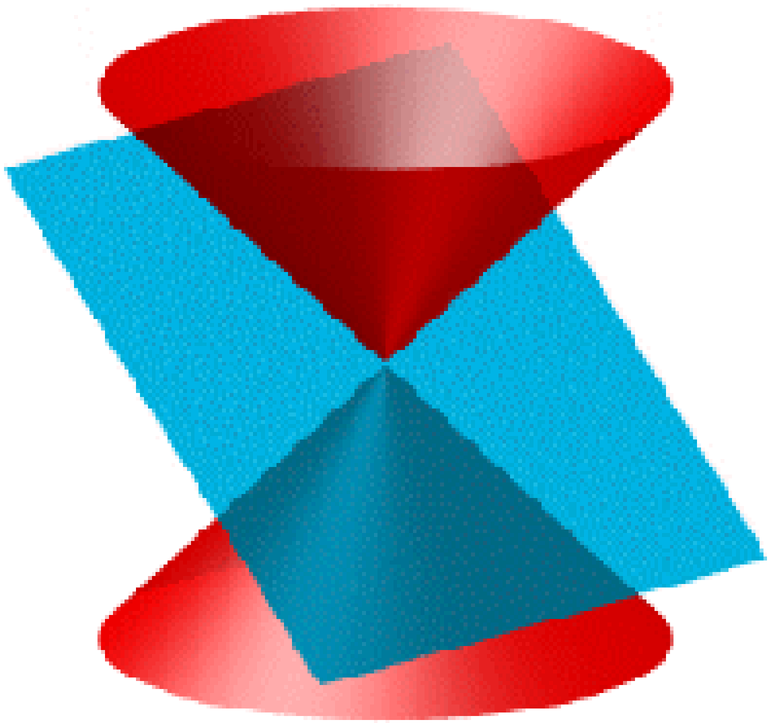
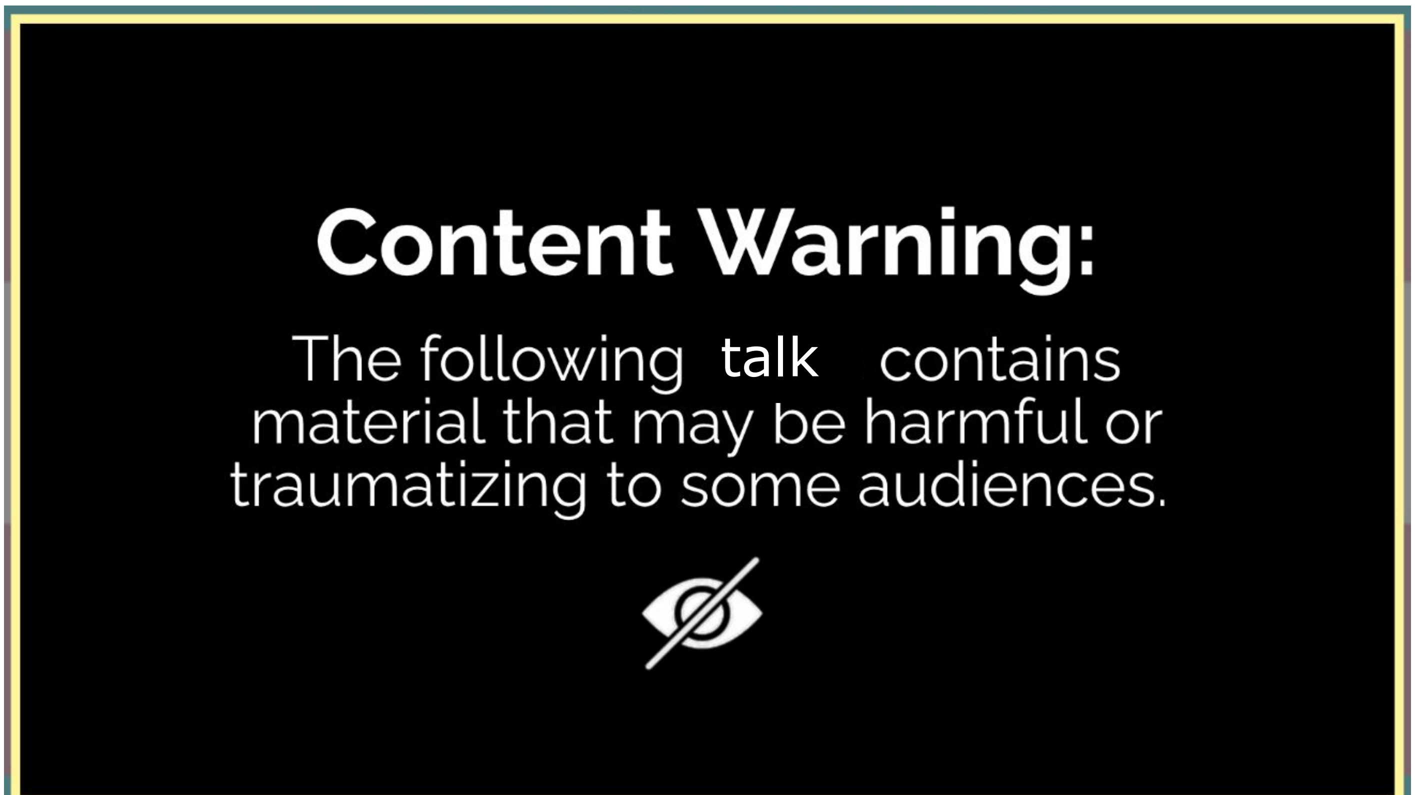
Light-Front Quantization (LFQ): Religion vs. (In)effective Theory



