BK21FOUR Lecture Series

Chueng-Ryong Ji
North Carolina State University

Two Lectures (2nd Day)

- **Light-Front QCD in Hadron Physic I**

a) Introduction of QCD
b) Color Confinement and Chiral Symmetry
c) Dirac's Proposition for Relativistic Dynamics

- **Light-Front QCD in Hadron Physic II**

a) Distinguished Features of Light-Front Dynamics
b) Large N_C QCD in 1+1 dim. ('tHooft Model)
c) Application to Hadron Phenomenology

July 7-8, KNU Physics

QED Example

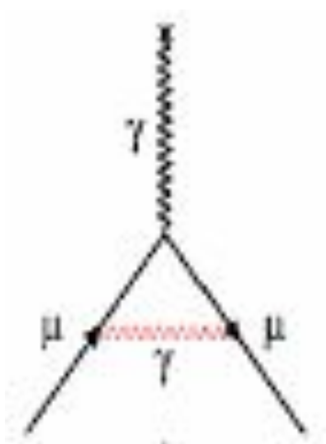
Anomalous Magnetic Moment

- Magnetic moment of a particle is related to its spin.

$$\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$$

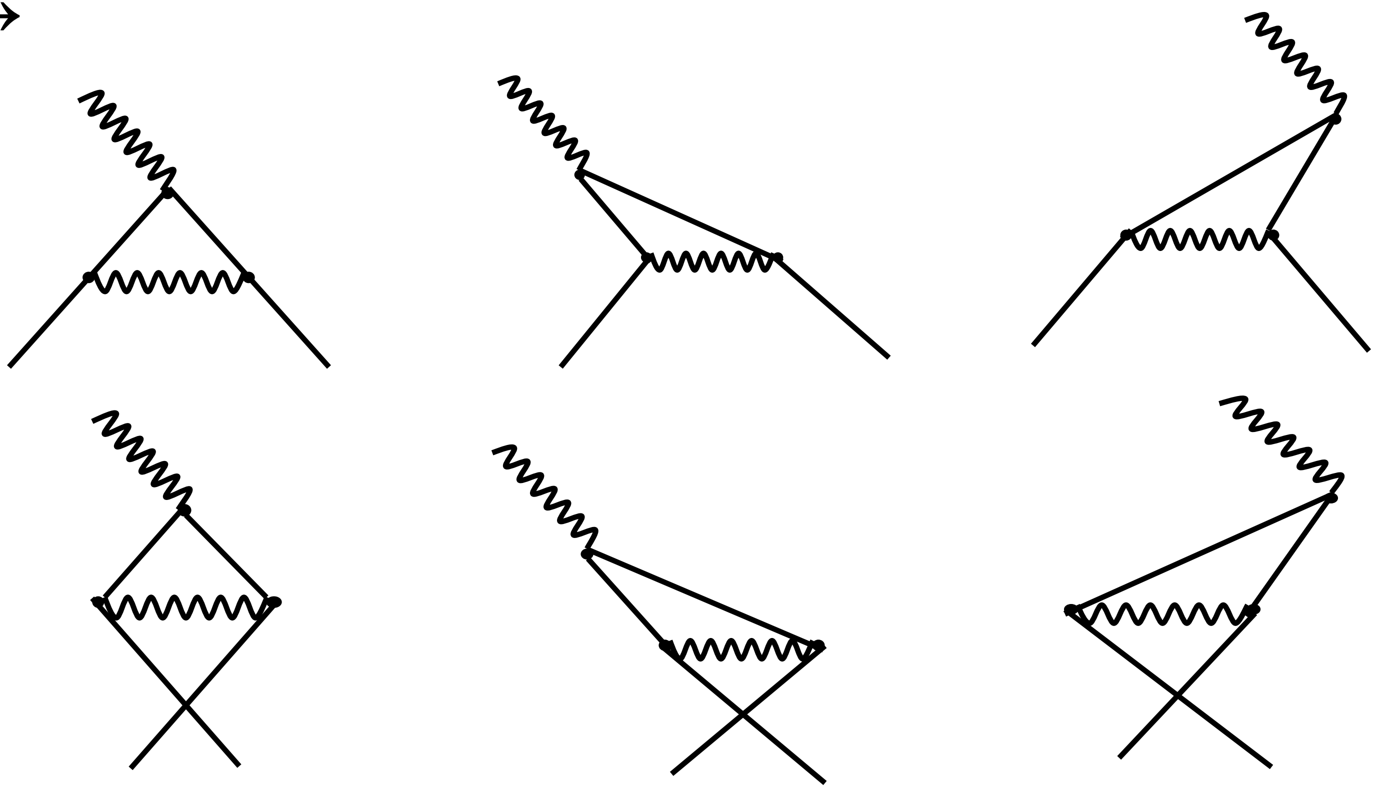
- For Dirac pointlike particle, g=2. However, the loop correction in QFT yields the non-zero g-2,

i.e. anomalous magnetic moment.



