BK21FOUR Lecture Series

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Two Lectures (1st Day)

- **Light-Front QCD in Hadron Physic I**

a) Introduction of QCD
b) Color Confinement and Chiral Symmetry
c) Dirac's Proposition for Relativistic Dynamics

- **Light-Front QCD in Hadron Physic II**

a) Distinguished Features of Light-Front Dynamics
b) Large N_C QCD in 1+1 dim. ('tHooft Model)
c) Application to Hadron Phenomenology

July 7-8, KNU Physics

Quantum Electrodynamics (QED)

$$\mathcal{L}_{\mathrm{QED}} = \overline{\psi}\,(i\,\not{D} - m)\,\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$(i\hbar\gamma^{\mu}\partial_{\mu} - \frac{e}{c}\gamma^{\mu}A_{\mu}(x) - mc)\psi(x) = 0 \qquad J^{\mu} = \overline{\psi}(x)\gamma^{\mu}\psi(x)$$

$$\varepsilon_{\mu\nu\alpha\beta}\partial^{\mu}F^{\alpha\beta} = 0,\ \partial_{\mu}F^{\mu\nu} = \frac{4\pi}{c}J^{\nu} \qquad F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

The invariance under local gauge transformation leads to the current conservation.

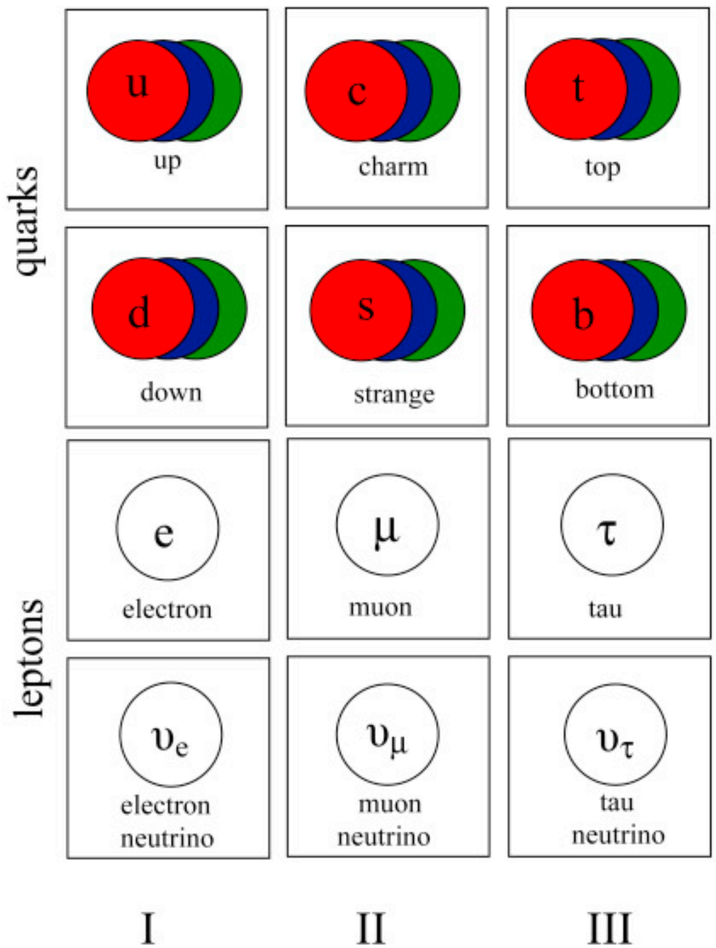
$$A_{\mu}(x) \rightarrow A'_{\mu}(x) = A_{\mu}(x) + \partial_{\mu}X(x) \longrightarrow \partial_{\mu}J^{\mu} = 0$$

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{i,j=1}^{3}\overline{q_i}\,(i\,\not{D} - m_q)_{ij}\,q_j - \frac{1}{4}\sum_{\alpha=1}^{8}G^{\alpha}_{\mu\nu}G^{\alpha,\mu\nu}$$

Standard Model

$$Q_f$$



$$3 \times 2/3$$

$$3 \times -1/3$$

$$-1$$

$$0$$

Anomaly Free Condition

$$\sum_f Q_f = 0$$

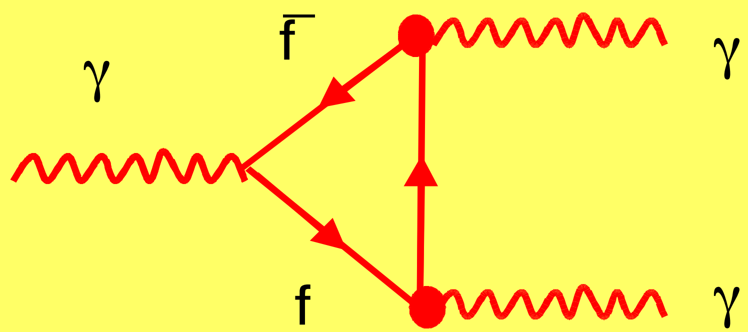
Evidences of $N_C = 3$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)}$$ indicates fractional charges and N_C=3

Δ^{++} (Ω_s) statistic problem: $|\Delta^{++}\rangle = \frac{1}{\sqrt{6}} \varepsilon_{ijk} |u_i \uparrow u_j \uparrow u_k \uparrow\rangle$

Triangle anomaly

Divergent fermion loops



Divergence

$$\sim \sum_f Q_f = \underbrace{(-1)+(-1)+(-1)}_{\text{leptons}} + \underbrace{N_C \cdot \left[\left(\frac{2}{3} - \frac{1}{3}\right) \cdot 3 \right]}_{\text{quarks}}$$

3 generations of u/d-type quark

➡ cancels if $N_C = 3$

Quantum Chromodynamics – SU(3) Theory

Lagrangian is constructed with quark wave functions $\psi = \begin{pmatrix} \psi_R \\ \psi_G \\ \psi_B \end{pmatrix}$

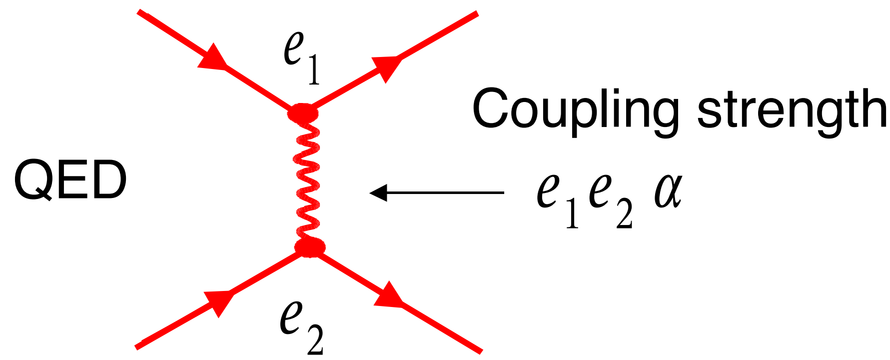
Invariance of the Lagrangian under Local SU(3) Gauge Transformation

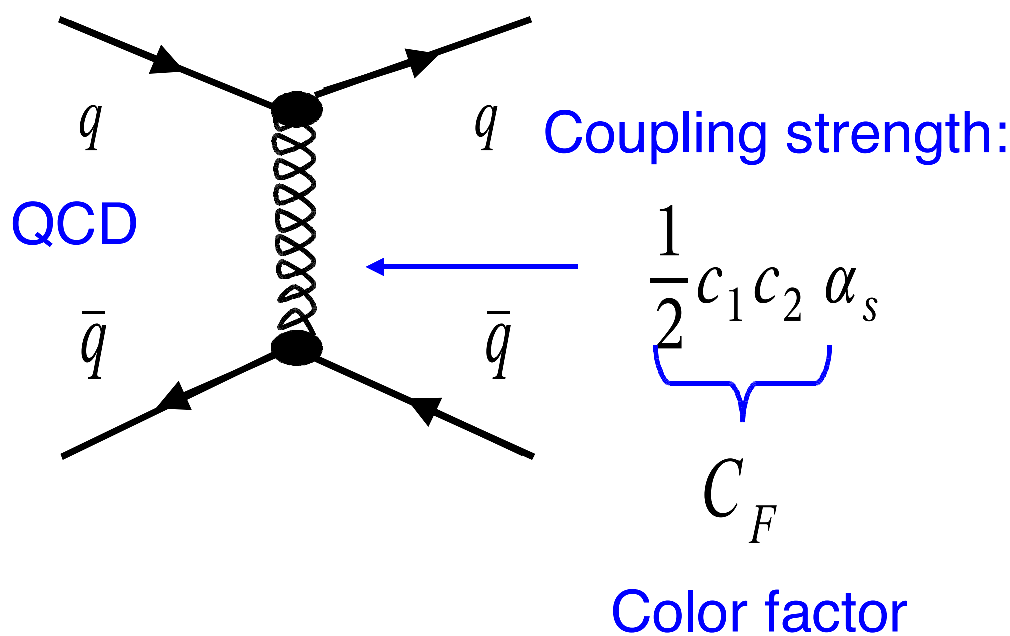
$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x) = e^{i\frac{\alpha_k(x)}{2}\lambda_k}\psi(x)$$

