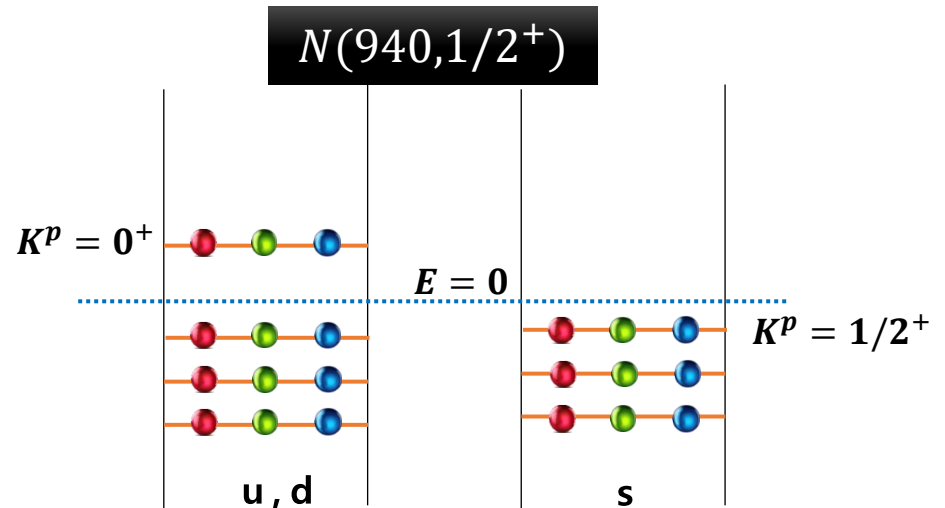
: Quantizing SU(3) meson fields rotated in flavor and spin space
 → Collective Hamiltonian, model baryon states

Hedgehog Ansatz:

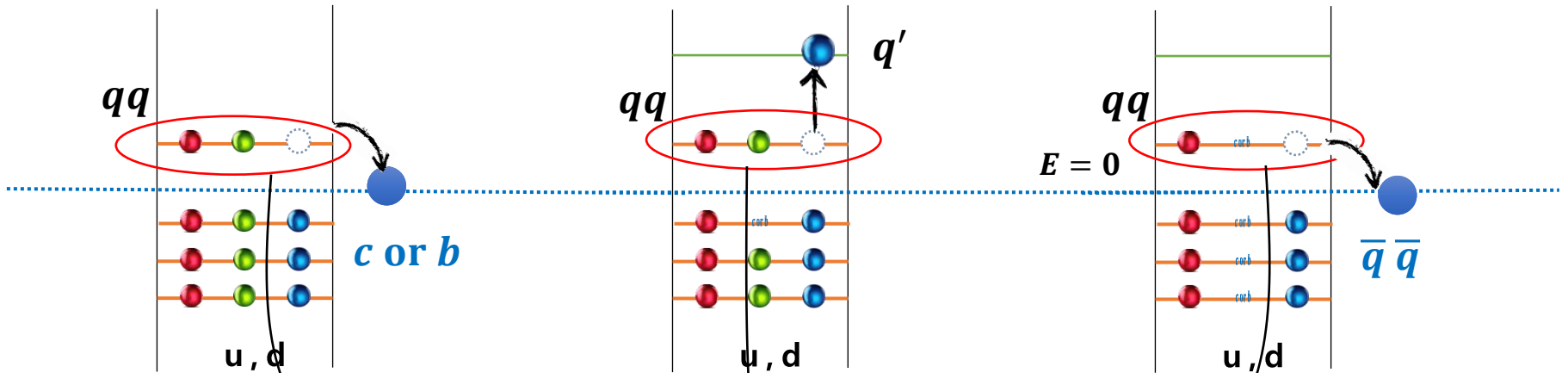
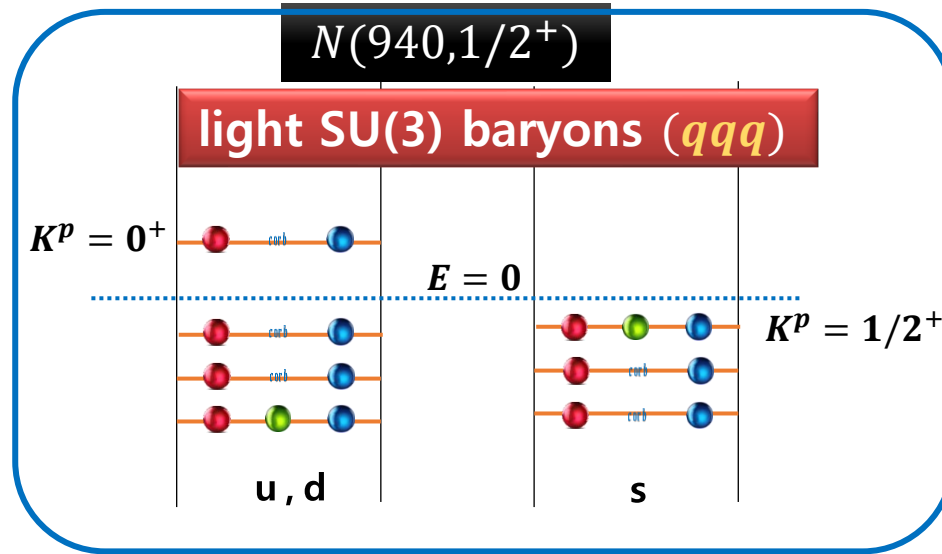
$$U_0 = \begin{bmatrix} e^{in \cdot \tau P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

SU(2) Witten imbedding

into SU(3): SU(2) X U(1)



- Motivation



heavy baryons (qqQ)

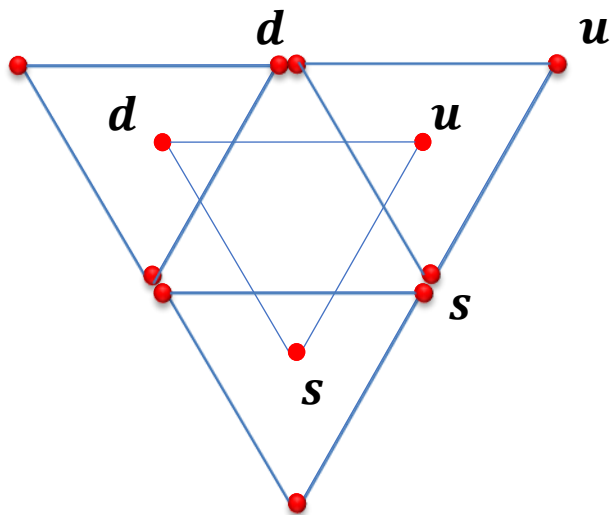
excited baryons (qqq')

tetraquarks ($qq\bar{q}\bar{q}$)

Colored mean field (soliton) consisting of $N_c - 1$ valence quarks



$$3 \otimes 3 = \bar{3} \oplus 6$$



$$[3] \otimes [3]$$

||

$(udc) \Lambda_c^+$

$(ddc) \quad (udc) \quad (uuc)$

Σ_c^{*0}

Σ_c^{*+}

Σ_c^{*++}

\oplus

(dsc)

Ξ_c^0

(usc)

Ξ_c^+

$(dsc) \Xi_c^{\prime 0} \quad \Xi_c^{\prime +} (usc)$

Ξ_c^{*0}

Ξ_c^{*+}

\oplus

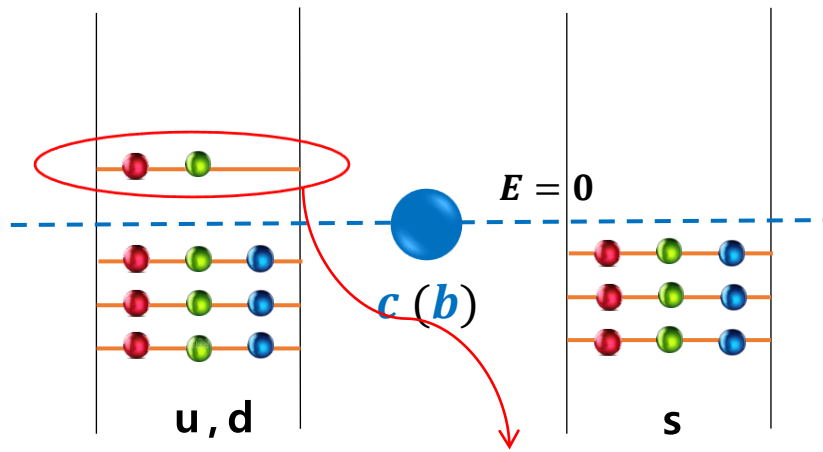
$\Omega_c^0 (ssc)$

Ω_c^{*0}

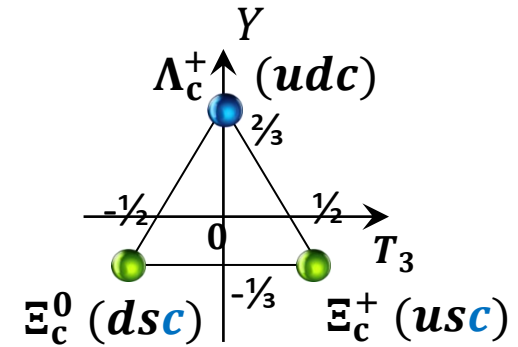
$[\bar{3}]$

$[6_{J=1/2}]$

$[6_{J=3/2}]$



Mean meson field : $N_c - 1$ valence quarks



For $S_z = +1/2$

$$\left\{ \begin{array}{l} \Psi_{B_Q}(\mathcal{R}_0^{1/2}) = \boxed{\chi_\uparrow} \psi_{B(-\frac{2}{3}, 0, 0)}, \quad : \psi \text{ from light baryons} \\ \Psi_{B_Q}(\mathcal{R}_1^{1/2}) = -\sqrt{\frac{1}{3}} \boxed{\chi_\uparrow} \psi_{B(-\frac{2}{3}, 1, 0)} + \sqrt{\frac{2}{3}} \boxed{\chi_\downarrow} \psi_{B(-\frac{2}{3}, 1, 1)} \\ \Psi_{B_Q}(\mathcal{R}_1^{3/2}) = \sqrt{\frac{2}{3}} \boxed{\chi_\uparrow} \psi_{B(-\frac{2}{3}, 1, 0)} + \sqrt{\frac{1}{3}} \boxed{\chi_\downarrow} \psi_{B(-\frac{2}{3}, 1, 1)} \end{array} \right.$$