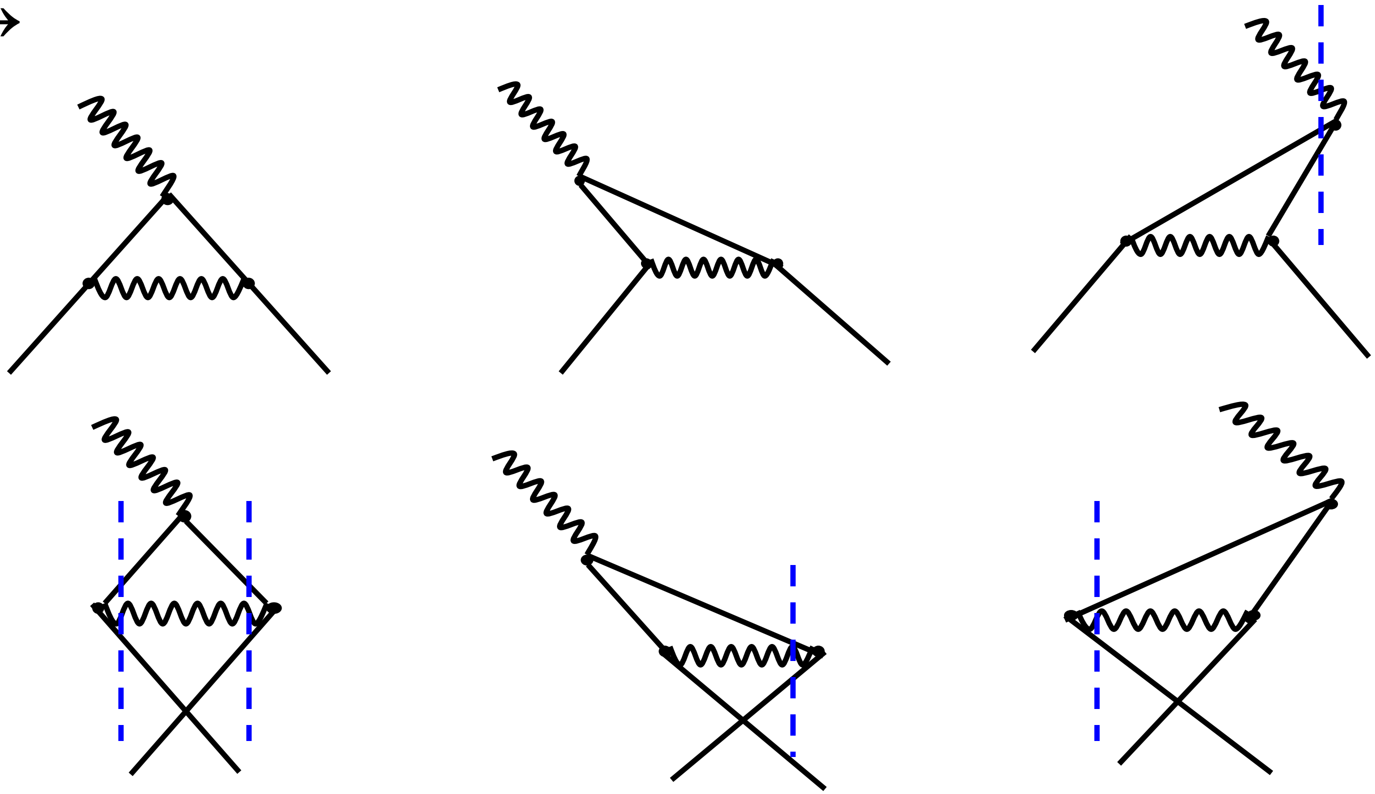
g-2 calculation

t→

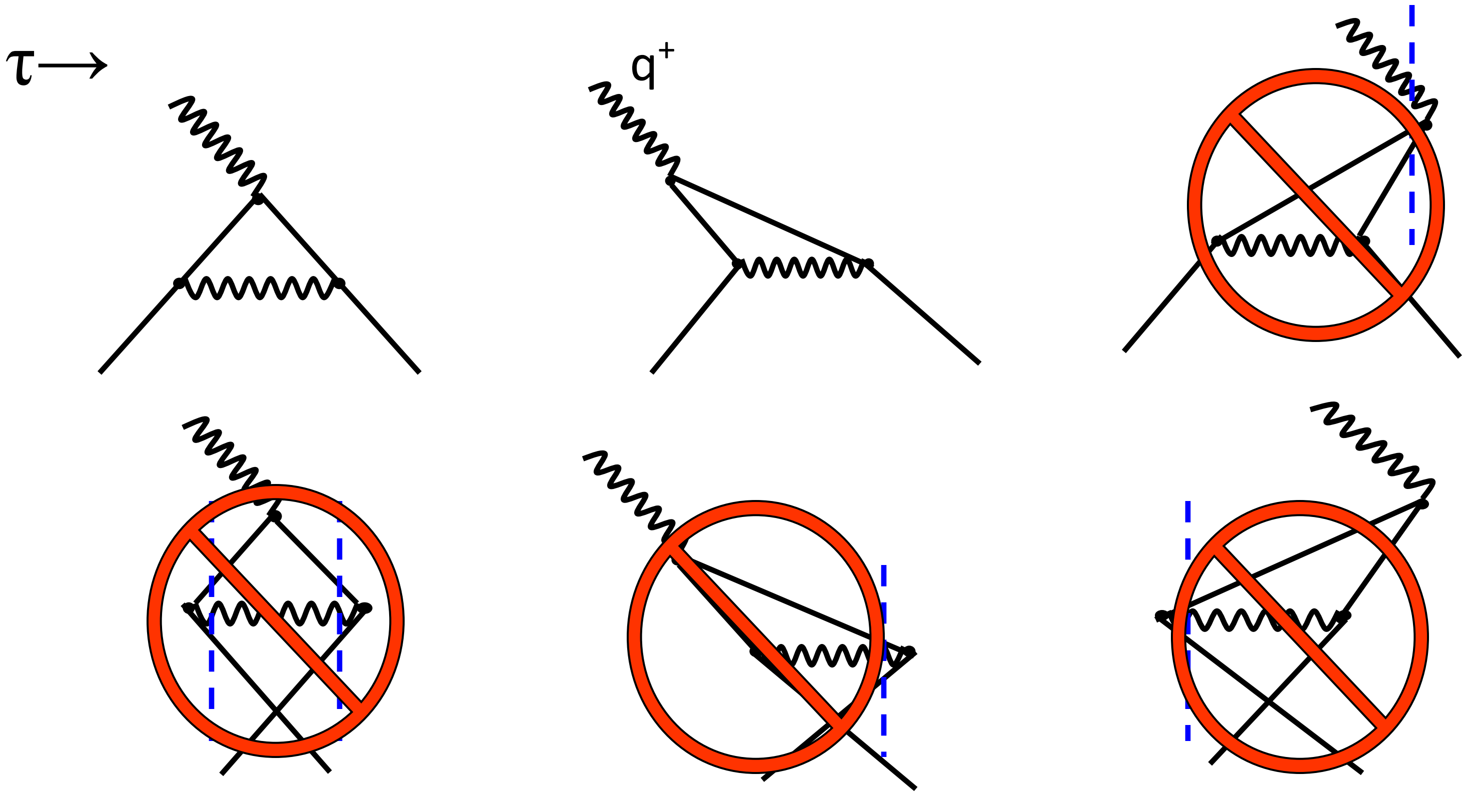


g-2 calculation

t→

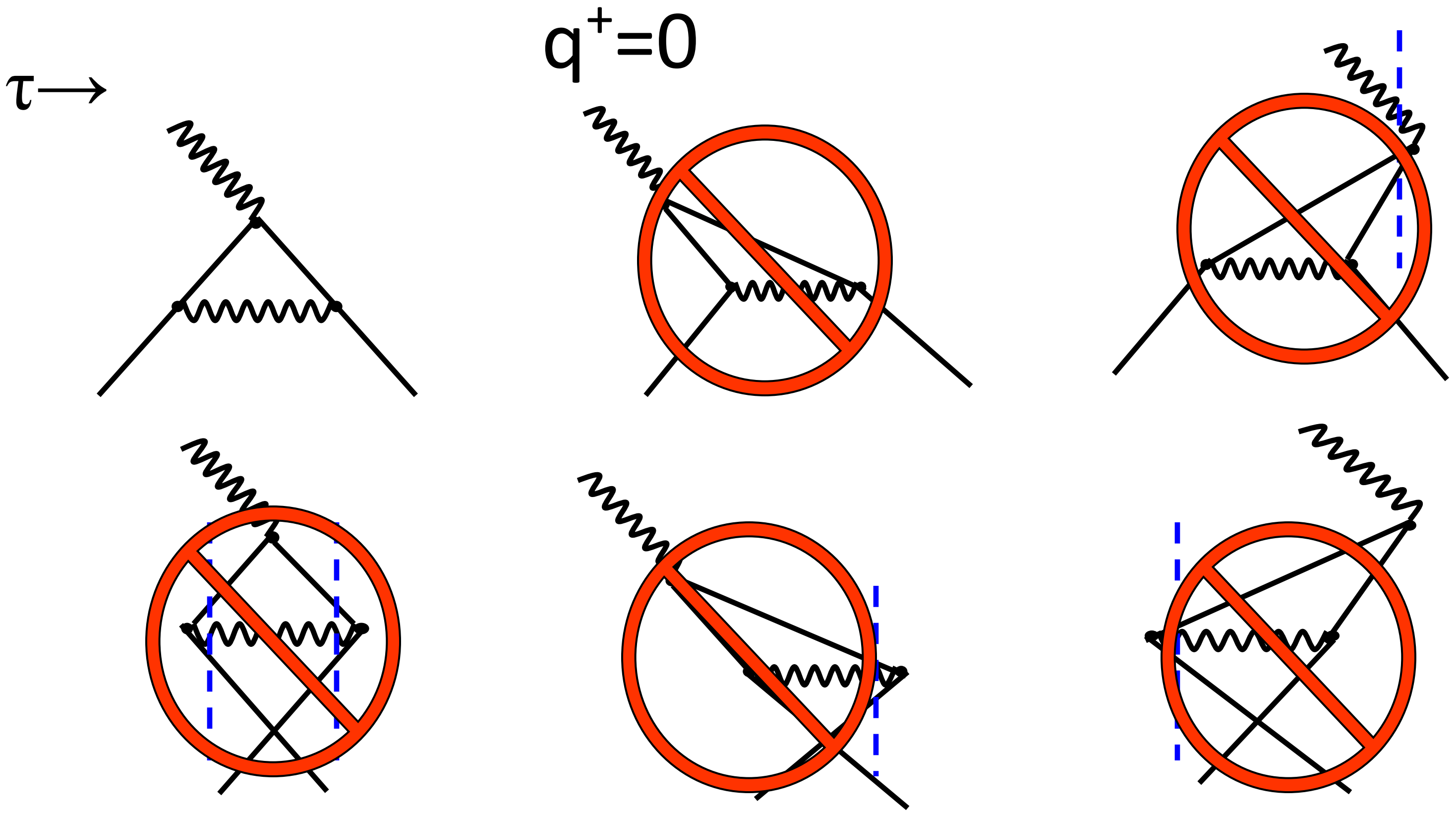


g-2 calculation



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

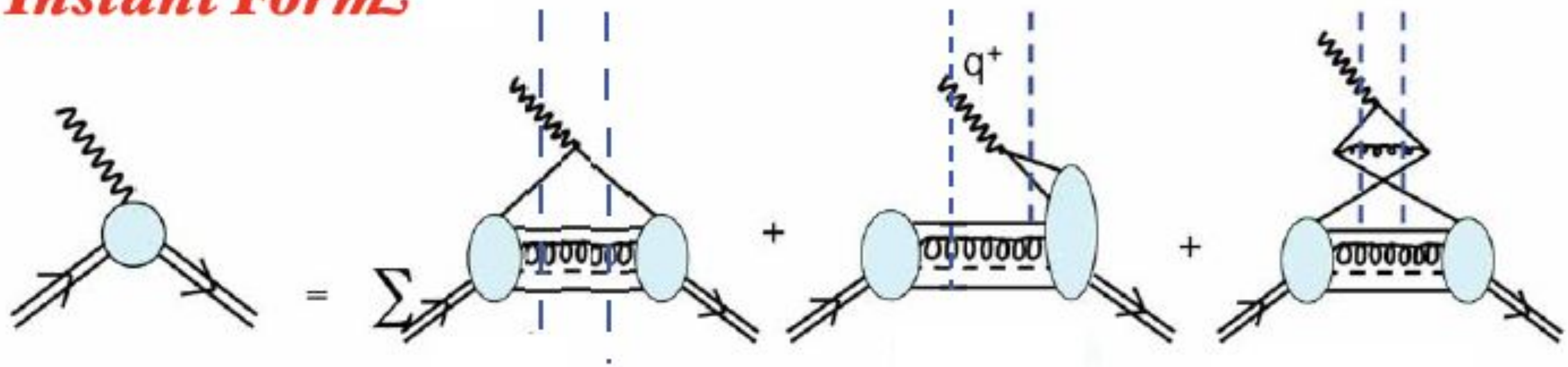
g-2 calculation



- Vacuum fluctuations are suppressed in LFD and clean hadron phenomenology is possible.

Calculation of Form Factors in Equal-Time Theory

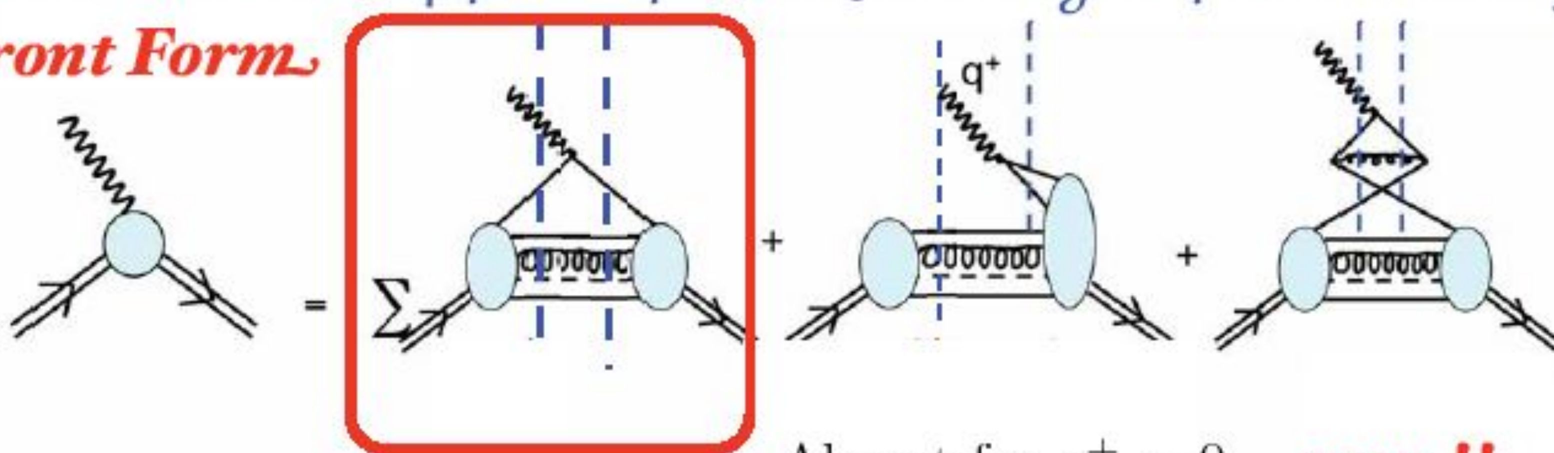
Instant Form



Need vacuum-induced currents

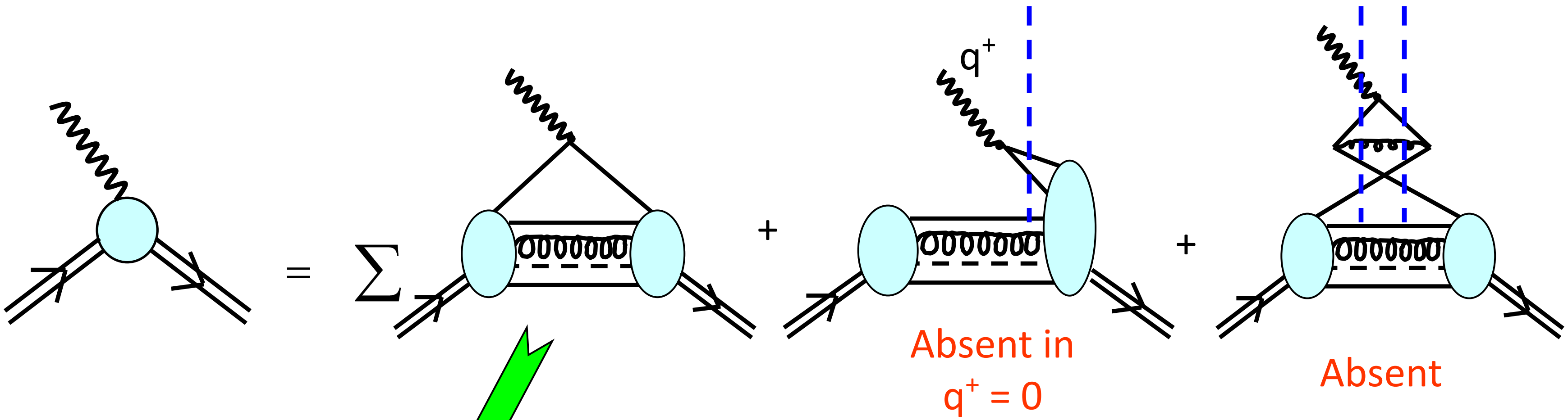
Calculation of Form Factors in Light-Front Theory

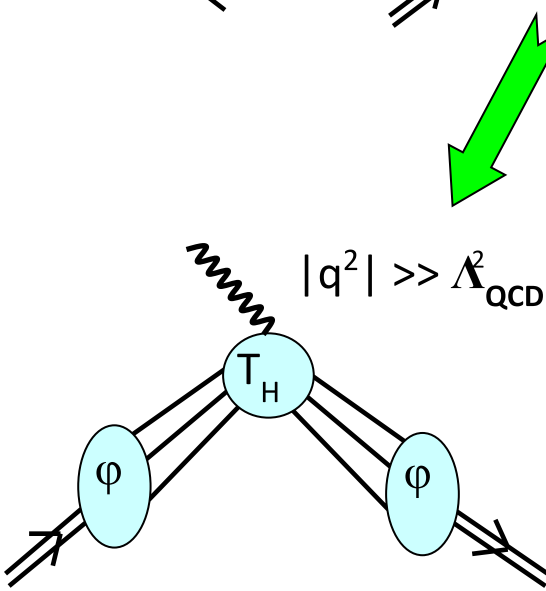
Front Form

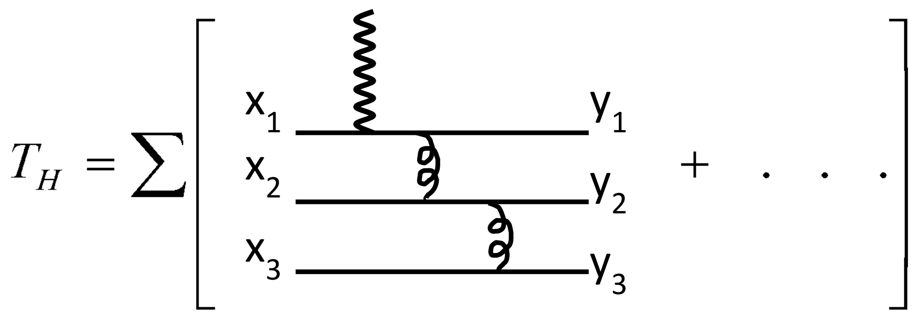


Absent for $q^+ = 0$ **zero !!**

LFD in Exclusive Processes



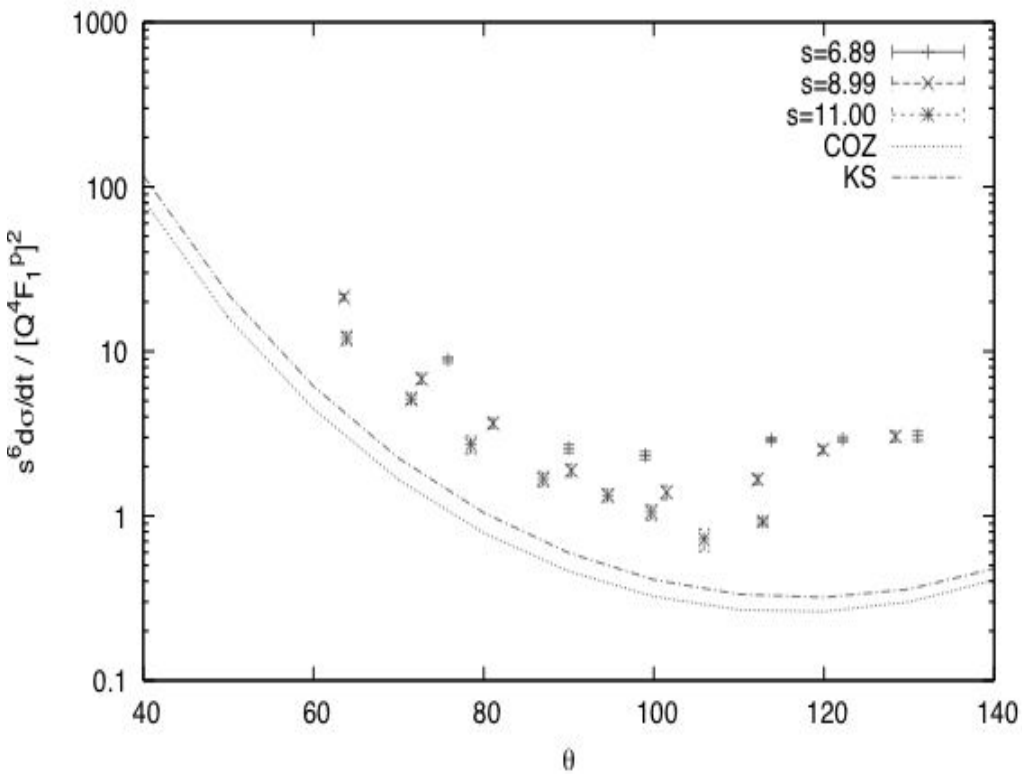


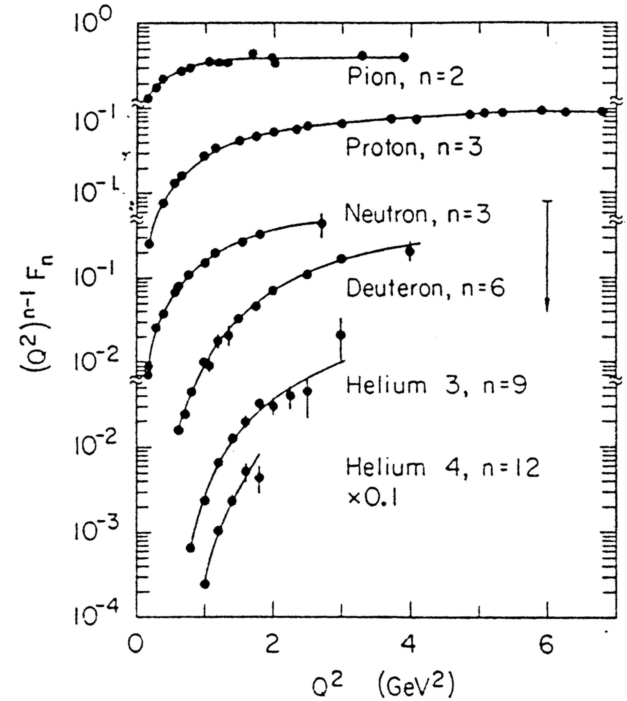


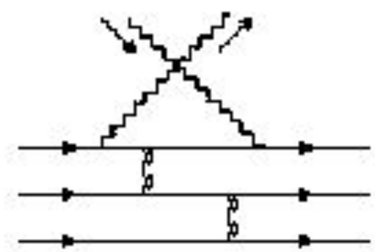
$$= \frac{\alpha_s^2}{Q^4} f(x_i, y_i)$$

Factorization of the hard part from the soft part is much clearer in LFD than in IFD.