Xiangdong Ji

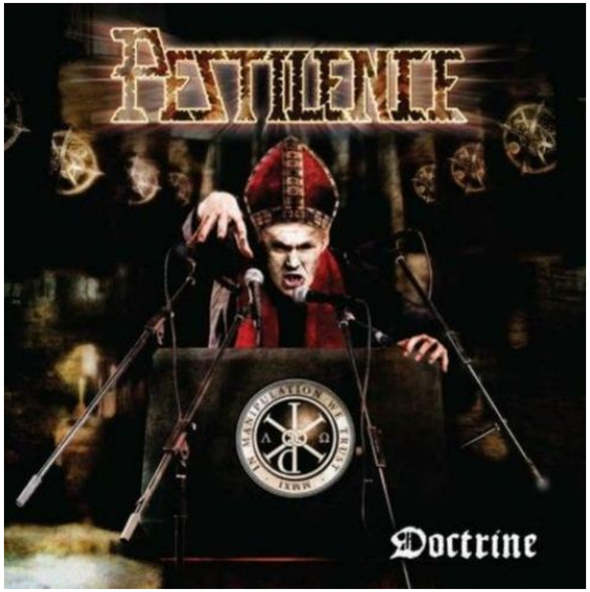
ILCAC seminar, Dec. 15, 2021

- This is not a politically-motivated talk.
- Nor an attack on the LC community, of which I consider myself as a fidel member.
- It is a result of spending numerous torturing hours to find a "non-null path" on the light cone since Wilson's time.



Outline

- The standard doctrine
- The technical problems
- An (in)EFT view of LFQ & partons
- Computing partons in QCD: Feynman said…
- Final comments



The standard doctrine

Three selling points for LFQ

- Light-front quantization is an alternative approach (to instant form) to quantize a field theory
- The vacuum in LFQ theories is simple
- Hadron structure in QCD can be solved through time-independent Schrodinger equation.

Light-front Schrodinger Eq.

Eigenstate problem:

(Brodsky, Pauli & Pinsky, Phys. Rep. 301, 1998))

$$\hat{P}^-|\Psi_n\rangle = \frac{M_n^2}{2P^+}|\Psi_n\rangle$$

Using Fock state expansion

$$|P\rangle = \sum_{m,n}^{\infty} \int \Pi_{i=1}^{n}[dx_i d^2\vec{k}_{i\perp}]\Pi_{i=1}^{m}[d^2\vec{k}_{i\perp}]$$
$$\times \delta(x_1 + ... + x_n - 1)\delta^2(\vec{k}_{1\perp} + ... + \vec{k}_{n+m\perp})$$
$$\phi(x_1, \vec{k}_{1\perp}, ..., 0, \vec{k}_{n+m\perp}) a^\dagger_{x_1,\vec{k}_{1\perp}} ... a^\dagger_{x_n,\vec{k}_{n\perp}} |\mathbf{0}_{nm}\rangle$$

$$a_{k\lambda}|0\rangle = b_{p\sigma}|0\rangle = d_{p\sigma}|0\rangle = 0 \ .$$

What can one do with it?

- Mass & light-front wave function amplitudes are obtained from matrix diagonalization.
- Calculate physical properties with LFWF amplitudes

PDFs:

$$G_{f/h}(x;Q) = \sum_n \int d[\mu_n] \left|\Psi^{(Q)}_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2 \sum_i \delta(x - x_i)\, \delta_{i,f}$$

Distribution amplitudes:

$$\Phi_{f/h}(x;Q) = \sum_n \int d[\mu_n]\, \Psi^{(Q)}_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i) \sum_i \delta(x - x_i)\, \delta_{i,f} \delta_{n,\text{valence}}\, \Theta\left(\vec{k}^2_{\perp i} \le Q^2\right)$$

QFTs & Lorentz symmetry

- Relativistic field theories as formulated by Feynman diagrams or path integrals are independent of the quantization plane: they are Lorentz invariant.
- Therefore, LFQ distinguishes itself through Hamiltonian formulation, which is not the most popular formalism of QFTs.

Constraints from Lorentz symmetry

- Since the vacuum of QFT is independent of frame, the properties of the vacuum, including various condensates, must be reproduced by any quantization formalism. For interacting QFT, the vacuum cannot be trivial.
- Lorentz symmetry demands that all scalars are independent of frames, thus the masses or couplings must be the same.

Main points for LFQ

- Light-front quantization is an alternative approach (to instant form) to quantize a field theory

- The vacuum in LFQ theories is simple
- Hadron structure in QCD can be solved through time-independent Schrodinger equation.

A partial theory

- Since LFQ choose a particular direction (z) to team up with time t, it emphasizes the modes travelling with infinite momentum along the z direction.
- Thus it is not convenient to describe particles traveling with infinite momentum along other directions (x, y or −z)
- Thus LFQ is best be considered as a part of a full theory.

Main points for LFQ

Light-front quantization is an alternative approach (to instant form) to quantize a field theory

? The vacuum in LFQ theories is simple

- Hadron structure in QCD can be solved through time-independent Schrodinger equation.

Interesting progress in LFQ in simple theories and models

- 2D theories
  - t' Hooft model, ϕ^4 theory, sin-Gordon, … Burkardt, Jia et al.
  - Discrete light-cone quantization, Brodsky et al.
- 4D models
  - AdS/CFT models, Brodsky et al.
  - Other LF models, Miller et al., Shuryak, Zahed,…
  - Basis LFQ, Vary et al.
- …

But, not a single approximated calculation of hadrons in non-perturbative QCD.