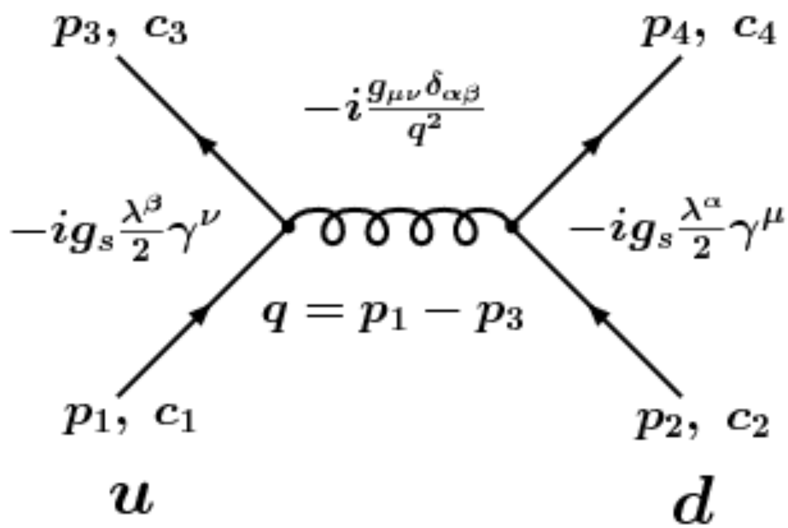
with any unitary (3 x 3) matrix $U(x)$.

$U(x)$ can be given by a linear combination of
8 Gell-Mann matrices $\lambda_1 \ldots \lambda_8$ [SU(3) group generators]

requires interaction fields – 8 gluons corresponding to these matrices


QED
e_1
e_2
Coupling strength
$e_1 e_2 \alpha$


QCD
q
q
$\bar{q}$
$\bar{q}$
Coupling strength:
$\frac{1}{2} c_1 c_2 \alpha_s$
C_F
Color factor



$$-i\mathrm{M} = \left[\bar{u}(p_3)c_3^+\right]\left[-ig_s\frac{\lambda^\alpha}{2}\gamma^\mu\right]\left[u(p_1)c_1\right]\frac{-i}{q^2}$$

$$\times\left[\bar{u}(p_4)c_4^+\right]\left[-ig_s\frac{\lambda^\alpha}{2}\gamma_\mu\right]\left[u(p_2)c_2\right]$$

$$= ig_s^2\left[\bar{u}(p_3)\gamma^\mu u(p_1)\right]\frac{1}{q^2}\left[\bar{u}(p_4)\gamma_\mu u(p_2)\right]$$

$$\times \boxed{\frac{1}{4}\left(c_3^+\lambda^\alpha c_1\right)\left(c_4^+\lambda^\alpha c_2\right)},$$

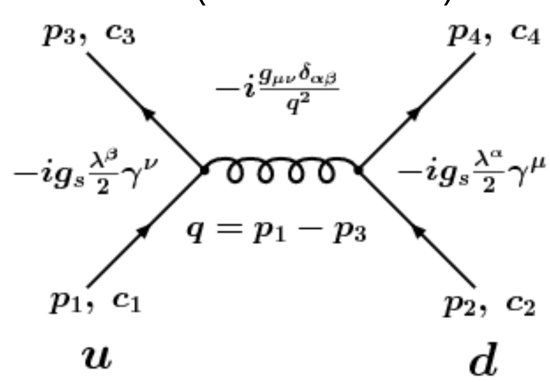
Homework: Compute the color factors between the two quarks and verify that the same colors repel and different colors attract each other.

Hint: Gell-Mann matrices in SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$



$$V_{qq}(r) = f \cdot \frac{\alpha_s \hbar c}{r},$$

where α_s is the QCD running coupling constant and f is given by

$$f = \frac{1}{4}(c_3{}^{\dagger}\lambda^{\alpha}c_1)(c_4{}^{\dagger}\lambda^{\alpha}c_2).$$

$$red(r) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \; blue(b) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } green(g) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

color factor between two red quarks going into two red quarks

$$c_1 = c_2 = c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{aligned} f &= \frac{1}{4}\left[(100)\lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right]\left[(100)\lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] \\ &= \frac{1}{4}\lambda_{11}^\alpha \lambda_{11}^\alpha \\ &= \frac{1}{4}\left[1 \times 1 + \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\right] \\ &= \frac{1}{3}. \end{aligned}$$

$$\{3\} \otimes \{3\} = \{6\} \oplus \{\bar{3}\}$$

$$\begin{array}{|c|}\hline \\ \hline\end{array} \otimes \begin{array}{|c|}\hline \\ \hline\end{array} \quad \begin{array}{|c|c|}\hline & \\ \hline\end{array} \oplus \begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}$$

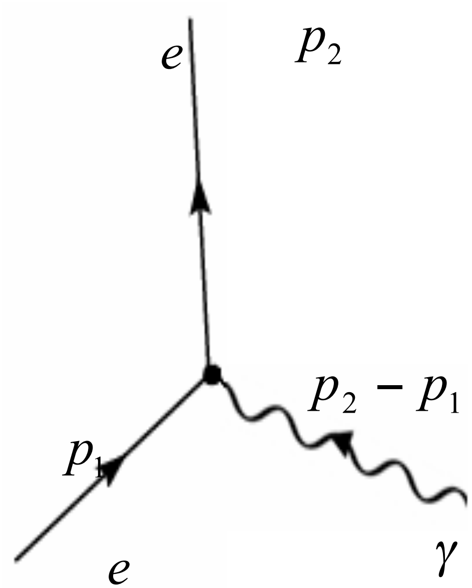
$$\begin{array}{|c|c|}\hline r & r \\ \hline\end{array} = rr, \quad \begin{array}{|c|c|}\hline b & b \\ \hline\end{array} = bb, \quad \begin{array}{|c|c|}\hline g & g \\ \hline\end{array} = gg, \quad \begin{array}{|c|c|}\hline r & b \\ \hline\end{array} = \frac{rb + br}{\sqrt{2}}, \quad \begin{array}{|c|c|}\hline b & g \\ \hline\end{array} = \frac{bg + gb}{\sqrt{2}}, \quad \begin{array}{|c|c|}\hline g & r \\ \hline\end{array} = \frac{gr + rg}{\sqrt{2}} \quad \rightarrow \quad f = \frac{1}{3}$$

$$\begin{array}{|c|}\hline r \\ \hline b \\ \hline\end{array} = \frac{rb - br}{\sqrt{2}}, \quad \begin{array}{|c|}\hline b \\ \hline g \\ \hline\end{array} = \frac{bg - gb}{\sqrt{2}}, \quad \begin{array}{|c|}\hline g \\ \hline r \\ \hline\end{array} = \frac{gr - rg}{\sqrt{2}} \quad \rightarrow \quad f = -\frac{2}{3}$$

$$V_{q\bar{q}} = \begin{cases} -\dfrac{4}{3}\dfrac{\alpha_s \hbar c}{r} \ \text{(color singlet)} \quad \text{(attractive force)} \\ \\ \dfrac{1}{6}\dfrac{\alpha_s \hbar c}{r} \ \text{(color octet)} \quad \text{(repulsive force)}. \end{cases}$$

Same colors repel and different colors attract each other.

QED Gauge Invariance



$$-i\mathrm{M} = \overline{u}(p_2) i g_e \gamma^{\mu} u(p_1) \varepsilon_{\mu}(p_2 - p_1),$$

$$A_{\mu} \longrightarrow A'_{\mu} = A_{\mu} + \partial_{\mu}\chi$$

$$\varepsilon_{\mu}(p) \longrightarrow \varepsilon'_{\mu}(p) = \varepsilon_{\mu}(p) + \frac{p_{\mu}}{i\hbar}\widetilde{\chi}.$$

$$\overline{u}(p_2)(\not{p}_2 - \not{p}_1)u(p_1) = \overline{u}(p_2)(mc - mc)u(p_1)$$

$$= 0.$$

Current Conservation

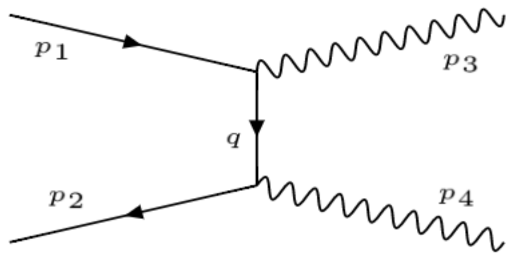
$$J^{\mu} = \overline{u}(p_2)\gamma^{\mu}u(p_1)$$

