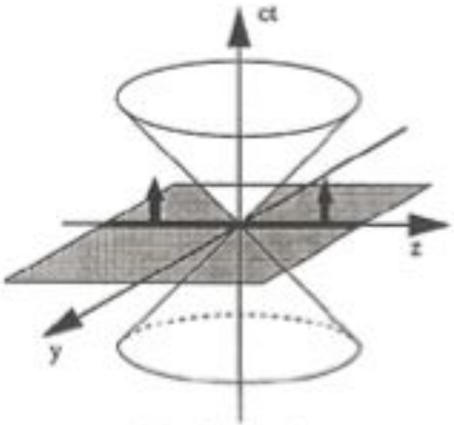
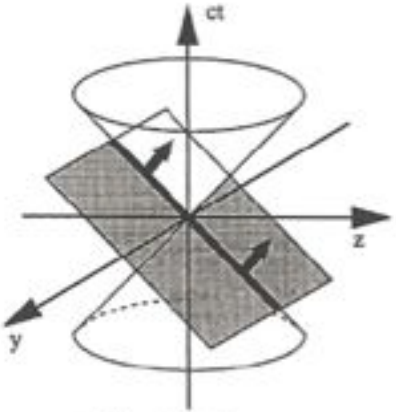
Interpolating instant form dynamics and light-front dynamics

Chueng-Ryong Ji

North Carolina State University



The instant form



The front form

APCTP Focus Program in NP2021 Part II

July 22, 2021

Motivation

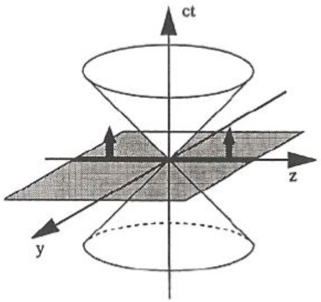
- Vigorous hadron structure studies in the forthcoming EIC programs motivate the review on the correspondence between the instant form dynamics (IFD) and the light-front dynamics (LFD), e.g. quasi-PDFs in IFD vs. PDFs in LFD.
- Possibilities of utilizing not only the reference frame dependence but also the interpolation between IFD and LFD emerge in recent hadron structure studies, e.g. QCD(1+1) in large Nc (‘tHooft model).

B.Ma&C.Ji, arXiv:2105.09388v1[hep-ph],PRD in press.

Outline

- Dirac’s Proposition for Relativistic Dynamics
- Infinite Momentum Frame (IMF) approach vs. LFD
- Link between IFD and LFD
- QED interpolation
- QCD(1+1) in large Nc (‘tHooft Model) interpolation
- Quasi-PDFs in IFD vs. PDFs in LFD
- Summary and Outlook

Dirac's Proposition for Relativistic Dynamics

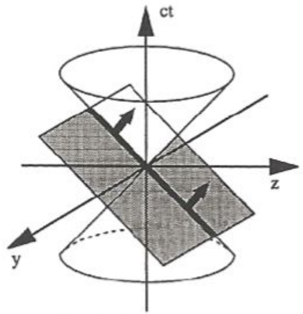


The instant form

1949

$k_1^+ = k_2^+ = 0$

Equal t		*Equal τ*
p^0	$\leftrightarrow$	$p^- = p^0 - p^3$
(p^1, p^2)	$\leftrightarrow$	$\vec{p}_\perp$
p^3	$\leftrightarrow$	$p^+ = p^0 + p^3$

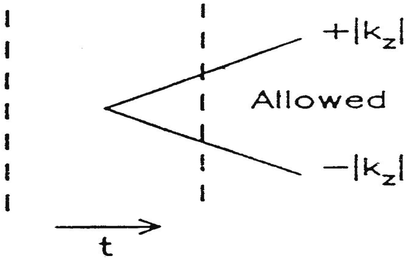


The front form

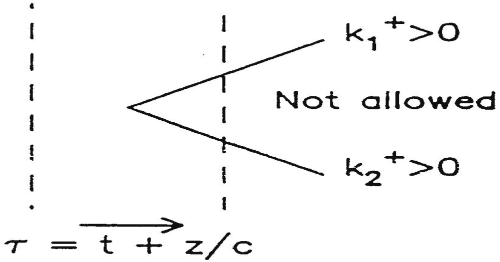
Energy-Momentum Dispersion Relations

$$p^0 = \sqrt{\vec{p}^{\,2} + m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



(a)



(b)

Except zero-modes

$k_1^+ = k_2^+ = 0$

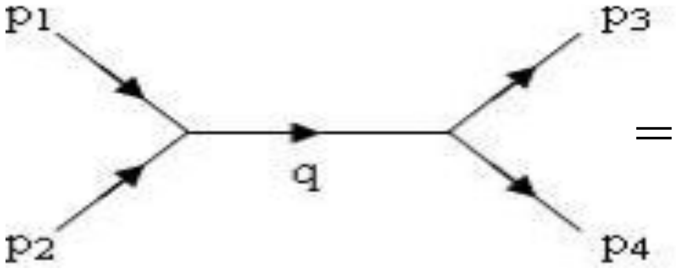
IFD

Instant Form Dynamics

LFD

Light-Front Dynamics

"$e^+e^- \to \mu^+\mu^-$"



$$= \frac{1}{q^2 - m^2} = \frac{1}{s - m^2} \qquad q^2 = (p_1 + p_2)^2 \neq m^2$$

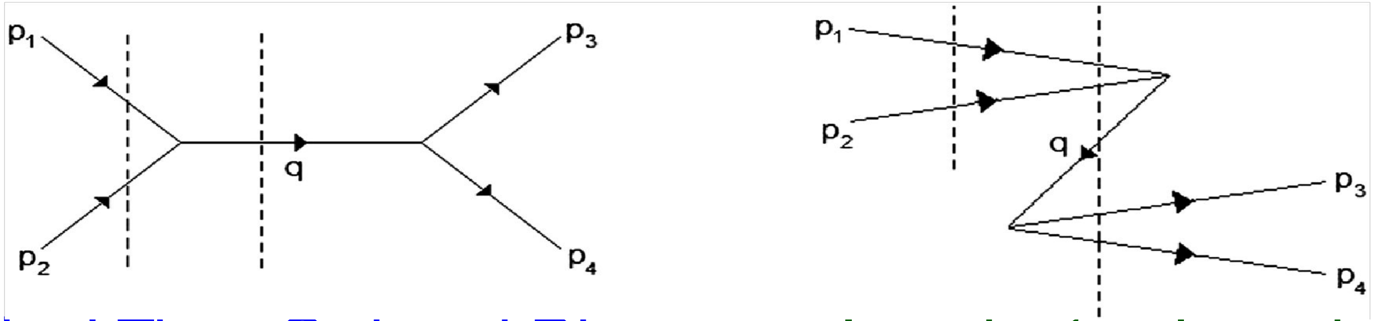
Four-momentum conservation but off-mass-shell

Feynman Diagram: Invariant under all 10 Poincaré generators

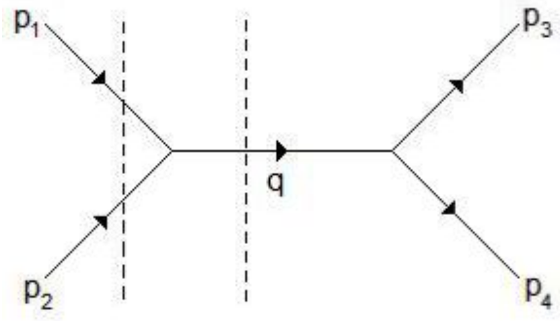
t⟶ (time evolution; time ordered process in QFT; Energy is not conserved within Δt)

$$(\Delta E)(\Delta t) \sim \hbar$$

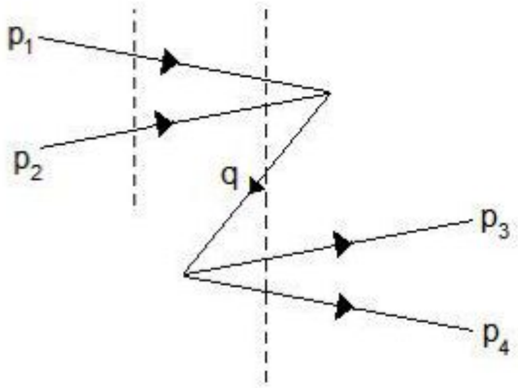
Three-momentum conservation but on-mass-shell



Individual Time-Ordered Diagrams: Invariant only under translation and rotation (6 kinematic generators)



(a)



(b)

$$\Sigma^a_{\rm IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0}\right) \qquad \Sigma^b_{\rm IFD} = -\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\Sigma^a_{IFD} + \Sigma^b_{IFD} = \frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$= \frac{1}{(p_1^0 + p_2^0)^2 - (q^0)^2}$$

$$= \frac{1}{\{(p_1^0 + p_2^0)^2 - (\vec{p}_1 + \vec{p}_2)^2\} - \{(q^0)^2 - \vec{q}^{\,2}\}}$$