The Quark Counting Rule and PQCD Predictions of Exclusive Processes







6x7x8=336

Pang & Ji, J.Comp.Phys.115,267(94)

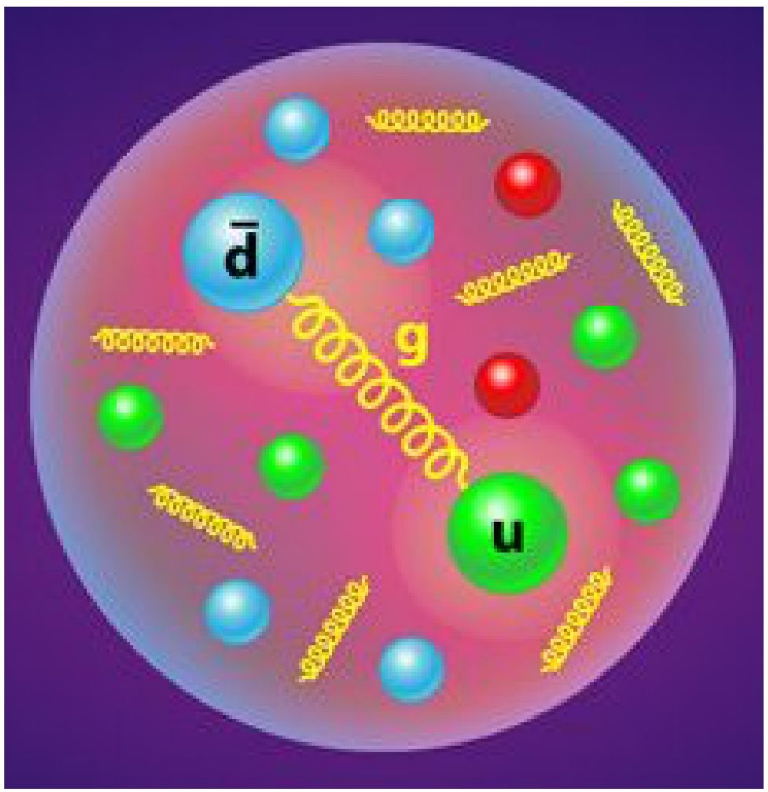
R. Thomson, A. Pang and C.Ji, PRD73,054023(2006)

JLab Hall A Collaboration, PRL98, 152001(2007);

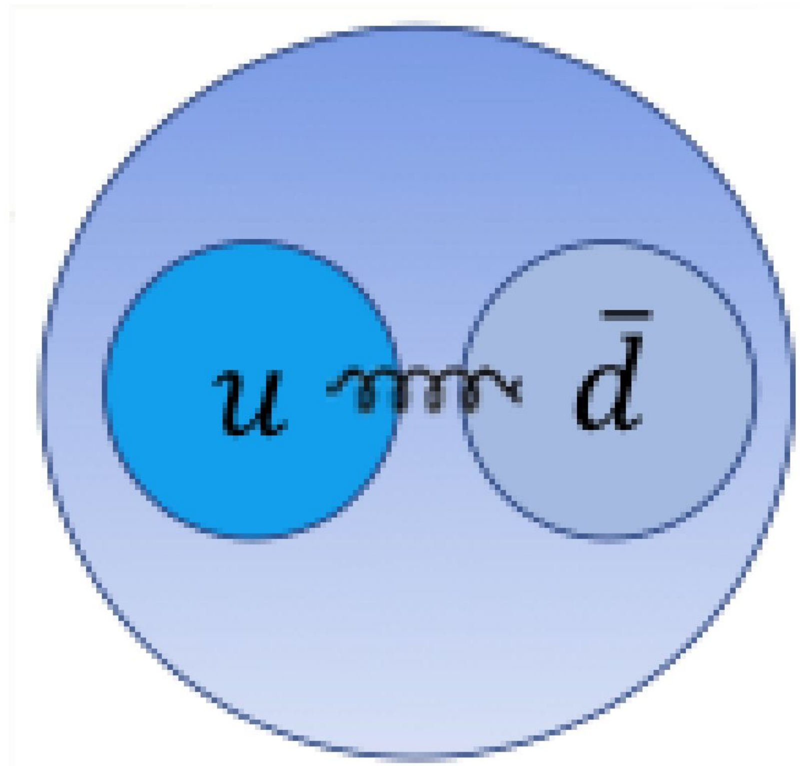
More data forthcoming from 12 GeV upgraded JLab.

How do we understand the Quark Model in Quantum Chromodynamics?

Take advantage of LFD and Construct the Light-Front Quark Model (LFQM)



QCD



LFQM

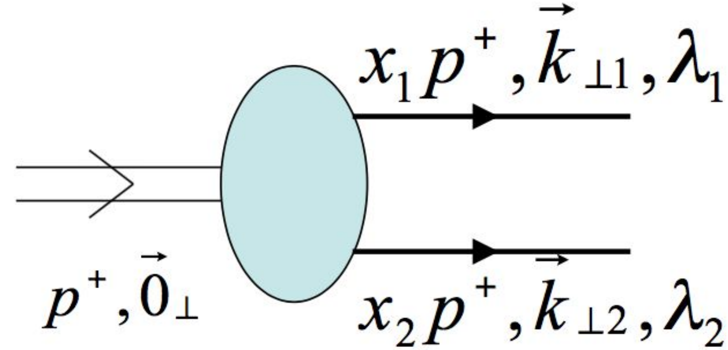
Effective Constituent Quark Model for Low Q^2

$$
\begin{aligned}
|Meson\rangle &= \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \dots \\
&\approx \Psi_{Q\bar{Q}}|Q\bar{Q}\rangle,
\end{aligned}
$$

where

$$
|Q\rangle = \psi_q^Q|q\rangle + \psi_{qg}^Q|qg\rangle + \dots
$$

$$
|\bar{Q}\rangle = \psi_{\bar{q}}^{\bar{Q}}|\bar{q}\rangle + \psi_{\bar{q}g}^{\bar{Q}}|\bar{q}g\rangle + \dots
$$



$$
\Psi_{Q\bar{Q}}(x_i, \vec{k}_{\perp i}, \lambda_i) = \Phi(x_i, \vec{k}_{\perp i})\chi(x_i, \vec{k}_{\perp i}, \lambda_i)
$$

Radial

(Dependent on the model potential)

H = T + V

V includes Coulomb, Confinement, Spin-Spin, Spin-Orbit interactions.

Spin-Orbit

(Interaction independent Melosh transformation)

$$
\begin{aligned}
J^{PC} &= 0^{++}(f_0, a_0, \dots) \\
&\quad 0^{-+}(\pi, K, \eta, \eta', \dots) \\
&\quad 1^{--}(\rho, K^*, \omega, \phi, \dots) \\
&\quad \dots
\end{aligned}
$$

PHYSICAL REVIEW C **92**, 055203 (2015)

Variational analysis of mass spectra and decay constants ···

Ho-Meoyng Choi,[1] Chueng-Ryong Ji,[2] Ziyue Li,[2] and Hui-Young Ryu[1]

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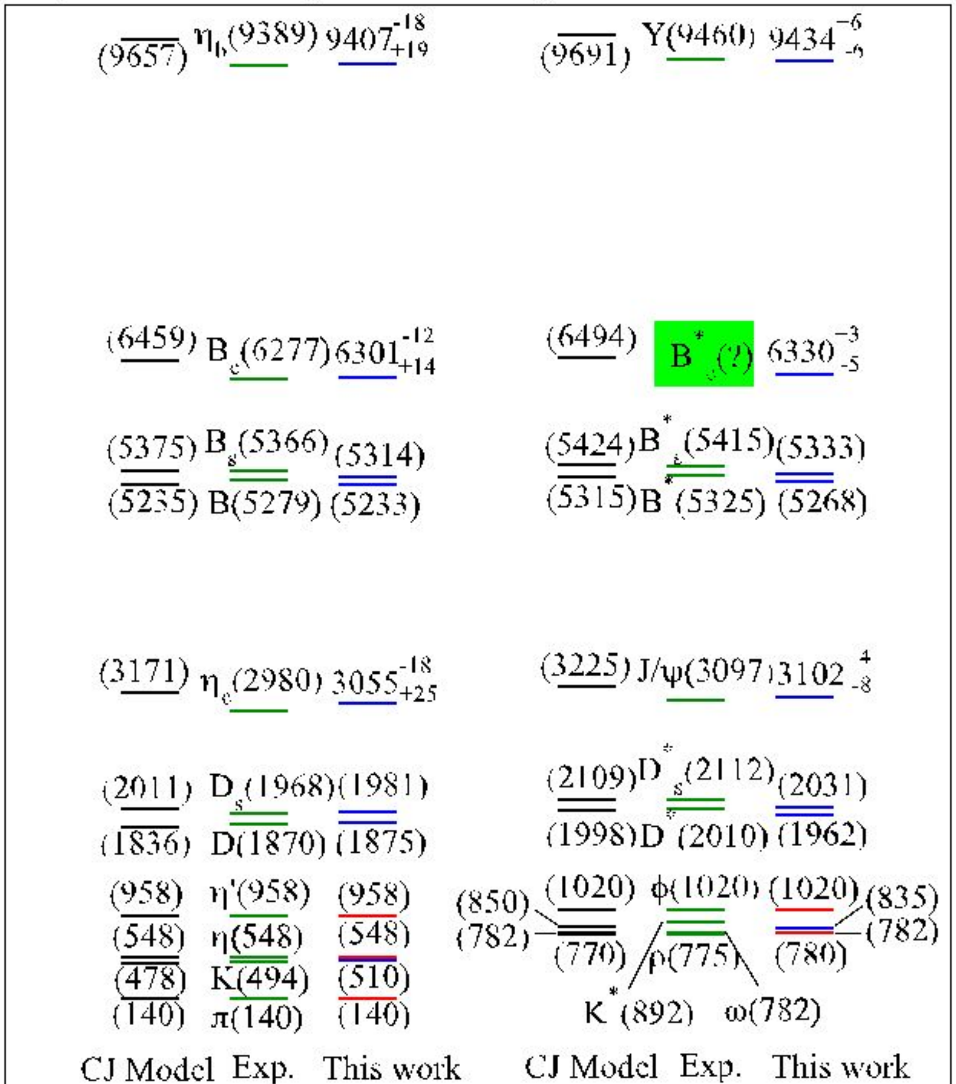
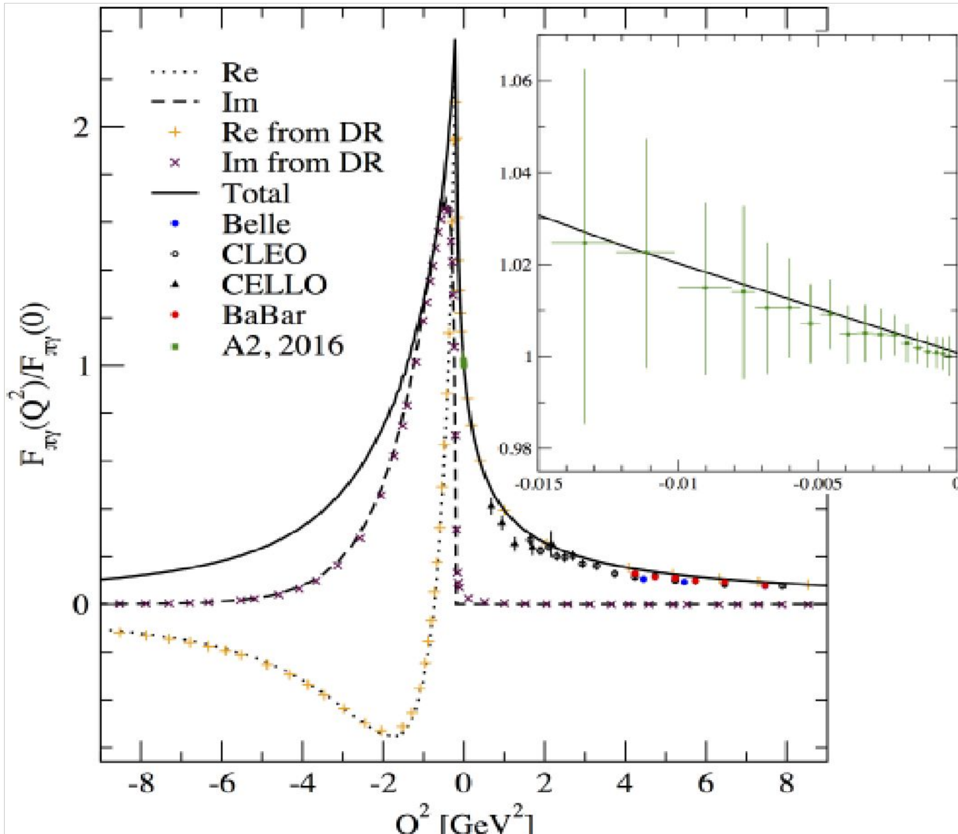
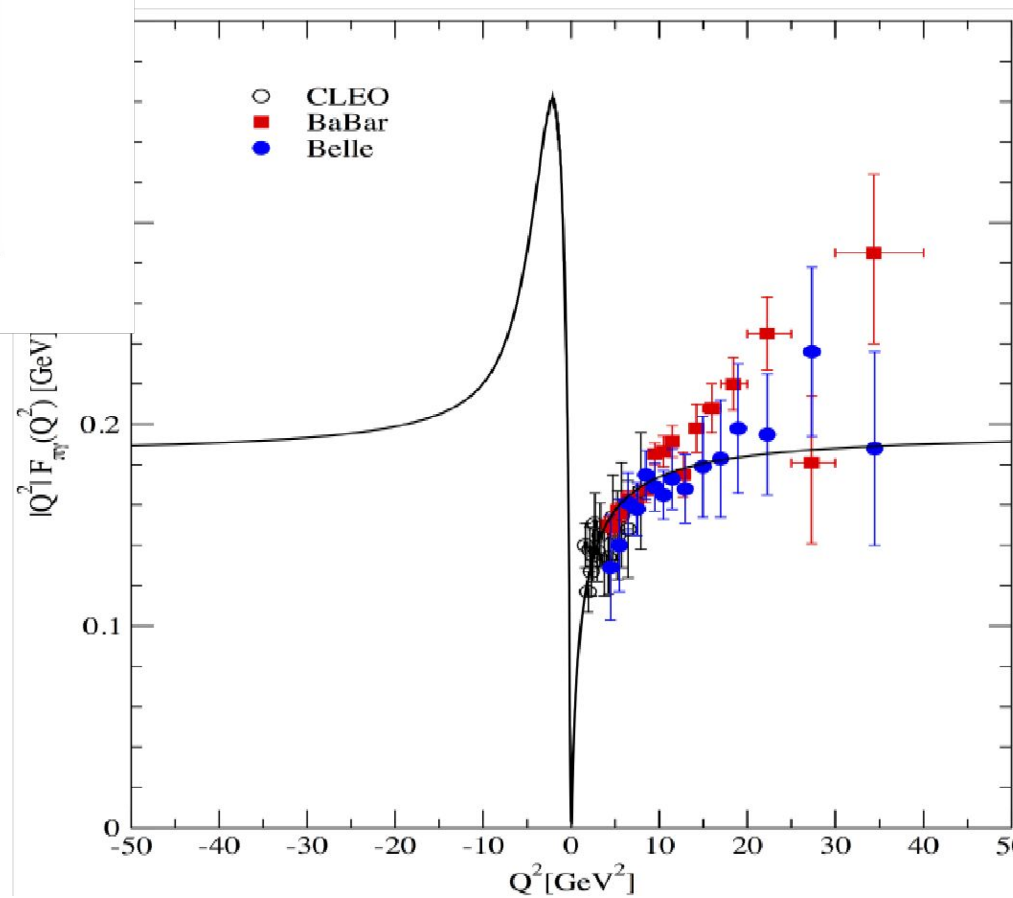