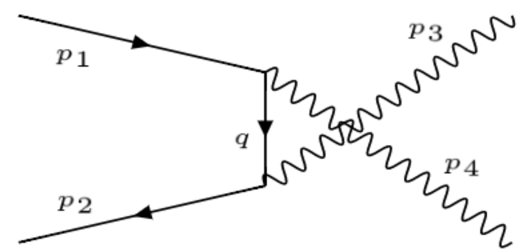
$$\partial_{\mu}J^{\mu} = 0$$

$$(p_2 - p_1)_{\mu}\overline{u}(p_2)\gamma^{\mu}u(p_1)$$
$$= \overline{u}(p_2)(\not{p}_2 - \not{p}_1)u(p_1)$$
$$= 0.$$

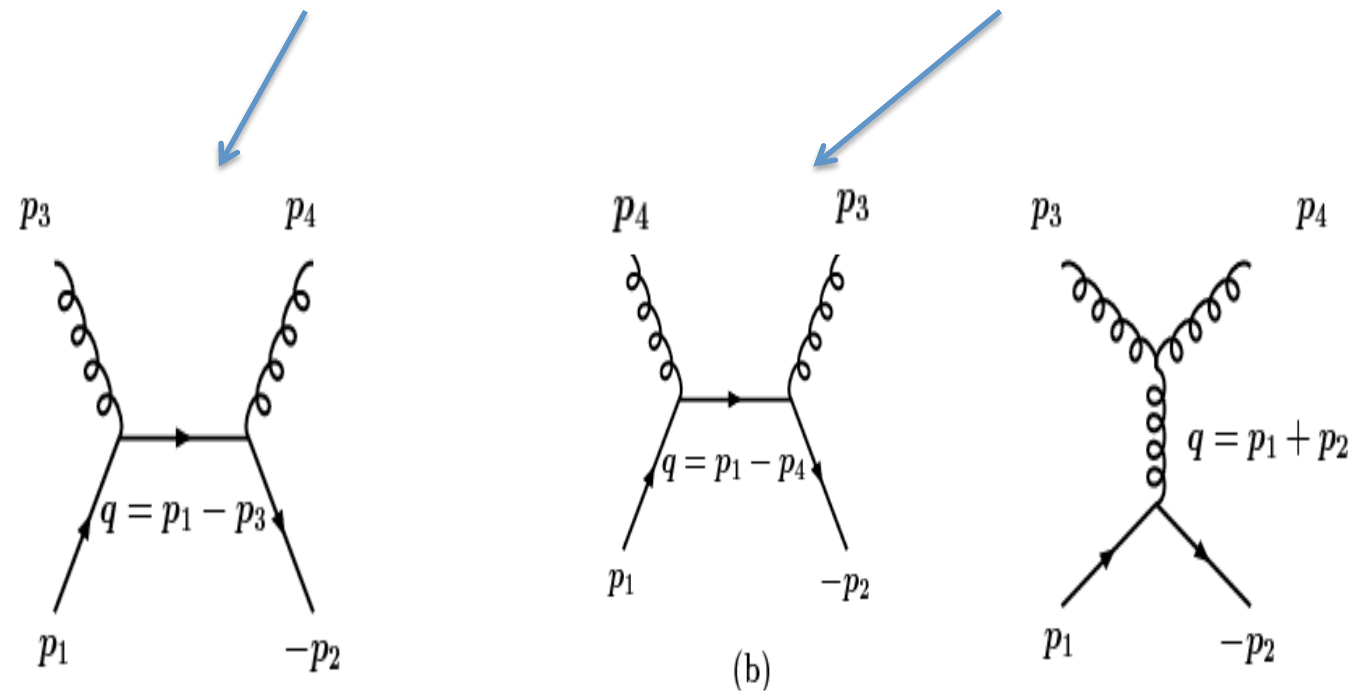
QED

$$e^+e^- \to \gamma\gamma$$





$$q\bar{q} \to gg$$

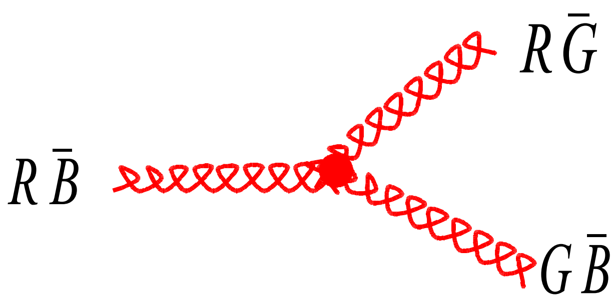


QCD

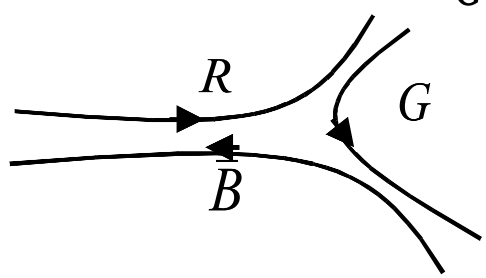
$$[\lambda^\alpha, \lambda^\beta] = 2if^{\alpha\beta\gamma}\lambda^\gamma,$$

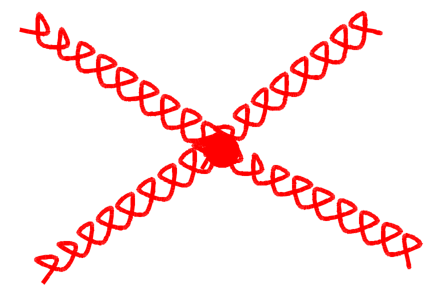
Triple and quadruple gluon Vertex

Gluons carry color charges: important feature of SU(3)

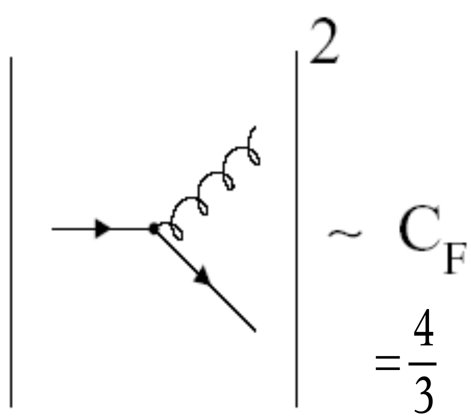


Color flow

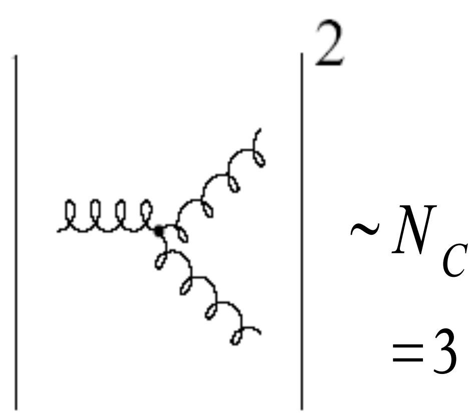




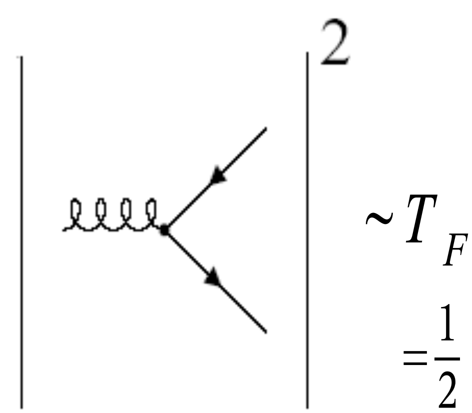
Color factors



$q \to qg$



$g \to gg$

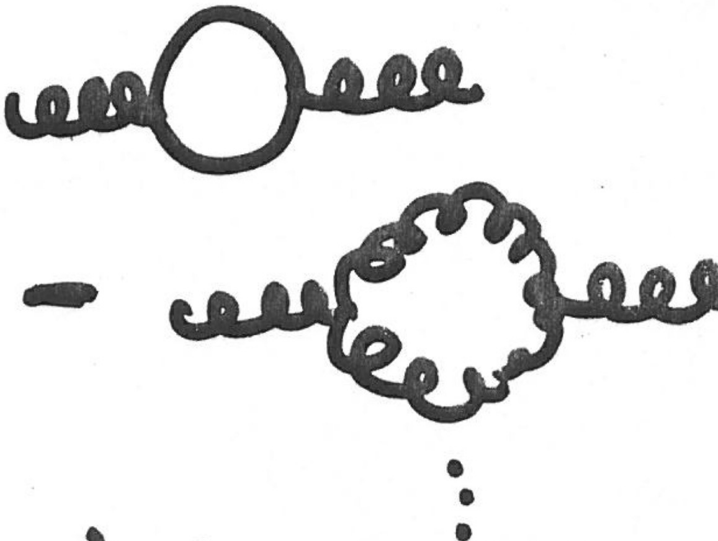


$g \to q\bar{q}$

Renormalization of coupling const.