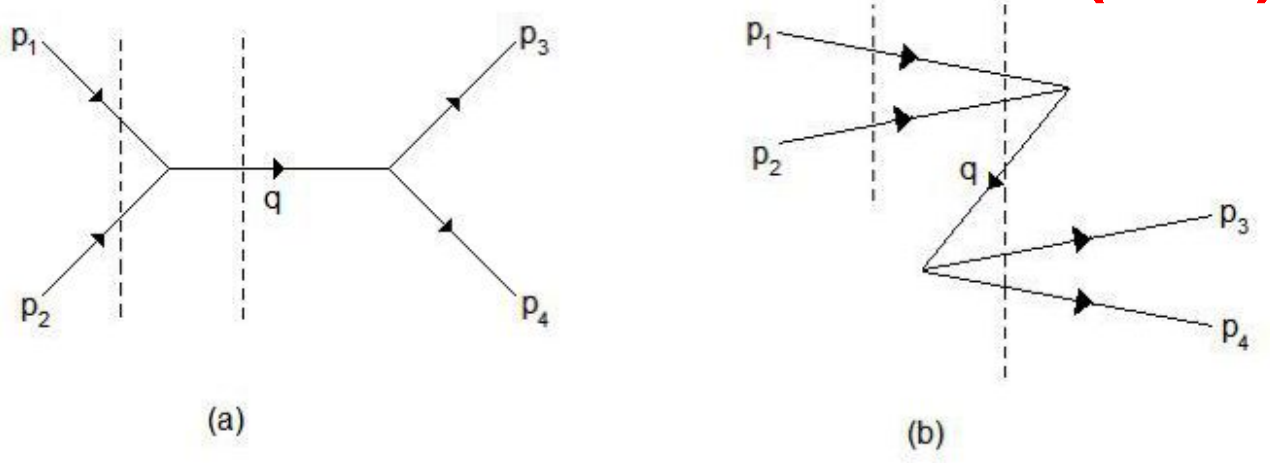
: Three-momentum conservation

$$= \frac{1}{(p_1 + p_2)^2 - q^2}$$

: $q^2 = m^2$; on -mass-shell

$$= \frac{1}{s - m^2}$$

Infinite Momentum Frame (IMF) Approach



$$\frac{1}{E_1 + E_2 - Eq}$$

$$-\frac{1}{Eq + E_3 + E_4}$$

$$= -\frac{1}{Eq + E_1 + E_2}$$

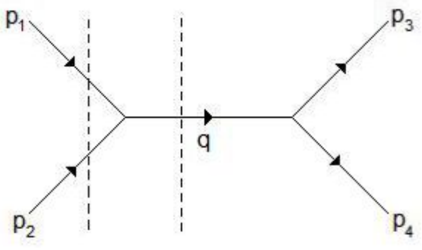
$$\to \quad 0$$

S.Weinberg, PR158,1638(1967)
"Dynamics at Infinite Momentum"

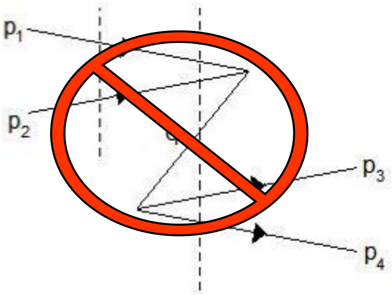
Note that this is still in the instant form (IFD).

However, in LFD, (b) drops for any reference frame (not just for IMF)

τ (= t+z/c) →



(a)



(b)

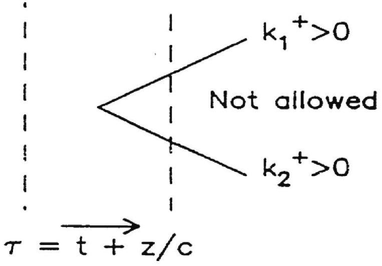
$$\Sigma^a_{LFD} + \Sigma^b_{LFD} = \frac{1}{q^+}\left(\frac{1}{p_1^- + p_2^- - q^-} + 0\right)$$

$$= \frac{1}{q^+\left(\frac{(p_1+p_2)^2 + (\vec{p}_{1\perp} + \vec{p}_{2\perp})^2}{(p_1+p_2)^+} - \frac{m^2 + \vec{q}_\perp^{\,2}}{q^+}\right)}$$

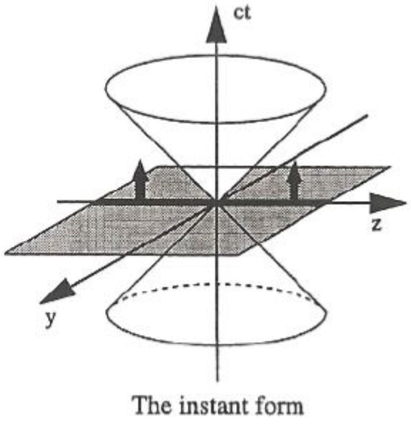
$$= \frac{1}{(p_1+p_2)^2 - m^2}$$

$$= \frac{1}{s - m^2}$$

$$p^- = \frac{\vec{p}_\perp^{\,2} + m^2}{p^+}$$



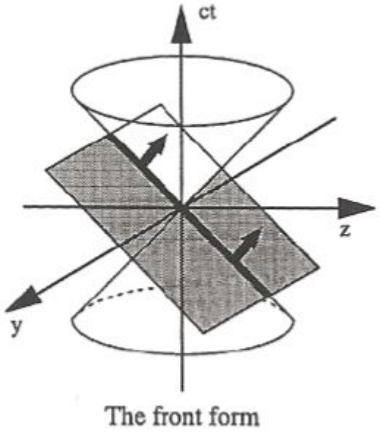
Can IFD and LFD be linked?



1949

Yes, they can!

1992~2021



Traditional approach
evolved from NR dynamics

Close contact with
Euclidean space

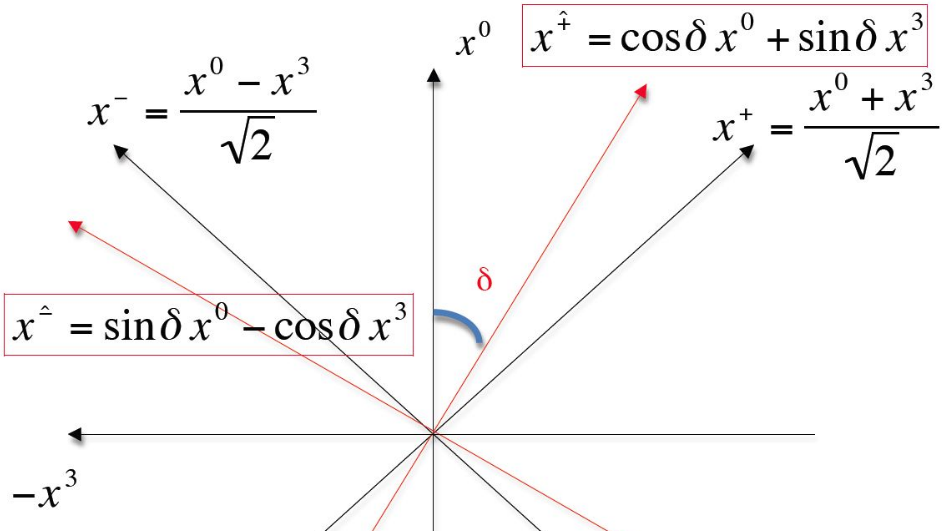
T-dept QFT, LQCD, IMF, etc.

Innovative approach
for relativistic dynamics

Strictly in Minkowski space

DIS, PDFs, DVCS, GPDs, etc.

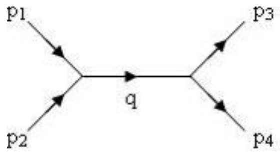
Interpolation between IFD and LFD



$$(IFD) \quad 0 \le \delta \le \frac{\pi}{4} \quad (LFD)$$

$$1 \ge \quad C \equiv \cos(2\delta) \quad \ge 0$$

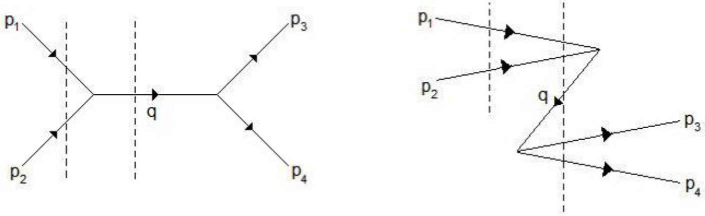
K. Hornbostel, PRD45, 3781 (1992) – RQFT
C.Ji and S.Rey, PRD53,5815(1996) – Chiral Anomaly
C.Ji and C. Mitchell, PRD64,085013 (2001) – Poincare Algebra
C.Ji and A. Suzuki, PRD87,065015 (2013) – Scattering Amps
C.Ji, Z. Li and A. Suzuki, PRD91, 065020 (2015) – EM Gauges
Z.Li, M. An and C.Ji, PRD92, 105014 (2015) – Spinors
C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017(2018) – QED
B.Ma and C.Ji, arXiv:2105.09388v1[hep-ph], PRD in press – QCD_{1+1}



$$\delta = 0$$
$$p_0 = p^0$$
$$-p_3 = p^3$$

$$0 < \delta < \pi/4$$
$$p_{\hat{+}} = p^0 \cos\delta - p^3 \sin\delta$$
$$p_{\hat{-}} = p^0 \sin\delta + p^3 \cos\delta$$

$$\delta = \pi/4$$
$$p_+ = p^-$$
$$p_- = p^+$$



(a) (b)

$$\frac{1}{2q^0}\left(\frac{1}{p_1^0 + p_2^0 - q^0} - \frac{1}{p_1^0 + p_2^0 + q^0}\right)$$

$$\frac{1}{2\omega_q}\left(\frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} - \omega_q}{\mathbb{C}}} - \frac{1}{P_{\hat{+}} + \frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}}}\right)$$

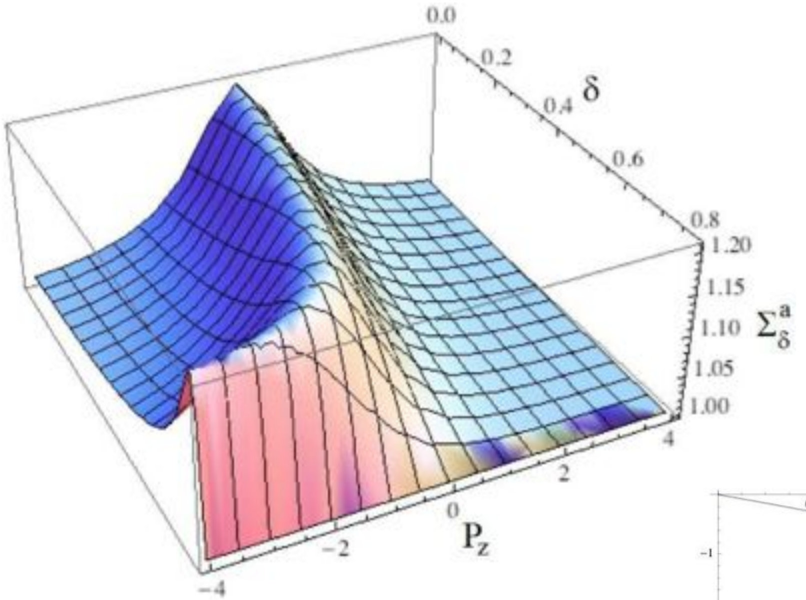
$$\omega_q = \sqrt{q_{\hat{-}}^2 + \mathbb{C}\left(\vec{\mathbf{q}}_\perp^2 + m^2\right)}$$

