Relativistic hadron densities on the light front

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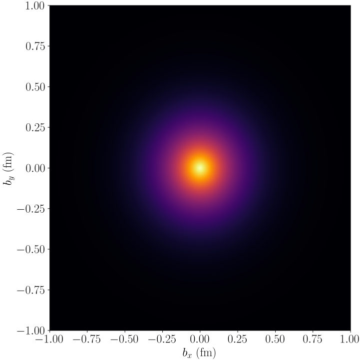
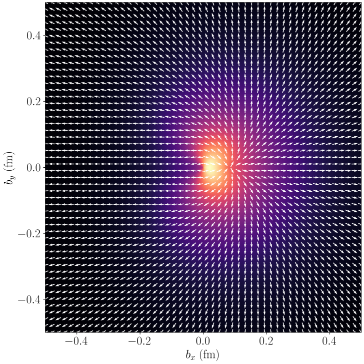
Introduction

- Main question: **how are mass/momentum energy and forces spatially distributed in hadrons?**
- Hadrons are **manifestly relativistc**: need a relativistic description.
- **Light front densities** provide that description.



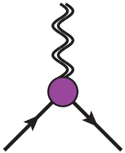


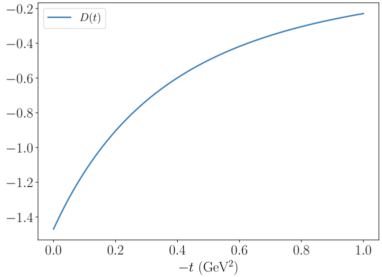
Based on work in:

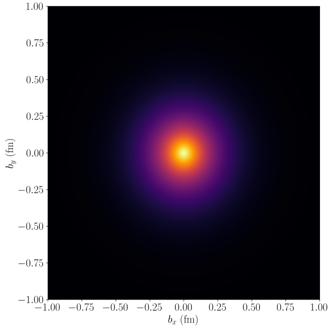
- AF & Gerald Miller, PRD**103**, 094023
- AF & Gerald Miller, PRD**104**, 014024
- AF & Gerald Miller, arxiv:2108.03301

The energy-momentum tensor

- The **energy-momentum tensor** (EMT) is an operator characterizing the distribution and flow of energy and momentum.
- Matrix elements between hadronic states characterize coveted properties of hadrons:
  - The distribution & decomposition of mass.
  - The distribution & decomposition of angular momentum.
  - The distribution & decomposition of forces, including shear and pressure.







The EMT from spacetime symmetry

- The EMT is obtained by applying Noether's theorem to spacetime translation symmetry:

$$T^{\mu\nu}_{\mathrm{QCD}}(x) = \sum_q \left\{ \frac{1}{2}\bar{q}(x) i\gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} q(x) - g^{\mu\nu}\bar{q}(x)\big(i\overleftrightarrow{\not{D}} - m_q\big)q(x) \right\} - \mathrm{Tr}\left[G^{\mu\lambda}G^{\nu}{}_{\lambda}\right] + \frac{1}{2}g^{\mu\nu}\mathrm{Tr}\left[G^{\lambda\sigma}G_{\lambda\sigma}\right]$$

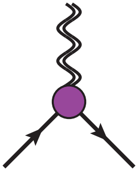
  - It is conserved: $\nabla_\mu T^{\mu\nu}(x) = 0$.
  - It is **symmetric and gauge invariant**.
- Regarding the Noether EMT being symmetric, see:
  - Gerardo Muñoz (1996), American Journal of Physics 64, 1153
  - Ricardo E Gamboa Saraví (2002), J. Phys. A: Math. Gen. 35 9199

Form factors of the EMT

- EMT matrix elements give **gravitational form factors** (GFFs).
  - EMT is what gravitates, after all.
- For a spin-zero hadron:

$$\langle p' | \hat{T}^{\mu\nu}(0) | p \rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) D(t)$$

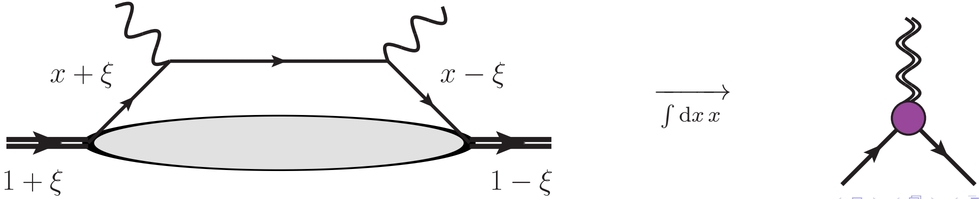
- $A(t)$ encodes momentum density (mass in NR limit)
- $D(t)$ encodes force distributions.