Main points for LFQ

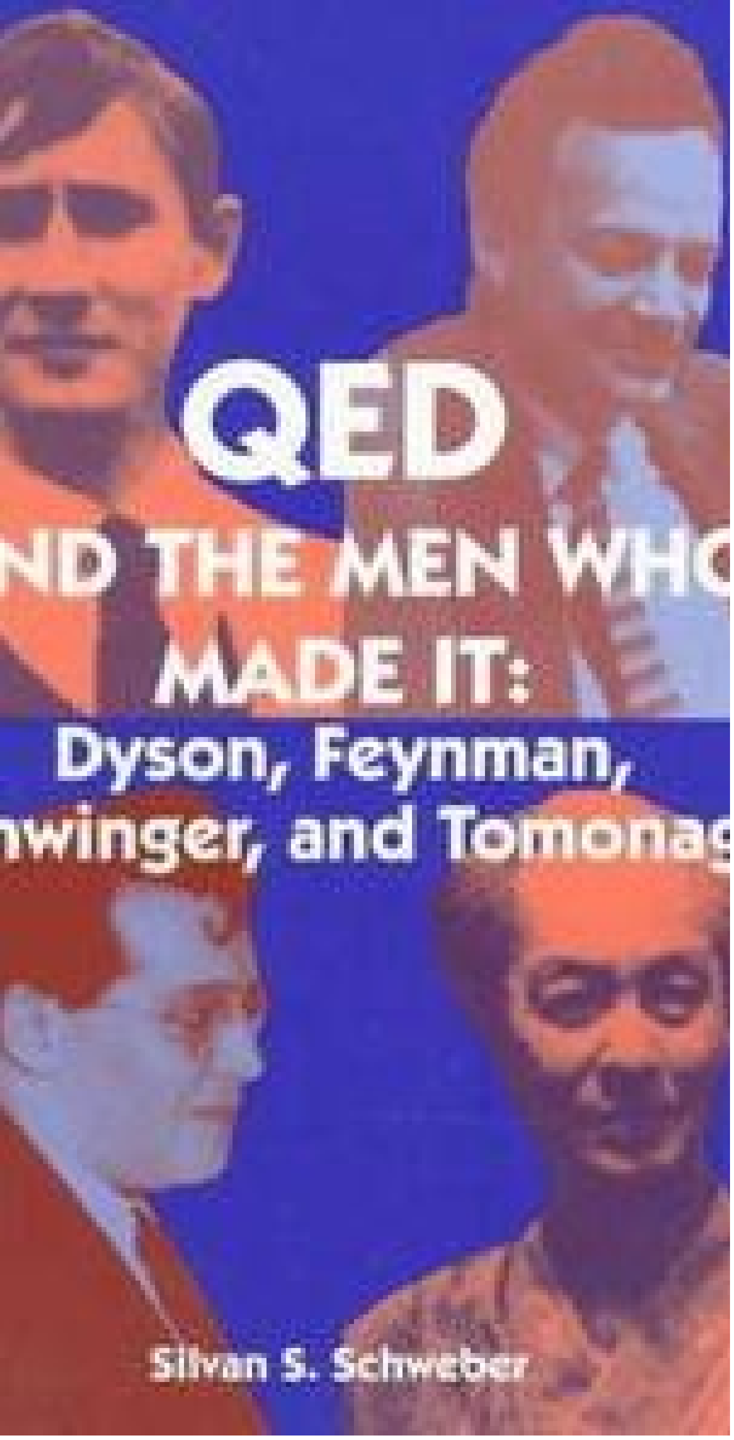
? Light-front quantization is an alternative approach (to instant form) to quantize a field theory ?

? The vacuum in LFQ theories is simple

Hadron structure in QCD can be solved through time-independent Schrodinger equation.....

The technical problems

“That is not a good idea” to break manifest Lorentz symmetry in QFT with ultra-violet (UV) divergences



Space–time symmetry

- Maintaining good space-time symmetry in QFTs formulation worth a Nobel prize.
- Space and time shall be treated in equal terms.

❖ First one-loop Lamb shift calculation was made in Hamiltonian pert. theory by French and Weisskopf.

Useful UV regulators, to all orders in PT

- Pauli-Villars
  - Covariant but does not work for non-Abelian gauge theories
- t' Hooft's dimensional regularization
  - Maintain d-dim Lorentz symmetry
- Wilson's lattice regularization
  - Do not have proper symmetry, but it is believed that the Lorentz symmetry is restored in the continuum limit

There is virtually no other PRACTICAL one.

Gauge choices:

- Covariant gauge: Lorentz symmetric

  Lorenz gauge, Feynman & Landau gauges, ….

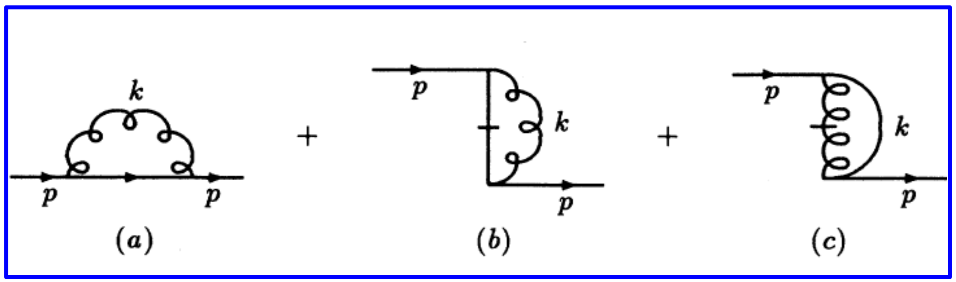
  non-physical degrees of freedom, but all orders

- Non-covariant gauge: break Lorentz symmetry
  - Axial gauge, $A^0 = 0, A^3 = 0$
  - Coulomb gauge, $\nabla \cdot \vec{A} = 0$

No one has shown how the non-covariant gauge work to all orders in PT!