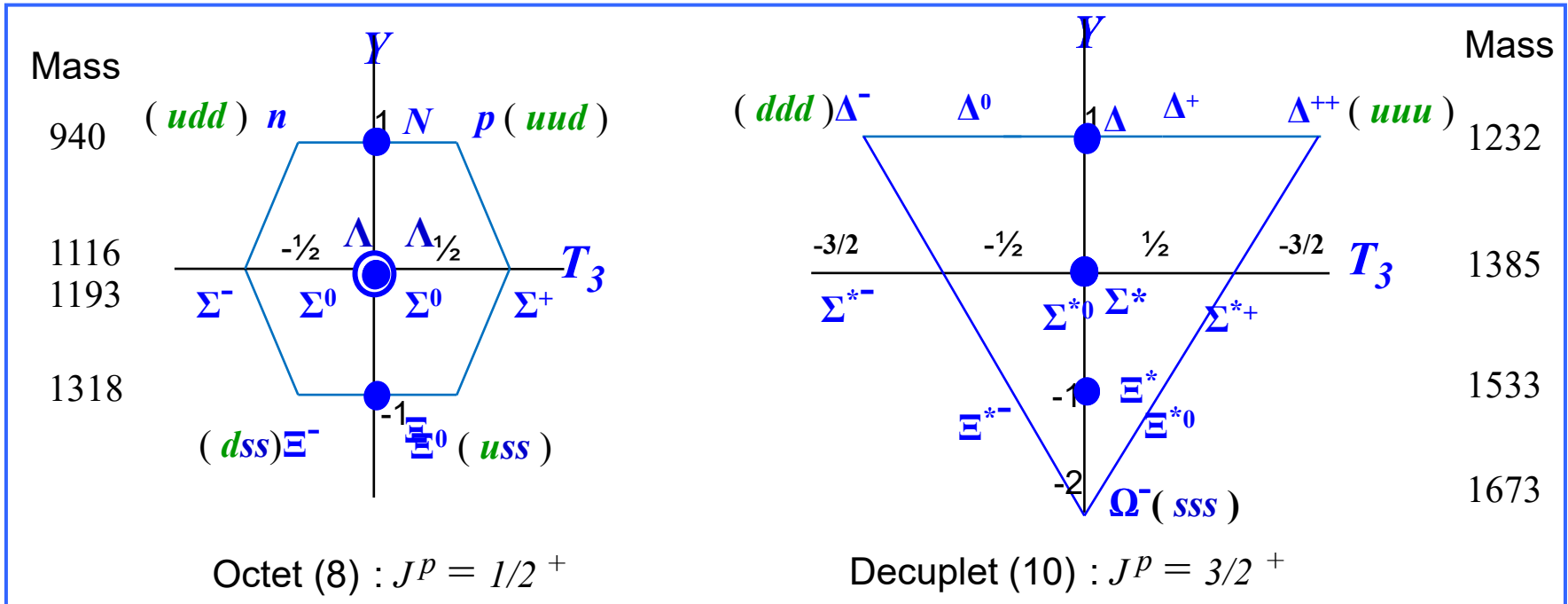
$$\bar{\Psi} \gamma^\mu \frac{\lambda^a}{2} \Psi A_\mu^a \sim \Psi^\dagger \frac{\lambda^a}{2} \Psi A_0^a - \cancel{\frac{1}{m_Q}} \Psi^\dagger \vec{\sigma} \frac{\lambda^a}{2} \Psi \cdot (\vec{\nabla} \times A^a)$$

Heavy quark spin-flip is not taken into account : Heavy quark (spin) symmetry

- Theoretical framework: Chiral soliton model (Collective hamiltonian)**

$$H_{\text{sym}} = M_{\text{cl}} + \frac{1}{2} \left(\frac{1}{I_1} - \frac{1}{I_2} \right) \sum_{i=1}^3 R_i R_i + \frac{1}{2} \frac{1}{I_2} \sum_{i=1}^8 R_i R_i - \frac{3}{8I_2}$$

$$H_{\text{sb}} = (m_s - \hat{m}) \left(\alpha D_{88}^{(8)}(\mathcal{R}) + \beta \hat{Y} + \frac{1}{\sqrt{3}} \gamma \sum_{i=1}^3 D_{8i}^{(8)}(\mathcal{R}) \hat{J}_i \right) \\ + (m_d - m_u) \left(\frac{\sqrt{3}}{2} \alpha D_{38}^{(8)}(\mathcal{R}) + \beta \hat{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^3 D_{3i}^{(8)}(\mathcal{R}) \hat{J}_i \right)$$

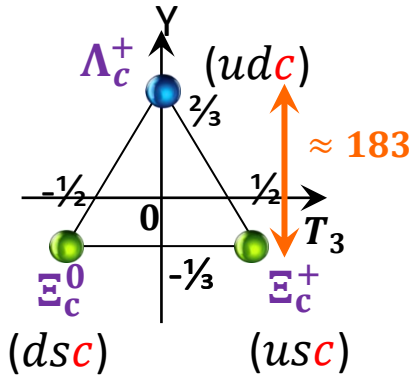


SU(3) flavor symmetry breaking

+ Isospin symmetry breaking

Recent results for heavy baryons in a chiral soliton model

➤ Mass splittings from SU(3) flavor symmetry breakings



$$[\bar{3}] = D(0,1),$$

$$J = 1/2$$

$$H_{\text{sb}}^{\text{SU}(3)} = (m_s - \hat{m}) \left(\alpha D_{88}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} \sum_{i=1}^3 D_{8i}^{(8)} \hat{J}_i \right)$$

where

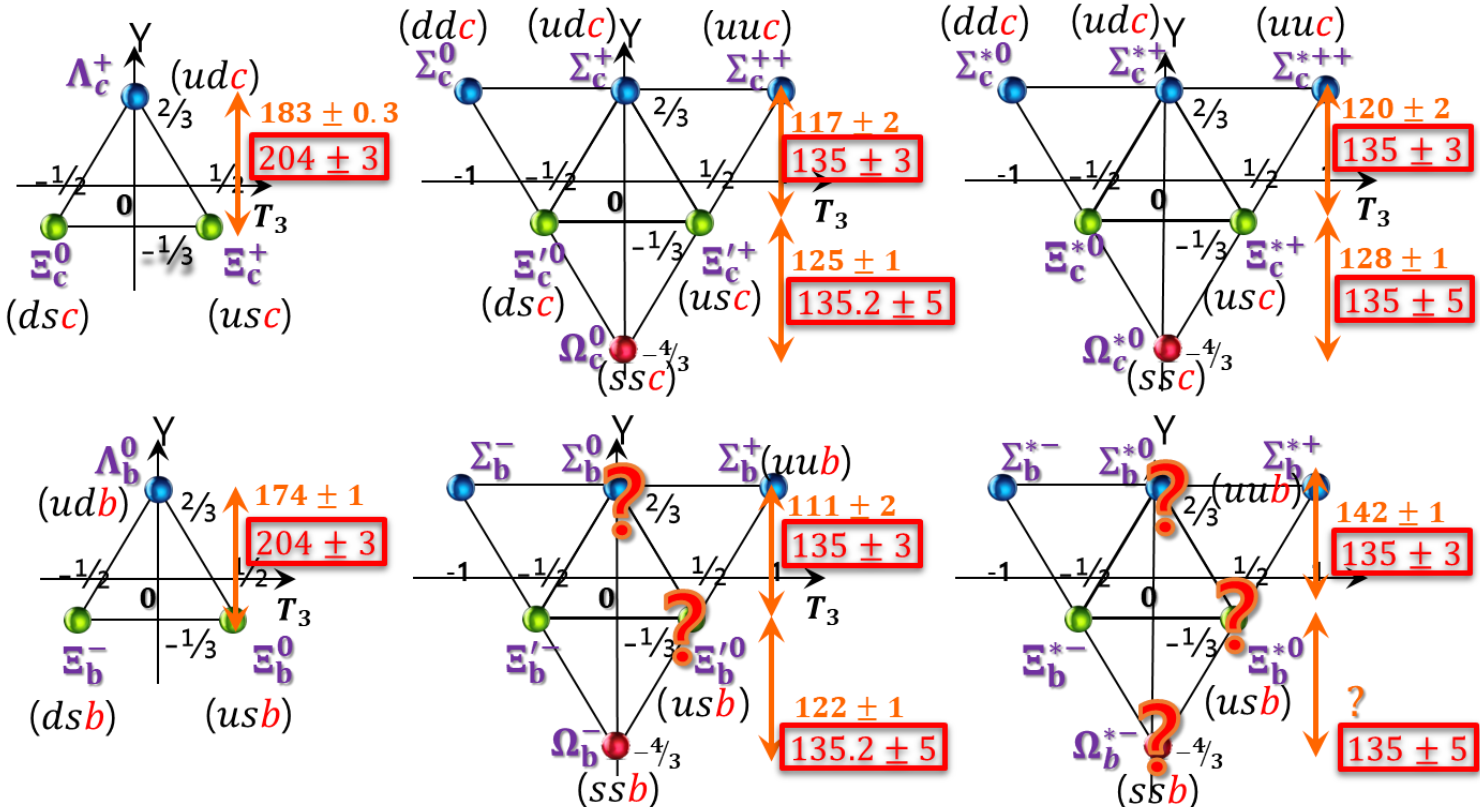
$$\alpha = -\frac{2}{3} \sigma - \beta Y', \quad \Rightarrow \quad \tilde{\alpha} = \frac{N_c - 1}{N_c} \left(-\frac{2}{3} \sigma - \beta Y' \right),$$

$$\beta = -\frac{K_2}{I_2}, \quad \Rightarrow \quad \beta = -\frac{K_2}{I_2},$$

$$\gamma = \frac{2K_1}{I_1} + 2\beta. \quad \Rightarrow \quad \gamma = \frac{2K_1}{I_1} + 2\beta.$$

For $a, b = 4, 5, 6, 7$

$$K_2 = K_{ab}^{\text{val}} = \frac{N_c}{2} \sum_{n \neq \text{val}} \frac{\langle n | \lambda_a | \text{val} \rangle \langle \text{val} | \lambda_b \gamma_0 | n \rangle}{E_n - E_{\text{val}}}$$



This results are from the soliton for the light baryons with a factor $(N_c - 1)/N_c$

\mathcal{R}_J^Q	B_c	Mass	Experiment [17]	Deviation ξ_c
$\bar{3}_c$	Λ_c	2272.5 ± 2.3	2286.5 ± 0.1	-0.006
	Ξ_c	2476.3 ± 1.2	2469.4 ± 0.3	0.003
	Σ_c	2445.3 ± 2.5	2453.5 ± 0.1	-0.003
6_c	Ξ'_c	2580.5 ± 1.6	2576.8 ± 2.1	0.001
	Ω_c	2715.7 ± 4.5	2695.2 ± 1.7	0.008
	Σ_c^*	2513.4 ± 2.3	2518.1 ± 0.8	-0.002
$6_{3/2}^c$	Ξ_c^*	2648.6 ± 1.3	2645.9 ± 0.4	0.001
	Ω_c^*	2783.8 ± 4.5	2765.9 ± 2.0	0.006

\mathcal{R}_J^Q	B_b	Mass	Experiment [17]	Deviation ξ_b
$\bar{3}_b$	Λ_b	5599.3 ± 2.4	5619.5 ± 0.2	-0.004
	Ξ_b	5803.1 ± 1.2	5793.1 ± 0.7	0.002
	Σ_b	5804.3 ± 2.4	5813.4 ± 1.3	-0.002
6_b	Ξ'_b	5939.5 ± 1.5	5935.0 ± 0.05	0.001
	Ω_b	6074.7 ± 4.5	6048.0 ± 1.9	0.004
	Σ_b^*	5824.6 ± 2.3	5833.6 ± 1.3	-0.002
$6_{3/2}^b$	Ξ_b^*	5959.8 ± 1.2	5955.3 ± 0.1	0.001
	Ω_b^*	6095.0 ± 4.4	-	-