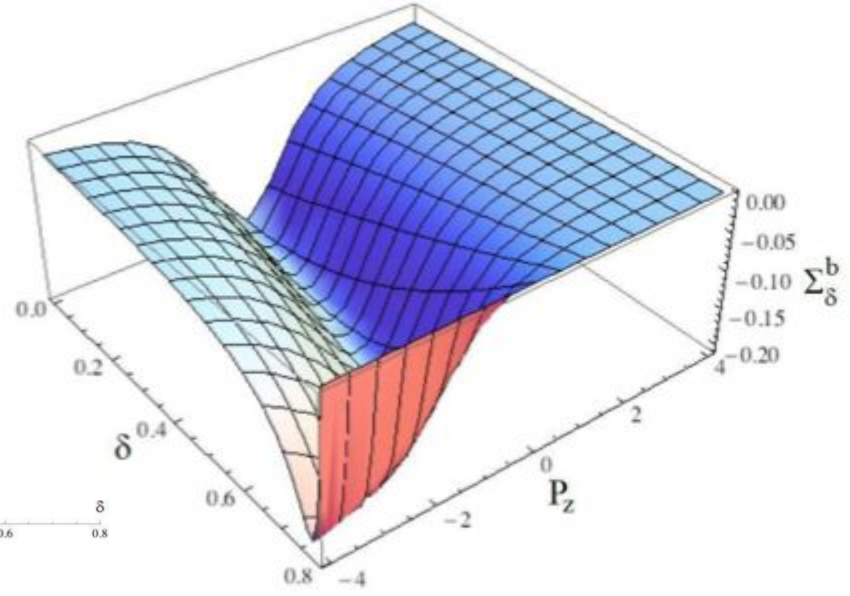
$$\mathbb{C} = \cos 2\delta$$

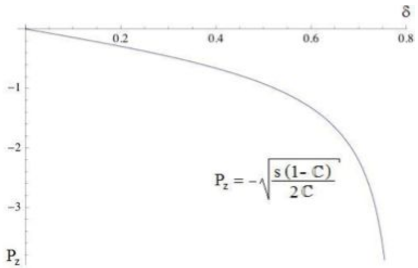
$$\mathbb{S} = \sin 2\delta$$

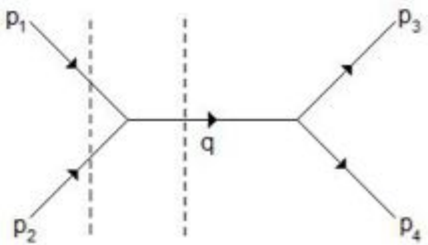
$$\frac{1}{P^+}\frac{1}{\left\{P^- - \frac{(\vec{\mathbf{P}}_\perp^2 + m^2)}{2P^+}\right\}}$$

$$\frac{\mathbb{S}q_{\hat{-}} + \omega_q}{\mathbb{C}} \rightarrow \frac{2}{\mathbb{C}} - \frac{\vec{\mathbf{q}}_\perp^2 + m^2}{2q_{\hat{-}}} + \mathcal{O}(\mathbb{C})$$
$$\rightarrow \infty \quad \text{as } \mathbb{C} \rightarrow 0$$





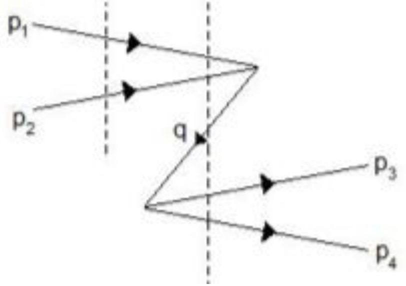




$\Sigma(a)+\Sigma(b)=1/(s-m^2)$; s=2 GeV^2, m=1GeV

J-shape peak & valley : $P_z = -\sqrt{\frac{s(1-C)}{2C}}$; $C = \cos(2\delta)$

As C → 0, $P^+ = P^0+P_z$ →0 leads to LF Zero-modes.



$$A^{\hat{+}} = 0,\ \partial_{\hat{-}} A_{\hat{-}} + \boldsymbol{\partial}_\perp \mathbf{A}_\perp \mathbb{C} = 0 \quad (\mathbb{C} = \cos 2\delta)$$

$\delta \to 0$ ($\mathbb{C} \to 1$)

$\delta \to \pi/4$ ($\mathbb{C} \to 0$)

C.Ji, Z. Li, and A. T. Suzuki, PRD91, 065020(2015)

$$A^0 = 0,\ \nabla \cdot \mathbf{A} = 0$$

Coulomb Gauge

$$A^+ = 0$$

Light-front Gauge

$$\sum_{\lambda=\pm} \epsilon^*_{\hat{\mu}}(\lambda)\epsilon_{\hat{\nu}}(\lambda) = -g_{\hat{\mu}\hat{\nu}} + \frac{(q\cdot n)(q_{\hat{\mu}} n_{\hat{\nu}} + q_{\hat{\nu}} n_{\hat{\mu}})}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{\mathbb{C} q_{\hat{\mu}} q_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2} - \frac{q^2 n_{\hat{\mu}} n_{\hat{\nu}}}{\mathbf{q}_\perp^2 \mathbb{C} + q_{\hat{-}}^2}$$

LFD

$$-g_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2}$$

P.Srivastava and S. Brodsky, PRD64,045006(2001)

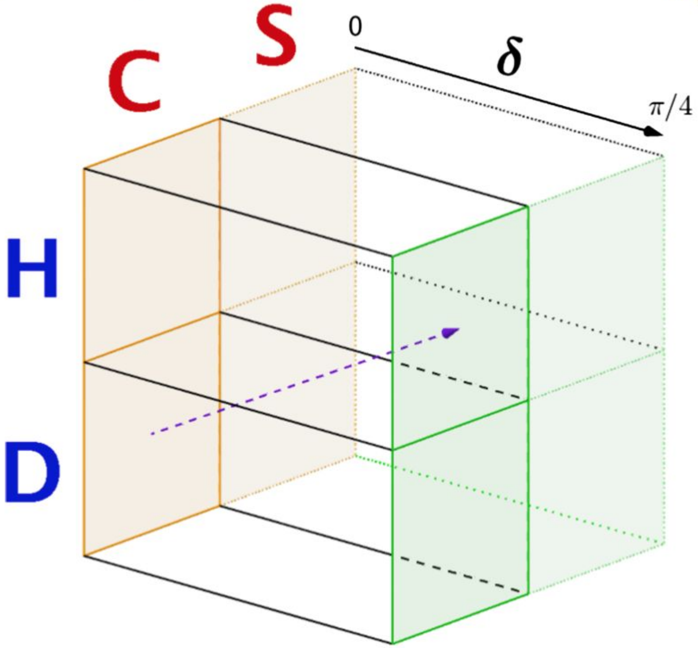
IFD

$$-\eta_{\mu\nu} + \frac{(q\cdot n)(q_\mu n_\nu + q_\nu n_\mu)}{(q\cdot n)^2 - q^2} - \frac{q_\mu q_\nu}{(q\cdot n)^2 - q^2} - \frac{q^2 n_\mu n_\nu}{(q\cdot n)^2 - q^2}$$

$$S = S^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix} I & I \\ I & -I \end{pmatrix}$$

$$u_S(p) = S u_C(p),$$
$$u_C(p) = S^\dagger u_S(p),$$

Z.Li,M.An&C.Ji, PRD92,105014 (2015)



$$\mathbf{A} = \frac{1}{2}(\mathbf{J} + i\mathbf{K}),$$

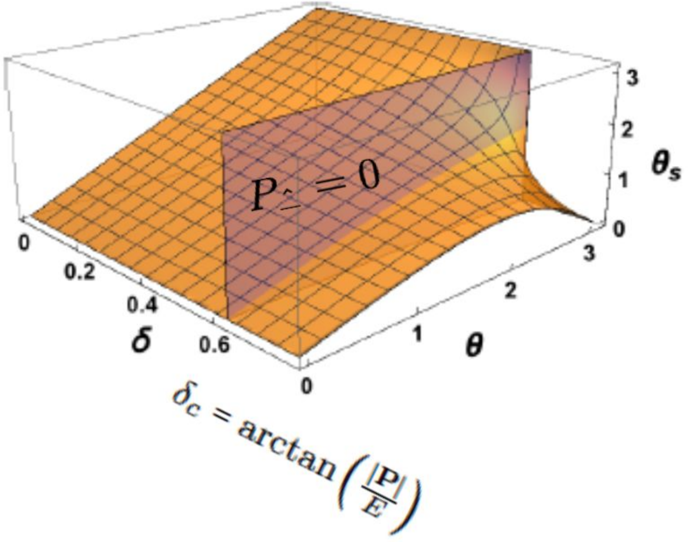
$$\mathbf{B} = \frac{1}{2}(\mathbf{J} - i\mathbf{K}),$$

$$[A_i, A_j] = i\epsilon_{ijk}A_k,$$
$$[B_i, B_j] = i\epsilon_{ijk}B_k,$$
$$[A_i, B_j] = 0, \quad (i, j, k = 1, 2, 3)$$