How to get the GFFs

- **Hard exclusive reactions** are used to measure GFFs—not gravitational experiments.
  - Deeply virtual Compton scattering (DVCS) to probe quark structure.
  - Deeply virtual meson production (DVMP), *e.g.*, J/ψ or Υ to probe gluon structure.
  - . . . and more!
- Related to GPDs—spin-zero example:

$$\int_{-1}^{1} \mathrm{d}x \, x H_{q,g}(x, \xi, t) = A_{q,g}(t) + \xi^2 D_{q,g}(t)$$



GFFs and densities

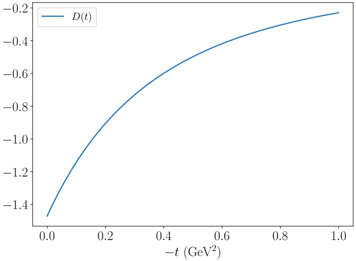
- So GFFs are given by:

$$\langle p'|\hat{T}^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A(t) + \frac{1}{2}(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu})D(t)$$

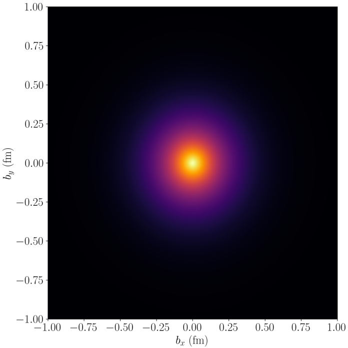
- And densities are given by:

$$T^{\mu\nu}(\mathbf{r}, t=0) = \langle \psi|\hat{T}^{\mu\nu}(\mathbf{r}, t=0)|\psi\rangle$$

- How are densities related to GFFs, then?



$\longrightarrow$



?

Fourier transforms and densities

- Fourier transforms always work at **amplitude level**:

$$\langle \mathbf{r}|\psi\rangle = \int \frac{\mathrm{d}^3\mathbf{p}}{2E_{\mathbf{p}}(2\pi)^3}\langle \mathbf{p}|\psi\rangle e^{-i(\mathbf{p}\cdot\mathbf{r})}$$

- Densities are defined at a **probability level**:

$$T^{\mu\nu}(\mathbf{r}, t=0) = \langle\psi|\hat{T}^{\mu\nu}(\mathbf{r}, t=0)|\psi\rangle$$

- Densities thus involve **two momentum integrals**:

$$T^{\mu\nu}(\mathbf{r}, t=0) = \int \frac{\mathrm{d}^3\mathbf{p}}{2E_{\mathbf{p}}(2\pi)^3}\int \frac{\mathrm{d}^3\mathbf{p}'}{2E_{\mathbf{p}'}(2\pi)^3}\psi^*(\mathbf{p}')\psi(\mathbf{p})\langle\mathbf{p}'|\hat{T}^{\mu\nu}(\mathbf{r}, t=0)|\mathbf{p}\rangle$$

Wave packet dependence

$$T^{\mu\nu}(\mathbf{r}, t=0) = \int \frac{d^3\mathbf{P}}{(2\pi)^3} \int \frac{d^3\mathbf{\Delta}}{(2\pi)^3} \psi^*(\mathbf{p}')\psi(\mathbf{p}) \frac{\langle \mathbf{p}'|\hat{T}^{\mu\nu}(0)|\mathbf{p}\rangle}{4E_{\mathbf{p}}E_{\mathbf{p}'}} e^{-i\mathbf{\Delta}\cdot\mathbf{r}}$$

$$\mathbf{P} = \frac{1}{2}(\mathbf{p}+\mathbf{p}') \qquad \mathbf{\Delta} = \mathbf{p}' - \mathbf{p}$$

- $\psi(\mathbf{p})$ is **external** wave function.
  - How fast is the hadron, where is it, etc.
- This density mixes internal structure with wave function spread.
- $\mathbf{P}$ dependence encodes wave function spread. **Need to get rid of P integral.**
- Use a **Gaussian wave function**:

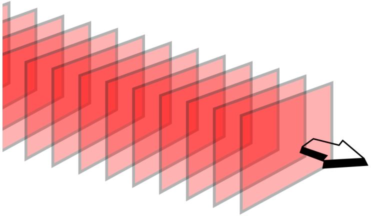
$$\psi(\mathbf{p}) = \sqrt{2E_p}\,(2\pi)^{3/4}(2R)^{3/2}e^{-R^2\mathbf{p}^2}$$

(See `Miller:2018ybm`.)

The two limits

$$T^{\mu\nu}(\mathbf{r}) = (2\pi)^{3/2}(2R)^3 \int \frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3} e^{-2R^2\mathbf{P}^2} \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} \frac{\langle \mathbf{p}' | T^{\mu\nu}(0) | \mathbf{p} \rangle}{\sqrt{4E_p E_p'}} e^{-\frac{R^2}{2}\boldsymbol{\Delta}^2} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} .$$

- $R \to \infty$ gives **Breit frame**
  - Initial and final state plane waves.
- $R \to 0$ gives **spatially localized state**.



Breit frame fallability

- Breit frame density:

$$T^{\mu\nu}_{\text{Breit}}(\mathbf{r}) = \lim_{R\to\infty} \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3} \frac{\langle \mathbf{p}'|T^{\mu\nu}(0)|\mathbf{p}\rangle}{\sqrt{4E_pE'_p}} e^{-\frac{R^2}{2}\boldsymbol{\Delta}^2} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}\Bigg|_{\mathbf{P}=0}$$

  - Most literature using "Breit frame" *erroneously* drops **red** factor.
- All radii are **infinite** in the Breit frame, e.g.

$$\langle r^2\rangle(\text{energy}) = -\boldsymbol{\nabla}^2_{\boldsymbol{\Delta}}\left[\frac{\langle \mathbf{p}'|T^{00}(0)|\mathbf{p}\rangle}{\sqrt{4E_pE'_p}}\right]\Bigg|_{\mathbf{P}=0,\boldsymbol{\Delta}=0} + \lim_{R\to\infty} 3MR^2$$

  - Again, most "Breit frame" literature *erroneously* drops **red** term.
- Happens because of **uncertainty principle**.