Recent results for heavy baryons in a chiral soliton model

➤ Strong decay widths of heavy baryons

$$\hat{g}_1^{(0)} = a_1 D_{\varphi 3}^{(8)} + a_2 d_{3bc} D_{\varphi b}^{(8)} \hat{J}_c + \frac{a_3}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{J}_3,$$

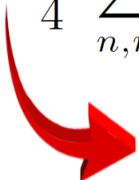
$$= \left[M_3 - \frac{2iQ_{12}}{I_1} \right] D_{X3}^{(8)} + \left[-\frac{4M_{44}}{I_2} \right] d_{pq3} D_{Xp}^{(8)} \hat{J}_q + \left[-\frac{2M_{83}}{I_1} \right] \frac{1}{\sqrt{3}} D_{X8}^{(8)} \hat{J}_3 + \dots$$

where

$$M_{3, \text{val}} = N_c \langle v | \gamma_0 \gamma_3 \gamma_5 \lambda_3 | v \rangle,$$

$$Q_{bc, \text{val}} = \frac{N_c}{2} \sum_n \frac{\langle n | \sigma_3 \lambda_b | v \rangle \langle v | \lambda_c | n \rangle}{E_n - E_v} \text{sign} E_n,$$

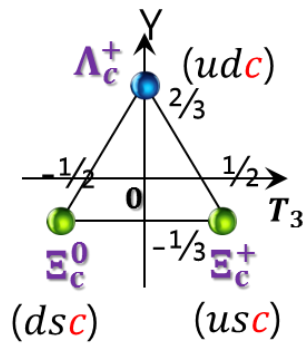
$$M_{bc} = \frac{N_c}{4} \sum_{n,m} \langle n | \sigma_3 \lambda_b | m \rangle \langle m | \lambda_c | n \rangle \frac{1}{2} \frac{\text{sign}(E_n - \mu) - \text{sign}(E_m - \mu)}{E_n - E_m}$$

 **$N_c - 1$**

For heavy baryons

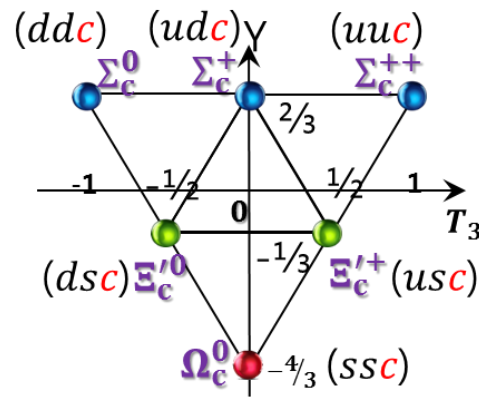
$$M_{3, \text{val}} = \frac{N_c - 1}{N_c} \boxed{N_c \langle v | \gamma_0 \gamma_3 \gamma_5 \lambda_3 | v \rangle} \quad \text{from octet baryons}$$

➤ Strong decay widths of charmed baryons



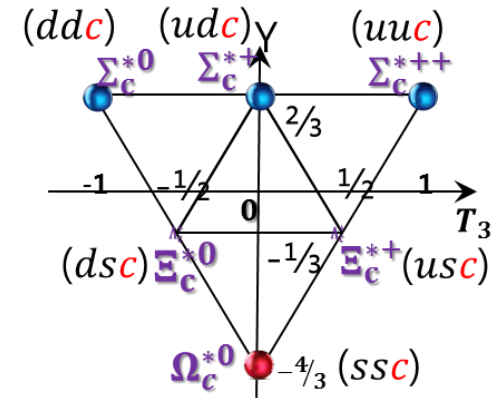
$$[\bar{3}] = D(0,1),$$

$$J = 1/2$$



$$[6] = D(2,0),$$

$$J = 1/2$$

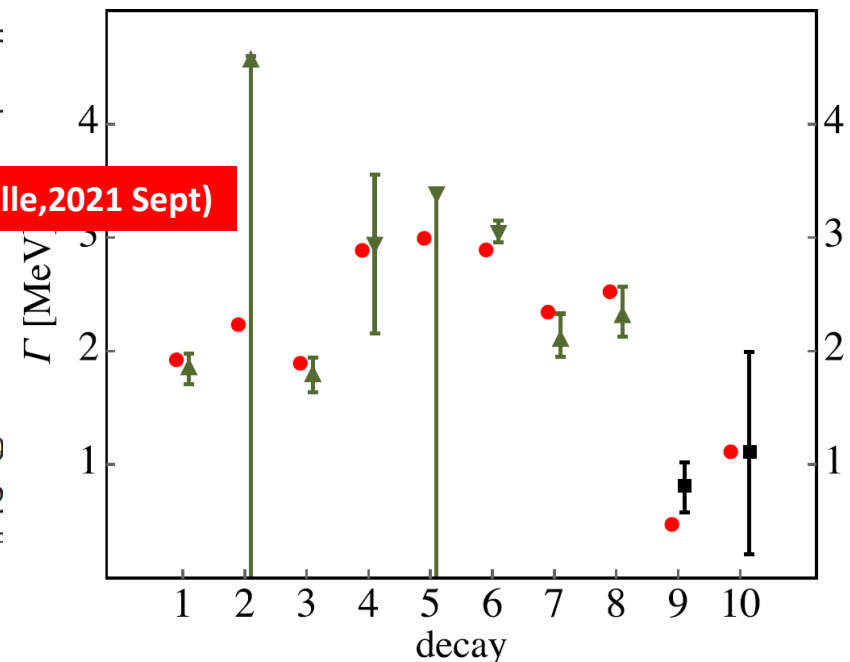


$$[6] = D(2,0),$$

$$J = 3/2$$

#	Decay	This work	Exp.
1	$\Sigma_c^{*+}(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	1.93	$1.89^{+0.09}_{-0.18}$
2	$\Sigma_c^+(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	2.24	2.3 ± 0.3 (Belle, 2021 Sept)
3	$\Sigma_c^0(\mathbf{6}_1, 1/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	1.90	$1.83^{+0.11}_{-0.19}$
4	$\Sigma_c^{*+}(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^+$	14.47	$14.78^{+0.30}_{-0.19}$
5	$\Sigma_c^+(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^0$	15.02	<17
6	$\Sigma_c^0(\mathbf{6}_1, 3/2) \rightarrow \Lambda_c^+(\bar{\mathbf{3}}_0, 1/2) + \pi^-$	14.49	$15.3^{+0.4}_{-0.5}$
7	$\Xi_c^+(\mathbf{6}_1, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.35	2.14 ± 0.19
8	$\Xi_c^0(\mathbf{6}_1, 3/2) \rightarrow \Xi_c(\bar{\mathbf{3}}_0, 1/2) + \pi$	2.53	2.35 ± 0.22

Yang, et al., Phys.Rev.D 96 (2017) 094021



Light quarks govern their structure of singly heavy baryons.

**Measurement of the masses and widths of the $\Sigma_c(2455)^+$
and $\Sigma_c(2520)^+$ baryons**

(Belle Collaboration)

Measurements of the masses of all members of the two isotriplets allow tests of models of isospin mass splittings. In the model of Yang and Kim [4], for instance, the mass splittings from the following four sources add: the electromagnetic corrections due to the light quarks, the differences of the masses of the u and d quarks, the hyperfine interactions between the light quarks, and the Coulomb interactions between the soliton and charm quark. Most

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Isospin mass differences of singly heavy baryons

Ghil-Seok Yang^a, Hyun-Chul Kim^{b,c,*}

^a Department of Physics, Soongsil University, Seoul 06978, Republic of Korea

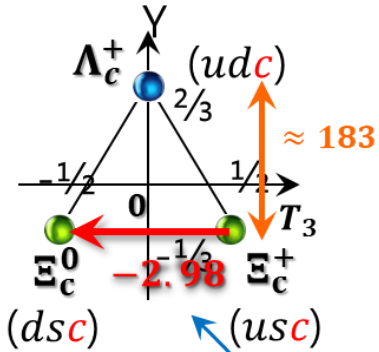
^b Department of Physics, Inha University, Incheon 22212, Republic of Korea

^c School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, Republic of Korea



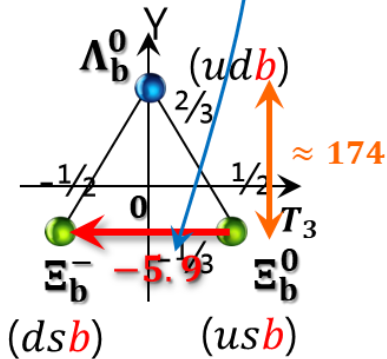
Motivation for isospin mass differences of singly heavy baryons

Why positive ?

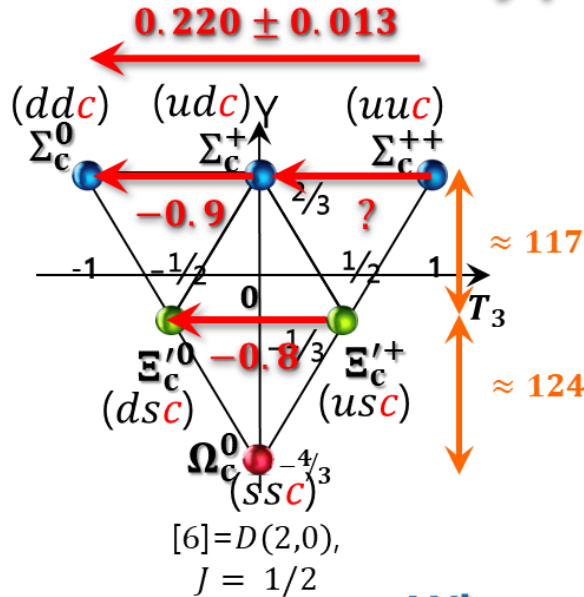


$[\bar{3}] = D(0,1),$
 $J = 1/2$

twice ?