If taking these lessons seriously...

- (LC) Time-ordered diagrams are more divergent than the covariant sum



$$\delta p_1^- = \frac{g^2 C_f}{[p^+]} \int \frac{d^2\kappa_\perp}{16\pi^3} \int_0^1 \frac{dx}{[1-x]} \times \frac{\left(4\frac{1}{[x]^2} - 4\frac{1}{[x]} + 2\right)\boldsymbol{\kappa}_\perp^2 + 2m^2x^2}{x(1-x)p^2 - \boldsymbol{\kappa}_\perp^2 - xm^2}$$

$$\delta p_2^- = 2\frac{g^2 C_f}{[p^+]} \int \frac{d^2\kappa_\perp}{16\pi^3} \int_0^\infty \frac{dx}{[x][1-x]},$$

$$\delta p_3^- = 2\frac{g^2 C_f}{[p^+]} \int \frac{d^2\kappa_\perp}{16\pi^3} \int_0^\infty dx \left\{ \frac{1}{[1-x]^2} - \frac{1}{(1+x)^2} \right\}$$

- LC gauge introduces extra light-cone singularities ($k^+ = 0$), which cancel for GI quantities

$$D^{\mu\nu} = \left(\frac{-i\delta^{ab}}{k^2 + i\epsilon}\right)\left(g^{\mu\nu} - \frac{n^\mu k^\nu + n^\nu k^\mu}{n \cdot k}\right)$$

Light-front PT & LC gauge

- No one has ever come up with
  - a regulator in LFPT which works to all order in PT, with a result consistent with covariant one.
- No one has ever come up with
  - a way to regulate the light-cone divergences so that the covariant perturbation theory in light-cone gauge can be calculated to all orders in PT.

Essential LC singularities

- Some quantities introduced in LFQ have essential singularities due to LC gauge links

$$W_n^{\pm}(\xi) = \mathcal{P}\exp\left[-ig\int_0^{\pm\infty} d\lambda n \cdot A(\xi + \lambda n)\right]$$

  - This LC singularities cannot be regulated by anything that we know already.
  - Hamiltonian: non-local in coordinate space. Cannot be defined without regularization
  - LC wave function amplitudes: the same
  - Transverse-momentum dependent quantities: yes

Regularizing the LC gauge-link divergences

- In the standard LFPT, these divergences are mixed with other non-essential divergences.
  - Regularization of these divergences introduces IR physics which cannot be calculated in pert. Destroys BPHZ power counting!
- Thus before solving the non-perturbative

$$\hat{P}^{-}|\Psi_n\rangle = \frac{M_n^2}{2P^+}|\Psi_n\rangle$$

We need to define P^- non-perturbatively (twist-4), with infinite number of renormalization constants. (ultimately tied with zero mode problem).

Non-perturbative solution

- Hamiltonian problem requires diagonalization of a really LARGE matrix.
- However, Fock state expansion might have a VERY slow convergence, and Fock states might be wrong language to solve non-pert. QCD.
- Lesson from lattice QCD: it is the QUANTUM FIELDS, not particles, the best for doing non-perturbative calculations!


LFQ for QCD?

An (in)EFT view of LFQ and partons

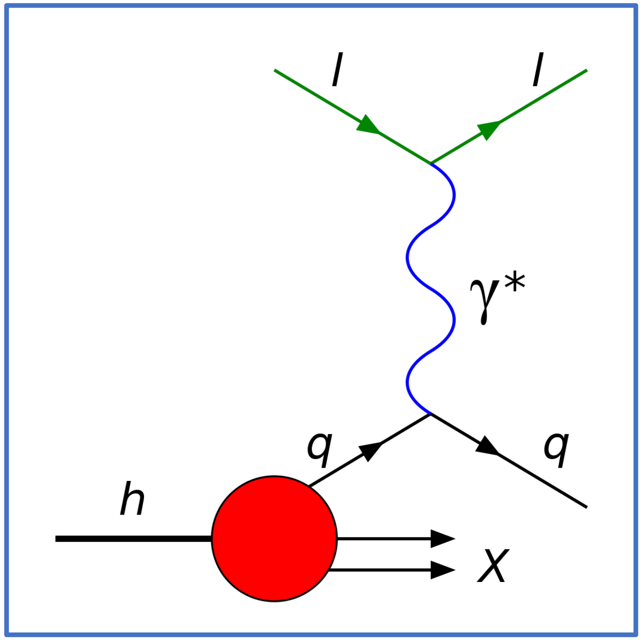
Need for LFQ?

- LFQ is not needed for
  - Calculating hadron masses
  - Calculating the QCD vacuum properties
  - Calculating Dirac & Pauli form factors
  - Or Calculating PQCD processes (see, however, A. Mueller, Kovchegov et al)
  - ….
- Lattice QCD & covariant PT are doing an increasingly impressive job, and LFQ is nowhere near a threat or complementary to them.

  there is no added value yet!

Partons!

- Feynman's parton model is fundamental to understanding high-energy scattering and hadron structure.



Parton are EFT objects

- Partons are Infinite-momentum collinear (& soft) modes in a pre-selected direction, to describe the quark and gluon structure of the hadrons travelling at the speed of light in the same direction.
  - All others are in soft modes.
- Partons in QFT are obtained by taking the infinite momentum limit before imposing a UV cut-off
  - A methodology adopted frequently in EFTs.
- Thus partons are EFT objects!