QED

QCD

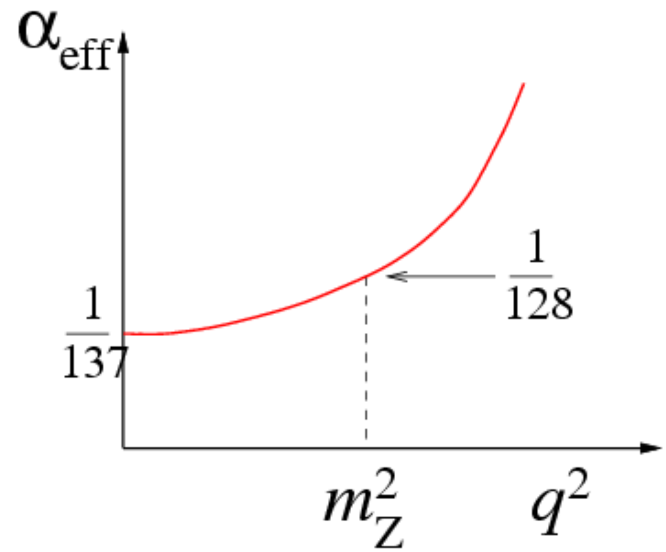


Screening

Anti screening

QED: Running coupling constant

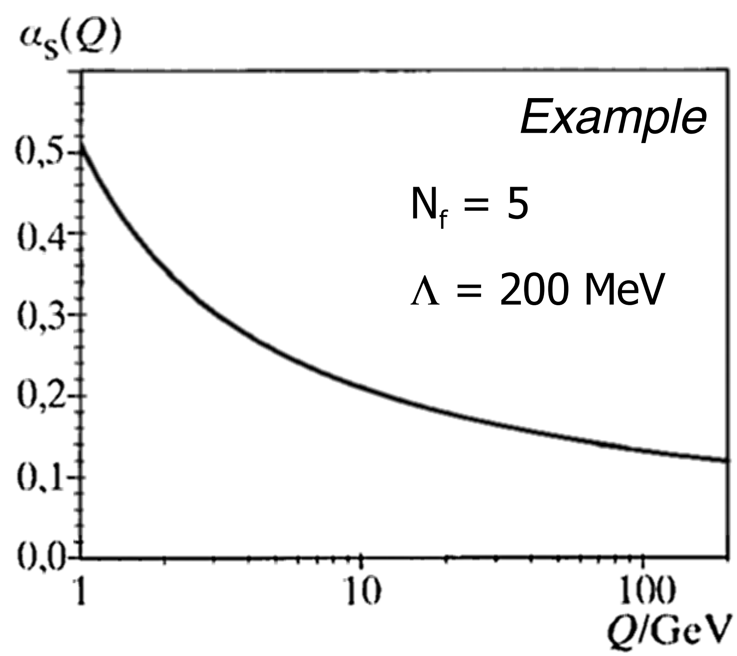
$$\alpha(q^2) = \frac{\alpha}{1 - \underbrace{\frac{\alpha}{3\pi} \cdot \sum_f Q_f^2 \cdot \log \frac{q^2}{m_f^2}}_{-\alpha\, \beta_0^{QED} \log \frac{Q^2}{\mu^2}}}$$



QCD Strong coupling constant α_s

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2) \underbrace{\frac{1}{12\pi}(33 - 2n_f)}\log \frac{Q^2}{\mu^2}}$$

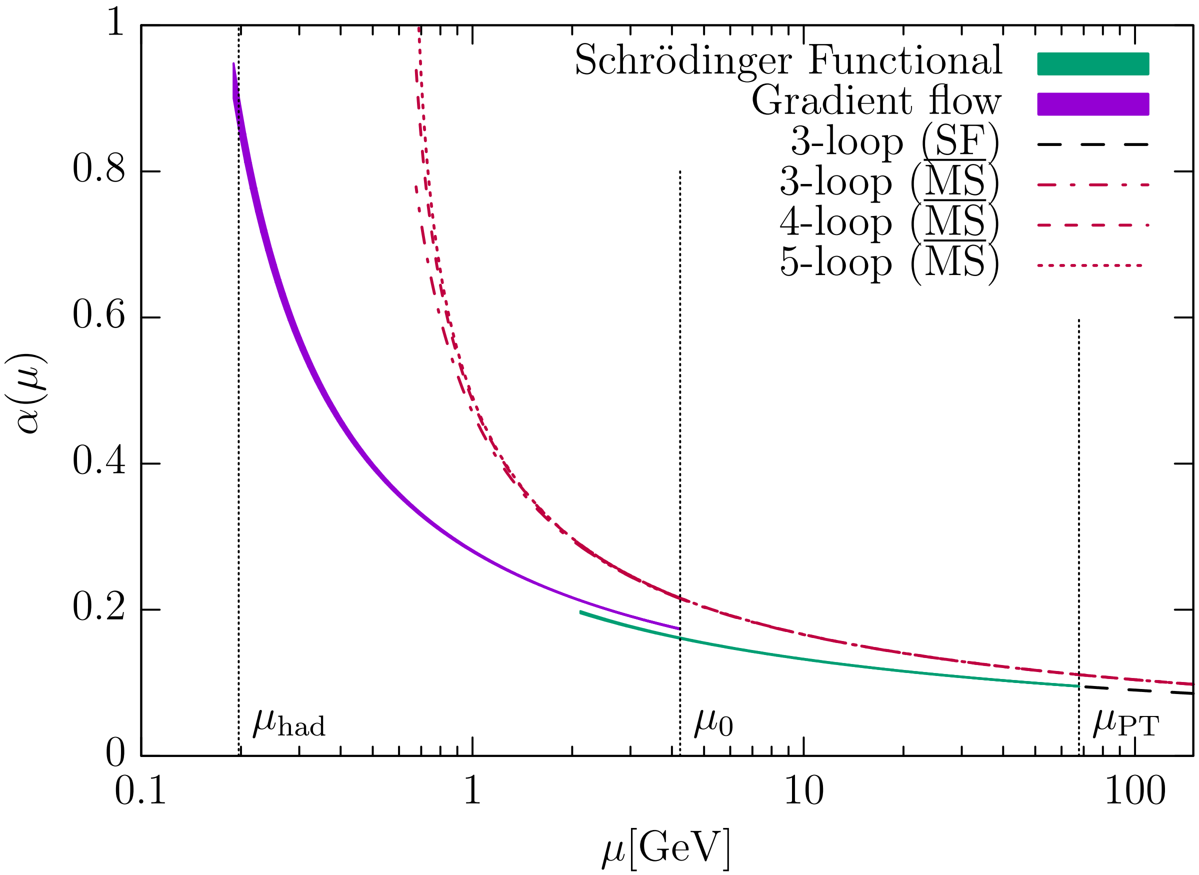
$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$



QCD Coupling from a Nonperturbative Determination of the Three-Flavor Λ Parameter

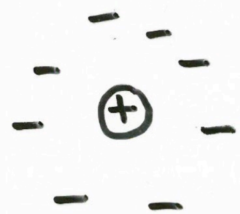
Mattia Bruno,[1] Mattia Dalla Brida,[2] Patrick Fritzsch,[3] Tomasz Korzec,[4] Alberto Ramos,[3] Stefan Schaefer,[5] Hubert Simma,[5] Stefan Sint,[6] and Rainer Sommer[5,7]

(ALPHA Collaboration)



$$\alpha^{(5)}_{\overline{\text{MS}}}(m_Z) = 0.11852(84)$$

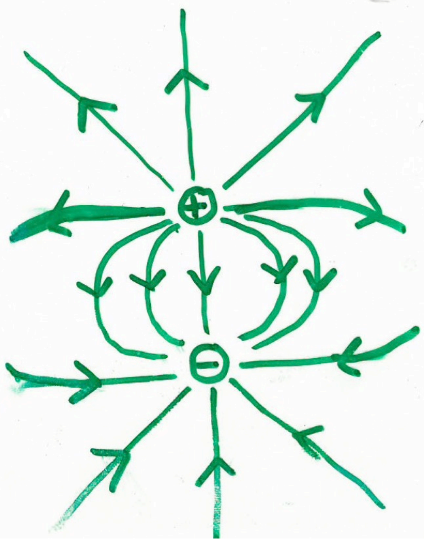
Phenomenological Interpretation of Antiscreening Effect.