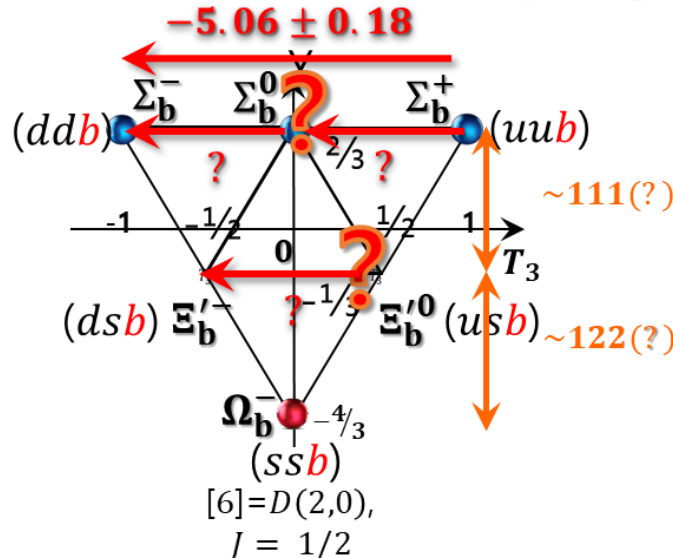


$[\bar{3}] = D(0,1),$
 $J = 1/2$

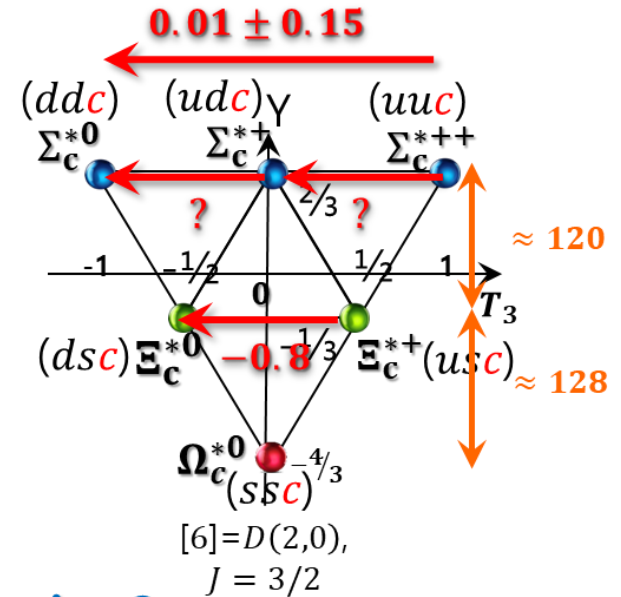


$[6] = D(2,0),$
 $J = 1/2$

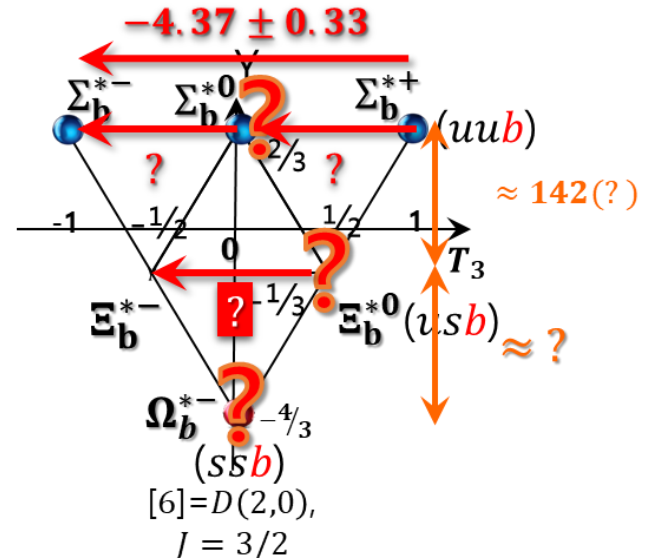
Why negative ?



$[6] = D(2,0),$
 $J = 1/2$

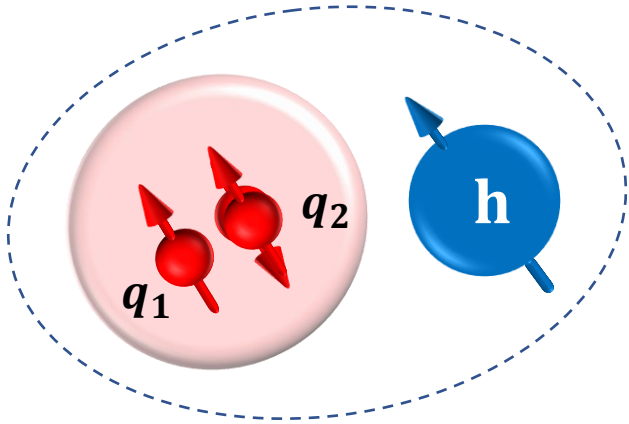


$[6] = D(2,0),$
 $J = 3/2$



$[6] = D(2,0),$
 $J = 3/2$

$$\Delta M_{B_h}^{\text{total}} = \Delta M_{\text{sol}}^{\text{EM}} + \Delta M_{\text{sb}}^{\text{iso}} + \Delta M_{\text{hf}} + \Delta M_{\text{sol-h}}^{\text{Coul}}$$



- EM self-energy for a soliton

$$\Delta M_{\text{sol}}^{\text{EM}} = \text{Diagram: A fermion line with momentum p enters from the left, emits a photon with momentum k, and continues with momentum p-k. The photon line is a wavy line. The fermion line ends with momentum p on the right. The diagram is labeled B(p) at both ends.$$

G. S. Yang, et al., Phys. Lett. B 695, 214 (2011)

- Hadronic contributions from $m_d - m_u$

$$\Delta M_{\text{sb}}^{\text{iso}} = H_{\text{sb}}^{\text{iso}} = (m_d - m_u) \left(\frac{\sqrt{3}}{2} \alpha D_{38}^{(8)} + \beta \hat{T}_3 + \frac{\gamma}{2} \sum_{i=1}^3 D_{3i}^{(8)} \hat{J}_i \right)$$

with $N_c - 1$

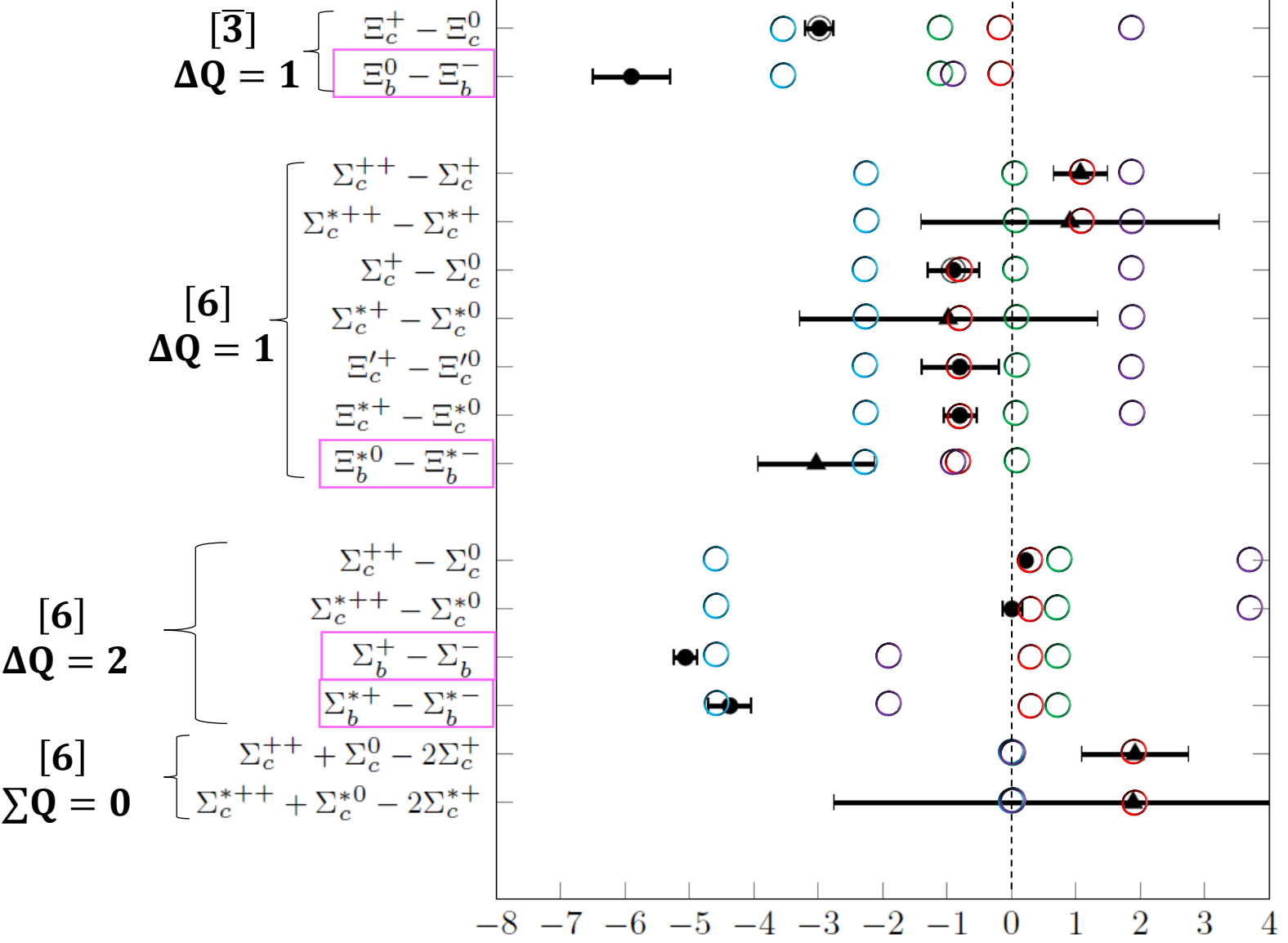
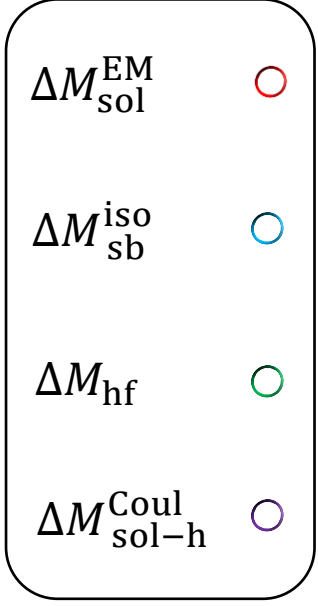
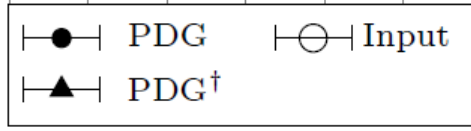
- Strong hyperfine interaction for the light quarks inside a heavy baryon