Both spacelike and timelike form factors can be computed in LFQM.





H.-M. Choi, H.-Y. Ryu, C.-R.Ji, PRD96, 056008 (2017)

PHYSICAL REVIEW D **102,** 036005 (2020)

Chiral anomaly and the pion properties in the light-front quark model

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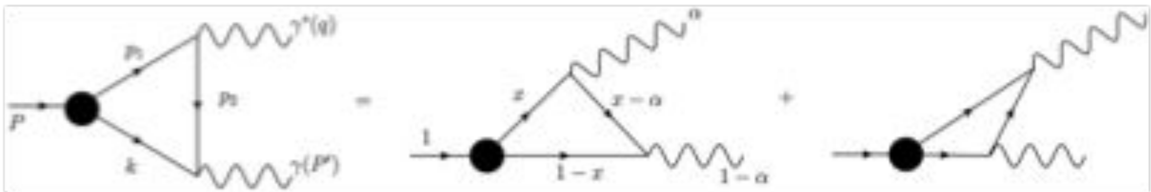


$$\Gamma_{\pi^0\to\gamma\gamma} = \frac{\pi}{4}\alpha_{\rm em}^2 M_\pi^3 |F_{\pi\gamma}(0)|^2$$

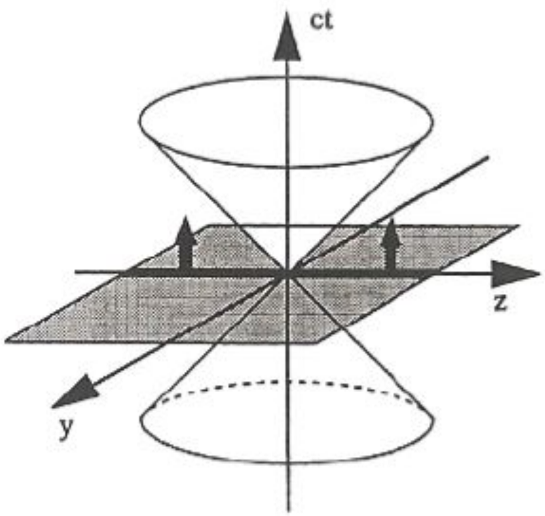
$$\Psi^\pi_{Q\bar{Q}} \equiv \Psi_\pi(x_i, \mathbf{k}_{i\perp}, \lambda_i) = \phi_R(x_i, \mathbf{k}_{i\perp})\chi(x_i, \mathbf{k}_{i\perp}, \lambda_i),$$

$$\chi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp) = \mathcal{N}\bar{u}_{\lambda_1}(k_1)\Gamma_\pi v_{\lambda_2}(k_2),$$

$$\Gamma_\pi = (A_\pi + B_\pi \not{P})\gamma_5$$

$$F_{\pi\gamma}(q^2) = \frac{e_u^2 - e_d^2}{\sqrt{2}}\frac{\sqrt{2N_c}}{4\pi^3}\int_0^1 \frac{dx}{(1-x)}\int d^2\mathbf{k}_\perp \frac{\psi_\pi(x, \mathbf{k}_\perp)}{M_0^2 - q^2}$$

Can IFD and LFD be linked?

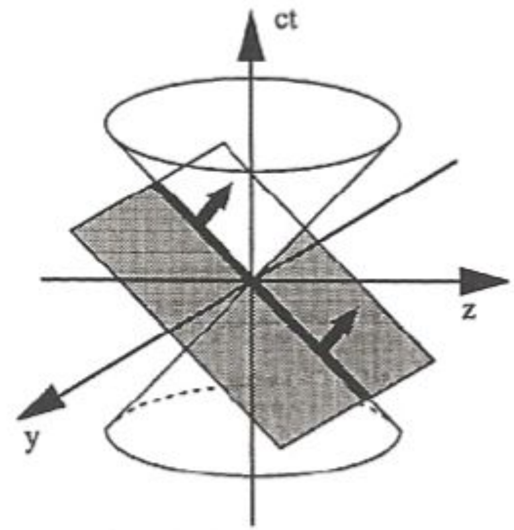


The instant form

1949

Yes, they can!

1992~2021



The front form

Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

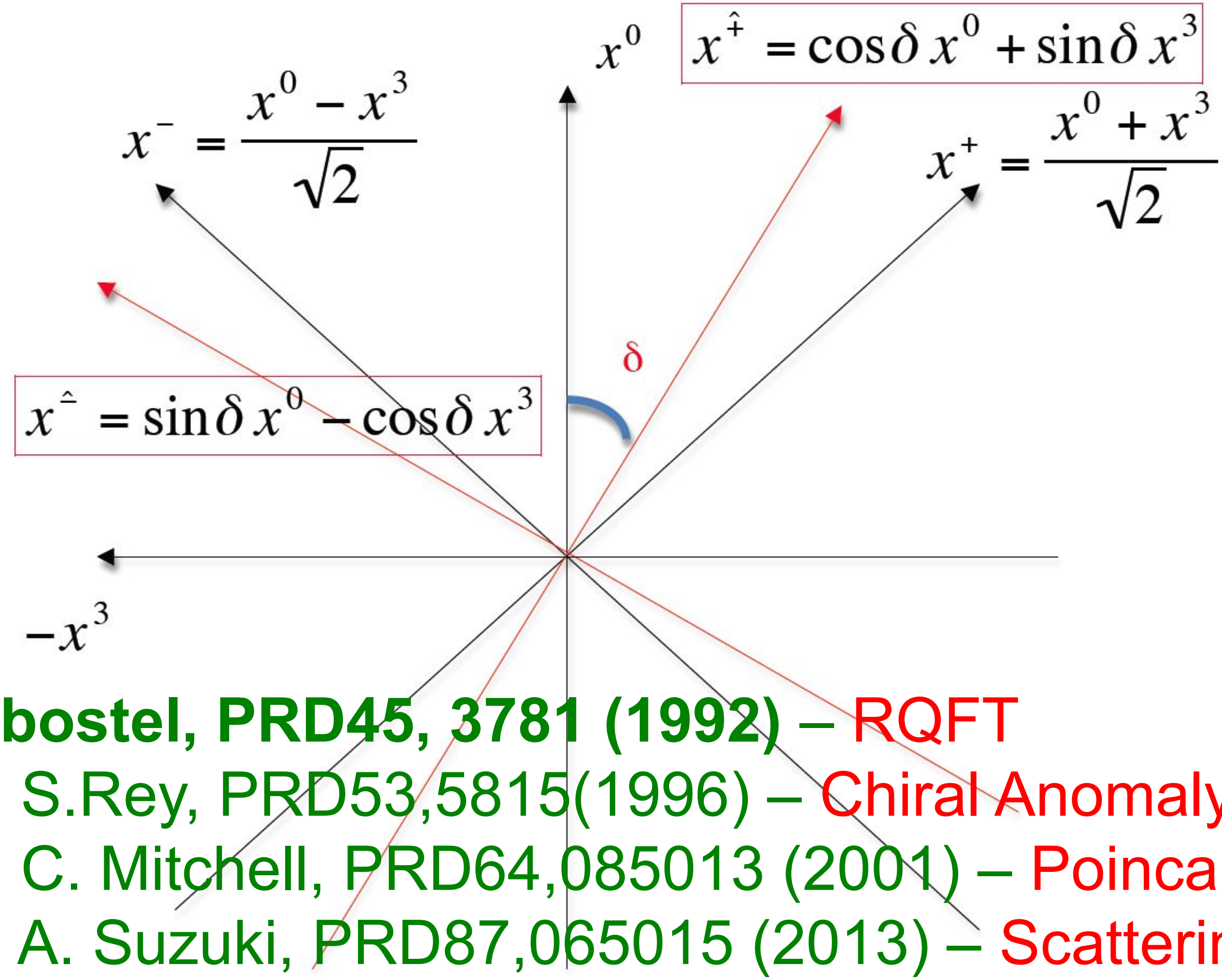
T-dept QFT, LQCD, IMF, etc.

Innovative approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

Interpolation between IFD and LFD



K. Hornbostel, PRD45, 3781 (1992) – RQFT

C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly

C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra

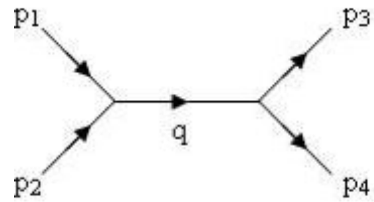
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps

C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges

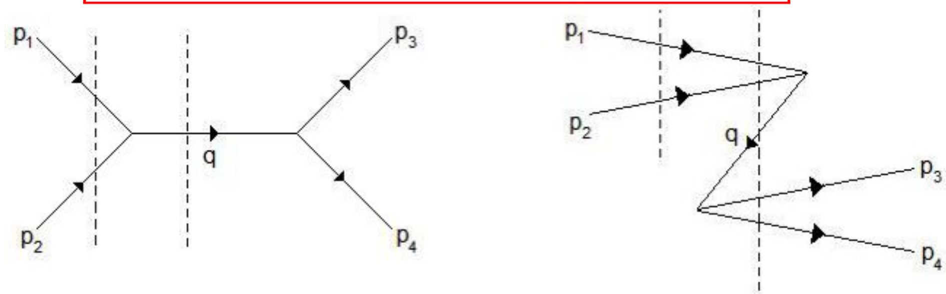
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors

C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED

B.Ma and C.Ji, arXiv:2105.09388v1[hep-ph], – QCD_{1+1}



$\delta = 0$	$0 < \delta < \pi/4$	$\delta = \pi/4$
$p_0 = p^0$	$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$	$p_+ = p^-$
$-p_3 = p^3$	$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$	$p_- = p^+$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2\right)}$$

$$\mathbb{C} = \cos 2\delta$$

$$\mathbb{S} = \sin 2\delta$$

$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_{\perp}^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \rightarrow \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_{\hat{\perp}}^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\rightarrow \infty \quad \text{as } \mathbb{C} \rightarrow 0$$

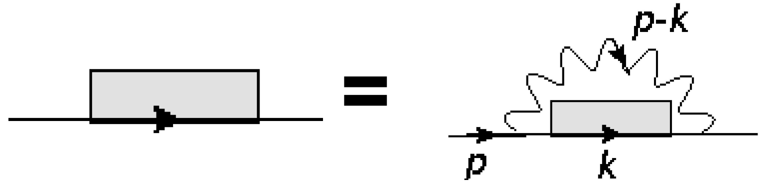
Large N_c QCD in 1+1 dim. ('tHooft Model)

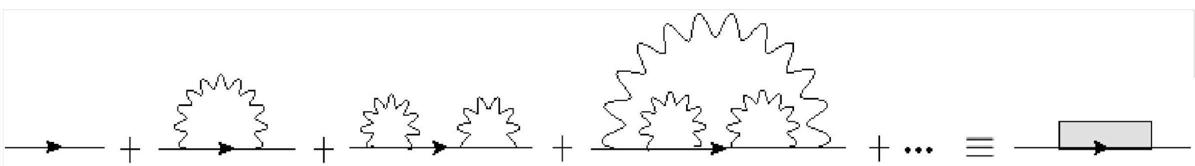
Interpolating 't Hooft model between instant and front forms