Helicity

IFD

LFD



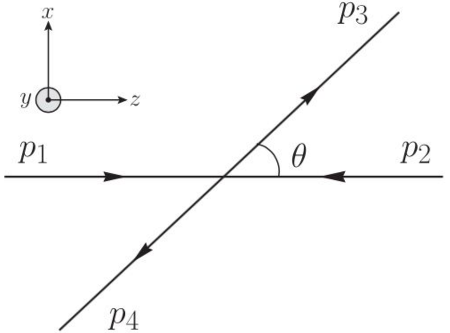
+1 -1

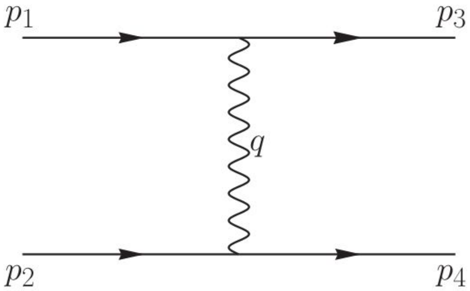
+1 +1

M. Jacob and G. Wick,
Ann. Phys., 7, 404 (1959)

C. Carlson and C.Ji,
PRD, 67, 116002 (2003)

K_z Dependent vs. K_z Independent



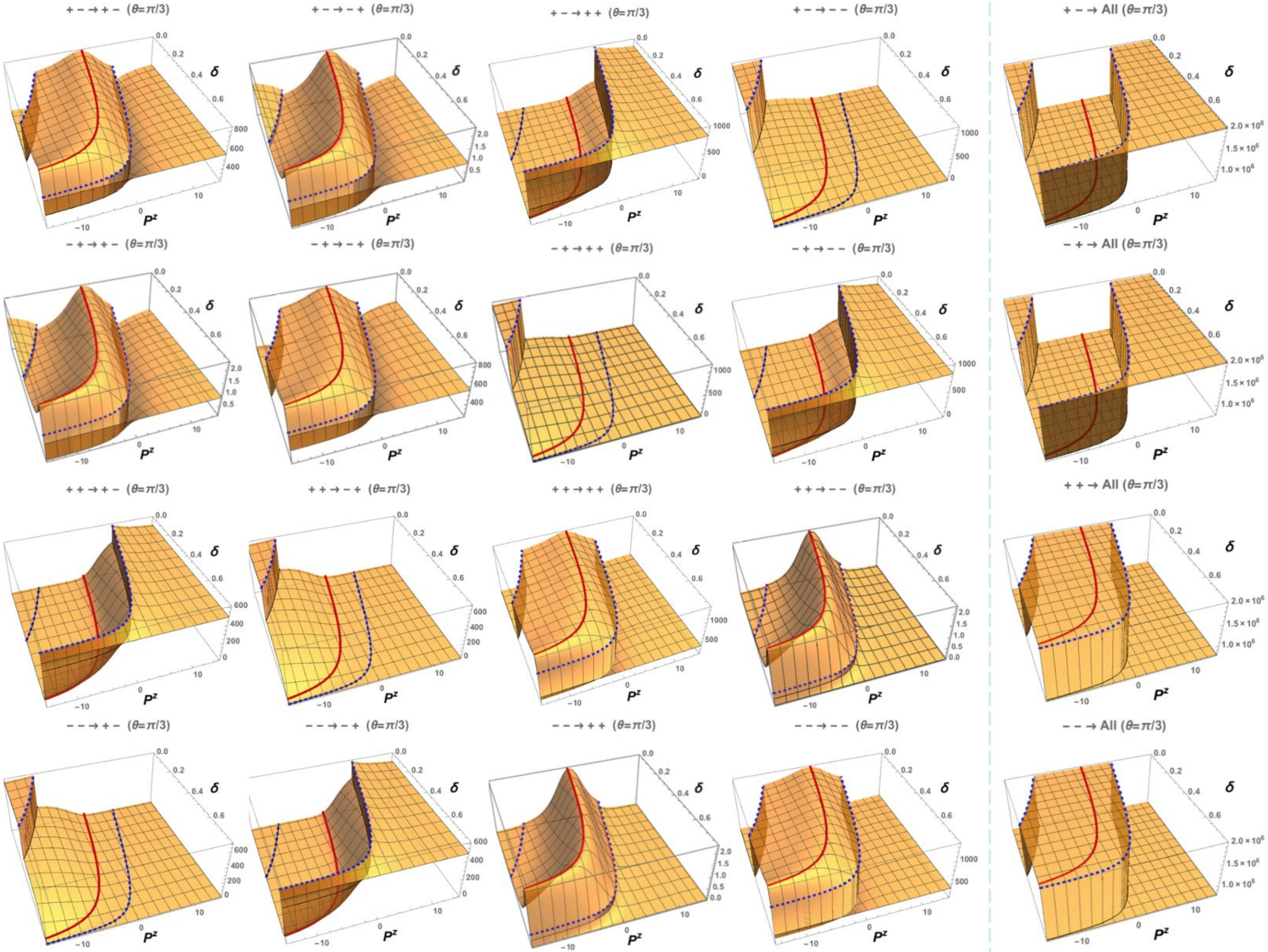


$$m_1 = 1 \text{ GeV}$$

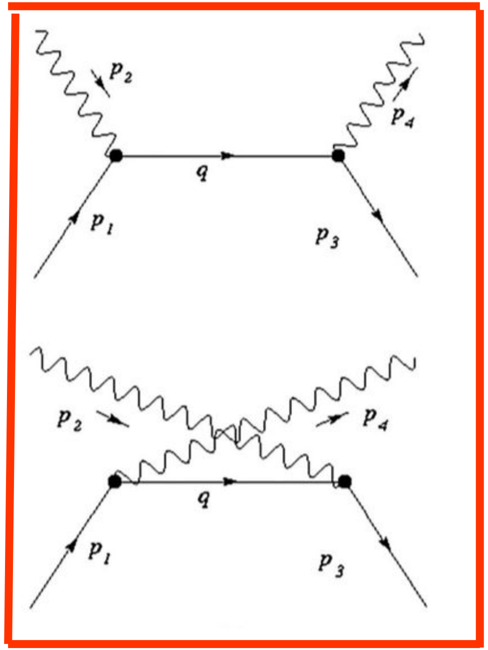
$$m_2 = 1.5 \text{ GeV}$$

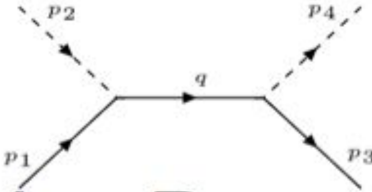
$$p = 2 \text{ GeV}$$

$$\theta = \pi/3$$

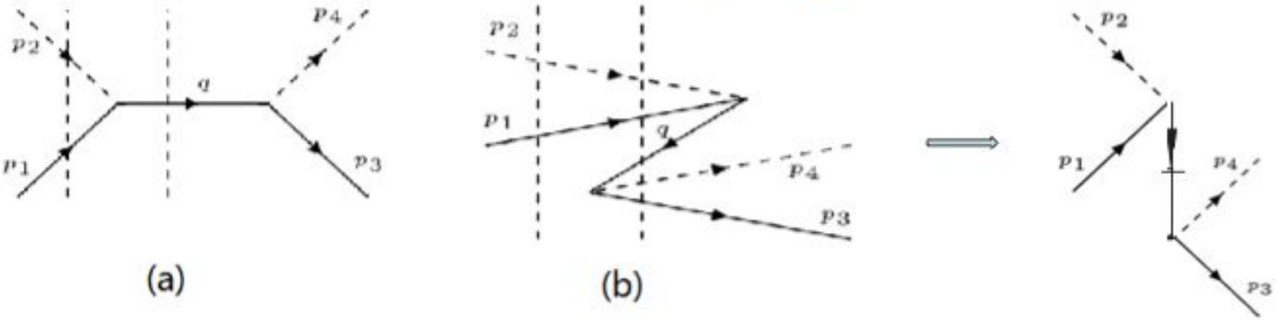


C.Ji, Z.Li, B.Ma and A.Suzuki, PRD98, 036017 (2018)





Fermion Propagator



(a) (b)

$$\Sigma_a^{\text{IFD}} + \Sigma_b^{\text{IFD}} = \frac{1}{2q^0_{on}}\left(\frac{\not{q}+m}{q^0 - q^0_{on}} - \frac{\not{q}+m}{q^0 + q^0_{on}}\right)$$
$$= \frac{1}{2q^0_{on}}\frac{2q^0_{on}(\not{q}+m)}{(q^0)^2 - (q^0_{on})^2}$$
$$= \frac{\not{q}+m}{q^2 - m^2}$$

$$\Sigma_{a,\delta\to\frac{\pi}{4}} = \frac{\not{q}_{on} + m}{q^2 - m^2}$$

$$\Sigma_{b,\delta\to\frac{\pi}{4}} = \frac{\gamma^+}{2q^+}$$

S.-J.Chang and T.-M.Yan, PRD7,1147(1973)

$$\frac{1}{\not{q} - m} = \frac{\sum_s u(q,s)\bar{u}(q,s)}{q^2 - m^2} + \frac{\gamma^+}{2q^+}$$

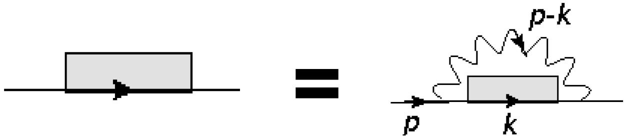
Large N_c QCD in 1+1 dim. ('tHooft Model)

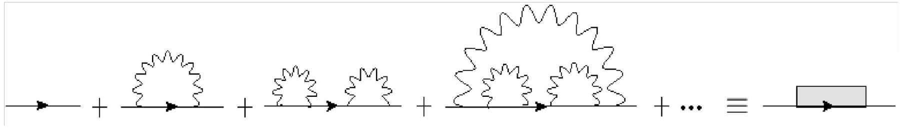
Interpolating 't Hooft model between instant and front forms

Bailing Ma and Chueng-Ryong Ji

arXiv:2105.09388v1 [hep-ph] 19 May 2021

MASS GAP EQUATION





$\equiv i\,\Sigma(p)$

$$\Sigma(p_{\hat{-}}) = i\frac{\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk^{\hat{+}}}{(p_{\hat{-}} - k_{\hat{-}})^2}\gamma^{\hat{+}}\frac{1}{\not{k} - m - \Sigma(k_{\hat{-}}) + i\epsilon}\gamma^{\hat{+}}$$