(Argument and slide title from `Miller:2018ybm`.)

Legends of localization

- Spatially localized hadron:

$$T^{\mu\nu}_{\text{loc}}(\mathbf{r}) = \lim_{R\to 0} \int \frac{\mathrm{d}^3\mathbf{U}}{(2\pi)^{3/2}} e^{-\frac{1}{2}\mathbf{U}^2} \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} \frac{\langle \mathbf{p}'|T^{\mu\nu}(0)|\mathbf{p}\rangle}{\sqrt{4E_pE'_p}}\bigg|_{\mathbf{P}=\mathbf{U}/(2R)} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}$$

  - Need finite matrix element at $\mathbf{P} \to \infty$.
- Spin-zero $T^{00}(0)$ element at large $\mathbf{P}$:

$$\frac{\langle \mathbf{p}'|T^{00}(0)|\mathbf{p}\rangle}{\sqrt{4E_pE'_p}} \approx |\mathbf{P}|A(t) + \mathcal{O}\left(\frac{M}{|\mathbf{P}|}\right) \xrightarrow[|\mathbf{P}|\to\infty]{} \infty$$

- Again happens because of uncertainty principle.
  - Energy increases with $\mathbf{P}$.

Fully non-relativistic limit

- We can at least use localized limit ($R \to 0$) in the **non-relativistic limit**.
  - **Fully non-relativistic** limit: $c \to \infty$.
  - Energy becomes mass; no $\mathbf{P}$ dependence:

$$\lim_{c\to\infty} \frac{1}{c^2} \frac{\langle \mathbf{p}' | T^{00}(0) | \mathbf{p} \rangle}{\sqrt{4E_p E_p'}} = M A(t)$$

  - No uncertainty principle problems!
- **Fully non-relativistic** mass density:

$$\rho^{(\mathrm{NR})}_{\mathrm{mass}}(\mathbf{r}) = M \int \frac{\mathrm{d}^3\mathbf{\Delta}}{(2\pi)^3} A(t) e^{-i\mathbf{\Delta}\cdot\mathbf{r}}$$

- Downside: it's non-relativistic.
  - Consistency requires taking NR limit into form factor.

Leading order relativistic corrections

- What if we expand in powers of v/c?

$$\rho_{\text{energy}}(\mathbf{r}) = \frac{\rho_{\text{mass}}(\mathbf{r})}{M}\left\{Mc^2 + \frac{\langle \mathbf{P^2}\rangle}{2M}\right\} + \mathcal{O}(1/c^2)\,,$$

where:

$$\rho^{(j=0)}_{\text{mass}}(\mathbf{r}) = M\int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3}\left\{A(t) + \frac{\boldsymbol{\Delta}^2}{8M^2c^2}\Big[A(t) + 2D(t)\Big]\right\} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} + \mathcal{O}(1/c^4)$$

$$\rho^{(j=1/2)}_{\text{mass}}(\mathbf{r}) = M\int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3}\left\{A(t) + \frac{\boldsymbol{\Delta}^2}{4Mc^2}\Big[A(t) + D(t) - 2J(t)\Big]\right\} e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} + \mathcal{O}(1/c^4)$$

- $\langle \mathbf{P}^2\rangle$ diverges for localized hadron.
- **Spin-half** agrees with standard “Breit frame” density.
  - At best a *leading order* relativistic density.

Beyond leading order?

- Is all-orders factorization is possible?

$$\rho_{\text{energy}}(\mathbf{r};R) = \frac{\rho_{\text{mass}}(\mathbf{r};R)}{Mc^2}\left\langle\sqrt{(Mc^2)^2+(\mathbf{P}c)^2}\right\rangle_R$$

  - Energy density depends on wave packet spread R.
  - Energy diverges when $R\to 0$, but mass doesn't?
- Need $\mathbf{P}$ and $\boldsymbol{\Delta}$ dependence to factorize in:

$$T^{\mu\nu}(\mathbf{r},t=0) = \int\frac{\mathrm{d}^3\mathbf{P}}{(2\pi)^3}\int\frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3}\psi^*(\mathbf{p}')\psi(\mathbf{p})\frac{\langle\mathbf{p}'|\hat{T}^{\mu\nu}(0)|\mathbf{p}\rangle}{4E_{\mathbf{p}}E_{\mathbf{p}'}}e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}$$

$$\mathbf{P} = \frac{1}{2}(\mathbf{p}+\mathbf{p}') \qquad \boldsymbol{\Delta} = \mathbf{p}'-\mathbf{p}$$

...but:

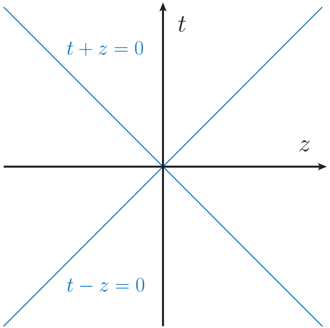
$$E_{\mathbf{p}}E_{\mathbf{p}'} = \sqrt{\Big((Mc^2)^2+(\mathbf{P}c)^2+(\boldsymbol{\Delta}c)^2/4\Big)^2-(\mathbf{P}\cdot\boldsymbol{\Delta})^4}$$

- **All-orders relativistic mass density is impossible**

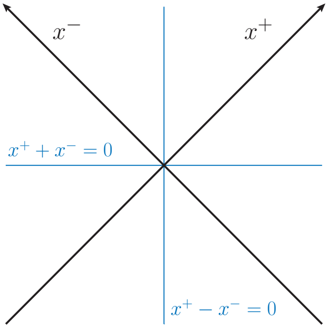
Light front coordinates

- Issues are resolved in **light front coordinates**.

$$x^{\pm} = \frac{t \pm z}{\sqrt{2}} \qquad \mathbf{x}_{\perp} = (x, y) \qquad \tau = x^{+} = \text{time}$$



Minkowski coordinates



Light front coordinates

Galilean subgroup

- Poincaré group has a $(2+1)$D **Galilean subgroup**.
  - x^+ is time and $\mathbf{x}_\perp$ is space under this subgroup.
  - x^- can be integrated out.
  - $P^+ = \frac{1}{\sqrt{2}}(E_\mathbf{p} + p_z)$ is the central charge.
- Basically, light front coordinates should give a **fully relativistic** 2D picture that looks like *non-relativistic* physics.
  - But with P^+ in place of M.

$$\frac{\mathrm{d}\mathbf{P}_\perp}{\mathrm{d}x^+} = P^+ \frac{\mathrm{d}^2\mathbf{x}_\perp}{\mathrm{d}x^{+2}}$$

$$H = P^- = H_{\text{rest}} + \frac{\mathbf{P}_\perp^2}{2P^+}$$

etc.

Light front densities

- Densities use *localized* wave function:

$$T^{\mu\nu}_{\mathrm{LF}}(\mathbf{b}_\perp) = \lim_{R\to 0} \int \frac{\mathrm{d}^2\mathbf{U}_\perp}{2\pi} e^{-\frac{1}{2}\mathbf{U}_\perp^2} \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \frac{\langle p'|T^{\mu\nu}(0)|p\rangle}{2P^+}\bigg|_{\mathbf{P}_\perp=\mathbf{U}_\perp/(2R)} e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

  - x^- has been integrated out.
  - Fixed light front time ($x^+ = 0$).
  - Matrix element must be finite when $\mathbf{P}_\perp \to \infty$.

Light front momentum density

- Spin-zero $T^{++}(0)$ element at all $\mathbf{P}_\perp$:

$$\frac{\langle p'|T^{++}(0)|p\rangle}{2P^+} = P^+ A(t)$$

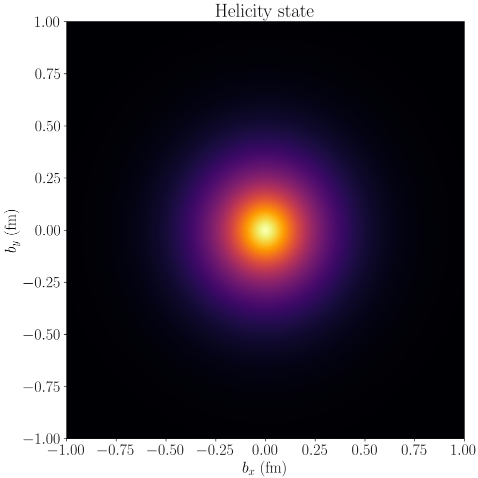
- Gives a P^+ density:

$$\rho^{(\mathrm{LF})}_{P^+}(\mathbf{b}_\perp) = P^+ \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} A(t) e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

  - **Fully relativistic.**

- Compare to NR mass density:

$$\rho^{(\mathrm{NR})}_{\mathrm{mass}}(\mathbf{r}) = M \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} A(t) e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}$$



(dipole model with $f_2(1270)$ pole)

Relativistic spin effects: transverse polarization

- Transverse polarization: superposition of helicity states.

$$|\mathbf{s}_\perp\rangle = \frac{|\lambda = +1\rangle + e^{i\phi_s}\,|\lambda = -1\rangle}{\sqrt{2}}\,.$$

- Matrix elements get helicity flip contribution:

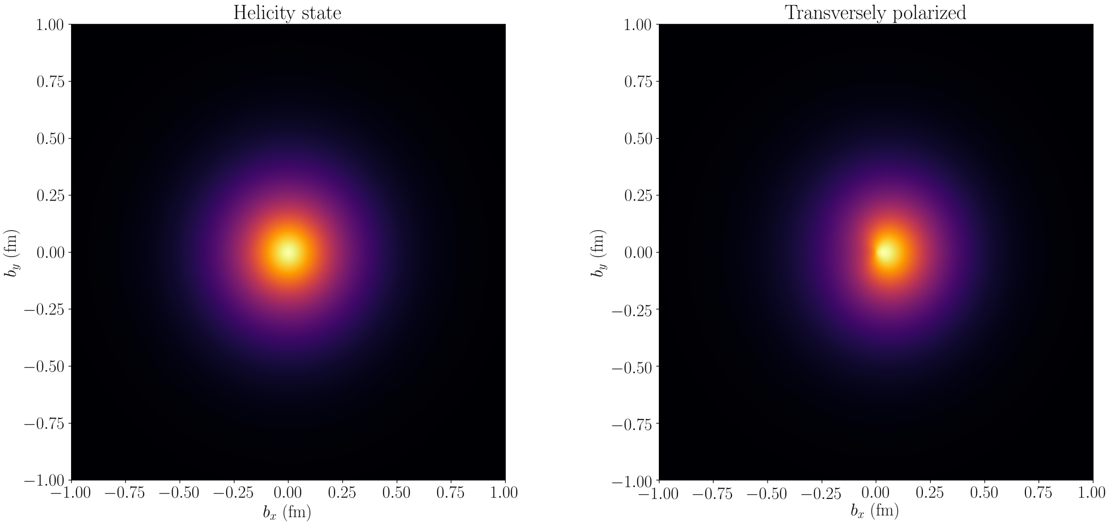
$$\langle \mathbf{s}_\perp|\hat{O}|\mathbf{s}_\perp\rangle = \frac{1}{2}\Big\{\langle +|\hat{O}|+\rangle + \langle -|\hat{O}|-\rangle + \langle +|\hat{O}|-\rangle e^{i\phi_s} + \langle -|\hat{O}|+\rangle e^{-i\phi_s}\Big\}\,,$$

- The P^+ density:

$$\rho^{(\mathrm{LF})}_{P^+,T}(\mathbf{b}_\perp,\mathbf{s}_\perp) = \rho^{(\mathrm{LF})}_{P^+}(b_\perp) + P^+\,\frac{\sin(\phi_b-\phi_s)}{2Mc}\,\frac{\mathrm{d}}{\mathrm{d}b_\perp}\int\frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}\Big(A(t)-2J(t)\Big)e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

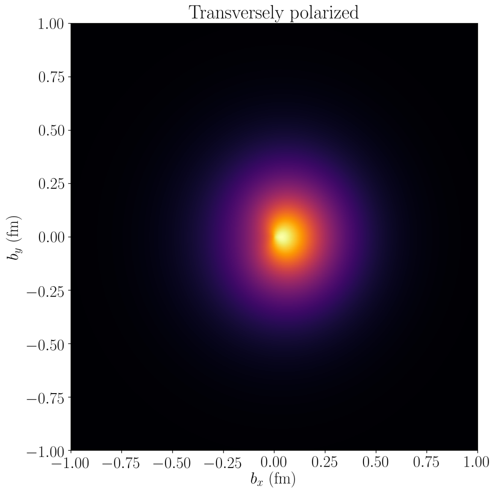
  - Now with spin dependence!

Spin dependence illustrated: P^+ density



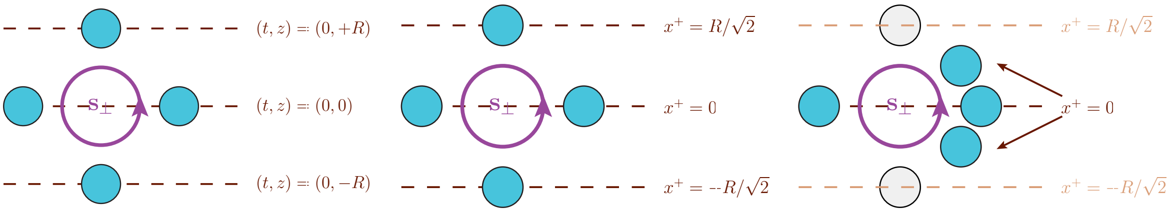
Model: $A(t)$ dipole & $J(t)$ tripole with $f_2(1270)$ mass at pole.

Features of transversely polarized state



- **Fully relativistic**
- Spin along positive y axis.
- **Not azimuthally symmetric**
  - $\sin(\phi)$ modulations if $A(t) \neq 2J(t)$
- **Cannot** obtain from spherically symmetric density by integrating out z.
  - **Cannot** apply inverse Abel transform!

Why density deforms for transverse polarization



- Short answer: **because light front time is not instant form time**
- Must evolve system from equal-t to get to equal-x^+.
  - Small z (closer) at fixed t: must evolve forward in time.
  - Big z (further) at fixed t: must evolve backward in time.

The stress tensor

- Spatial components of the EMT give the **stress tensor**.
- The **stress tensor** encodes distribution of forces.
- Recommended reading:
  - Polyakov and Schweitzer,
    International Journal of Modern Physics A23 (2018) 1830025
  - Lorcé, Moutarde and Trawinski,
    European Journal of Physics C79 (2019) 89

Stress tensor and hadron flow

- Stress tensor also encodes hadron flow.
  - *Includes wave function dispersion*
- In Galilean theory (non-relativistic or light front):

$$T^{ij}(\mathbf{x}, \mathbf{v}) = \mathbf{v}^i \mathbf{v}^j \rho(\mathbf{x}) + S^{ij}(\mathbf{x})$$

  - **Uncertainty principle**: $\mathbf{v}^i \mathbf{v}^j \rho(\mathbf{x})$ blows up when $R \to 0$.
  - $S^{ij}(\mathbf{x})$ remains finite.
- **Comoving stress tensor**: $S^{ij}(\mathbf{x}) = T^{ij}(\mathbf{x}, \mathbf{v} = 0)$.
  - Requires Galilean covariance.
  - Seen from perspective of comoving observer.