Bailing Ma and Chueng-Ryong Ji

arXiv:2105.09388v1 [hep-ph] 19 May 2021

MASS GAP EQUATION





$$\equiv i\,\Sigma(p)$$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k}-m-\Sigma(k_{\hat{-}})+i\epsilon}\gamma^{\hat{+}}$$

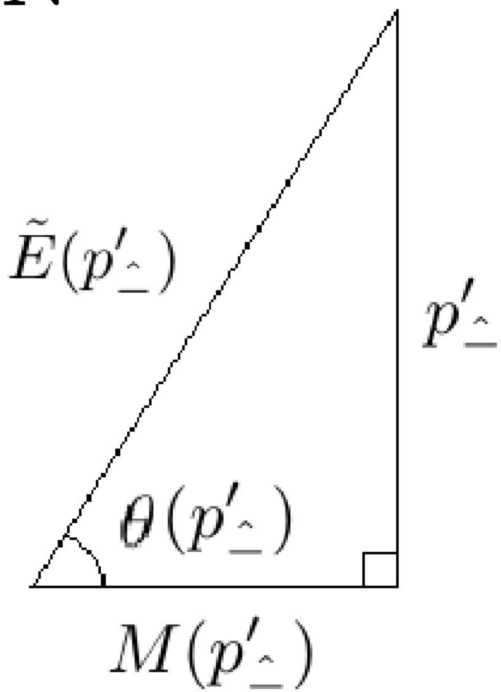
MASS GAP SOLUTION

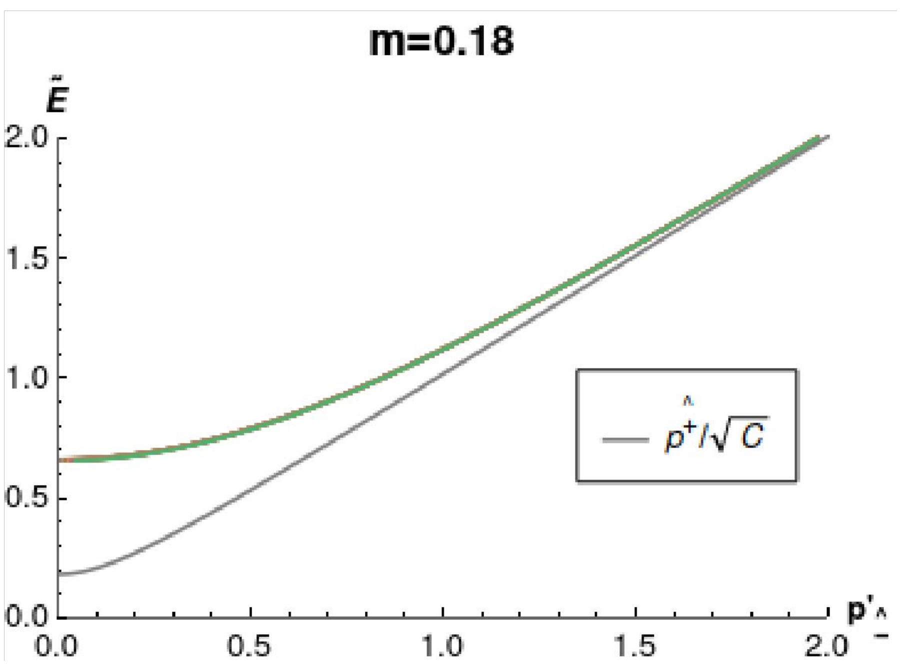
$$E = \sqrt{p_z^2 + m^2}\ ; \theta = \tan^{-1}(p_z / m)$$

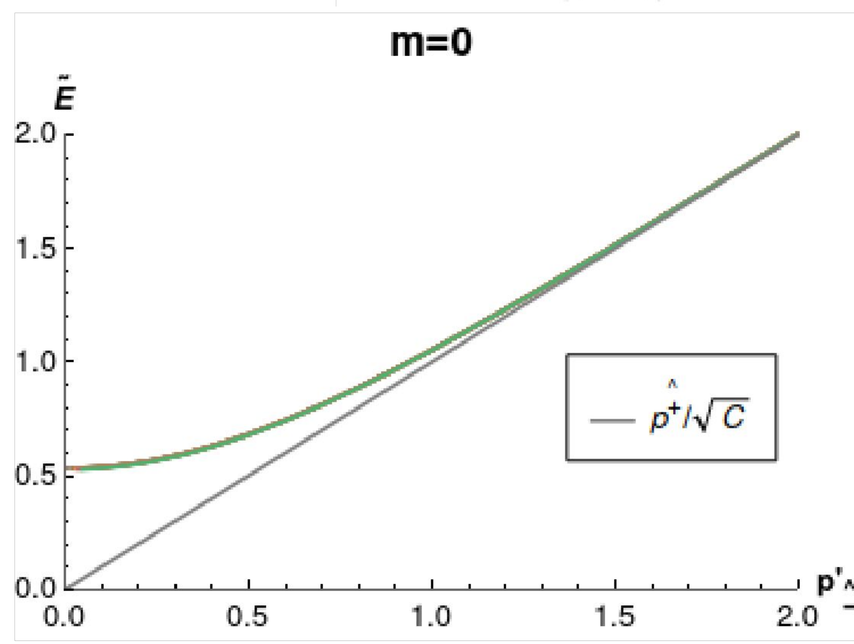
$$(E, p_z) \Rightarrow (p^{\hat{+}} / \sqrt{C}, p'_{\underline{\wedge}} = p_{\underline{\wedge}} / \sqrt{C})$$

$$m \Rightarrow M(p'_{\underline{\wedge}})$$

$$\tilde{E}(p'_{\underline{\wedge}}) = \sqrt{p'^2_{\underline{\wedge}} + M(p'_{\underline{\wedge}})^2}$$

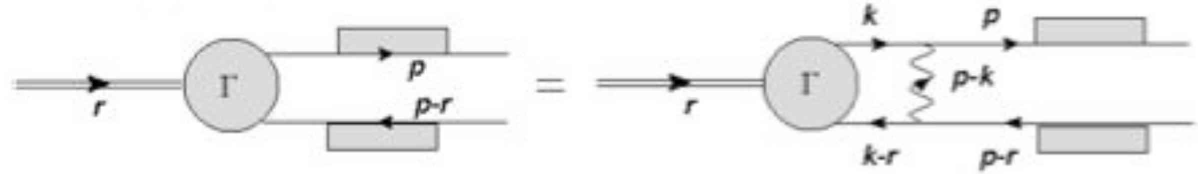






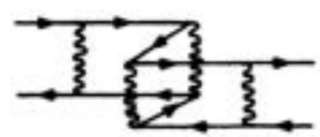
BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$



$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}})\right],$$

$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}} - r_{\hat{-}}) + E(p_{\hat{-}} - r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}}, p_{\hat{-}})$$
$$= \lambda \int \frac{dk_{\hat{-}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\left[C(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}}, k_{\hat{-}}) - S(p_{\hat{-}}, k_{\hat{-}}, r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}}, k_{\hat{-}})\right].$$

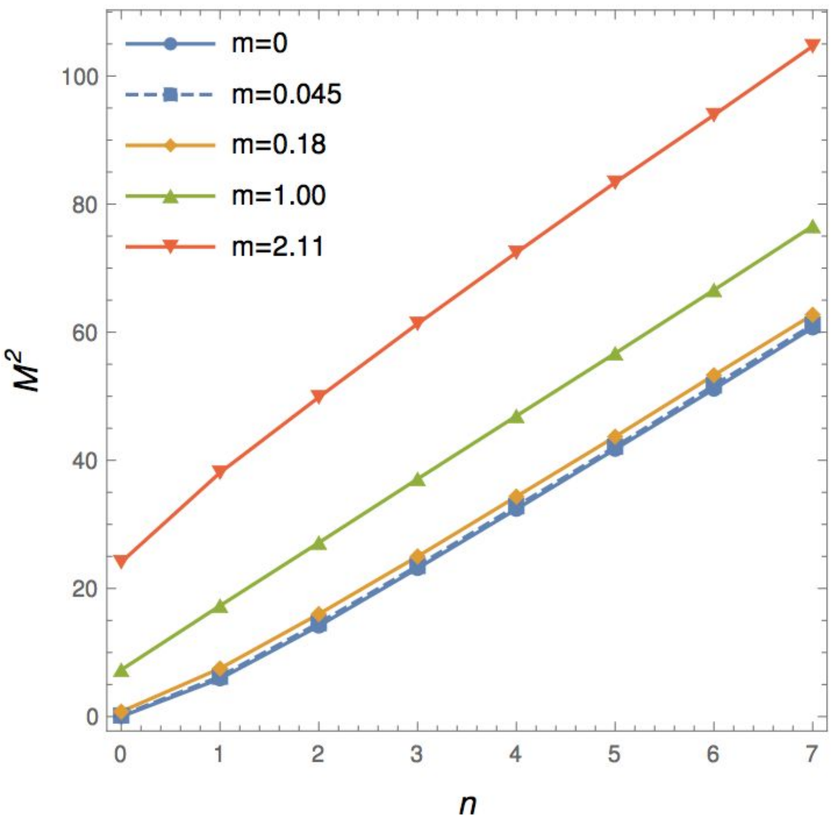


LFD $$\left[\mathcal{M}^2 - \frac{m^2 - 2\lambda}{x} - \frac{m^2 - 2\lambda}{1-x}\right]\phi(x) = -2\lambda \int_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

Meson Spectroscopy

LFD

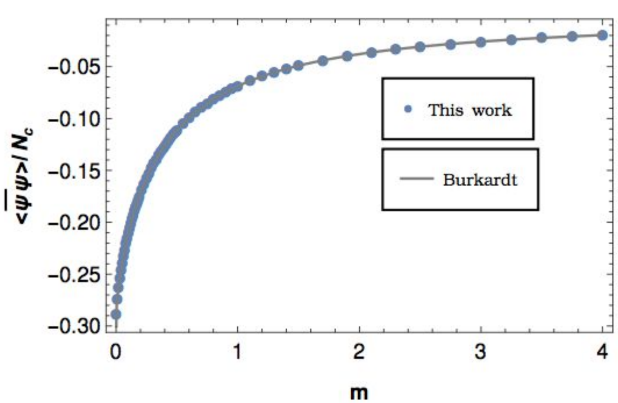
G.'tHooft, NPB75,461 (1974)



IFD

Li,Wilets,Birse JPG:NP13,915 (1987)

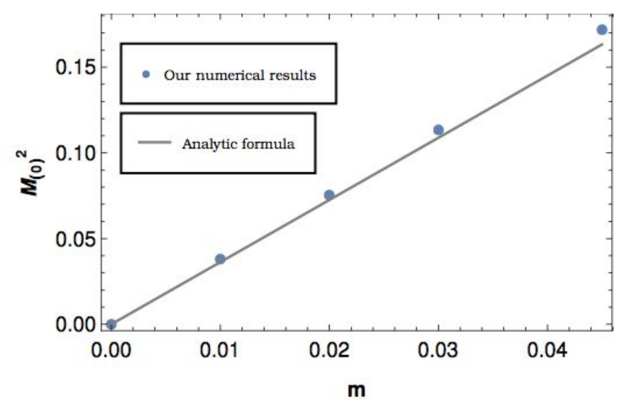
Jia,Liang,Xiong JHEP11,151 (2017)



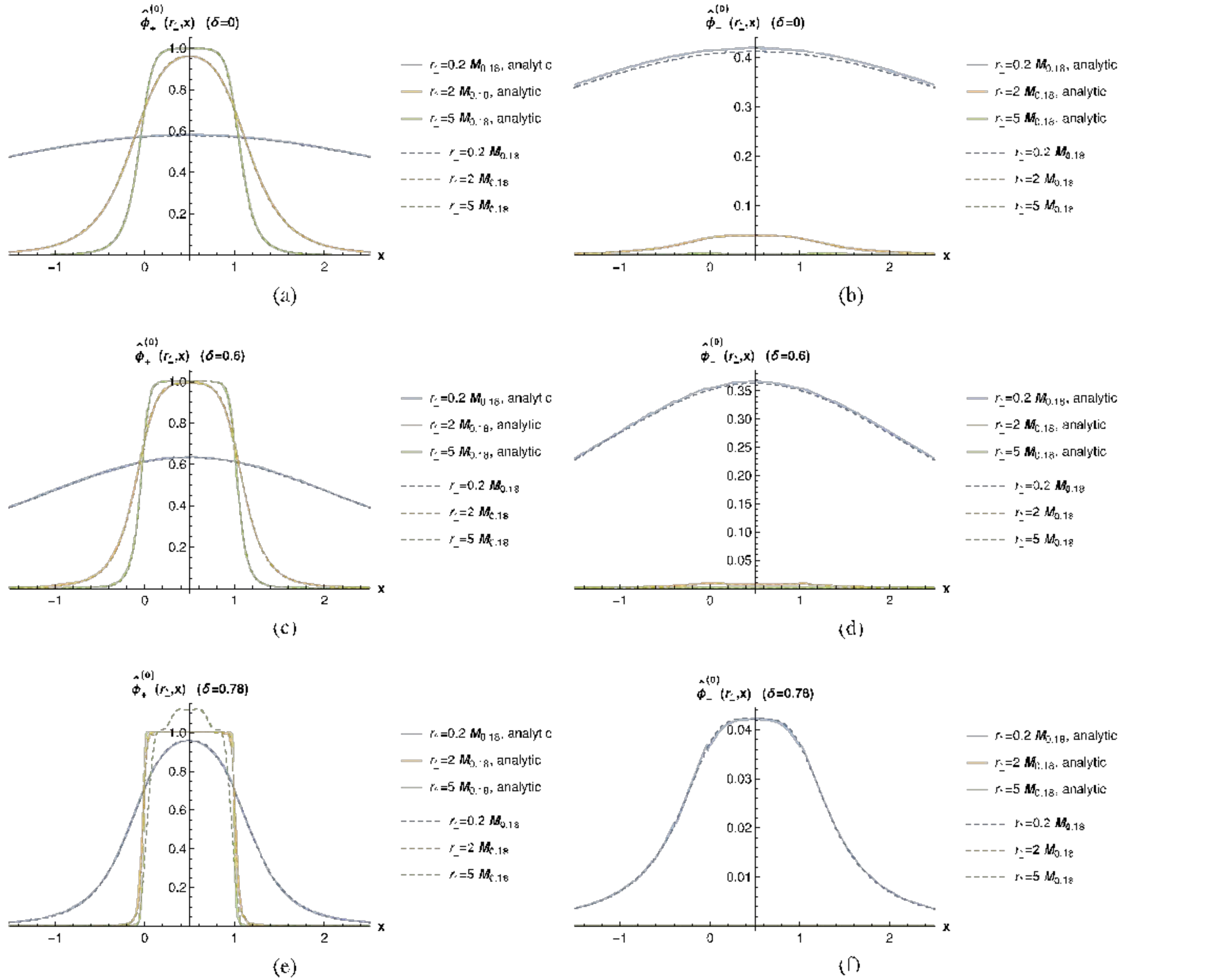
$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

$$< \bar{\psi}\psi >= -N_c/\sqrt{12}$$



Meson Wavefunctions



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-) \gamma^+ \mathcal{W}[\xi^-, 0] \psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