Fermion Propagator

Free Propagator

$$S_f(p) = \frac{1}{\not{p} - m + i\varepsilon}$$

$$\Sigma(p) = \Sigma_s(p) + \Sigma_v(p)\not{p}$$

Interacting Propagator

$$S(p) = \frac{1}{\not{p} - m - \Sigma(p) + i\varepsilon}$$

$$= \frac{F(p)}{\not{p} - M(p) + i\varepsilon}$$

$$F(p) = \left(1 - \Sigma_v(p)\right)^{-1}$$ "Wave function renormalization factor"

$$M(p) = \frac{m + \Sigma_s(p)}{1 - \Sigma_v(p)}$$ "Renormalized fermion mass function"

Energy-Momentum Dispersion Relation

Free particle | **Interacting particle**

$$E=\sqrt{p_z^2+m^2}$$

$$\theta_f=\tan^{-1}(p_z/m)$$

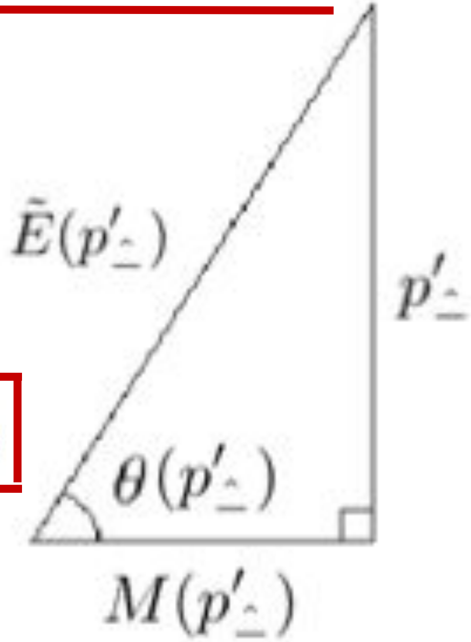
$$\beta=p_z/E$$
$$=\sin\theta_f$$
$$=\tanh\eta$$

$$\frac{F(p'_\perp)E(p'_\perp)}{\sqrt{C}}=\sqrt{p'^2_\perp+M(p'_\perp)^2}\equiv\tilde{E}(p'_\perp)$$

$$\theta(p'_\perp)=\theta_f(p'_\perp)+2\zeta(p'_\perp)$$

$$\begin{pmatrix} b^i(p'_\perp) \\ d^{+i}(p'_\perp) \end{pmatrix}=\begin{pmatrix} \cos\zeta(p'_\perp) & -\sin\zeta(p'_\perp) \\ \sin\zeta(p'_\perp) & \cos\zeta(p'_\perp) \end{pmatrix}\begin{pmatrix} b^i_f(p'_\perp) \\ d^{+i}_f(p'_\perp) \end{pmatrix}$$

$b^i_f|0>=0, d^i_f|0>=0$ **vs.** $b^i|\Omega>=0, d^i|\Omega>=0$



Interpolation

$$(E,p_z)\Rightarrow(p^+/\sqrt{C},p_\perp/\sqrt{C}\equiv p'_\perp)$$

Mass Gap Equation in Scaled Variables

$$\bar{p}'_{\perp} = \frac{\bar{p}_{\perp}}{\sqrt{\mathbb{C}}}, \ \bar{E}' = \frac{\bar{E}}{\sqrt{\mathbb{C}}}, \bar{p}_{\perp} = \frac{p_{\perp}}{\sqrt{2\lambda}}, \ \bar{E} = \frac{E}{\sqrt{2\lambda}}, \bar{m} = \frac{m}{\sqrt{2\lambda}}$$

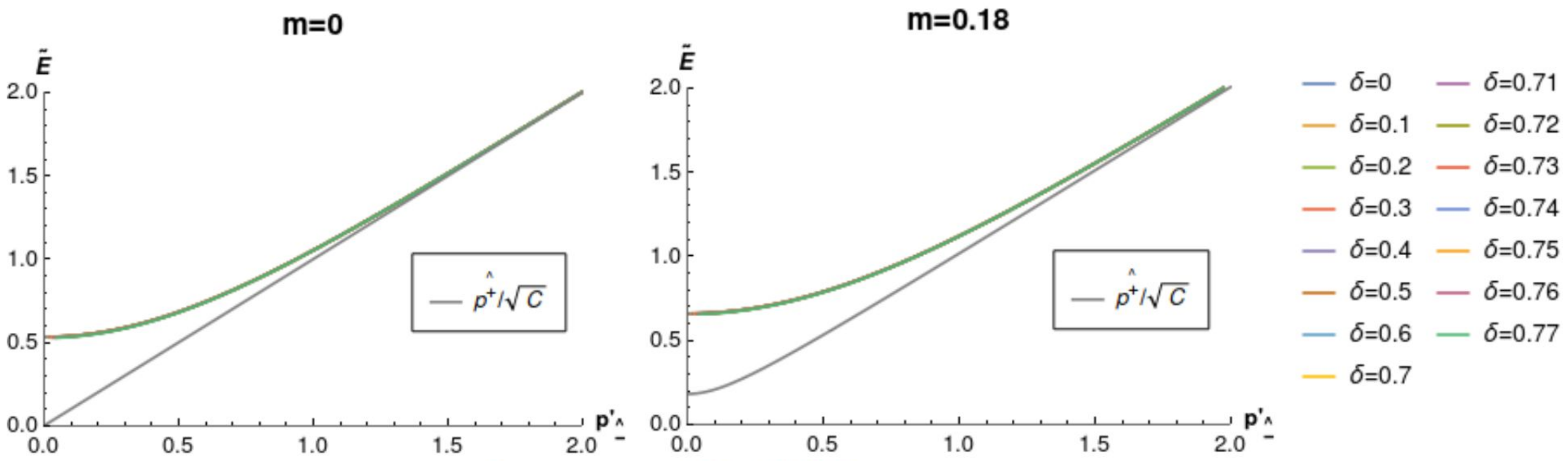
$$\bar{p}'_{\perp} \cos\theta(\bar{p}'_{\perp}) - \bar{m}\sin\theta(\bar{p}'_{\perp}) = \frac{1}{4} \int \frac{d\bar{k}'_{\perp}}{(\bar{p}'_{\perp} - \bar{k}'_{\perp})^2} \sin\left(\theta(\bar{p}'_{\perp}) - \theta(\bar{k}'_{\perp})\right)$$

$$\bar{E}'(\bar{p}'_{\perp}) = \bar{p}'_{\perp} \sin\theta(\bar{p}'_{\perp}) + \bar{m}\cos\theta(\bar{p}'_{\perp}) + \frac{1}{4} \int \frac{d\bar{k}'_{\perp}}{(\bar{p}'_{\perp} - \bar{k}'_{\perp})^2} \cos\left(\theta(\bar{p}'_{\perp}) - \theta(\bar{k}'_{\perp})\right)$$

$$\frac{p_{\perp}}{\mathbb{C}} \cos\theta(p_{\perp}) - \frac{m}{\sqrt{\mathbb{C}}} \sin\theta(p_{\perp}) = \frac{\lambda}{2} \int \frac{dk_{\perp}}{(p_{\perp} - k_{\perp})^2} \sin\left(\theta(p_{\perp}) - \theta(k_{\perp})\right)$$

$$E(p_{\perp}) = p_{\perp} \sin\theta(p_{\perp}) + \sqrt{\mathbb{C}} m \cos\theta(p_{\perp}) + \frac{\mathbb{C}\lambda}{2} \int \frac{dk_{\perp}}{(p_{\perp} - k_{\perp})^2} \cos\left(\theta(p_{\perp}) - \theta(k_{\perp})\right)$$