Momentum conservation and force balance

- Conservation law from Noether's theorem:

$$\partial_\mu T^{\mu\nu}(x) = 0$$

- Additional **force balance** equation:

$$\mathbf{F}(\mathbf{x}) = \nabla_i S^{ij}(\mathbf{x}) = 0$$

  - Applies to both NR and LF.
- **Force density acting on a hadron is everywhere zero.**
  - The hadron is in equilibrium.
  - The hadron is not being acted on by outside forces.
  - The torque density is also everywhere zero.

Non-relativistic vs light front

- Non-relativistic comoving stress tensor:

$$S^{ij}_{\mathrm{NR}}(\mathbf{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} \left(\frac{\boldsymbol{\Delta}^i\boldsymbol{\Delta}^j - \delta_{ij}\boldsymbol{\Delta}^2}{4M} \right) D(t) e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}}$$

- Light front comoving stress tensor (**spin zero or helicity case**):

$$S^{ij}_{\mathrm{LF}}(\mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \left(\frac{\boldsymbol{\Delta}^i_\perp\boldsymbol{\Delta}^j_\perp - \delta_{ij}\boldsymbol{\Delta}^2_\perp}{4P^+} \right) D(t) e^{-i\boldsymbol{\Delta}_\perp\cdot\mathbf{b}_\perp}$$

- Very similar! Differences:
  - 2 vs. 3 dimensions
  - M vs. P^+ (central charge of Galilean group)
  - Related by Abel transform, see Panteleeva and Polyakov, PRD**104**, 014008
  - Also see AF & Miller, arxiv:2108.03301

Abel transform caveats: Round 1

- Abel transform only connects non-relativistic & light front pressures.
- Standard "Breit frame" pressures identical to NR **only for spin-half**.
  - Spin-half:

$$S^{ij}_{\mathrm{BF}}(\mathbf{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} \left(\frac{\boldsymbol{\Delta}^i \boldsymbol{\Delta}^j - \delta_{ij}\boldsymbol{\Delta}^2}{4M} \right) D(t) e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} = S^{ij}_{\mathrm{NR}}(\mathbf{r})$$

  - Spin-zero:

$$S^{ij}_{\mathrm{BF}}(\mathbf{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} \left(\frac{\boldsymbol{\Delta}^i \boldsymbol{\Delta}^j - \delta_{ij}\boldsymbol{\Delta}^2}{4M\sqrt{1 - t/(4M^2c^2)}} \right) D(t) e^{-i\boldsymbol{\Delta}\cdot\mathbf{r}} \neq S^{ij}_{\mathrm{NR}}(\mathbf{r})$$

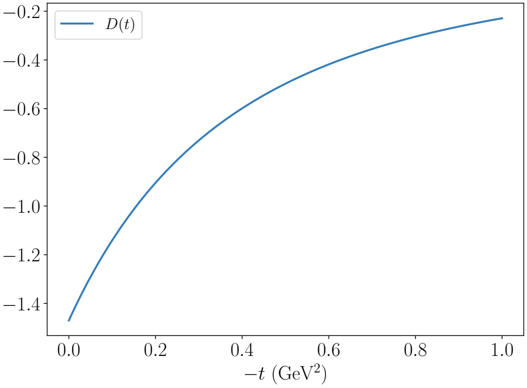
- **It is erroneous to say Abel transforms connect light front to Breit frame**
  - See AF & Miller, arxiv:2108.03301 for more detail

Experimental pressure data

- $D(t)$ experimentally found from DVCS.
- V.D. Burkert, L. Elouadrhiri, F.X. Girod, arxiv:2104.02031
- All plots I show use $D(t)$ from the above paper:

$$D(t) = \frac{D(0)}{(1 - t/\Lambda^2)^\alpha}$$

$$\begin{aligned} D(0) &= -1.47 \pm 0.06 \pm 0.14 \\ \Lambda^2 &= \quad 1.02 \pm 0.13 \pm 0.21 \text{ GeV}^2 \\ \alpha &= \quad 2.76 \pm 0.23 \pm 0.48 \end{aligned}$$



Radial and tangential eigenpressures

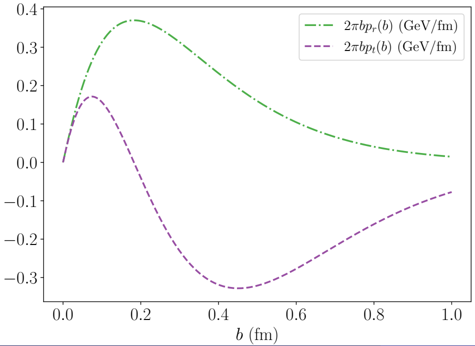
- Pressures are elements of **comoving stress tensor**:

$$p_v(b_\perp) = \hat{v}_i \hat{v}_j S^{ij}_{\mathrm{LF}}(\mathbf{b}_\perp)$$

- Has two eigenvectors with **eigenpressures**.