$A^+=0$ Gauge in LFD

Quasi-PDFs

$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi}\, \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}})\, \gamma_{\hat{-}}\, \mathcal{W}[x^{\hat{-}}, 0]\, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

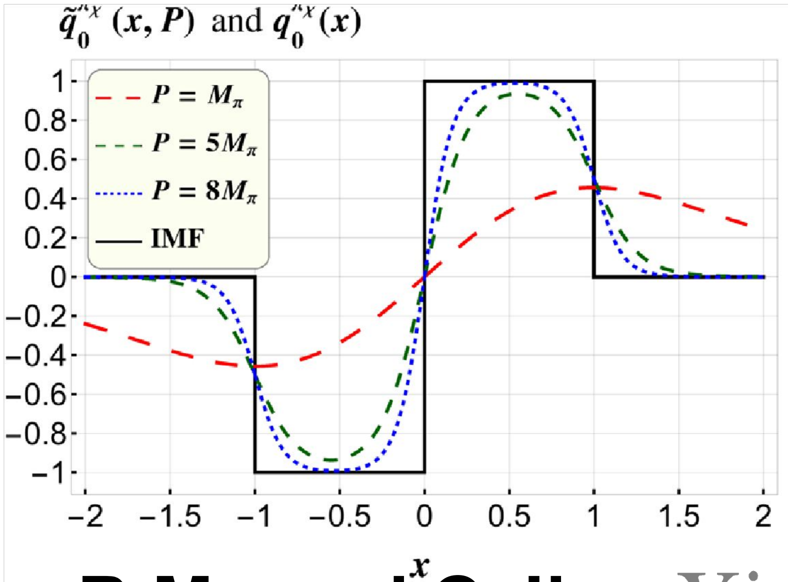
$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

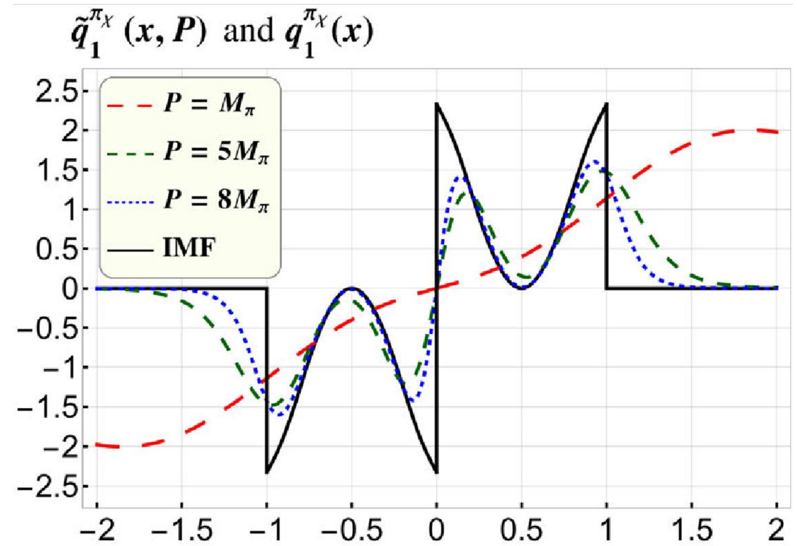
Interpolating dynamics

PHYSICAL REVIEW D **98,** 054011 (2018)

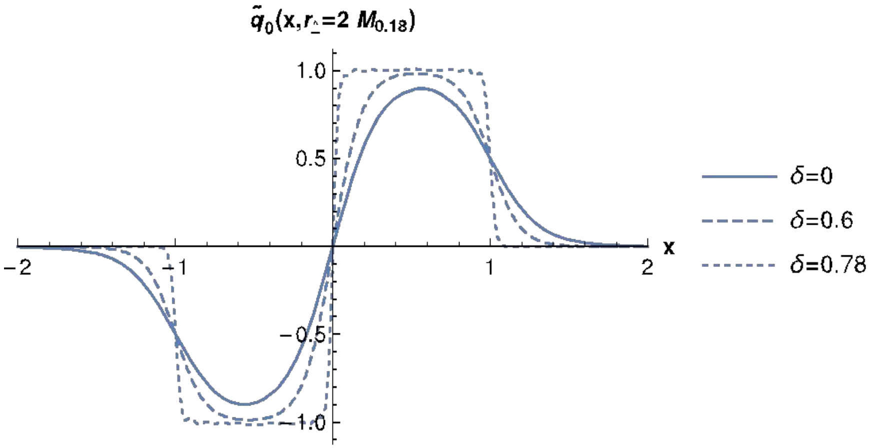
Partonic quasidistributions in two-dimensional QCD

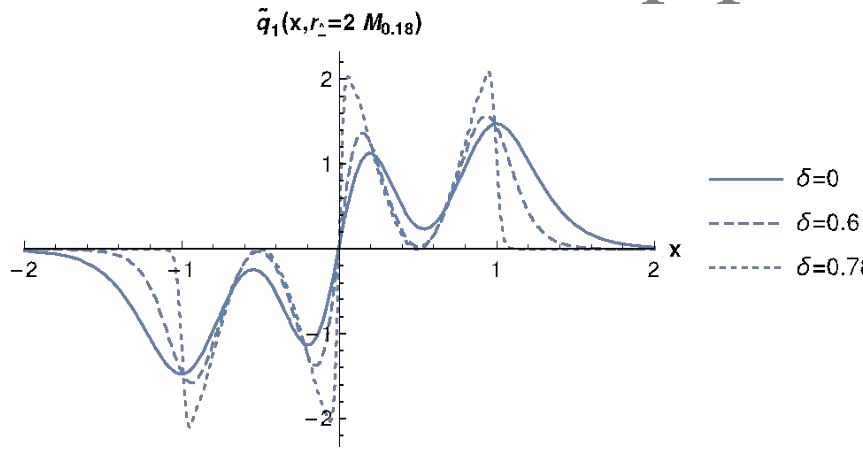
Yu Jia,[1,2,*] Shuangran Liang,[1,2,†] Xiaonu Xiong,[3,‡] and Rui Yu[1,2,§]



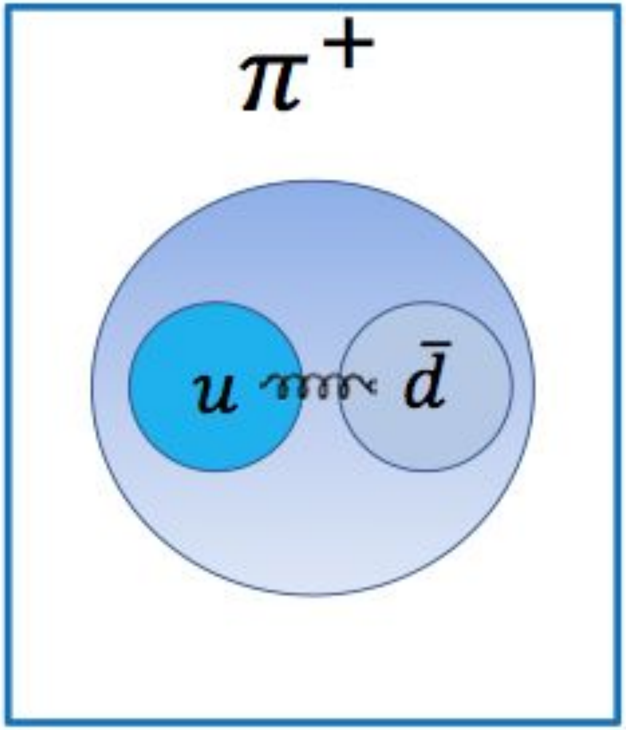


B.Ma and C.Ji, arXiv:2105.09388v1 [hep-ph]

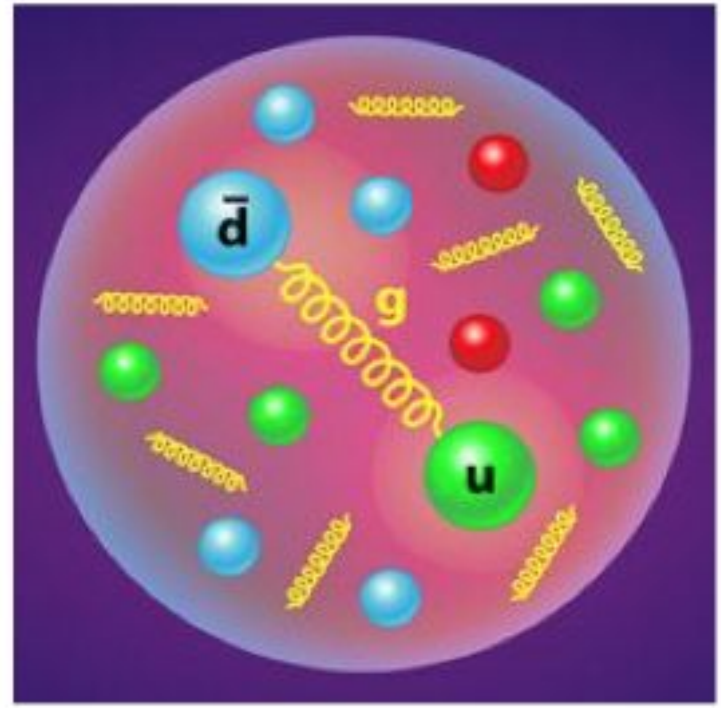




Pion's Dichotomy



vs.



Constituent Quark Model

$$M = m_1 + m_2 + A\frac{\vec{s}_1 \cdot \vec{s}_2}{m_1 m_2}$$

$$m_u = m_d = 310 MeV/c^2$$

$$A = \left(\frac{2m_u}{\hbar}\right)^2 160 MeV/c^2$$

Quantum Chromodynamics

Isospin symmetry

Chiral symmetry

$$\mathrm{SU}(2)_\mathrm{R} \times \mathrm{SU}(2)_\mathrm{L}$$

Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d)\langle 0|\bar{u}u|0\rangle$$

Effective field theory

Pion Properties

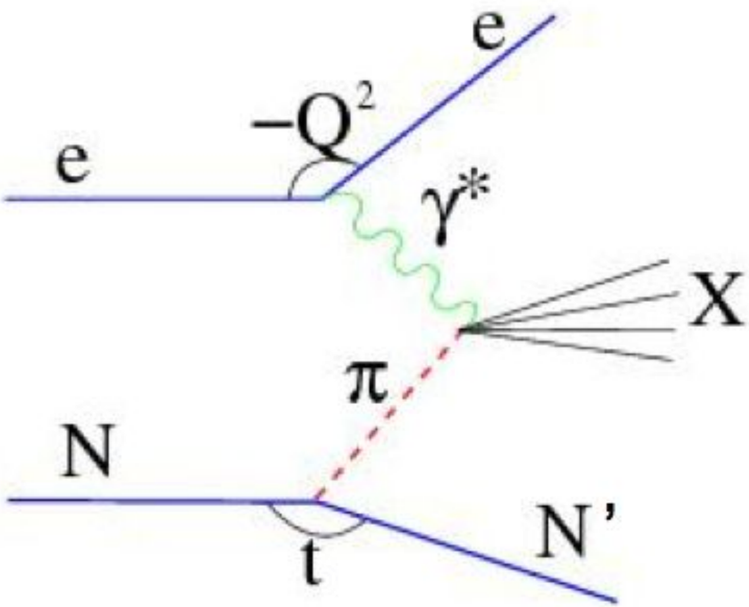
- Lightest bound state composed of quarks, antiquarks, and gluons
- Masses: $m_{\pi^\pm} = 139.57\ \mathrm{MeV}, m_{\pi^0} = 134.977\ \mathrm{MeV}$
- Lifetimes: $\tau_{\pi^\pm} = 2.603\times10^{-8}\ \mathrm{s}$, $\tau_{\pi^0} = 8.52\times10^{-17}\ \mathrm{s}$

Charged pions decay via weak interaction

Neutral pions decay via electromagnetic interaction, *i.e.* $\pi^0 \to \gamma\gamma$