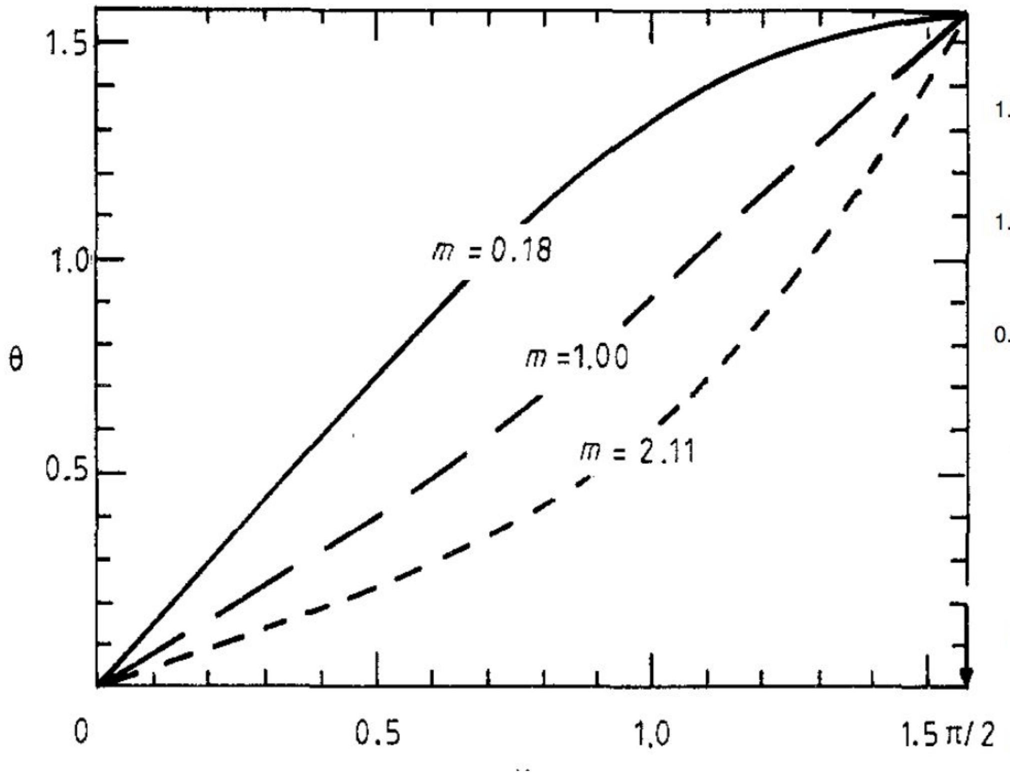
Mass Gap Solutions

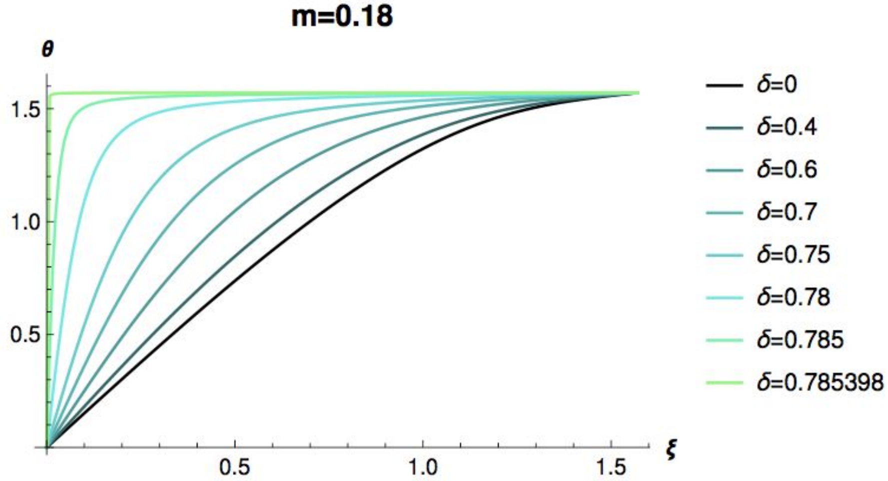


$$\tilde{E}(0) = \frac{F(0)E(0)}{\sqrt{\mathbb{C}}} = M(0)$$

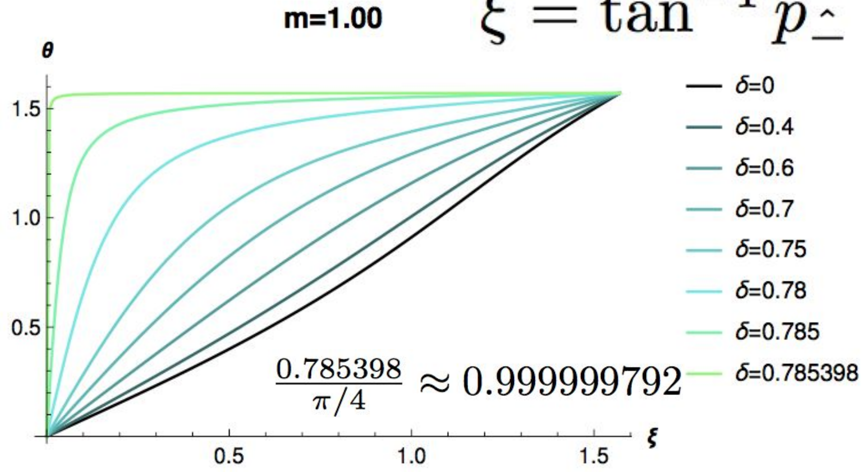
m	0	0.045	0.18	0.749	1.00	2.11	4.23
$M(0)$	0.532778	0.563644	0.659112	1.10105	1.31167	2.30969	4.34358
$F(0)$	−0.495173	−0.584175	−0.987673	4.11079	2.17976	1.22134	1.05526

$$m \lesssim 0.56$$



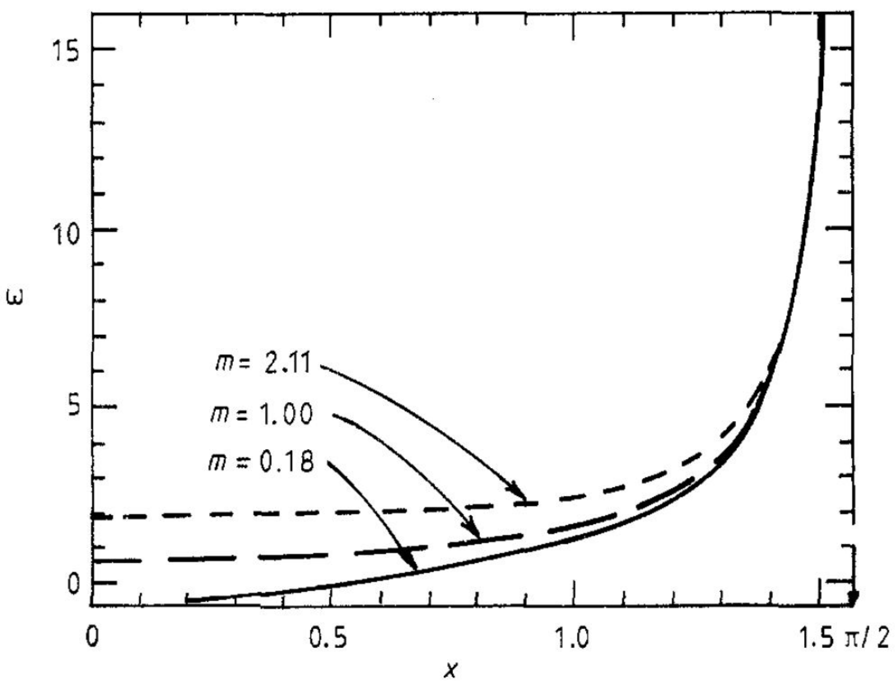


$$\xi = \tan^{-1} p_{\hat{-}}$$

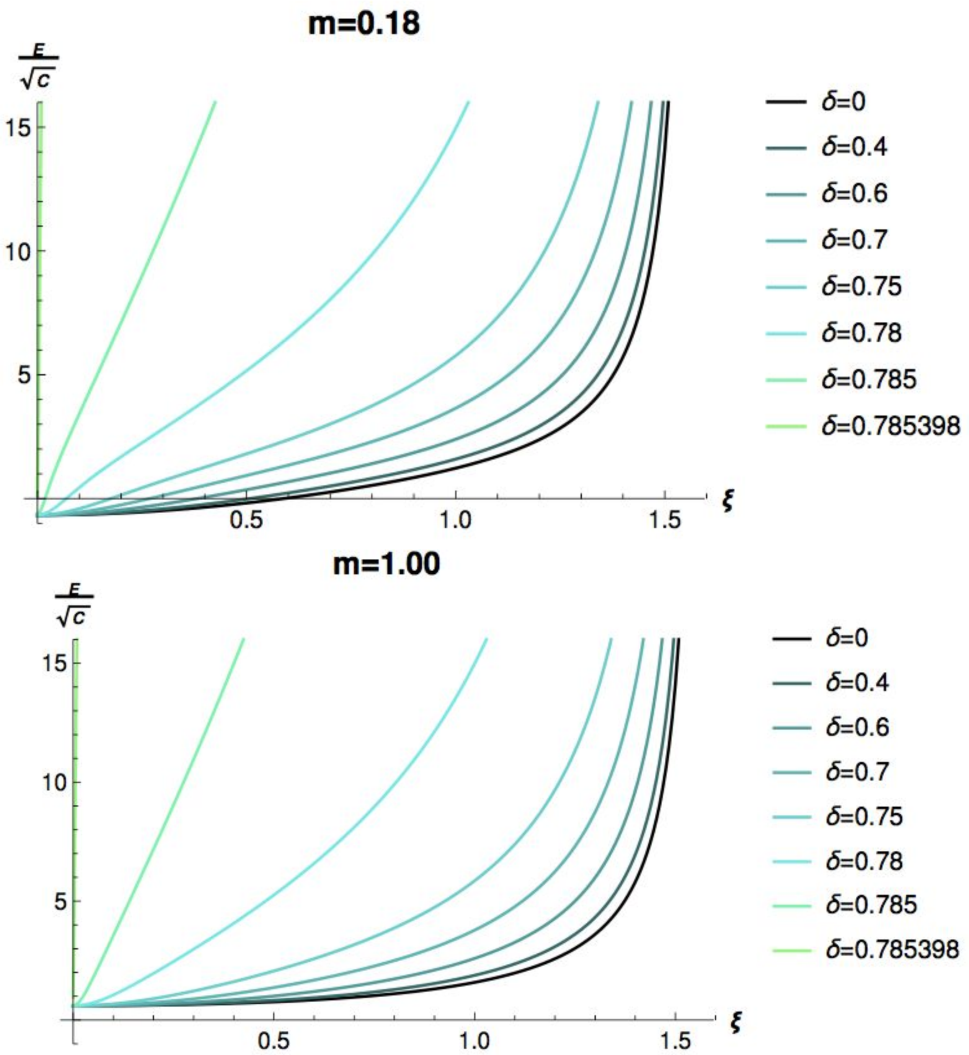


$$\frac{0.785398}{\pi/4} \approx 0.999999792$$

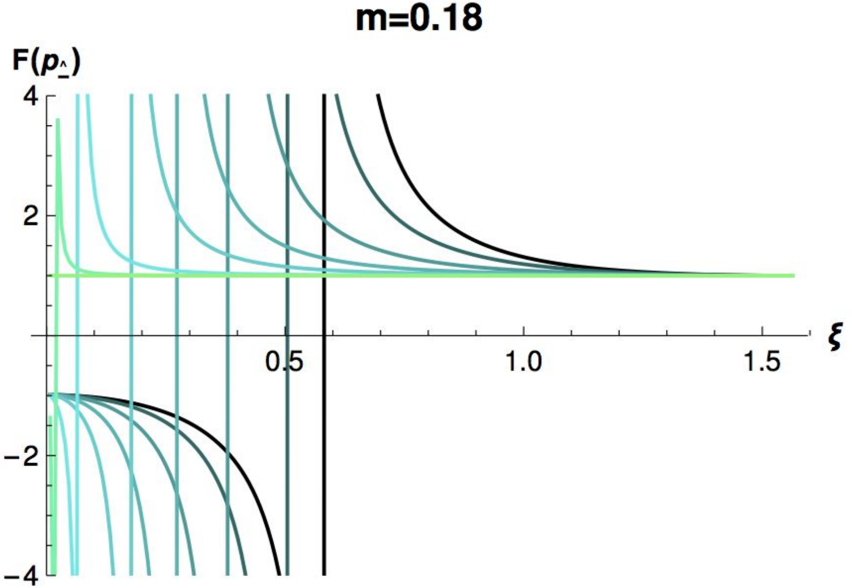
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)