Partons & LFQ

- Since its proposal in 1969, people quickly realize that the infinite momentum frame (IFM) and the physical pictures used by Feynman and Bjorken are related to LFQ proposed by Dirac in 1949.

  Weinberg '66

  Chang & Ma '69, Kogut & Soper, '69

LFQ as EFT

- LFQ is an effective theory (taking $P \to \infty$ disregarding UV), it is supposed to describe $x \neq 0$ particles. There is no need to worry about the soft (zero) mode, which can be integrated out of the EFT or by imposing cut-off $x \geq \epsilon > 0$.
  - Lift the burden of LFQ!
- Essential divergences in LFQ are the standard EFT problem which requires additional regularization (separating collinear from soft modes).
  - Almost all EFTs have new divergences.

EFT with ∞ Renormalization constants

- Due to rapidity divergences, LFQ now has ∞ renormalization constants.
- Thus the predictive power of LFQ is limited
  - Cannot solve the hadron masses.
  - Cannot describe the QCD vacuum.
  - Cannot fully describe higher-twist effects due to zero modes.
  - Cannot even give the full Hamiltonian
- Thus, LFQ is an ineffective field theory!

LFQ is too singular to be solved!

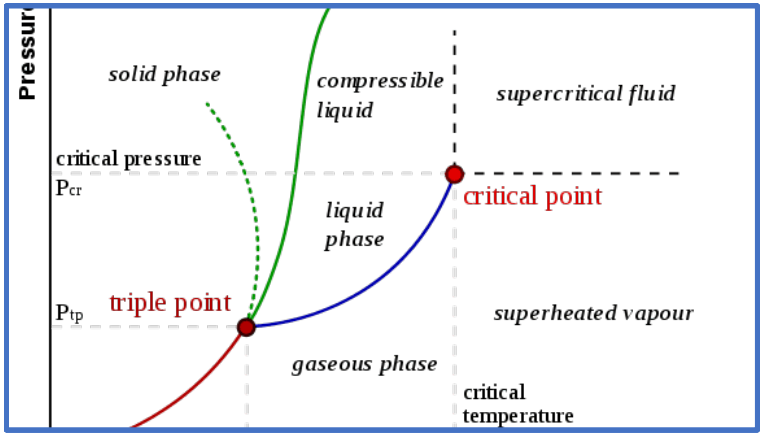
- Consider PDFs f(x), with a Reggie behavior at small x

$$f(x) \sim x^{-\alpha}$$

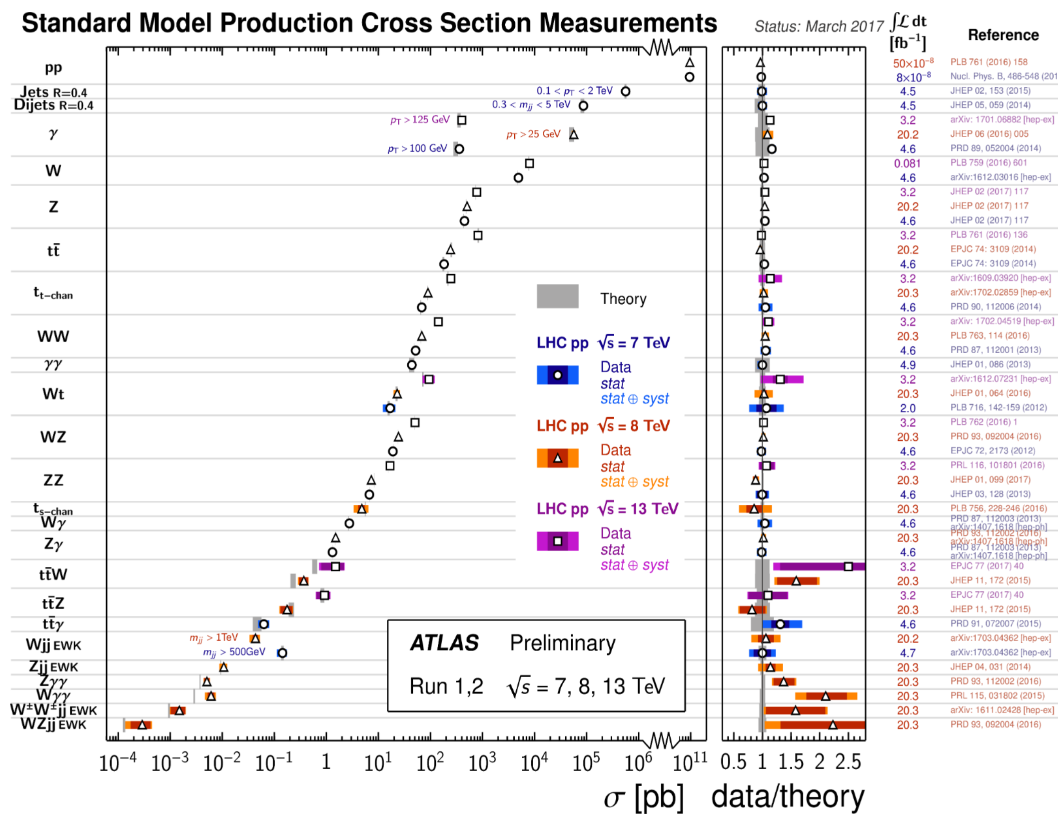
The correlation length in the coordinate space is infinite