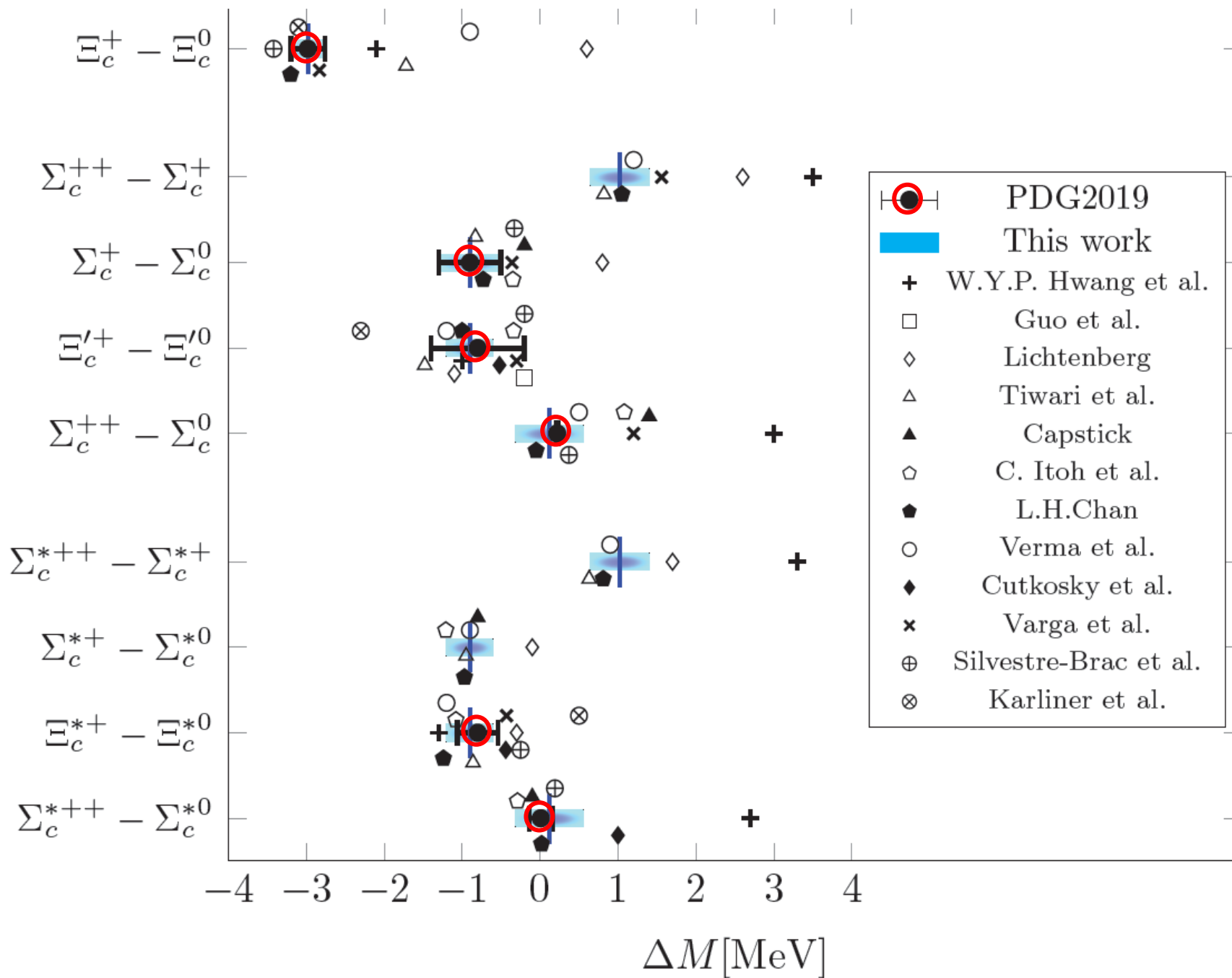
$$\Delta M_{\text{hf}} = \delta^{\text{hf}} \mathbf{S}_1 \cdot \mathbf{S}_2$$

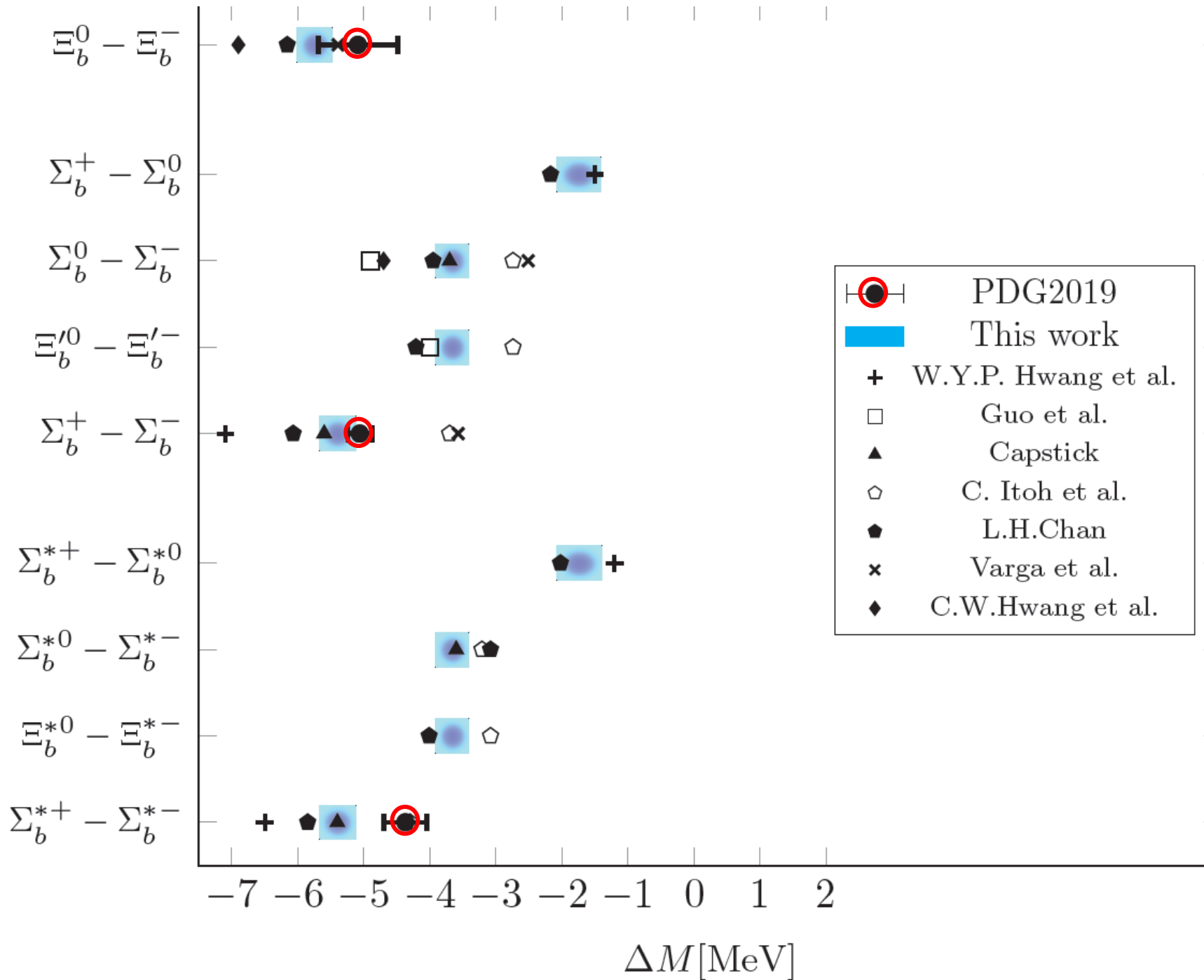
- Coulomb interaction between a soliton and a heavy quark

$$\Delta M_{\text{sol-h}}^{\text{Coul}} = \alpha_{\text{sol-h}} \hat{Q}_{\text{sol}} \hat{Q}_h$$

$$\Delta M_{B_h}^{\text{total}} = \Delta M_{\text{sol}}^{\text{EM}} + \Delta M_{sb}^{\text{iso}} + \Delta M_{hf} + \Delta M_{\text{sol-h}}^{\text{Coul}}$$

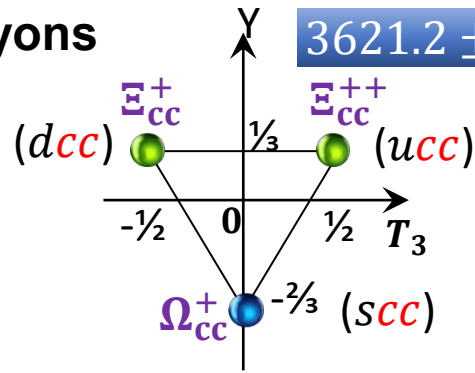




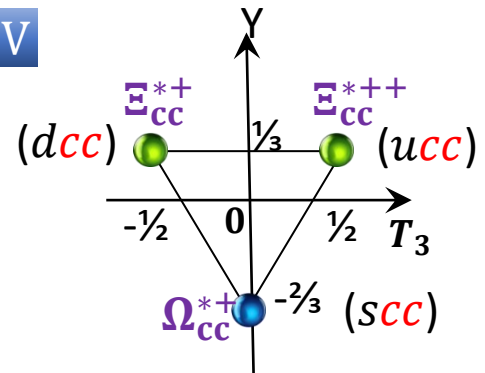


➤ Doubly heavy baryons

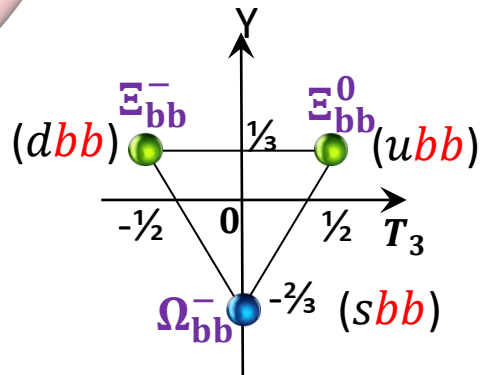
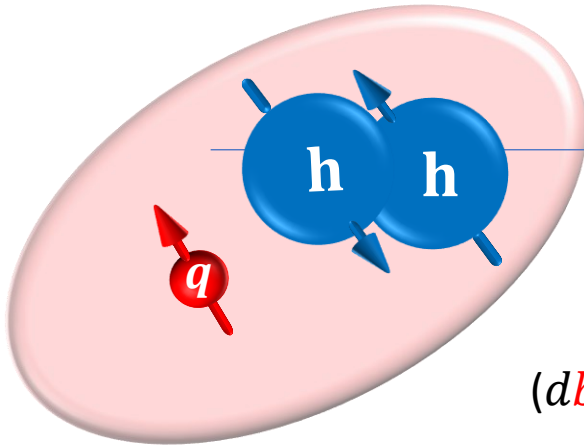
$3621.2 \pm 0.7 \text{ MeV}$



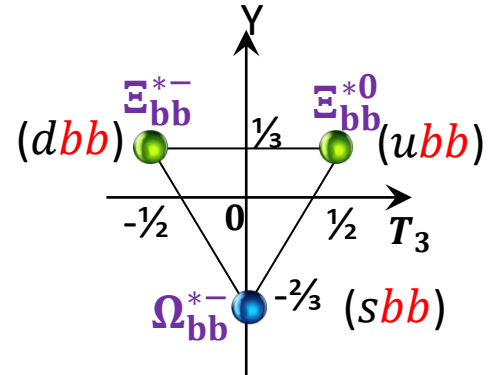
$[3]=D(1,0),$
 $J = 1/2$



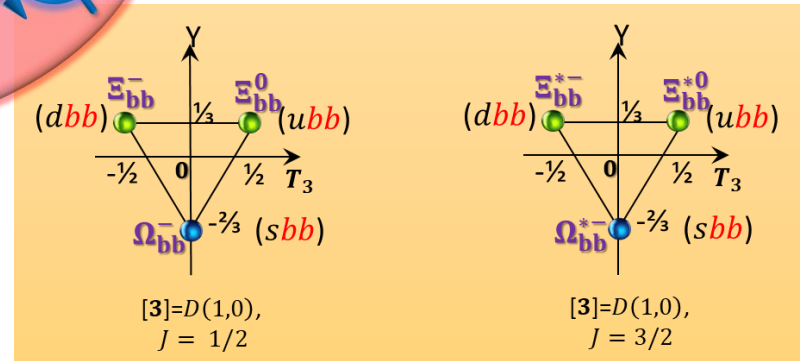
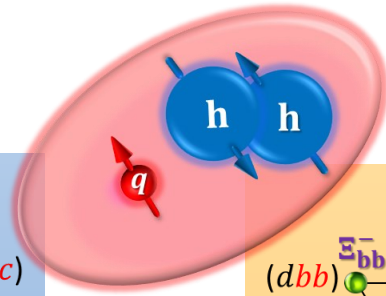
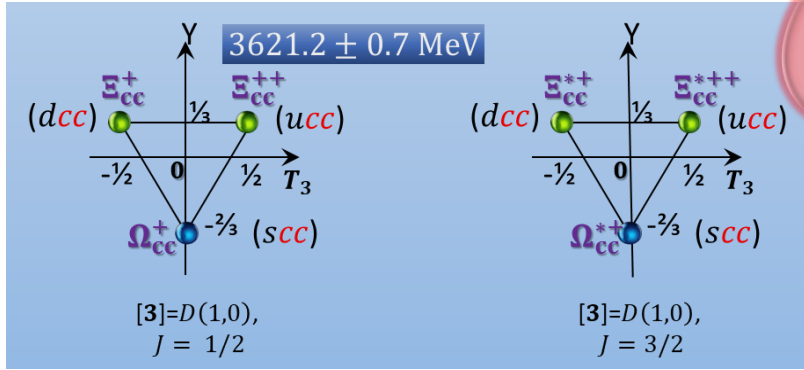
$[3]=D(1,0),$
 $J = 3/2$