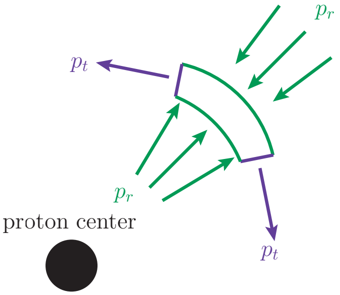
$$S^{ij}_{\mathrm{LF}}(\mathbf{b}_\perp)\hat{v}_j = \lambda_v(b_\perp)\hat{v}_i$$

- Eigenpressures are **radial** and **tangential**.



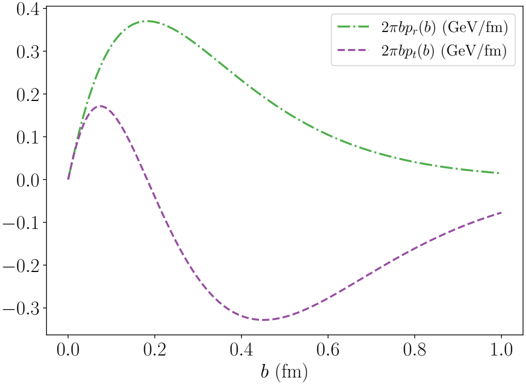
$$p_r(b_\perp) = \hat{r}_i \hat{r}_j S^{ij}(\mathbf{b}_\perp)$$

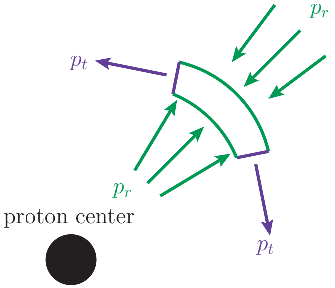
$$p_t(b_\perp) = \hat{\phi}_i \hat{\phi}_j S^{ij}(\mathbf{b}_\perp)$$



Net force on slab is zero

About the signs





Net force on slab is zero

- **Net force everywhere is zero**
- Positive means pressure is pushing from both sides.
- Negative means pressure is pulling from both sides.

Isotropic pressure and pressure anisotropy

- Comoving stress tensor can be written:

$$S^{ij}(\mathbf{b}) = \delta^{ij} p(b) + \left(\frac{b^i b^j}{b^2} - \frac{1}{2}\delta^{ij} \right) s(b)$$

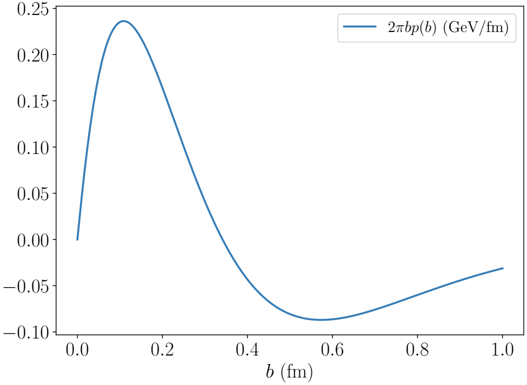
- $p(b)$ is the **isotropic pressure**.

$$p(b) = \frac{1}{2}\Big(p_r(b) + p_t(b)\Big)$$

- $s(b)$ is **pressure anisotropy**.

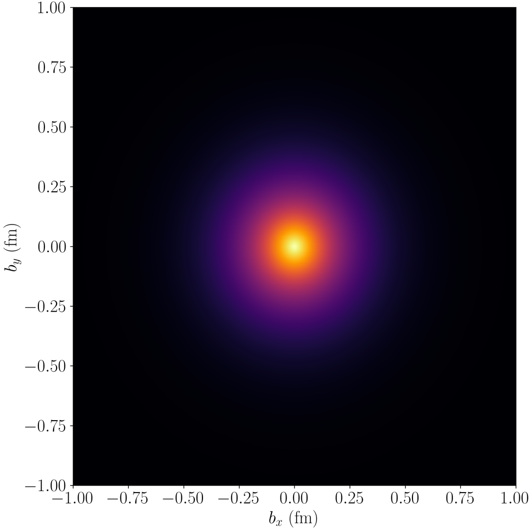
$$s(b) = p_r(b) - p_t(b)$$

  - Also called shear.



- Isotropic pressure plot.
- **Not a net force plot.**
- Says **nothing** about force towards origin.

Empirical proton pressure: helicity state



- Empirical proton $p_r(b)$.
- Helicity state.
- $p_r(b) > 0$
- Gives **mechanical radius**.

$$\sqrt{\langle b^2 \rangle_{\mathrm{mech}}} = 0.518 \pm 0.062 \pm 0.126 \text{ fm}$$

- See AF & Gerald Miller, PRD**104**, 014024

Transverse polarization and pressure

- Transverse polarization changes comoving stress tensor:

$$S_T^{ij}(\mathbf{b},\mathbf{s}_\perp) = \delta_{ij} p_T(b,\phi) + \left(\frac{b^i b^j}{b^2} - \frac{1}{2}\delta^{ij}\right) s_T(b,\phi) + \left(\frac{b^i \tilde{s}_\perp^j + b^j \tilde{s}_\perp^i}{b} - 2\sin(\phi)\frac{b^i b^j}{b^2}\right) v_T(b)$$