$$f(\lambda) \sim \lambda^{-1+\alpha}$$

- LF theory is at a critical point which is the most difficult to solve



A superbly effective language for factorization in high-energy scattering



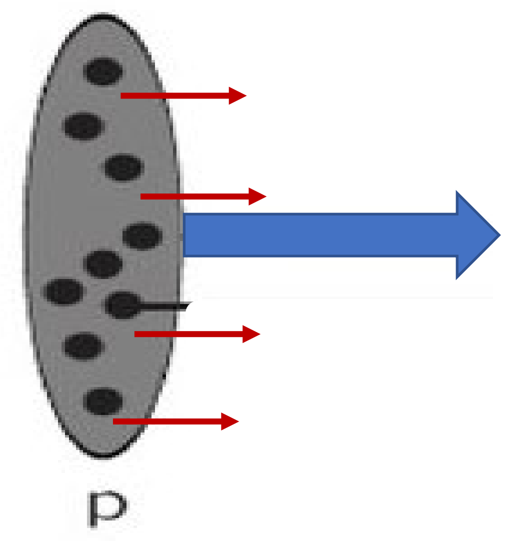
but not an effective method for solving the PDFs.

Computing partons in QCD & Feynman said

partons are Euclidean not Minkowskian

Feynman said...

- Parton distributions are the ordinary momentum distributions of constituents in a many-body system when boosted to the infinite momentum limit



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

  In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \to \infty$

Feynman's partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp \, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \rightarrow \infty$ limit,

$$f(k^z, P^z) \rightarrow_{p^z \rightarrow \infty} f(x) \quad with\ x = \frac{k^z}{P^z},$$

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Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

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(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Partons from a large-P expansion of Euclidean observables

- Assuming $P^z \to \infty$ limit exists, parton physics is obtained by expansion (Feynman, 1969)

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

Partons from a large-P expansion of Euclidean observables

- Assuming $P^z \to \infty$ limit exists, parton physics is obtained by expansion (Feynman, 1969)

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

- Account for subtlety of non-commuting limits of $\mathrm{P^z} \to \infty \;\&\; \Lambda_{UV} \to \infty$ in QFT (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\mathrm{QCD}}}{y^2(P^z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{(1-y)^2(P^z)^2}\Big),$$

Weinberg EFT expansion

- Approximate $P^z = \infty$ by a finite large P^z.

  We frequently do this in QCD

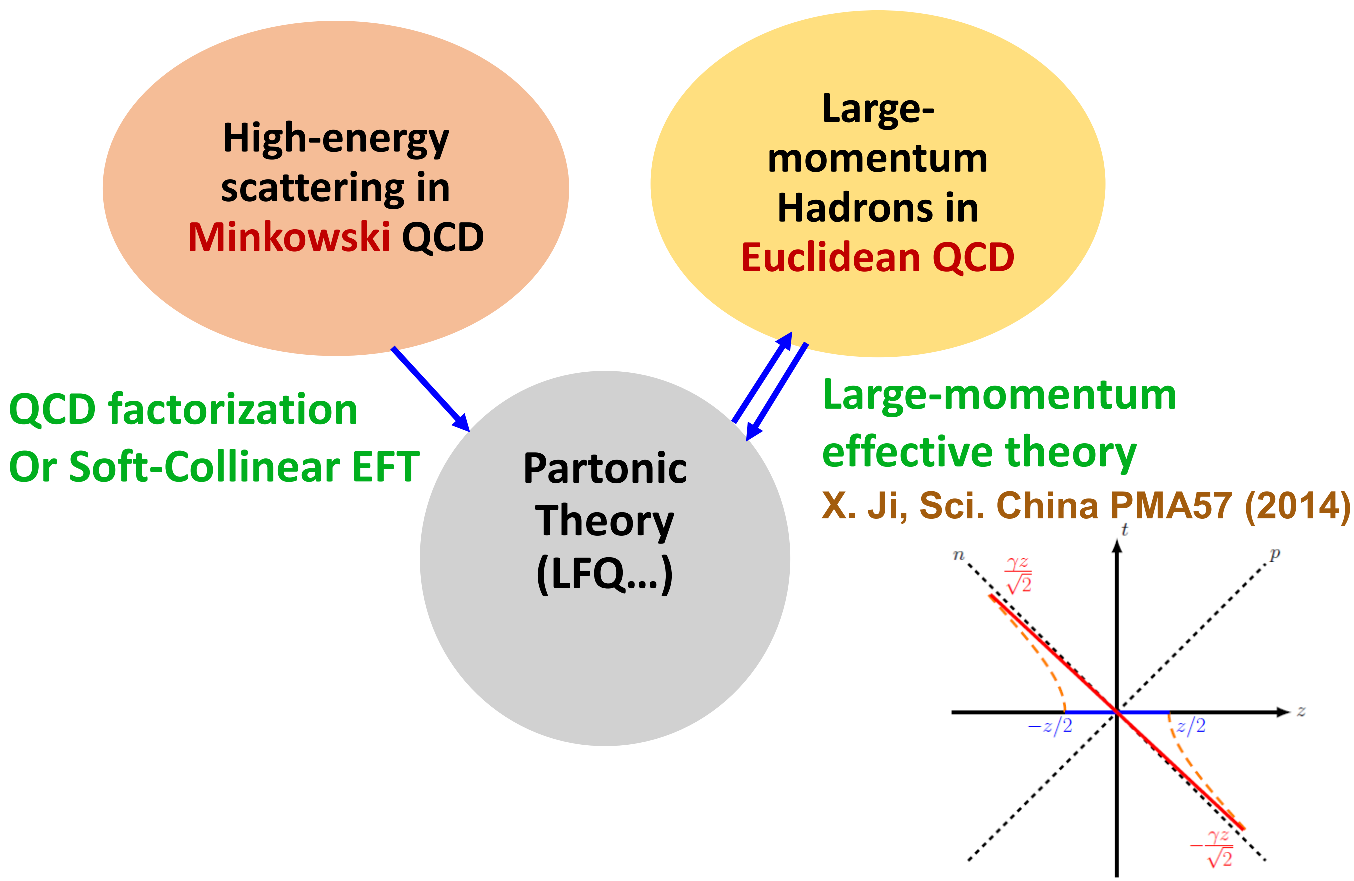
  Lattice QCD & HQET

- EFT expansion for PDFs in the spirit of Weinberg,

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \tilde{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{q}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right).$$

x-dependence of PDF can be calculated from QCD!
Not by fitting as in extracting PDFs from exp. data.