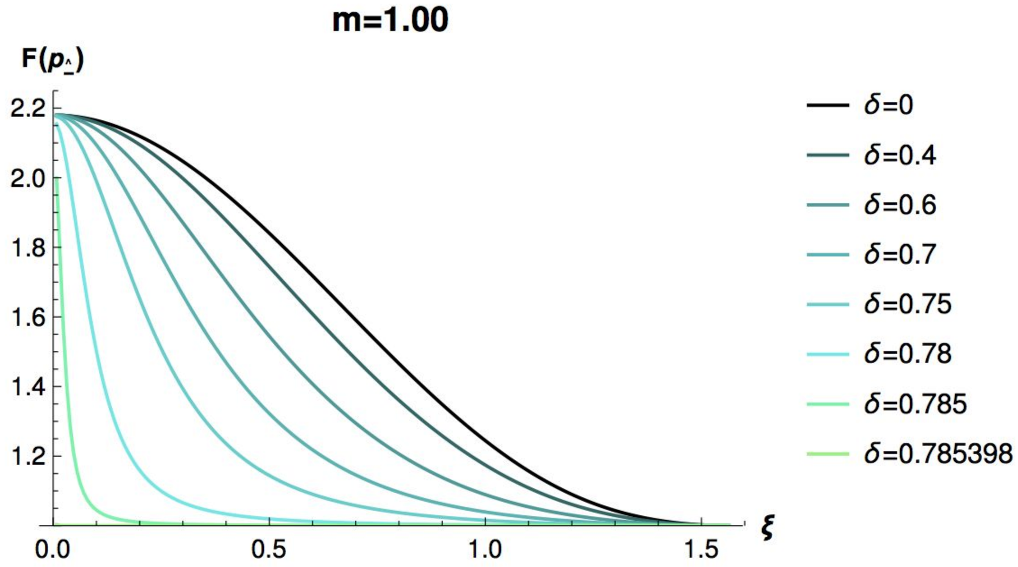


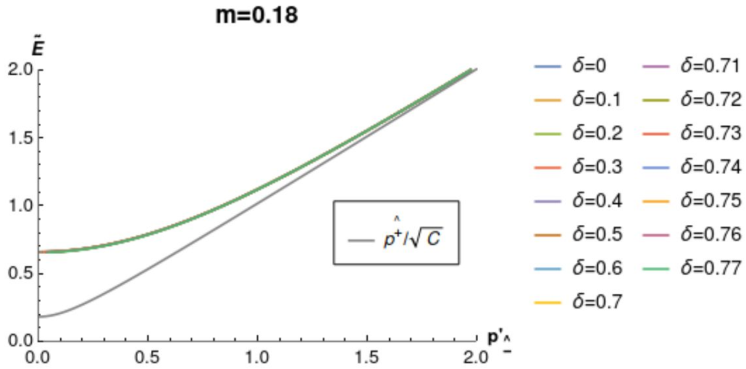
- M.Li, et al., JPG13, 915(87) - IFD(rest frame)



Wave Function Renormalization Factors



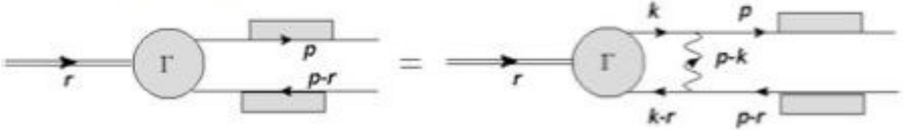




$$\frac{F(p'_{\hat{-}})E(p'_{\hat{-}})}{\sqrt{C}} = \sqrt{p'^2_{\hat{-}} + M(p'_{\hat{-}})^2} \equiv \tilde{E}(p'_{\hat{-}})$$

BOUND-STATE EQUATION

$$\Gamma(r,p) = \frac{i\lambda}{2\pi}\int \frac{dk_{\hat{-}}\,dk_{\hat{+}}}{(p_{\hat{-}}-k_{\hat{-}})^2} S(p)\gamma^{\hat{+}}\Gamma(r,k)\gamma^{\hat{+}}S(p-r)$$

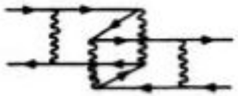


$$\left[-r_{\hat{+}} + \frac{-\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_+(r_{\hat{-}},p_{\hat{-}})$$

$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}})\right],$$

$$\left[r_{\hat{+}} + \frac{-\mathbb{S}(p_{\hat{-}}-r_{\hat{-}}) + E(p_{\hat{-}}-r_{\hat{-}})}{\mathbb{C}} + \frac{\mathbb{S}p_{\hat{-}} + E(p_{\hat{-}})}{\mathbb{C}}\right]\hat{\phi}_-(r_{\hat{-}},p_{\hat{-}})$$

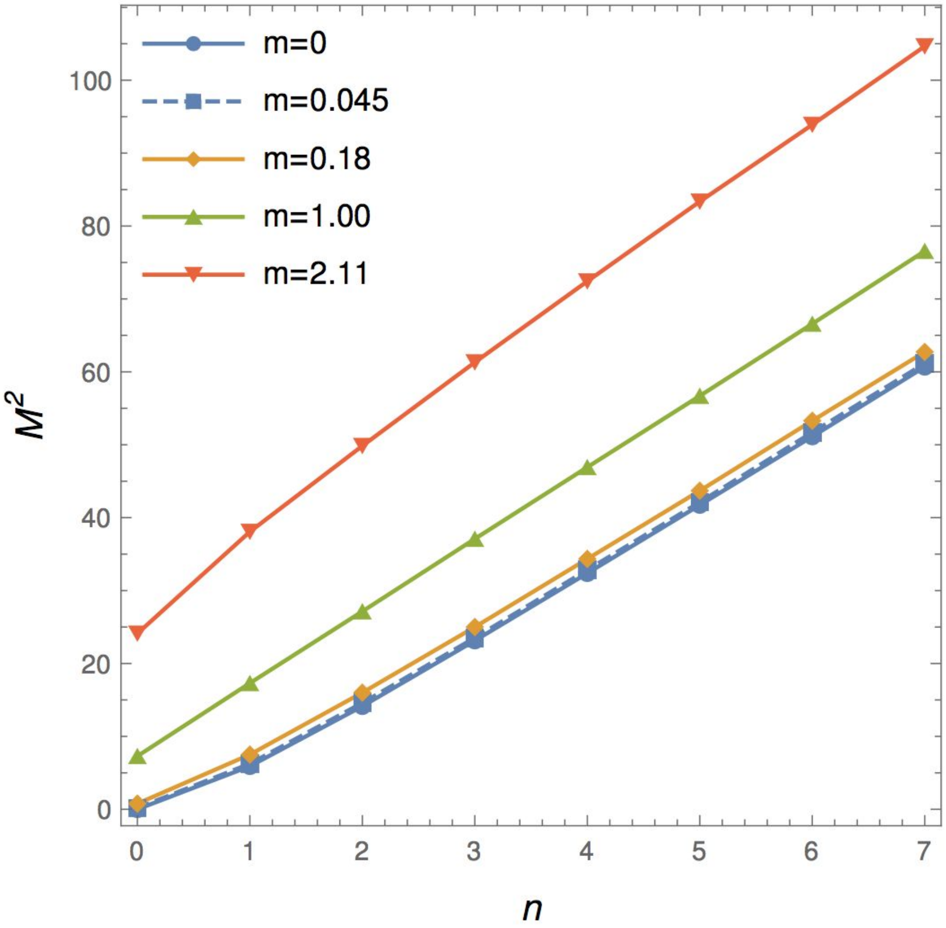
$$= \lambda \fint \frac{dk_{\hat{-}}}{(p_{\hat{-}}-k_{\hat{-}})^2}\left[C(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_-(r_{\hat{-}},k_{\hat{-}}) - S(p_{\hat{-}},k_{\hat{-}},r_{\hat{-}})\hat{\phi}_+(r_{\hat{-}},k_{\hat{-}})\right].$$