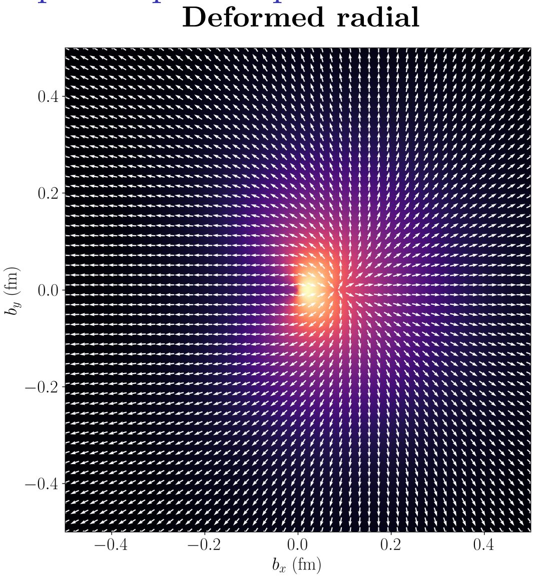
$$p_T(b,\phi) = p(b) + \frac{\sin(\phi)}{2M} p'(b)$$

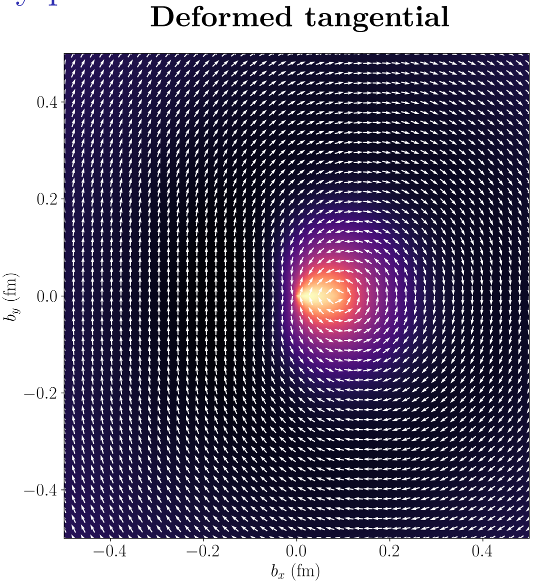
$$s_T(b,\phi) = s(b) + \frac{\sin(\phi)}{2M} s'(b)$$

$$v_T(b) = \frac{s(b)}{2Mb}$$

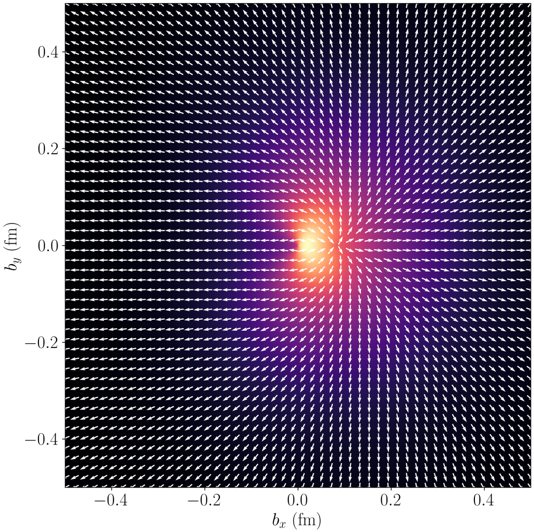
- New $v_T(b)$ structure
- Radial and tangential pressures **no longer eigenpressures**.

Empirical proton pressure: transversely polarized state





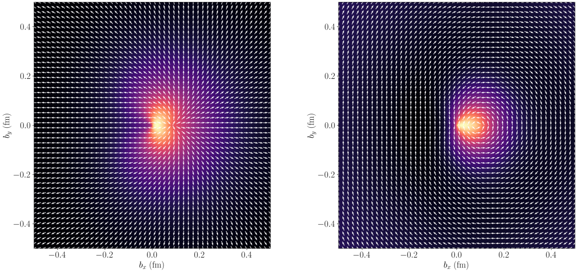
Features of transversely polarized pressure



- **Fully relativistic**
- Deformed from radial pressure
- Spin along positive y axis
- Eigenpressure **not azimuthally symmetric**
- Center shifted right

Full expressions in arxiv:2108.03301

Abel transform caveats: Round 2



- Abel transform only connects **azimuthally symmetric** & **spherically symmetric** functions
- For **transverse polarization**:
  - NR & "Breit frame" are spherically symmetric
  - Light front is **not azimuthally symmetric**

It is erroneous to say Abel transforms connect light front to Breit frame

- See AF & Miller, arxiv:2108.03301 for more detail

Summary

- Obtaining EMT densities requires **non-relativistic limit** or **light front coordinates**.
- **Light front coordinates** give a fully relativistic picture.
- **Galilean covariance** was needed to isolate comoving stress tensor.
- Transverse polarization introduces **relativistic spin effects** in momentum density & pressure.

The End

Thanks for your time!

Based on work in:

- AF & Gerald Miller, PRD**103**, 094023
- AF & Gerald Miller, PRD**104**, 014024
- AF & Gerald Miller, arxiv:2108.03301