EFT for partons: Full Euclidean QCD



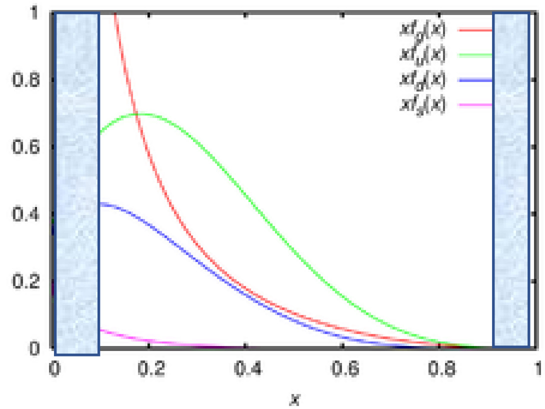
Main feature & limitation

- Large-momentum systematic expansion!
- LaMET expansion breaks done near

$$xP^z \sim \Lambda_{QCD}; \quad (1-x)P^z \sim \Lambda_{QCD}$$

where the collinear modes end. These are soft modes or zero modes.

For $P^z \sim 2-3\ GeV,$ max x-range: 0.1-0.9



LaMET and parton physics

LaMET 2021 Online

Dec 7 – 9, 2021
Zoom
US/Eastern timezone

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- Overview
- Call for Abstracts
- Timetable
- Contribution List
- Book of Abstracts
- Registration
- Participant List

Contact Dr. Yong Zhao

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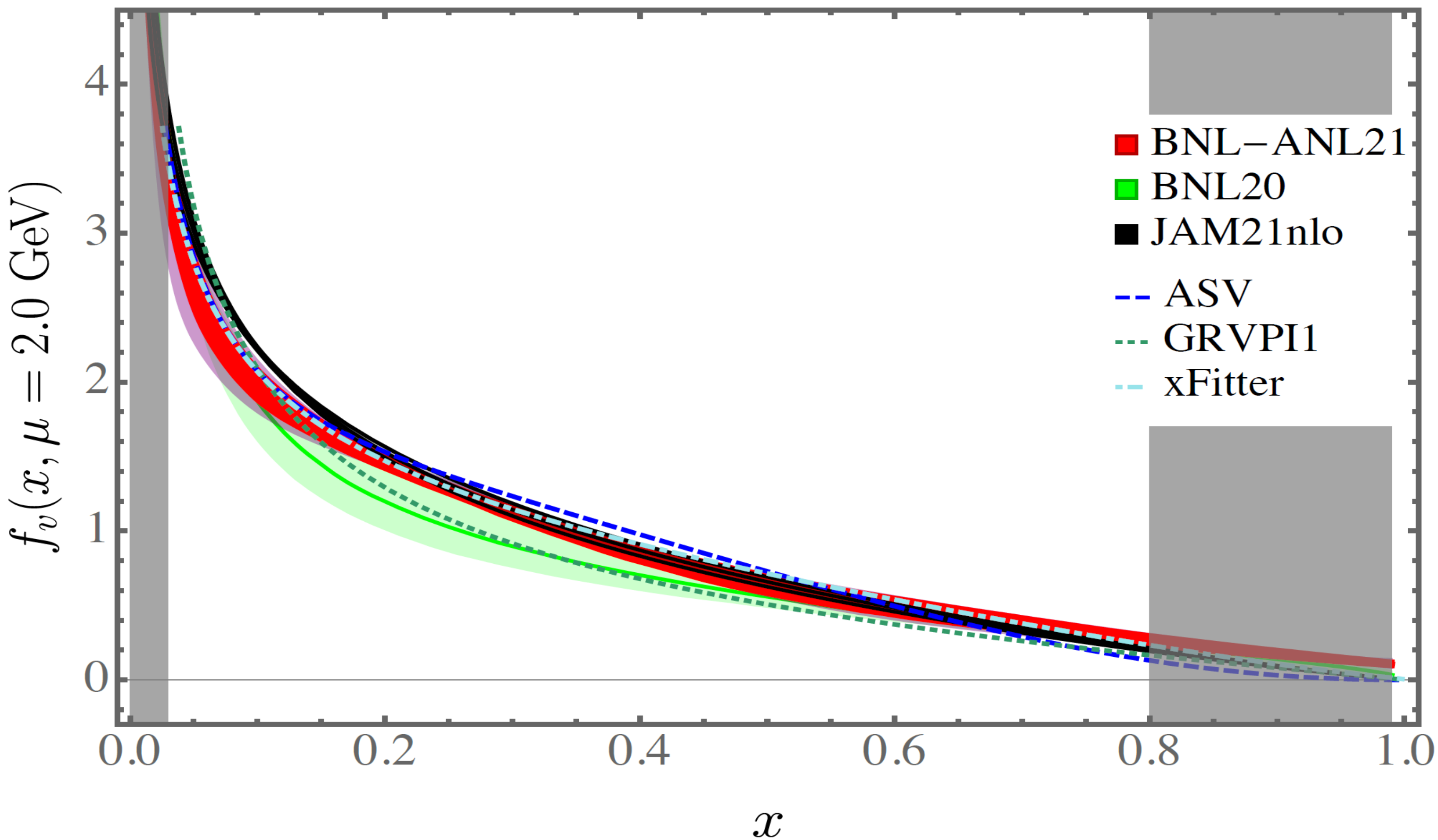
The **2021 Meeting on Lattice Parton Physics from Large-Momentum Effective Theory (LaMET 2021)** will be held online by Center for Nuclear Femtography (CNF) at SURA in Washington DC. The meeting will take place from Dec. 7-9, 2021. Registration is open now at https://indico.cern.ch/event/1082505/, and abstracts can be submitted for presentation. The deadline for abstract submission is 12:00 AM EST, Nov. 15, 2021.