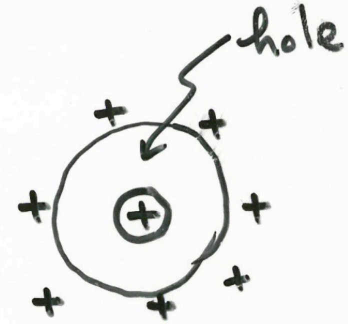


$$\vec{D} = \epsilon \vec{E}$$

$\epsilon > 1$

(Screening)



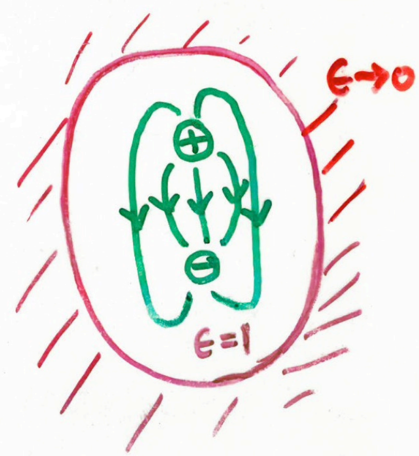


$\epsilon < 1$

(Antiscreening)

$\epsilon \to 0$

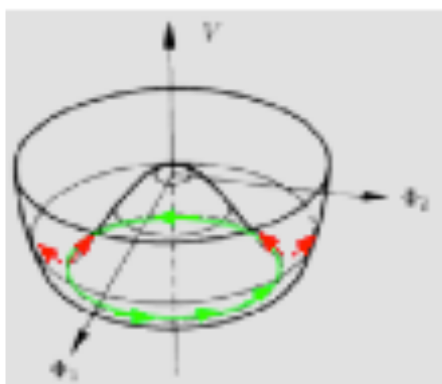
size of hole $\to \infty$



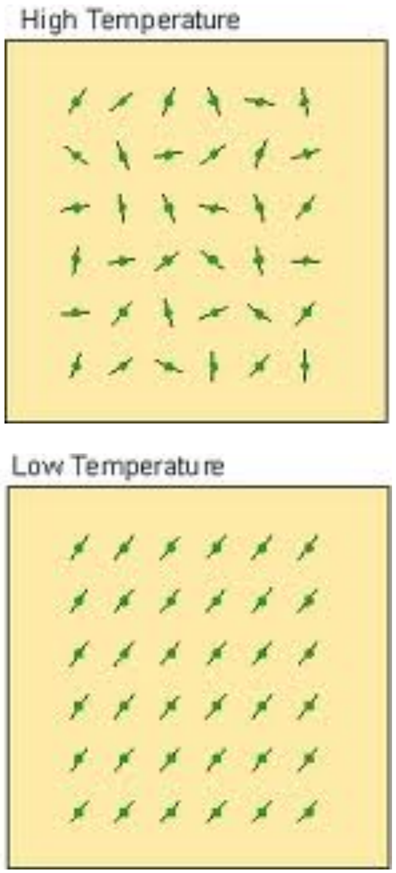
Meissner Effect of Superconductor

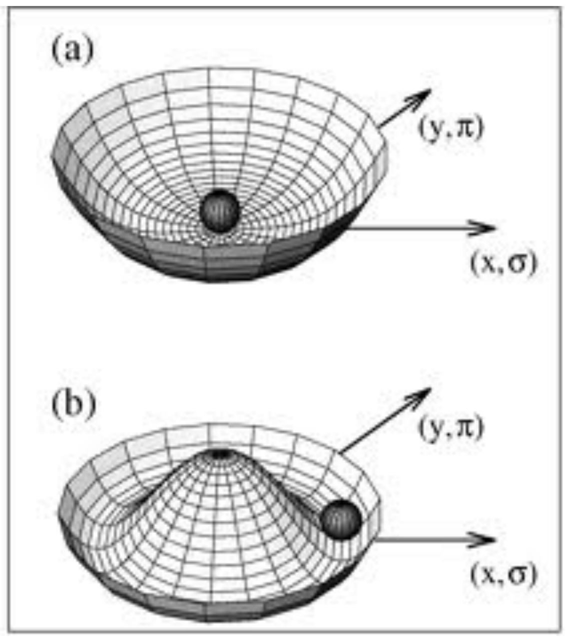
$$m\dot{\vec{x}} = \vec{p} - e\vec{A}$$

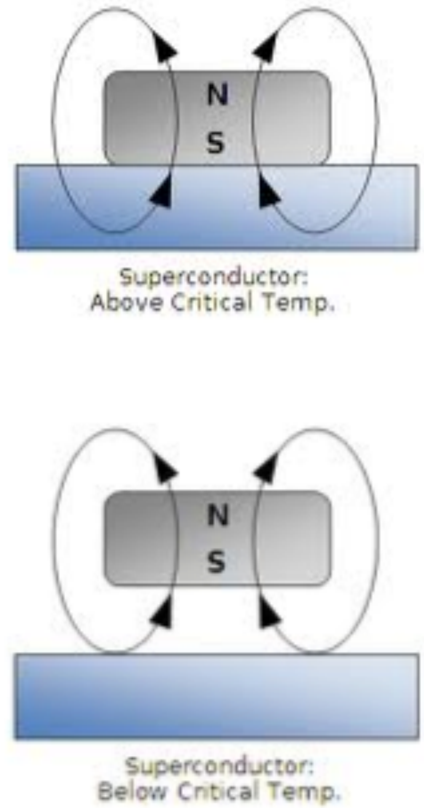
$$D_\mu \phi = \partial_\mu \phi - eA_\mu \phi$$



$$\phi = \begin{pmatrix} 0 \\ V + \eta \end{pmatrix} \Rightarrow m_\eta = \sqrt{\lambda V} \ , \ m_W = gV \ , \cdots$$


High Temperature
Low Temperature


(a)
(y, π)
(x, σ)
(b)
(y, π)
(x, σ)


N
S
Superconductor:
Above Critical Temp.
N
S
Superconductor:
Below Critical Temp.

If vacuum has condensation, vacuum symmetry is broken.

$\Rightarrow \exists$ Goldstone Boson. $(m=0)$

e.g.

Ferromagnetism

magnetization breaks the rotational symmetry

magnon: or spin wave

Superconductivity

Cooper pair condensation breaks the phase sym.

$$\psi \to e^{i\theta}\psi$$

$$v = \langle 0|\psi_\downarrow(x)\psi_\uparrow(x) + \psi_\uparrow^\dagger \psi_\downarrow^\dagger(x)|0\rangle \neq 0$$

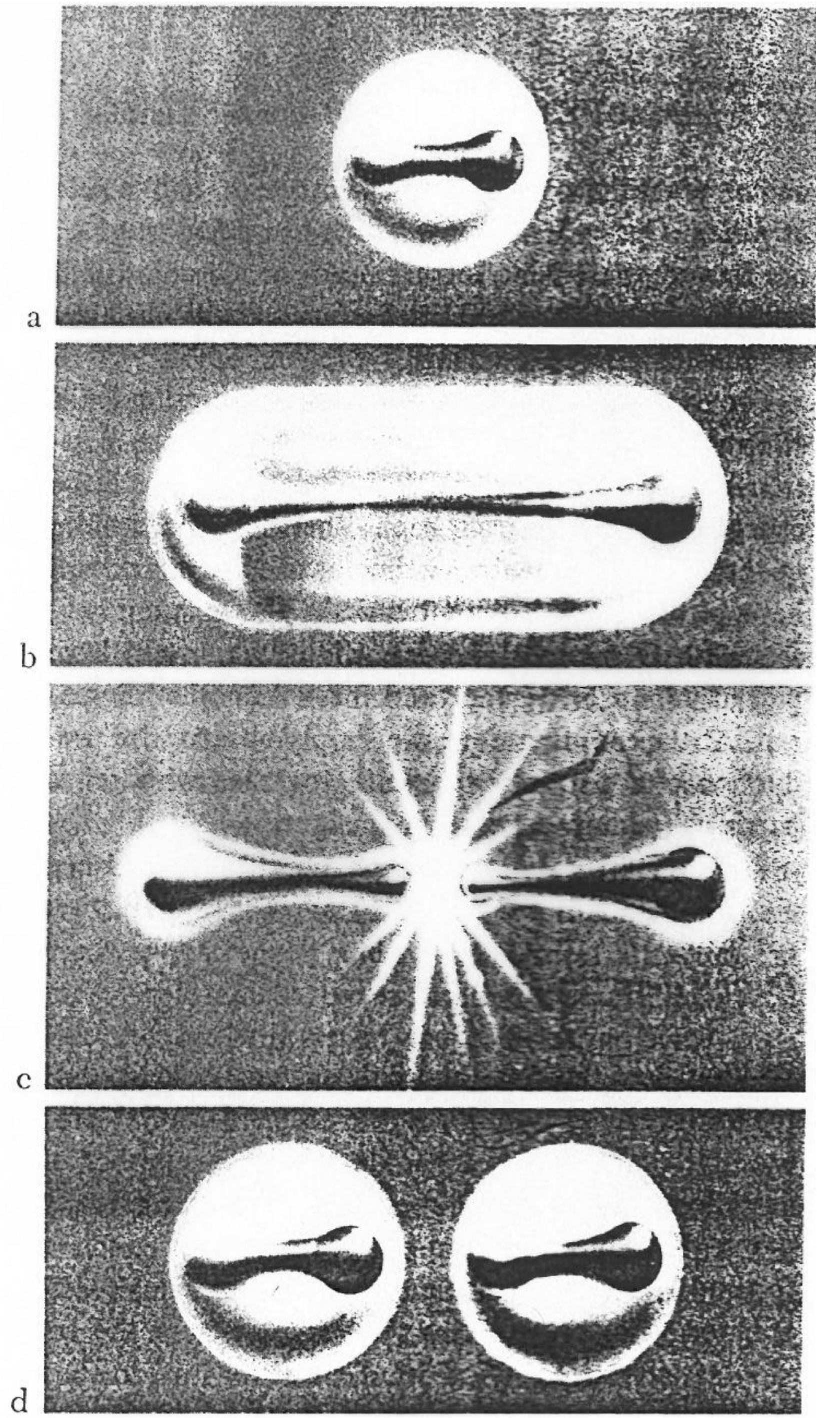
Phason: $\chi(x) = \psi_\downarrow(x)\psi_\uparrow(x) - \psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)$

$m_\chi = 0$. charge $-2e$.