$[3]=D(1,0),$
 $J = 1/2$

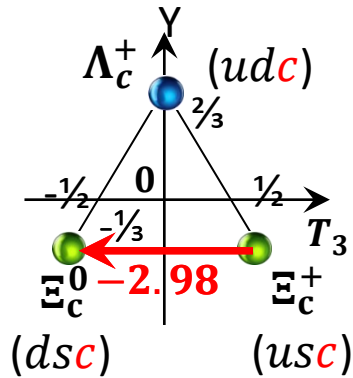


$[3]=D(1,0),$
 $J = 3/2$



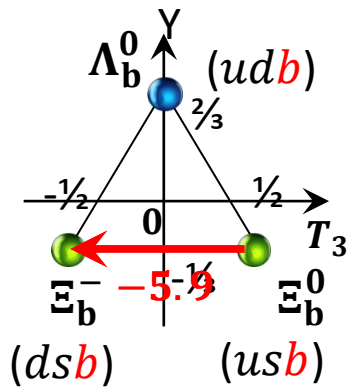
\mathcal{R}_J	ΔB_{hh}	ΔM_{sol}^{EM}	ΔM_{sb}^{iso}	ΔM_{hf}	ΔM_{sol-hh}^{Coul}
$\mathbf{3}_{1/2}$	$\Xi_{cc}^{++}(ucc) - \Xi_{cc}^+(dcc)$	$3c^{(8)}/8$	δ_3^{iso}	.	$4\alpha_{sol-h}/3$ 4.48 ± 0.91
	$\Xi_{bb}^0(ubb) - \Xi_{bb}^-(d bb)$				$-2\alpha_{sol-h}/3$ -2.24 ± 0.45
$\mathbf{3}_{3/2}$	$\Xi_{cc}^{*++}(ucc) - \Xi_{cc}^{*+}(dcc)$	-0.06 ± 0.09	-2.20 ± 0.01		$4\alpha_{sol-h}/3$ 4.48 ± 0.91
	$\Xi_{bb}^{*0}(ubb) - \Xi_{bb}^{*-}(d bb)$				$-2\alpha_{sol-h}/3$ -2.24 ± 0.45

where $\delta_3^{iso} = \frac{3}{16} (m_d - m_u) \left(\bar{\alpha} + \frac{16}{3}\beta - \frac{3}{2}\gamma \right)$ in which $\bar{\alpha} = \left(\frac{N_c - 2}{N_c} \right)$.



$$[\bar{3}] = D(0,1),$$

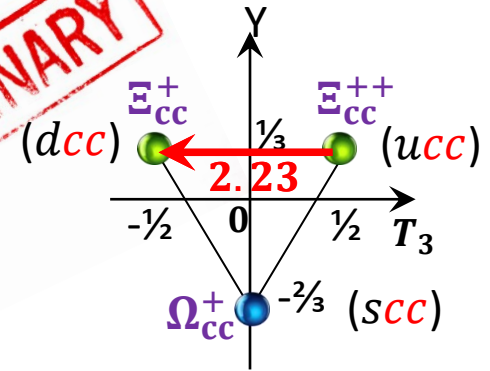
$$J = 1/2$$



$$[\bar{3}] = D(0,1),$$

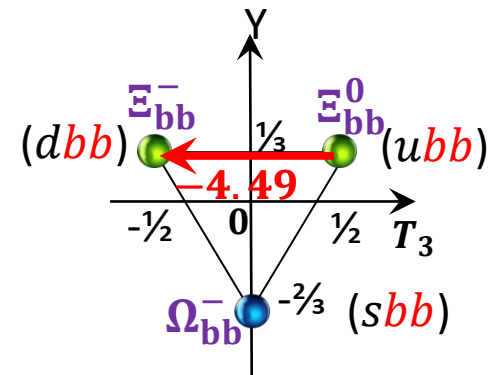
$$J = 1/2$$

PRELIMINARY



$$[3] = D(1,0),$$

$$J = 1/2$$



$$[3] = D(1,0),$$

$$J = 1/2$$

➤ **Excited SU(3) baryons**

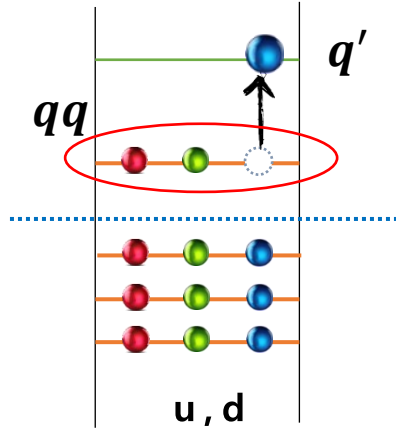
Full expression with the grand spin K is

$$M = M_{\text{sol}} + \Delta\mathcal{E}(J_i \rightarrow J_f) + \mathcal{E}_K + M_{\text{sb}}^K.$$

$$H_{\text{sb}}^K = \alpha \mathcal{D}_{88}^{(8)}(\mathcal{A}) + \beta Y - \frac{\gamma}{\sqrt{3}} \sum_i \mathcal{D}_{8i}^{(8)}(\mathcal{A}) \tilde{T}_i - \frac{\delta}{\sqrt{3}} \sum_i \mathcal{D}_{8i}^{(8)}(\mathcal{A}) \hat{K}_i,$$

where $K = \tilde{T} + \tilde{J}$ and $\tilde{Y}_K = N_c/3$.

D. Diakonov et al. PhysRev D88 074030, 2013



Excited baryons (qqq')

$$\alpha \rightarrow (N_c - 1)/N_c \alpha \equiv \bar{\alpha},$$

$$\tilde{Y}_K = N_c/3 \rightarrow (N_c - 1)/3,$$

$$I_{1,2} \rightarrow (N_c - 1)/N_c I_{1,2} \equiv \bar{I}_{1,2},$$

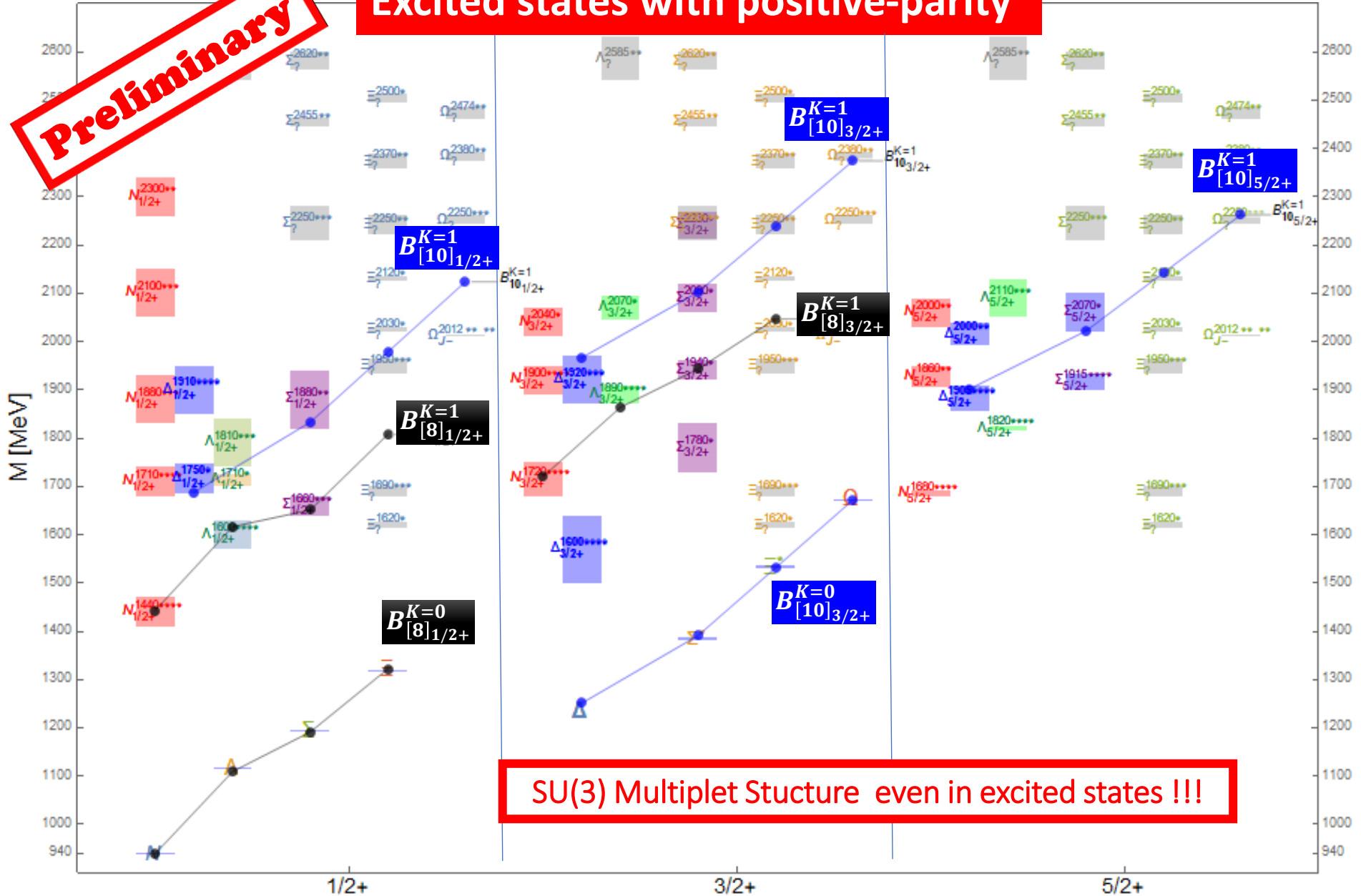
for one-quark (u, d) excited baryons

$$\mathcal{E}_{\mathcal{R}_J}^K = \frac{\mathcal{C}_2(\mathcal{R}) - \tilde{T}(\tilde{T} + 1) - \frac{3}{4}\tilde{Y}_K^2}{2I_2} + \frac{1}{2I_1} \left[\tilde{a}_K J(J + 1) + (1 - \tilde{a}_K)\tilde{T}(\tilde{T} + 1) - \tilde{a}_K(1 - \tilde{a}_K)K(K + 1) \right],$$

$$\Psi_K^{(\mathcal{R})}(\mathcal{A}, \mathcal{S}, \chi) = \sqrt{\frac{\dim(\mathcal{R})(2J + 1)}{2K + 1}} \sum_{\tilde{T}, \tilde{T}_3, \tilde{J}_3} C_{\tilde{T}\tilde{T}_3; J\tilde{J}_3}^{KK_3} \mathcal{D}_{\tilde{Y}\tilde{T}\tilde{T}_3; YTT_3}^{(\mathcal{R})}(\mathcal{A}^+) \mathcal{D}_{\tilde{J}_3; J_3}^J(\mathcal{S}^+) \chi_{K_3},$$