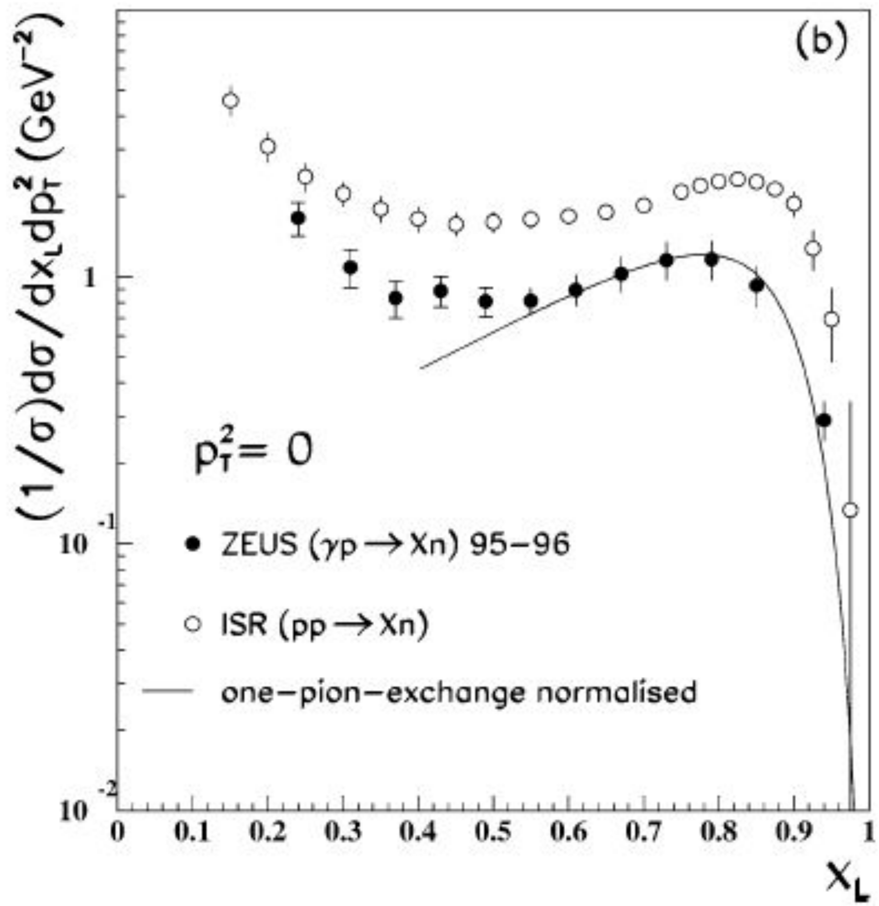
Measurement of Tagged Deep Inelastic Scattering (TDIS)

C.Keppel (Contact person)



$$e + p(or\ n) \rightarrow e' + p + X$$

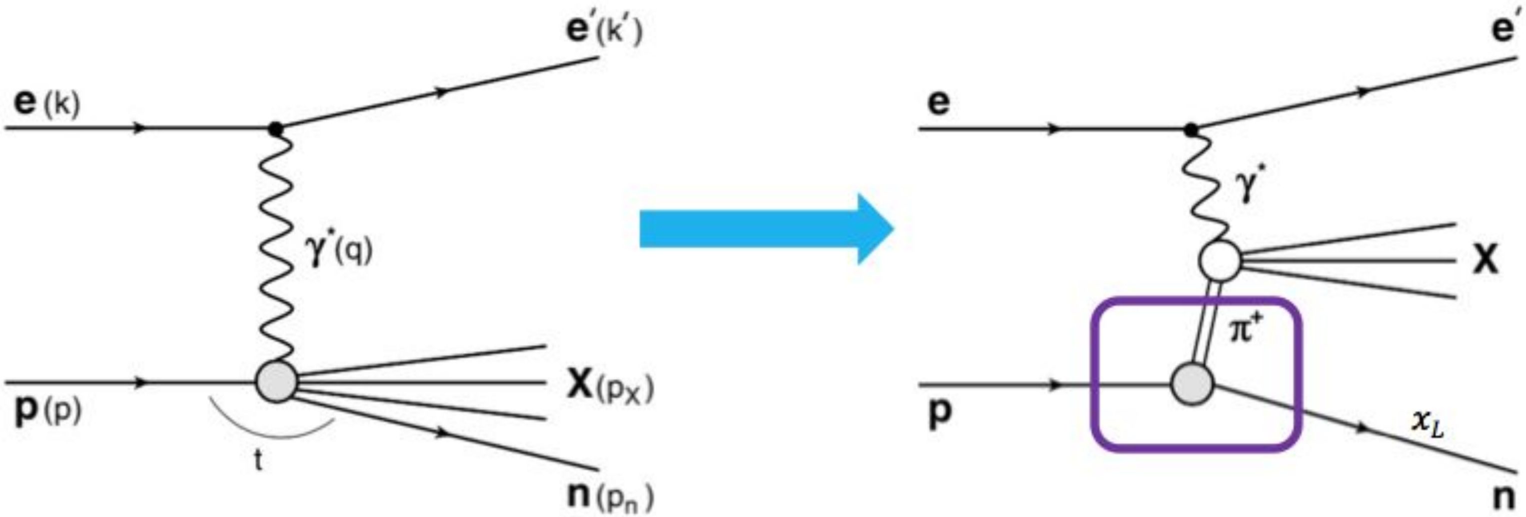
$$e + D \rightarrow e' + p + p + X$$



Leading neutron production in e^+p collisions at HERA

ZEUS Collaboration, NPB 637 (2002) 3–56

Convolution with Chiral Effective Theory

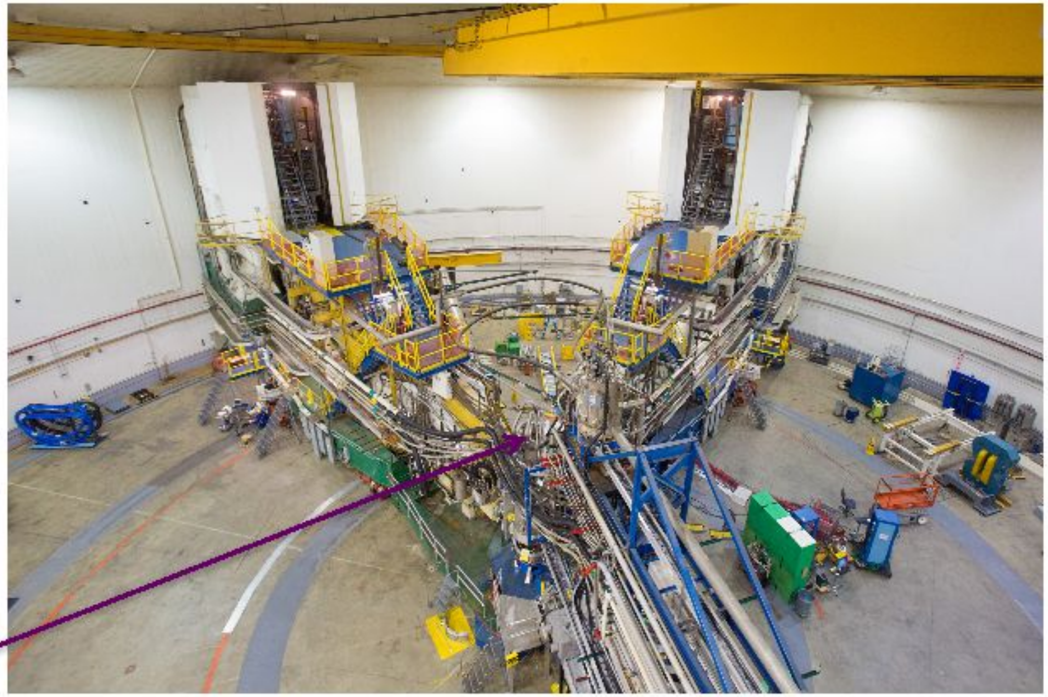


Denotes the splitting of *e.g.*, $p \rightarrow \pi^+ n$
Notationally: $f_\pi(\bar{x}_L = 1 - x_L)$

$$(\bar{d} - \bar{u})(x) = \frac{2}{3} \int_x^1 \frac{dy}{y}\, f_\pi(y)\, \bar{q}^\pi(x/y)$$

pion light-cone momentum distribution in nucleon

JLab Hall A TDIS Experiment



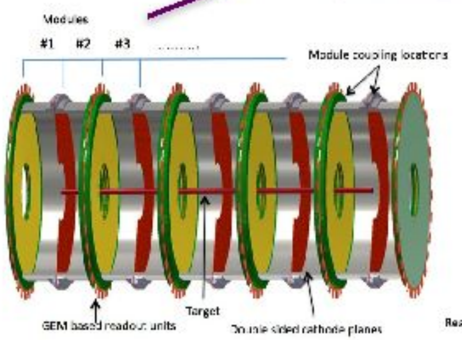
proton tag detection in GEM-based mTPC at pivot

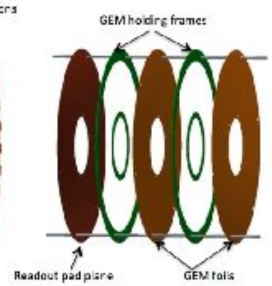
Hall A with SBS:

- High luminosity,
  50 ∝Amp,
  $L = 3x10^{36}/cm^2 s$
- Large acceptance
  ~70 msr

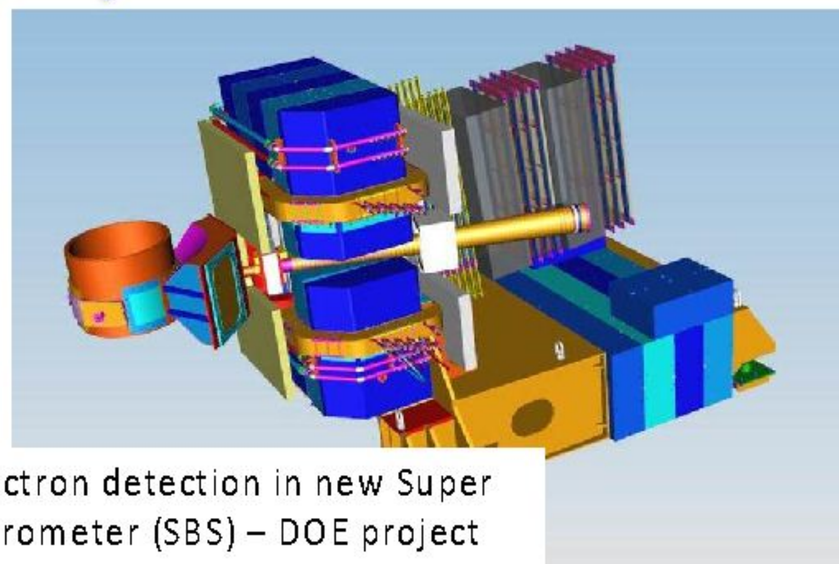
Important for small cross sections

e- beam





mTPC inside superconducting solenoid



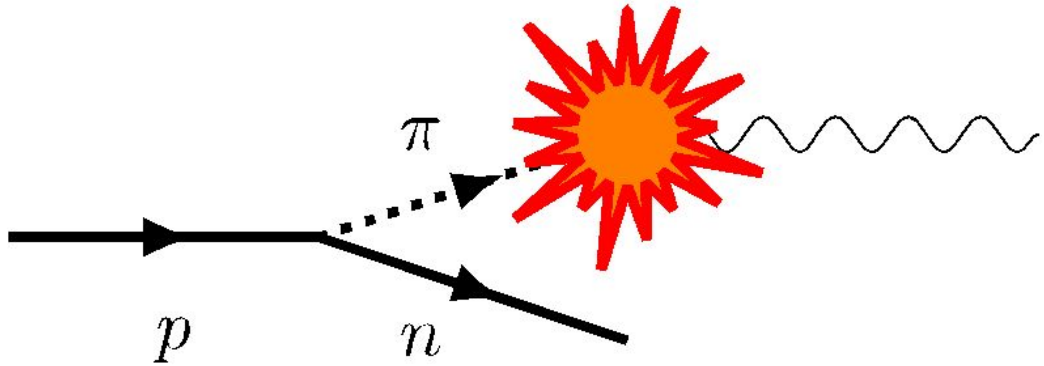
Scattered electron detection in new Super Bigbite Spectrometer (SBS) – DOE project complete

First global Monte Carlo analysis of pion PDFs

P. Barry, N. Sato, W. Melnitchouk, C.Ji

PRL121,152001(2018) Featured in Physics

- **How to probe pion structure**
  - $\pi + A \to l\bar{l} + X$ **(Drell-Yan)**
  - $\pi + A \to \gamma + X$ (prompt photons)
  - $e + p \to e' + n + X$ **(SIDIS)** $\to$ small x_π gluon PDF



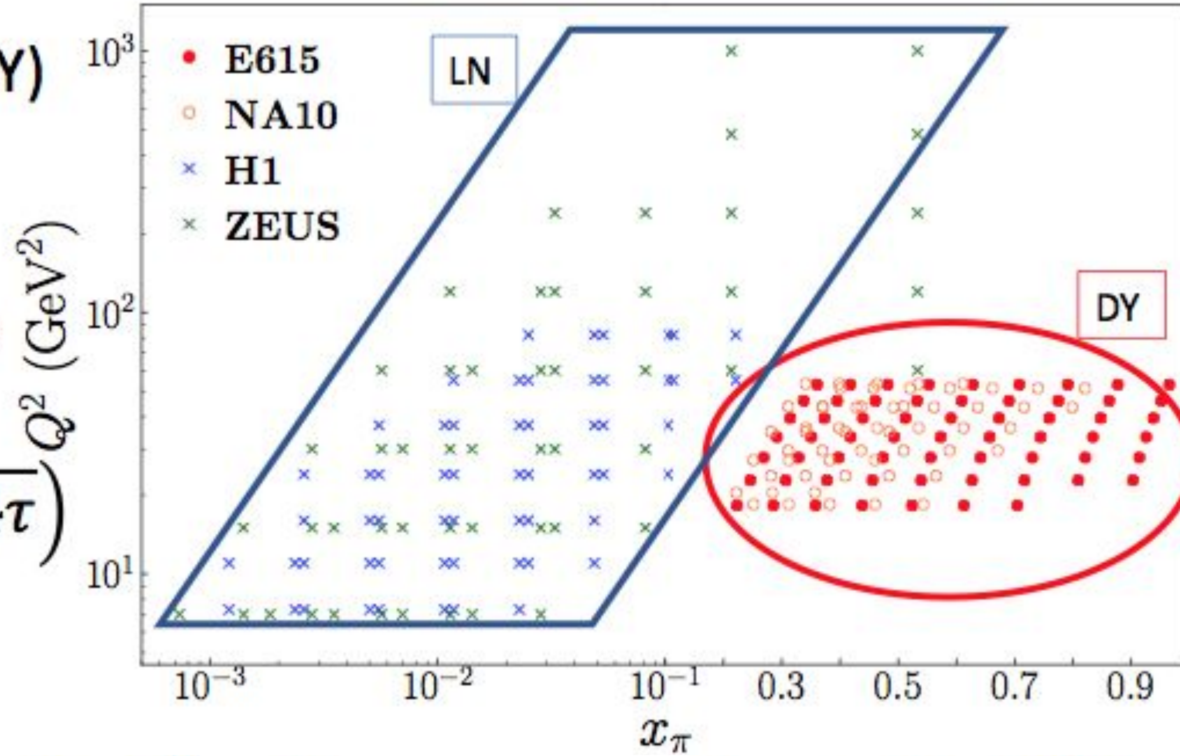
Datasets vs. Kinematics

- Large x_π -- Drell-Yan (DY)
- Small x_π -- Leading Neutron (LN)
- Not much data overlap
- In DY:

$$x_\pi = \frac{1}{2}\left(x_F + \sqrt{x_F^2 + 4\tau}\right)$$

- In LN:

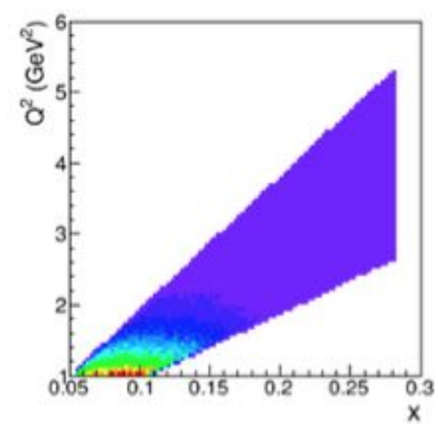
$$x_\pi = x_B / \bar{x}_L$$

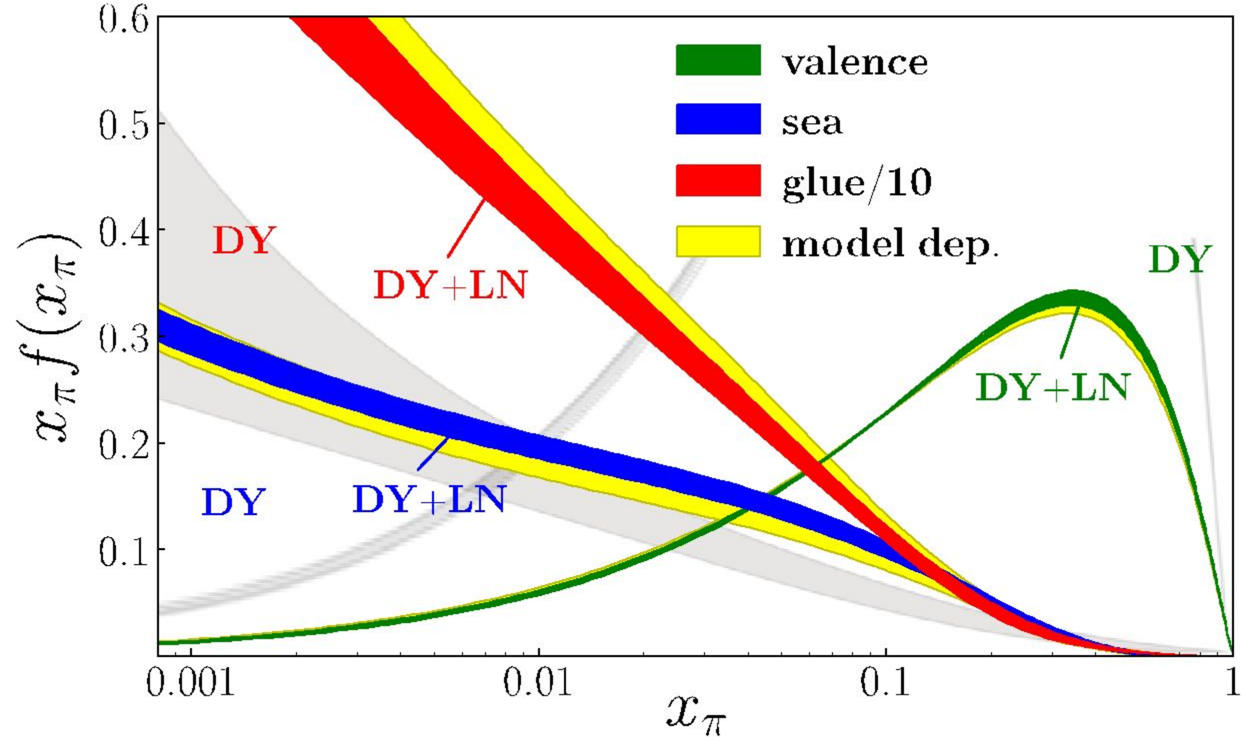


- JLab can reach much smaller Q^2 and larger x range than in the HERA

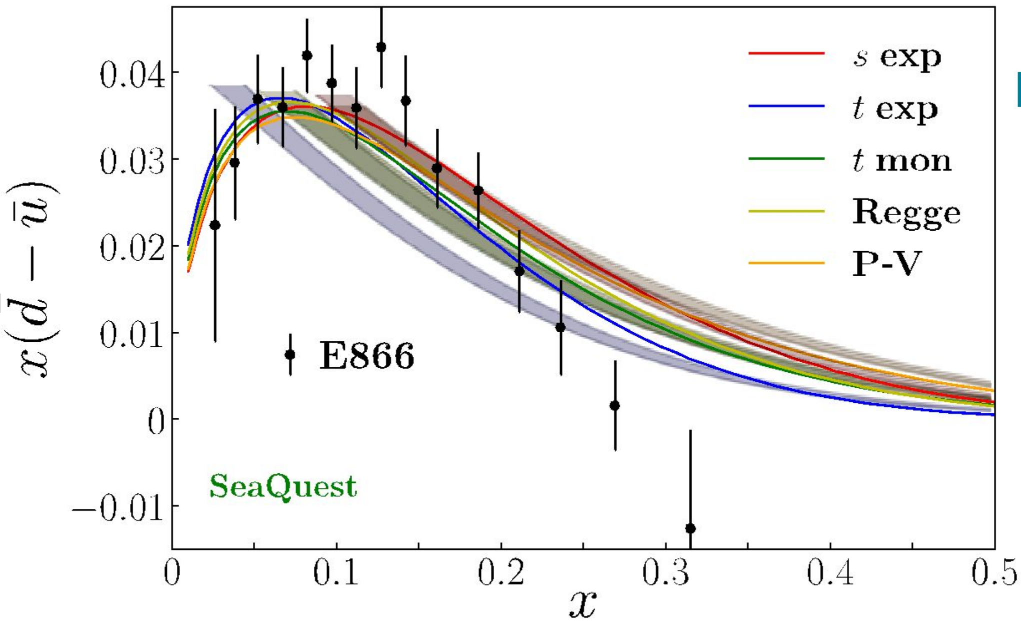
EIC Impact on Pion PDFs

- $s = 5400$ GeV^2, 1.2% systematic uncertainty, integrated $\mathcal{L} = 100\text{fb}^{-1}$

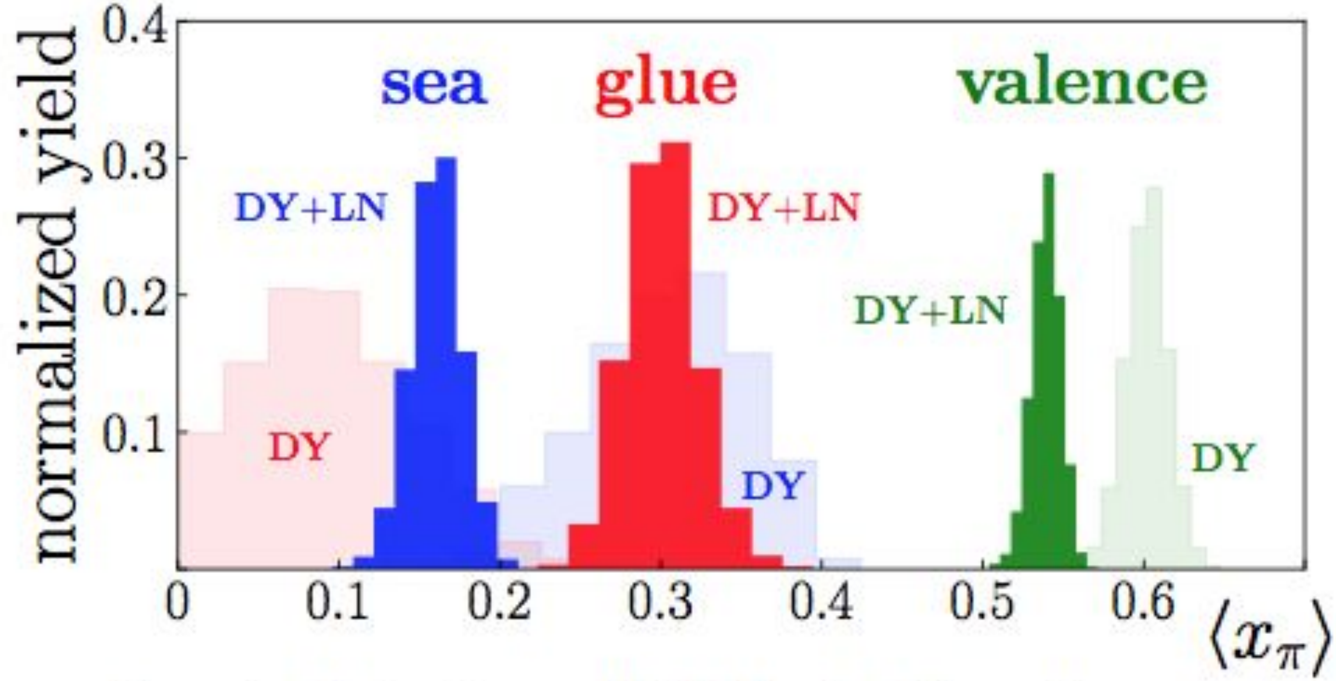




- Significant reduction of the uncertainties
- Non-overlapping uncertainties æ tensions among the data
- Accuracy will be improved with future TDIS (JLab12/EIC)



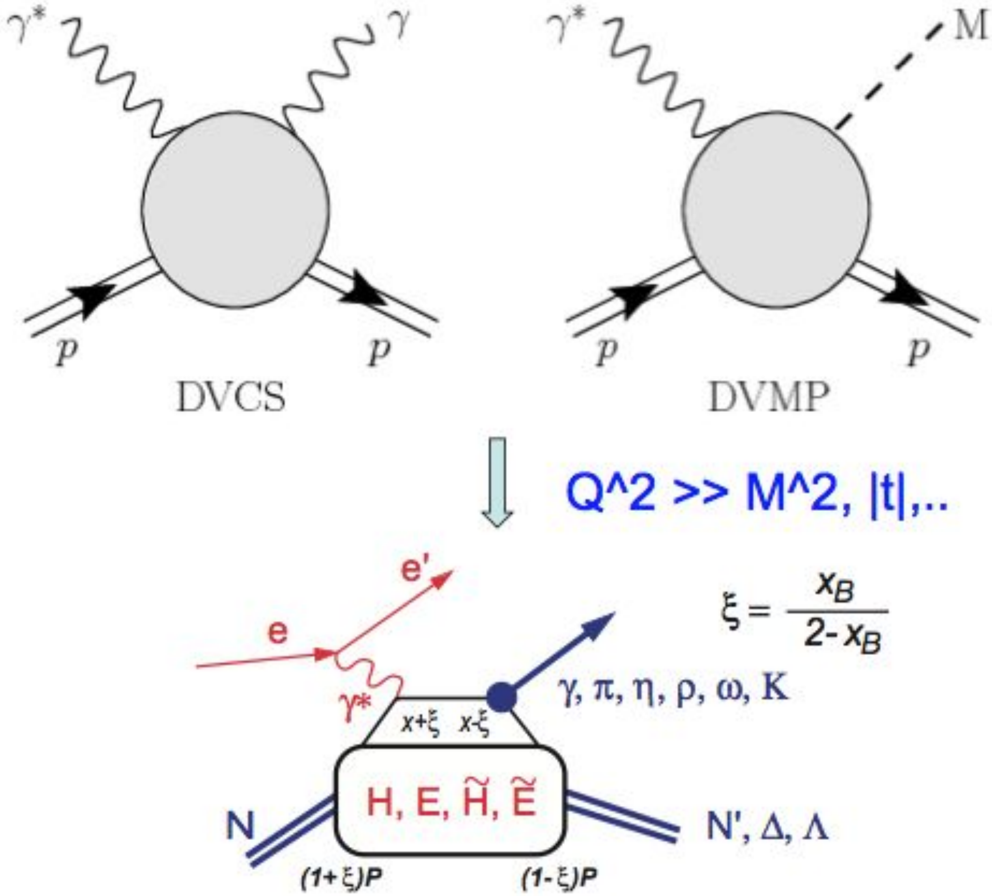
- We performed an additional analysis of LN+DY+E866
  æ good description of E866 data except for large x



- Constraints from HERA significantly increase $\langle x_\pi^g \rangle$.
  The role of the glue is more important than suggested by DY alone
- In contrast, the strength of the sea is reduced
- Due to momentum sum rule $\langle x_\pi^{\text{valence}} \rangle$ decreases

Hadron Physics and QCD

Phenomenology with 12 GeV Upgrade of Jefferson Laboratory



H, E - unpolarized, $\widetilde{H}, \widetilde{E}$ - polarized GPD

The GPDs Define Nucleon Structure

Outlook

- LFD provides a useful tool to study highly nontrivial quatum chromodynamic phenomenology taking advantage its distinguished features such as the boost invariance and the cleaner vacuum properties.
- Corresponding the LFD results with the IFD and its IMF approach is useful to understand the complicate the confinement mechanism and the chiral symmetry aspects of QCD and the associate hadron phenomenology.
- Vigorous experimental measurements, e.g. 12 GeV upgrade of Jefferson Lab, future Electron Ion Collider projects, etc. are encouraging to provide deeper understanding on the nature of hadrons.