Large-momentum effective theory (LaMET) is based on the field theoretical realization of Feynman parton model in which PDFs are momentum distributions of quarks and gluons in an infinite momentum hadron state. One can start from the momentum distributions in a hadron with finite but large momentum, which can be calculated in Euclidean approaches such as lattice QCD, and then expand the results systematically. The leading term is the PDFs after proper field-theoretical matching and running. Over the past years, LaMET has enabled much progress in the lattice QCD calculation of PDFs as well as GPDs and TMDs. The lattice data for LaMET calculations can also be analyzed in coordinate-space factorization approaches to get moments of PDFs or x-distributions through phenomenological parametrizations.

A recent pion PDF hep-lat: 2112:02208, ANL&BNL

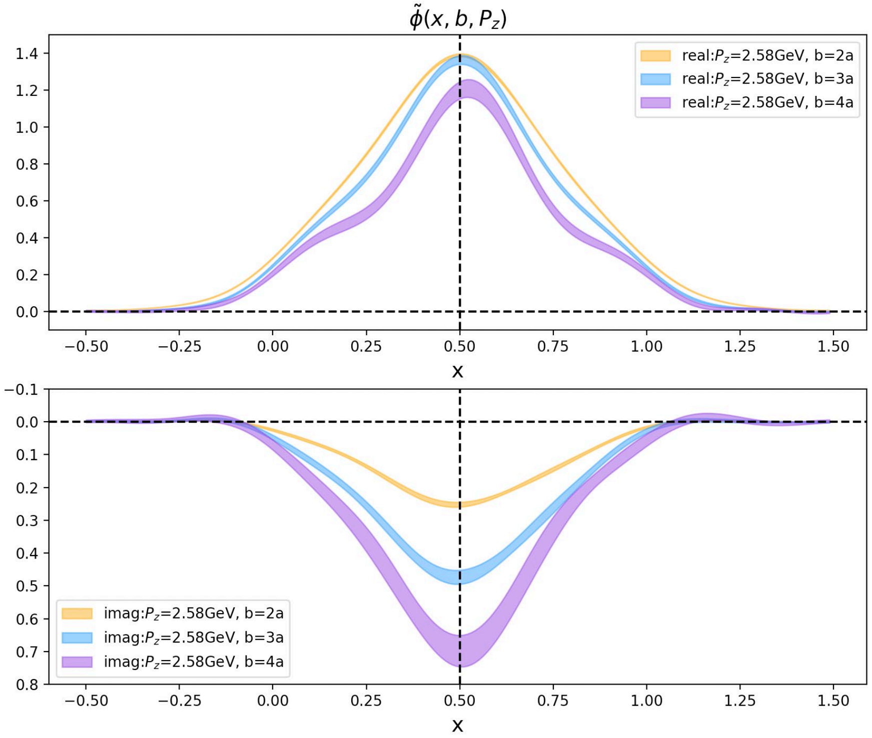
- Lattice spacing 0.04-0.06 fm, $m_\pi = 300$ MeV

$\mathrm{P} = 2.42$ MeV



A first example of QCD LFWF

- QCD meson LFWF (preliminary, LPC collaboration)



Physical effects of zero mode (Ji, 2003.04478)

- While zero modes are difficult to control in the QCD Hamiltonian and LFWFs directly, it is possible to calculate directly through laMET in physical observables: Mass, scalar charge, …
- Sivers function

$$f^{\mathrm{TMD}}(x, k_\perp, S_\perp) \sim (S_\perp \times \vec{k}_\perp)^z f_{1T}^\perp(x, k_\perp) + \ldots$$

$$f(x, \vec{k}_\perp) = \int \frac{d\lambda}{2\pi} \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\lambda x + i\vec{k}_\perp \cdot \vec{b}_\perp}$$

$$\times \langle P|\bar{\psi}(\lambda n + \vec{b}_\perp)\gamma^+ \mathcal{W}_n(\lambda n + \vec{b}_\perp)\psi(0)|P\rangle \ ,$$

where $\mathcal{W}_n(\lambda n + \vec{b}_\perp)$ is the staple-shaped gauge link,

$$\mathcal{W}_n(\xi) = W_n^\dagger(\xi) W_\perp^\dagger(\xi_\perp) W_\perp(0) W_n(0) \ ,$$

a zero-mode effect!

A well-defined procedure has been developed in LaMET to calculate this.

Conclusions

- Directly solving LFQ version of 3+1 QCD is very very hard, if not impossible.
- If considered as an effective theory, it has infinite number of renormalization constant, and thus is ineffective.
- Parton is a great language, but not intrinsically a Minkowskian concept. It can be accessed through effective Euclidean approach, such as lattice QCD.