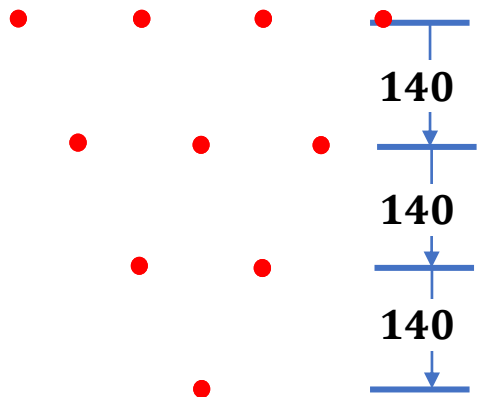
Excited states with positive-parity

Preliminary

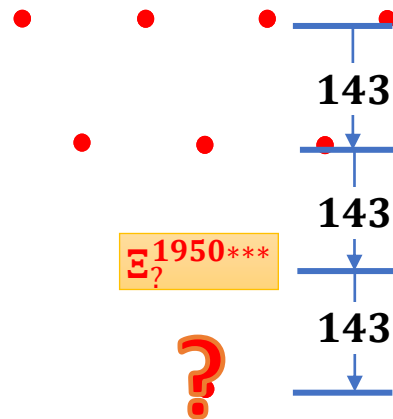


SU(3) Multiplet Structure even in excited states !!!

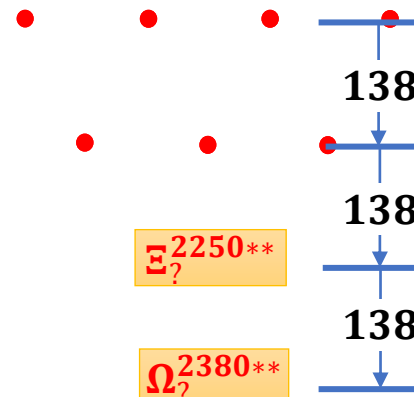
Study for negative-parity excited states are in progress



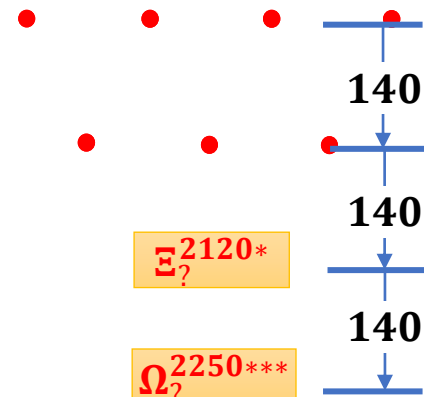
$$J^P(B_{10}^{K=0}) = 1/2^+$$



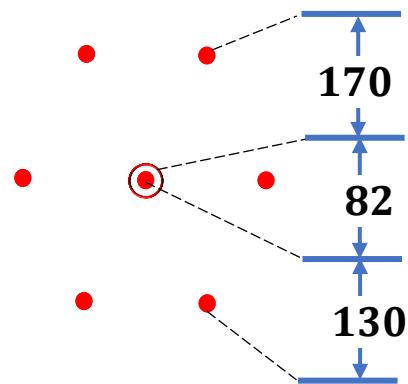
$$J^P(B_{10}^{K=1}) = 1/2^+$$



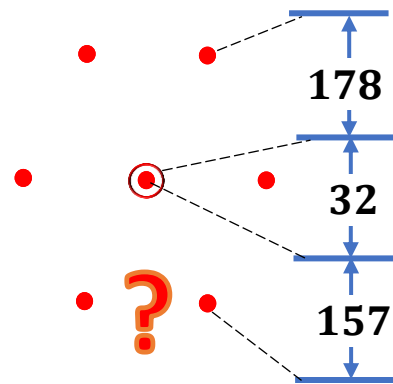
$$J^P(B_{10}^{K=1}) = 3/2^+$$



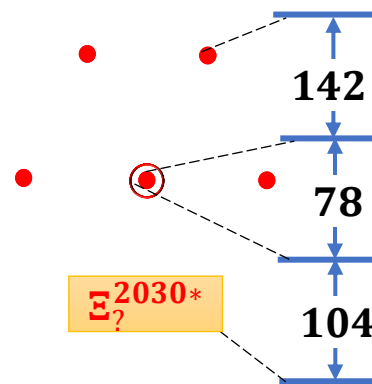
$$J^P(B_{10}^{K=1}) = 5/2^+$$



$$J^P(B_8^{K=0}) = 1/2^+$$



$$J^P(B_8^{K=1}) = 1/2^+$$



$$J^P(B_8^{K=1}) = 3/2^+$$



$$J^P(B_8^{K=1}) = 5/2^+$$

Ground States

Excited States (from u, d sector)

Summary

- Assuming that the valence quarks are bound by the pion mean fields, we can regard the nucleon as a chiral soliton.
- The present pion mean-field approach indeed explains consistently both the SU(3) light baryons and the heavy baryons.
- Dynamical parameters and flavor quantum numbers of the collective operators and wave functions are modified by $N_c - (\# \text{ of heavy quarks})$ mean field for the heavy baryons based on N_c mean field for the light baryons
- We have obtained excellent description of various physical observables (Masses from isospin and SU(3) flavor symmetry breakings, widths of strong and radiative decays, magnetic moments and transitions)
- Contributions from isospin symmetry breaking for a soliton are significant to describe the isospin mass differences of singly and doubly charmed and beauty baryons.
- It is shown that **light quarks govern their structure of heavy baryons.**
- **We can see the multiplet structure even in excited states as like octet, decuplet.**

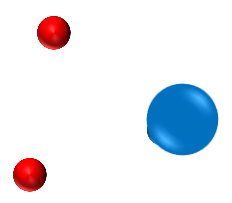
- Strong decay widths of decuplet baryons

Decay modes	$\Gamma_i^{(0)}$	$\Gamma_i^{(\text{total})}$	Γ	$\Gamma(\text{Exp.}) [2]$
$\Delta \rightarrow N\pi$	75.98 ± 1.01	88.58 ± 1.31		116–120
$\Sigma^{*+} \rightarrow \Sigma^0\pi^+$	2.59 ± 0.03	3.22 ± 0.06		
$\Sigma^{*+} \rightarrow \Sigma^+\pi^0$	3.17 ± 0.05	2.62 ± 0.05	36.25 ± 0.42	36.0 ± 0.7
$\Sigma^{*+} \rightarrow \Lambda\pi^+$	29.68 ± 0.26	30.41 ± 0.33		
$\Sigma^{*0} \rightarrow \Sigma^0\pi^0$	0	0		
$\Sigma^{*0} \rightarrow \Sigma^+\pi^-$	3.61 ± 0.11	2.98 ± 0.1	37.21 ± 0.69	36 ± 5
$\Sigma^{*0} \rightarrow \Sigma^-\pi^+$	2.78 ± 0.1	2.30 ± 0.09		
$\Sigma^{*0} \rightarrow \Lambda\pi^0$	31.15 ± 0.47	31.92 ± 0.52		
$\Sigma^{*-} \rightarrow \Sigma^-\pi^0$	3.50 ± 0.06	2.89 ± 0.06		
$\Sigma^{*-} \rightarrow \Sigma^0\pi^-$	3.64 ± 0.06	3.01 ± 0.06	38.18 ± 0.48	39.4 ± 2.1
$\Sigma^{*-} \rightarrow \Lambda\pi^-$	31.50 ± 0.30	32.28 ± 0.37		
$\Xi^{*0} \rightarrow \Xi^0\pi^0$	4.76 ± 0.05	4.33 ± 0.06		
$\Xi^{*0} \rightarrow \Xi^-\pi^+$	7.61 ± 0.08	6.93 ± 0.10	11.26 ± 0.17	9.1 ± 0.5
$\Xi^{*-} \rightarrow \Xi^-\pi^0$	4.76 ± 0.05	4.33 ± 0.06		
$\Xi^{*-} \rightarrow \Xi^0\pi^-$	8.20 ± 0.13	8.68 ± 0.16	13.01 ± 0.21	$9.9^{+1.7}_{-1.9}$

a_1	a_2	a_3	a_4	a_5	a_6
-3.509 ± 0.011	3.437 ± 0.028	0.604 ± 0.030	-1.213 ± 0.068	0.479 ± 0.025	-0.735 ± 0.040

Yang, et al. Rev. C **92** 035206 (2015)

Yang, et al, Phys. Lett. B **785** 434 (2018)



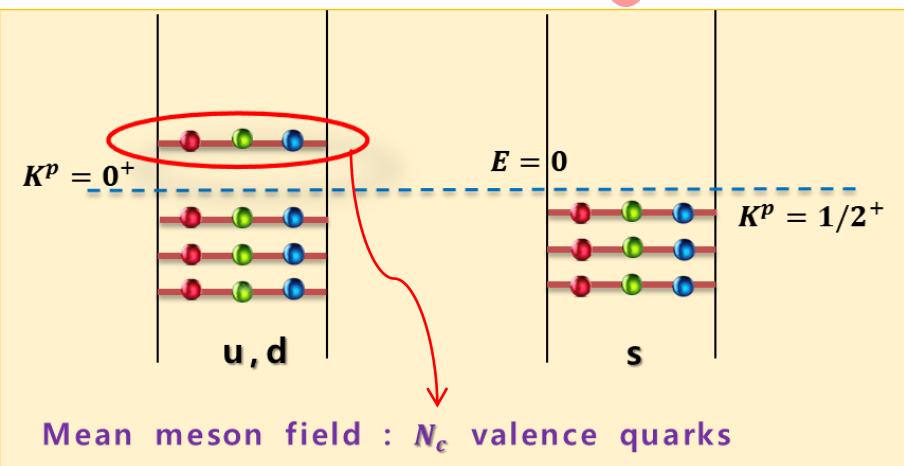
Heavy quarks are too heavy !

$$\Lambda_{QCD} (210 \sim 340 \text{ MeV}) \ll m_c (\sim 1.2 \text{ GeV}) \ll m_b (\sim 4.5 \text{ GeV})$$