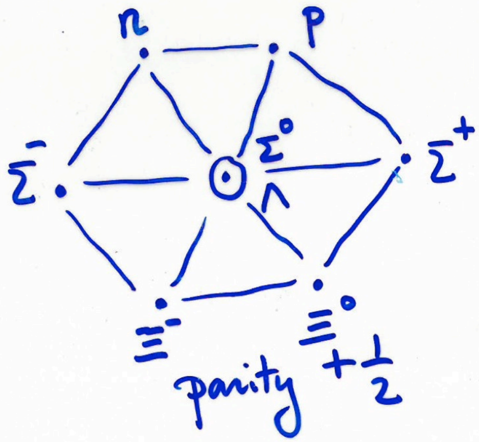
$\vec{J} = \sigma \vec{E} \quad \sigma \sim \frac{1}{m_\chi^2} \to \infty$

Confinement and
Vacuum Condensation are
intimately associated with
each other.

Vacuum Condensation
indicates that
the vacuum symmetry is
Broken due to
the condensation.



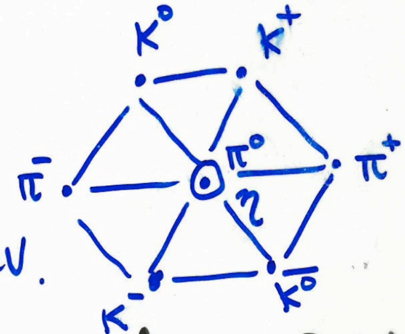
In QCD vacuum, chiral sym. is broken.



?

parity $-\frac{1}{2}$

Goldstone boson



$m_\pi \simeq 140\,\mathrm{MeV}$.

Low-energy Phenomenology : PCAC.

Phenomenological theory

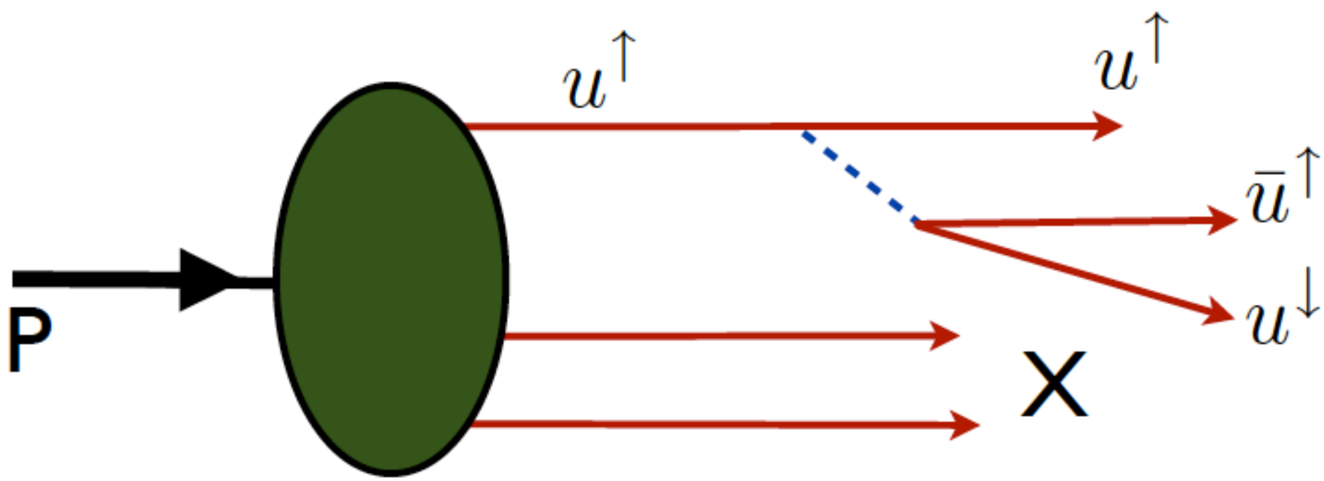
Landau - Ginzburg | Linear σ-model.
Nambu-Jona-Lassinio model.

Microscopic Derivation

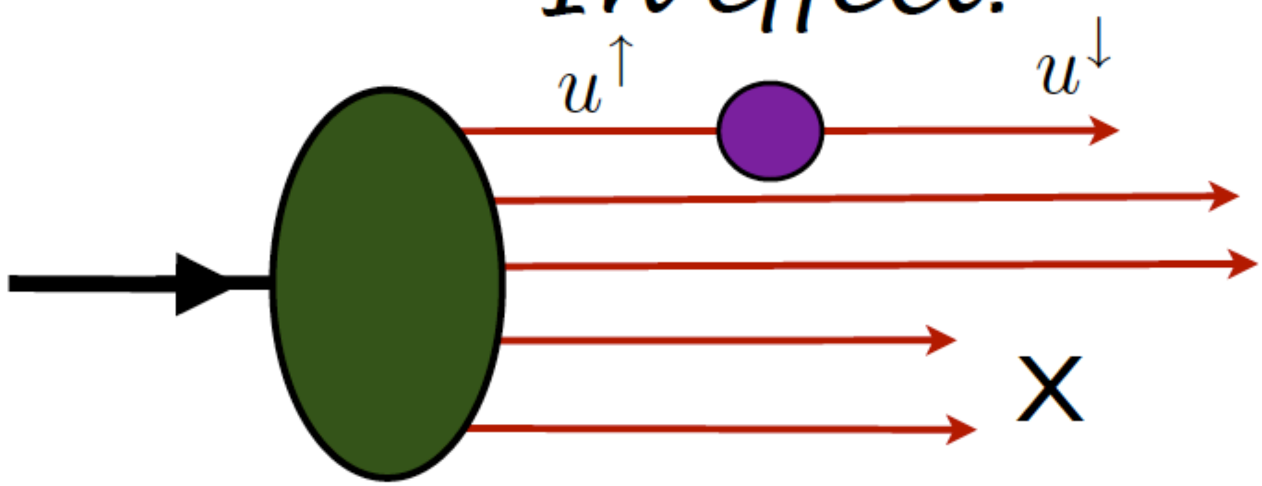
QED (BCS)

QCD (?)
Some attempt has been discussed

Chiral Symmetry in QCD

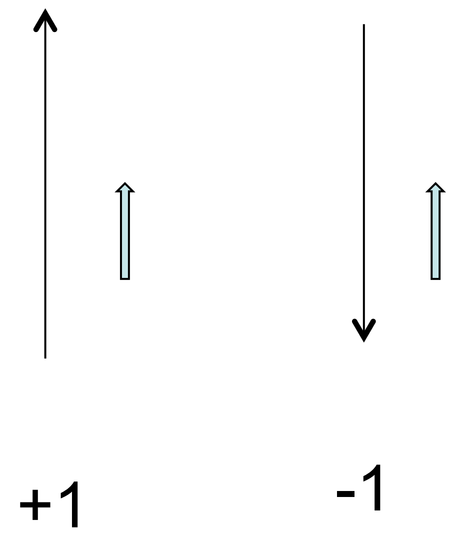


In effect:



Consistency with Relativistic Dynamics

Helicity is invariant for m=0



Helicity = Chirality for m=0

Chiral Symmetry Breaking due to dynamical mass generation

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{i,j=1}^{3} \overline{q_i} \left(i \not{D} - m_q \right)_{ij} q_j - \frac{1}{4} \sum_{\alpha=1}^{8} G^{\alpha}_{\mu\nu} G^{\alpha,\mu\nu}$$

Local SU(3)$_c$ Gauge Theory

interrelated with

Global Symmetry

Isospin symmetry

Chiral symmetry

$$\mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{SU}(2)_{\mathrm{L}}$$

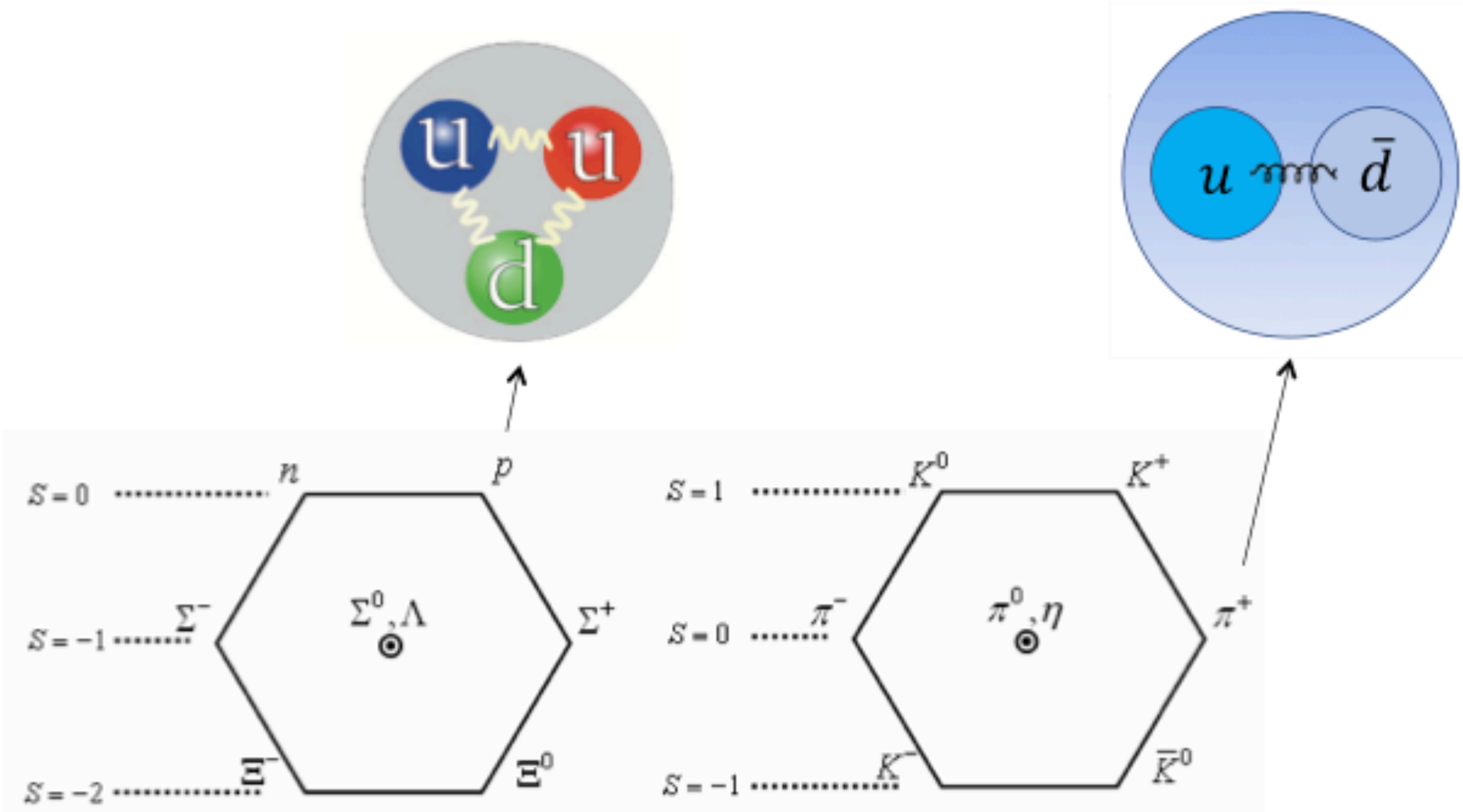
Spontaneous symmetry breakdown

Goldstone Bosons

$$F_\pi^2 M_\pi^2 = -(m_u + m_d) \langle 0 | \bar{u}u | 0 \rangle$$

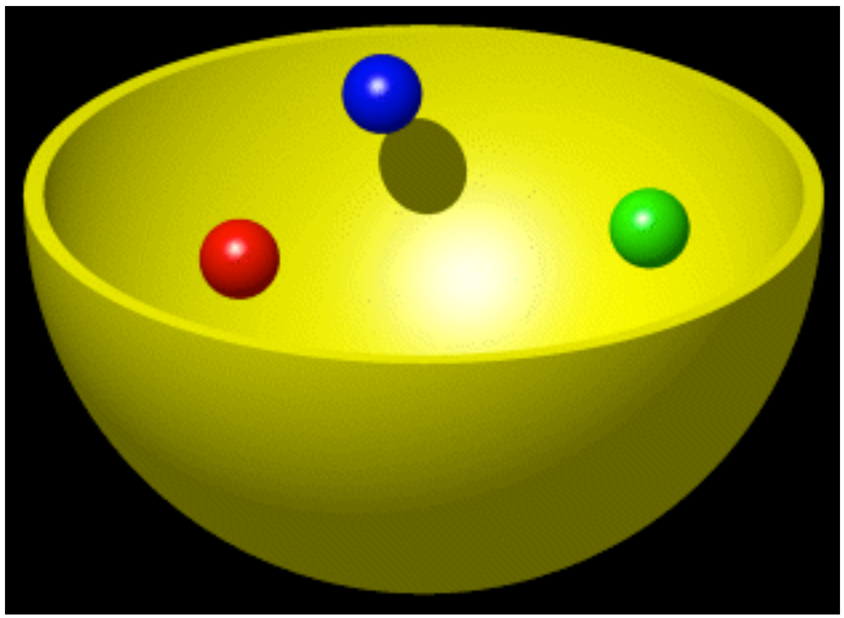
Effective field theory

How do we understand the Quark Model in Quantum Chromodynamics?



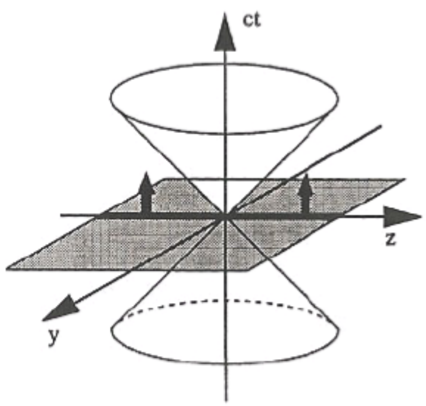
$$M_p = 938.272046 \pm 0.000021\, MeV$$

$$M_n = 939.565379 \pm 0.000021\, MeV$$



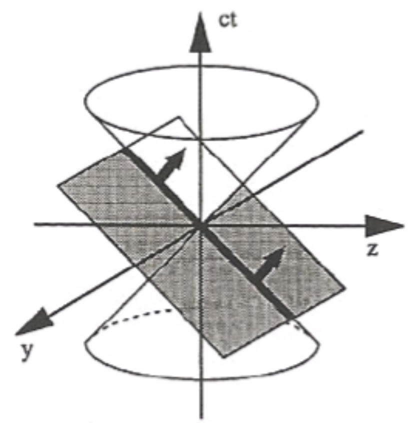
$$m_u = 2.3^{+0.7}_{-0.5}\, MeV \quad ; \quad m_d = 4.8^{+0.7}_{-0.3}\, MeV$$

Dirac's Proposition for Relativistic Dynamics



The instant form

1949

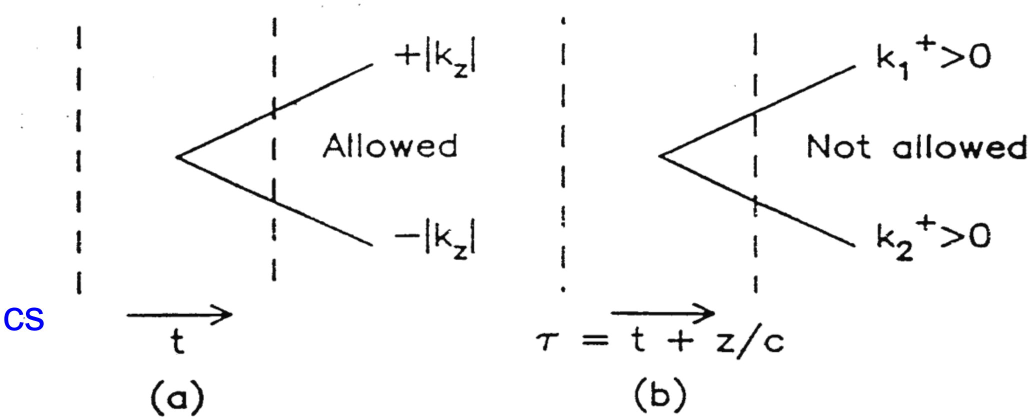


The front form

Equal t		*Equal τ*
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$

Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2} \qquad p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



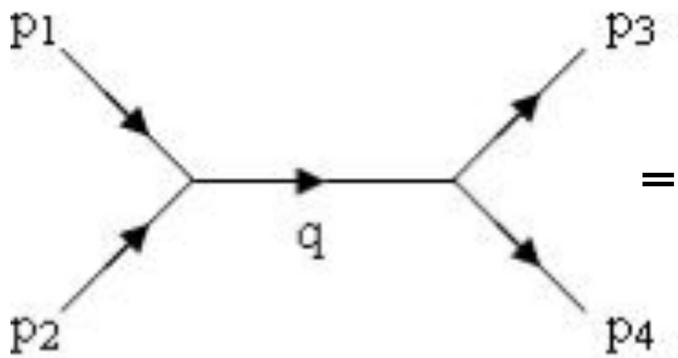
IFD

Instant Form Dynamics

LFD

Light-Front Dynamics

"$e^+e^- \to \mu^+\mu^-$"