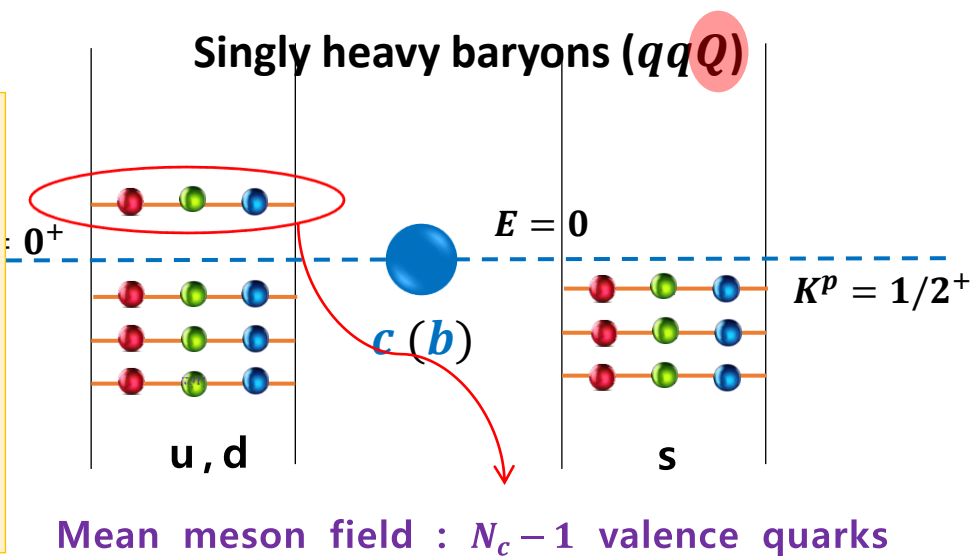


[M. Oka, Nucl.Phys A 914, 447 (2013)]

light baryons (qqq)



Singly heavy baryons (qqQ)



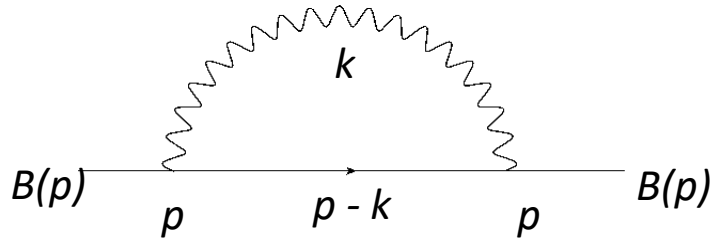
$$\delta^{\text{hf}} = (0.40 \pm 0.06) \text{ MeV}$$

$$\alpha_{\text{sol-h}} = (2.76 \pm 0.28) \text{ MeV}$$

\mathcal{R}_J	B_c	$\Delta M_{\text{sol}}^{\text{EM}}$	$\Delta M_{\text{sb}}^{\text{iso}}$	ΔM_{hf}	$\Delta M_{\text{sol-h}}^{\text{Coul}}$	ΔM^{total}	PDG [56]	PDG [†]
$\bar{3}_{1/2}$	$\Xi_c^+ - \Xi_c^0$	-0.11 ± 0.17	-3.51	-1.20 ± 0.18	1.84 ± 0.19	input	-2.98 ± 0.22	–
	$\Sigma_c^{++} - \Sigma_c^+$	1.10 ± 0.33	-2.33	0.40 ± 0.06	1.84 ± 0.19	1.02 ± 0.38	–	1.07 ± 0.42
$6_{1/2}$	$\Sigma_c^+ - \Sigma_c^0$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	1.84 ± 0.19	input	-0.9 ± 0.4	–
	$\Xi_c'^+ - \Xi_c'^0$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	1.84 ± 0.19	-0.90 ± 0.30	-0.8 ± 0.6	–
	$\Sigma_c^{++} - \Sigma_c^0$	0.29 ± 0.17	-4.66	0.80 ± 0.12	3.68 ± 0.37	0.12 ± 0.43	0.220 ± 0.013	–
	$\Sigma_c^{++} + \Sigma_c^0 - 2\Sigma_c^+$	1.92 ± 0.53	0	0	0	1.92 ± 0.53	–	1.92 ± 0.82
	$\Sigma_c^{*++} - \Sigma_c^{*+}$	1.10 ± 0.33	-2.33	0.40 ± 0.06	1.84 ± 0.19	1.02 ± 0.38	–	0.91 ± 2.31
$6_{3/2}$	$\Sigma_c^{*+} - \Sigma_c^{*0}$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	1.84 ± 0.19	-0.90 ± 0.30	–	-0.98 ± 2.31
	$\Xi_c^{*+} - \Xi_c^{*0}$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	1.84 ± 0.19	-0.90 ± 0.30	-0.80 ± 0.26	–
	$\Sigma_c^{*++} - \Sigma_c^{*0}$	0.29 ± 0.17	-4.66	0.80 ± 0.12	3.68 ± 0.37	0.12 ± 0.43	0.01 ± 0.15	–
	$\Sigma_c^{*++} + \Sigma_c^{*0} - 2\Sigma_c^{*+}$	1.92 ± 0.53	0	0	0	1.92 ± 0.53	–	1.89 ± 4.64
\mathcal{R}_J	B_b	$\Delta M_{\text{sol}}^{\text{EM}}$	$\Delta M_{\text{sb}}^{\text{iso}}$	ΔM_{hf}	$\Delta M_{\text{sol-h}}^{\text{Coul}}$	ΔM^{total}	PDG [56]	PDG [†]
$\bar{3}_{1/2}$	$\Xi_b^0 - \Xi_b^-$	-0.11 ± 0.17	-3.51	-1.20 ± 0.18	-0.92 ± 0.09	-5.74 ± 0.27	-5.9 ± 0.6	–
	$\Sigma_b^+ - \Sigma_b^0$	1.10 ± 0.33	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-1.74 ± 0.34	–	–
$6_{1/2}$	$\Sigma_b^0 - \Sigma_b^-$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-3.66 ± 0.25	–	–
	$\Xi_b'^0 - \Xi_b'^-$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-3.66 ± 0.25	–	–
	$\Sigma_b^+ - \Sigma_b^-$	0.29 ± 0.17	-4.66	0.80 ± 0.12	-1.84 ± 0.19	-5.40 ± 0.28	-5.06 ± 0.18	–
	$\Sigma_b^+ + \Sigma_b^- - 2\Sigma_b^0$	1.92 ± 0.53	0	0	0	1.92 ± 0.53	–	–
$6_{3/2}$	$\Sigma_b^{*+} - \Sigma_b^{*0}$	1.10 ± 0.33	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-1.74 ± 0.34	–	–
	$\Sigma_b^{*0} - \Sigma_b^{*-}$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-3.66 ± 0.25	–	–
	$\Xi_b^{*0} - \Xi_b^{*-}$	-0.81 ± 0.22	-2.33	0.40 ± 0.06	-0.92 ± 0.09	-3.66 ± 0.25	–	-3.03 ± 0.91
	$\Sigma_b^{*+} - \Sigma_b^{*-}$	0.29 ± 0.17	-4.66	0.80 ± 0.12	-1.84 ± 0.19	-5.40 ± 0.28	-4.37 ± 0.33	–
	$\Sigma_b^{*+} + \Sigma_b^{*-} - 2\Sigma_b^{*0}$	1.92 ± 0.53	0	0	0	1.92 ± 0.53	–	–

➤ Electromagnetic self-energy

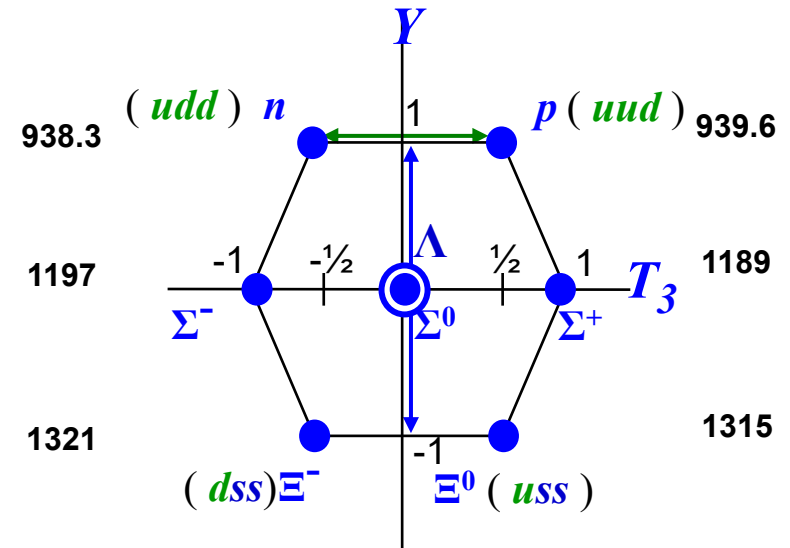
• EM mass corrections



Electromagnetic (EM) self-energy

EM [MeV]	Exp.
$(p - n)_{EM}$	0.76 ± 0.30
$(\Sigma^+ - \Sigma^-)_{EM}$	-0.17 ± 0.30
$(\Xi^0 - \Xi^-)_{EM}$	-0.86 ± 0.30

Gasser, Leutwyler, *Phys.Rep* **87**, 77 "Quark Masses"



$$\Delta M_B = M_{B_1} - M_{B_2} = (\Delta M_B)_H + (\Delta M_B)_{EM}$$

$$(p - n)_{\text{exp}} \sim -1.293 \text{ MeV}$$

$$(p - n)_{EM} \sim 0.76 \text{ MeV}$$

➤ Electromagnetic self-energy

In the ChSM, $(\Delta M_B)_{\text{EM}} = \langle B | J_\mu(x) J^\mu(0) | B \rangle = \langle B | \mathcal{O}_{\text{EM}} | B \rangle$

$$\begin{aligned} \mathcal{O}_{\text{EM}} &= -\frac{e^2}{2} \int d^3x d^3y D_\gamma(x, \mathbf{y}) \int \frac{d\omega}{2\pi} \text{tr} \left\langle \mathbf{x} \left| \frac{1}{\omega + iH} \gamma_\mu \lambda^a \right| \mathbf{y} \right\rangle \left\langle \mathbf{y} \left| \frac{1}{\omega + iH} \gamma_\mu \lambda^b \right| \mathbf{x} \right\rangle D_{Qa}^{(8)} D_{Qb}^{(8)} \\ &= \alpha_1 \sum_{i=1}^3 D_{Qi}^{(8)} D_{Qi}^{(8)} + \alpha_2 \sum_{p=4}^7 D_{Qp}^{(8)} D_{Qp}^{(8)} + \alpha_3 D_{Q8}^{(8)} D_{Q8}^{(8)} \end{aligned}$$

It can be further reduced to

$$\begin{aligned} \mathcal{O}^{\text{EM}} &= c^{(27)} \left(\sqrt{5} D_{\Sigma_2^0 \Lambda_{27}}^{(27)} + \sqrt{3} D_{\Sigma_1^0 \Lambda_{27}}^{(27)} + D_{\Lambda_{27} \Lambda_{27}}^{(27)} \right) \\ &+ c^{(8)} \left(\sqrt{3} D_{\Sigma^0 \Lambda}^{(8)} + D_{\Lambda \Lambda}^{(8)} \right) + c^{(1)} D_{\Lambda \Lambda}^{(1)} \end{aligned}$$

$$\mathbf{8} \otimes \mathbf{8} = \mathbf{1} \oplus \mathbf{8}_s \oplus \mathbf{8}_a \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

Because of Bose symmetry

$$c^{(27)} = \frac{1}{40} (\alpha_1 - 4\alpha_2 + 3\alpha_3),$$

$$c^{(8)} = \frac{1}{10} \left(\alpha_1 - \frac{2}{3}\alpha_2 - \frac{1}{3}\alpha_3 \right),$$

$$c^{(1)} = \frac{1}{8} \left(\alpha_1 + \frac{4}{3}\alpha_2 + \frac{1}{3}\alpha_3 \right)$$