$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

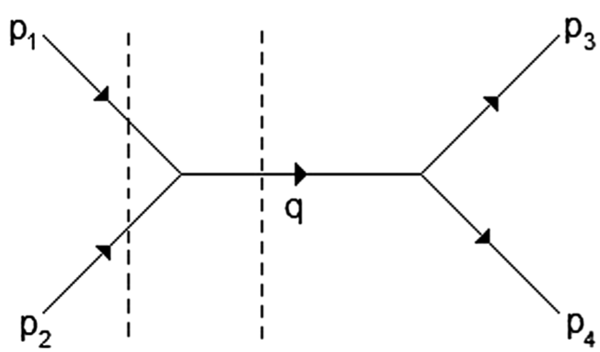
Four-momentum conservation but off-mass-shell

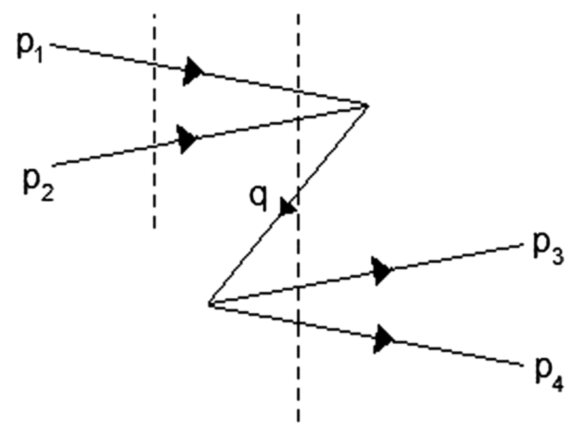
Feynman Diagram: Invariant under all 10 Poincaré generators

t→ (time evolution; time ordered process in QFT; Energy is not conserved within Δt)

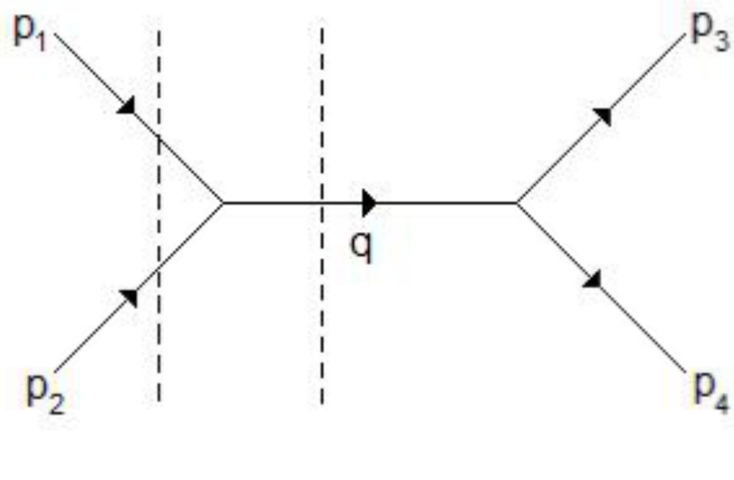
$(\Delta E)(\Delta t) \sim \hbar$

Three-momentum conservation but on-mass-shell

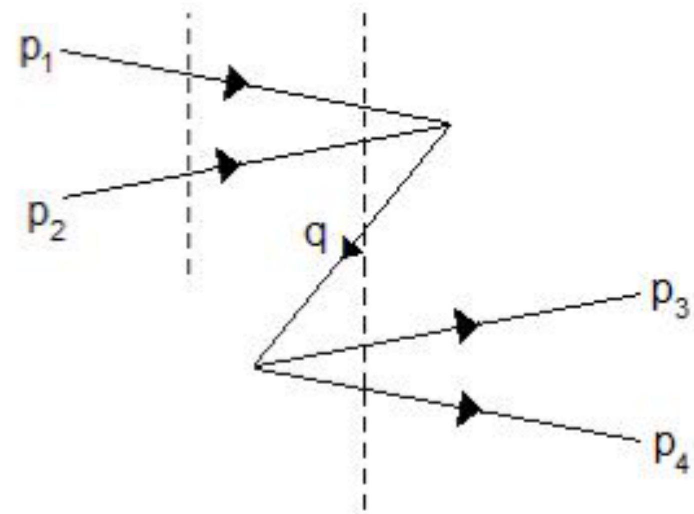




Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)



(a)



(b)

$$\Sigma^a_{\mathrm{IFD}} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0}\right)$$

$$\Sigma^b_{\mathrm{IFD}} = -\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\Sigma^a_{IFD} + \Sigma^b_{IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$= \frac{1}{(p_1^0 + p_2^0)^2 - (q^0)^2}$$

$$= \frac{1}{\{(p_1^0 + p_2^0)^2 - (\vec{p}_1 + \vec{p}_2)^2\} - \{(q^0)^2 - \vec{q}^{\,2}\}}$$

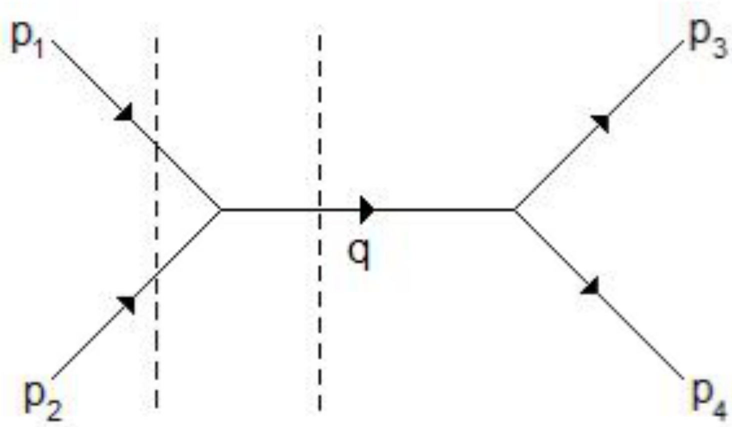
: Three-momentum conservation

$$= \frac{1}{(p_1 + p_2)^2 - q^2}$$

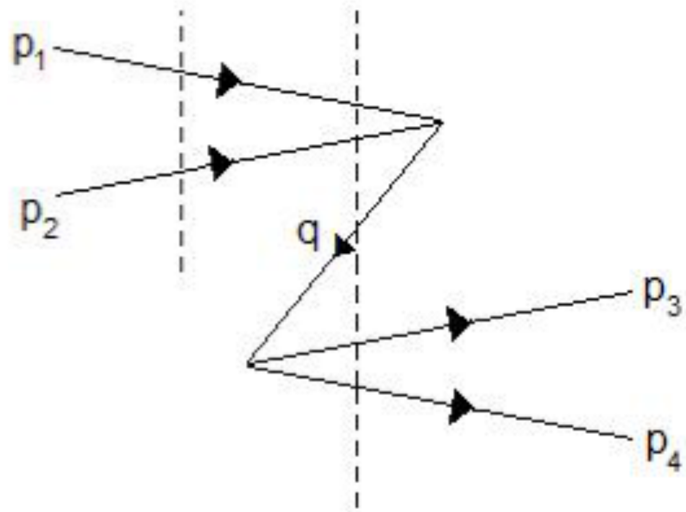
: $q^2 = m^2$; on -mass-shell

$$= \frac{1}{s - m^2}$$

Infinite Momentum Frame (IMF) Approach



(a)



(b)

$$\frac{1}{E_1 + E_2 - Eq}$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

$$-\frac{1}{Eq + E_3 + E_4}$$

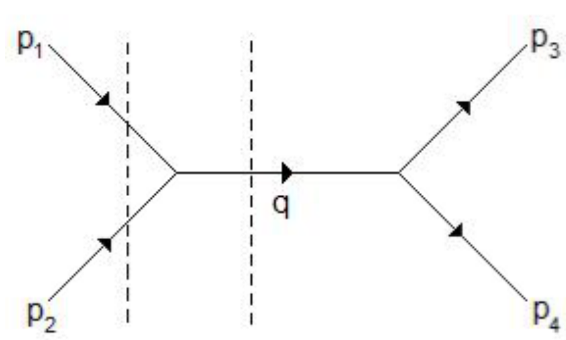
$$= -\frac{1}{Eq + E_1 + E_2}$$

$$\rightarrow \quad 0$$

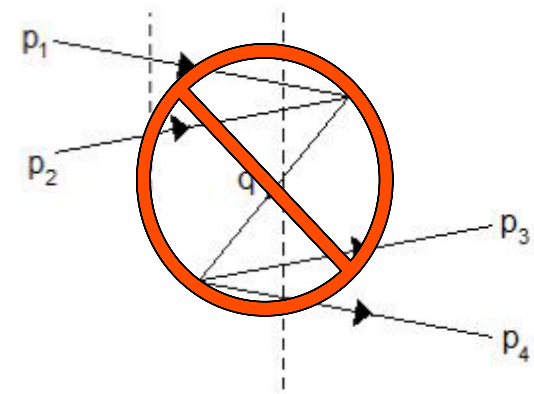
Note that this is still in the instant form (IFD).

However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →



(a)



(b)

$$\Sigma^a_{LFD} + \Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^- + p_2^- - q^-} + 0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2 + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2}{(p_1+p_2)^+} - \frac{m^2 + \vec{q}_\perp^{\,2}}{q^+}\right)}$$

$$= \frac{1}{(p_1+p_2)^2 - m^2}$$

$$= \frac{1}{s - m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$