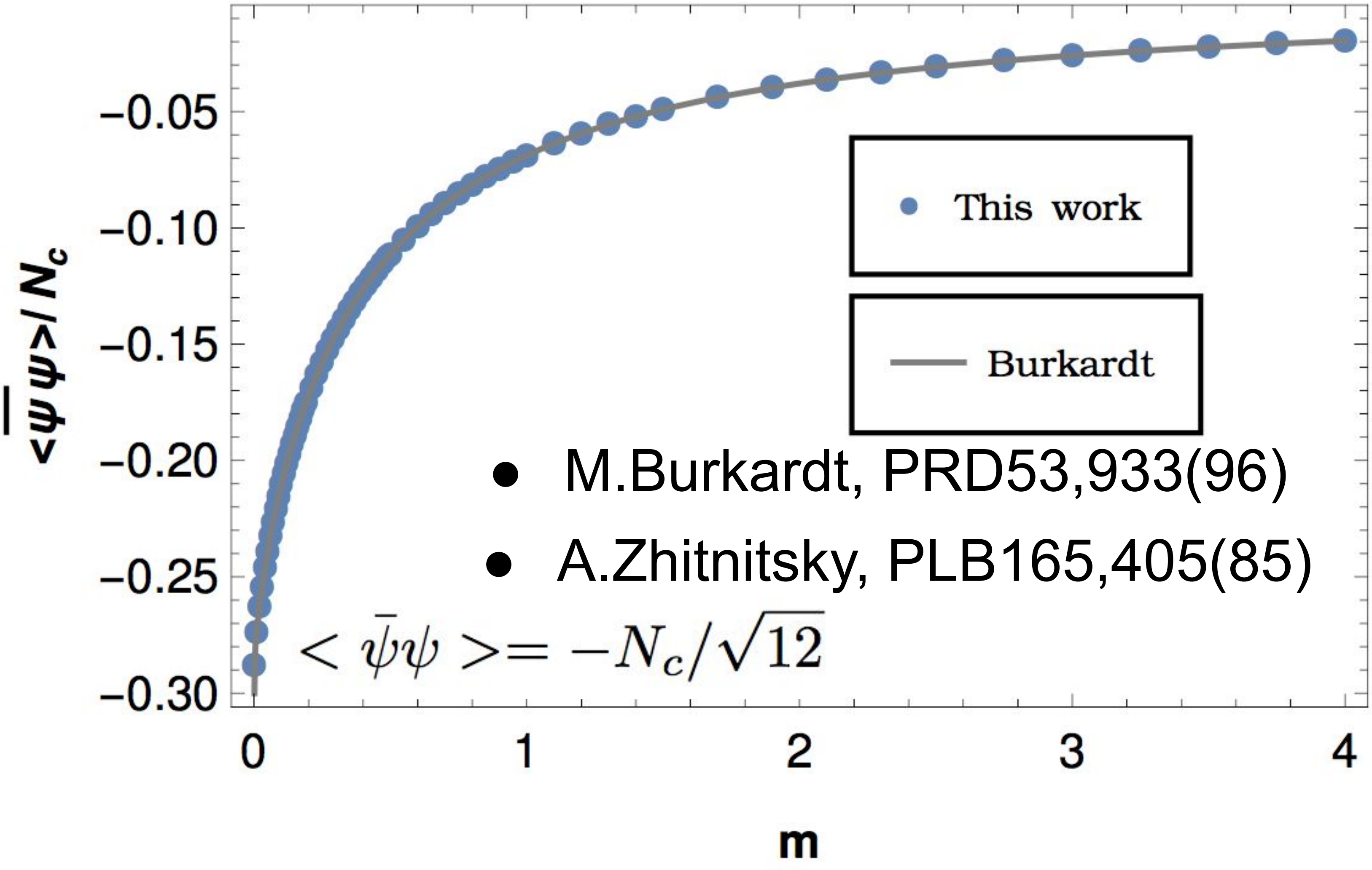
LFD $$\left[\mathcal{M}^2 - \frac{m^2-2\lambda}{x} - \frac{m^2-2\lambda}{1-x}\right]\phi(x) = -2\lambda \fint_0^1 \frac{dy}{(x-y)^2}\phi(y)$$

Meson Spectroscopy

- G.'tHooft, NPB75, 461(74) - LFD

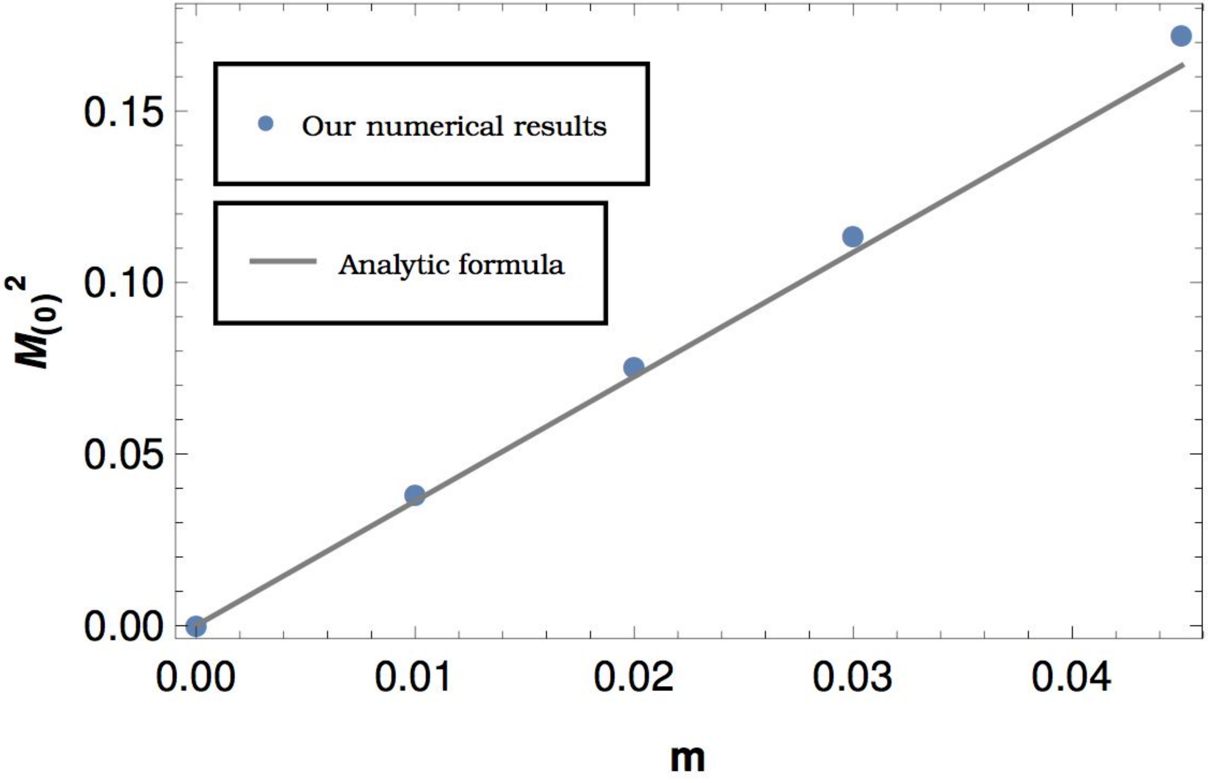


- M.Li, et al., JPG13, 915(87) - IFD (rest frame)
- Y. Jia, et al., JHEP11, 151('17) - IFD (moving frame)



- M.Burkardt, PRD53,933(96)
- A.Zhitnitsky, PLB165,405(85)

Gell-Mann - Oaks - Renner Relation



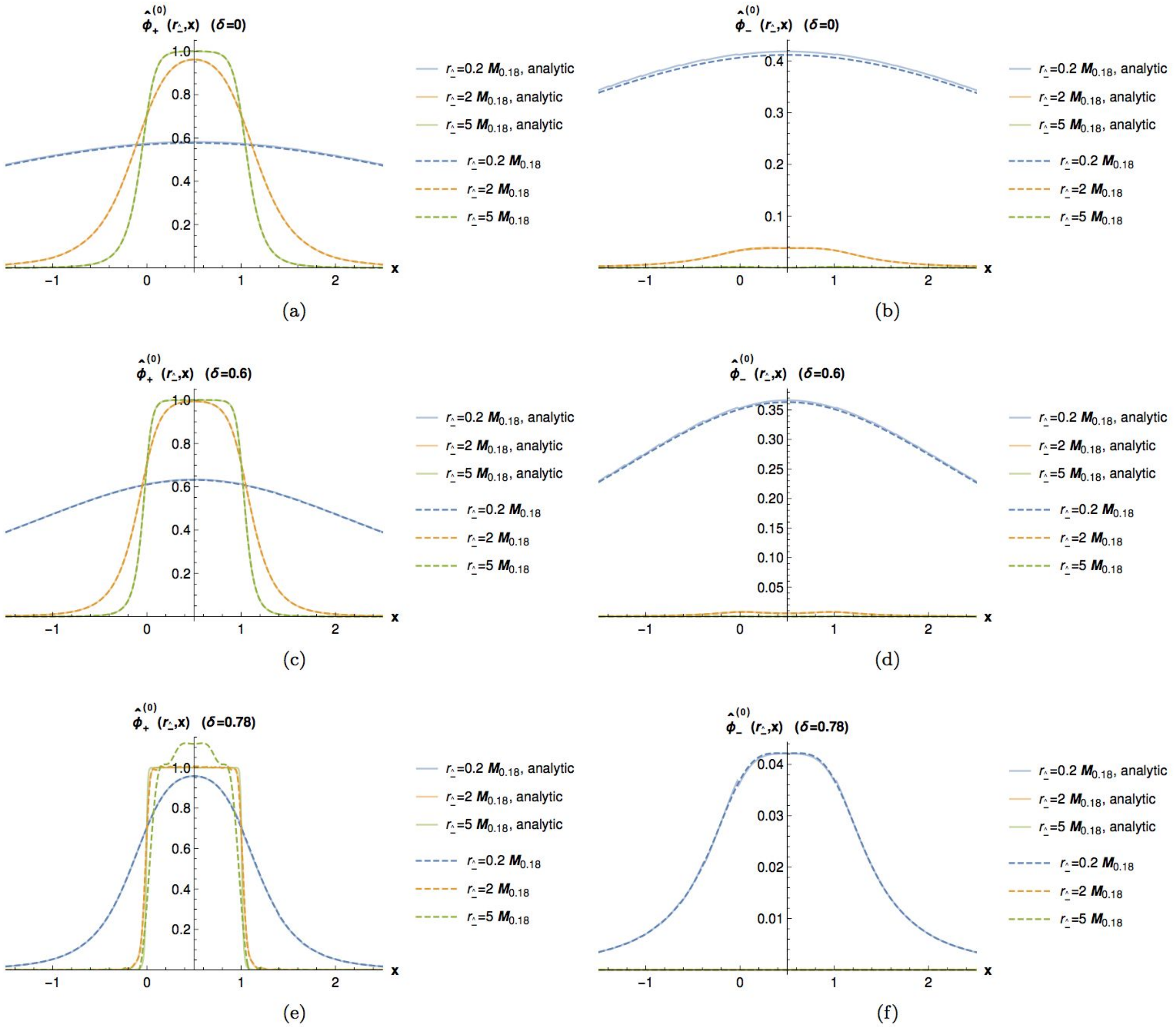
$$\mathcal{M}_\pi^2 = -\frac{4m < \bar{\psi}\psi >}{f_\pi^2} = \sqrt{\frac{8\pi^2 m^2 \lambda}{3}}$$

$$f_\pi = \sqrt{N_c/\pi}$$

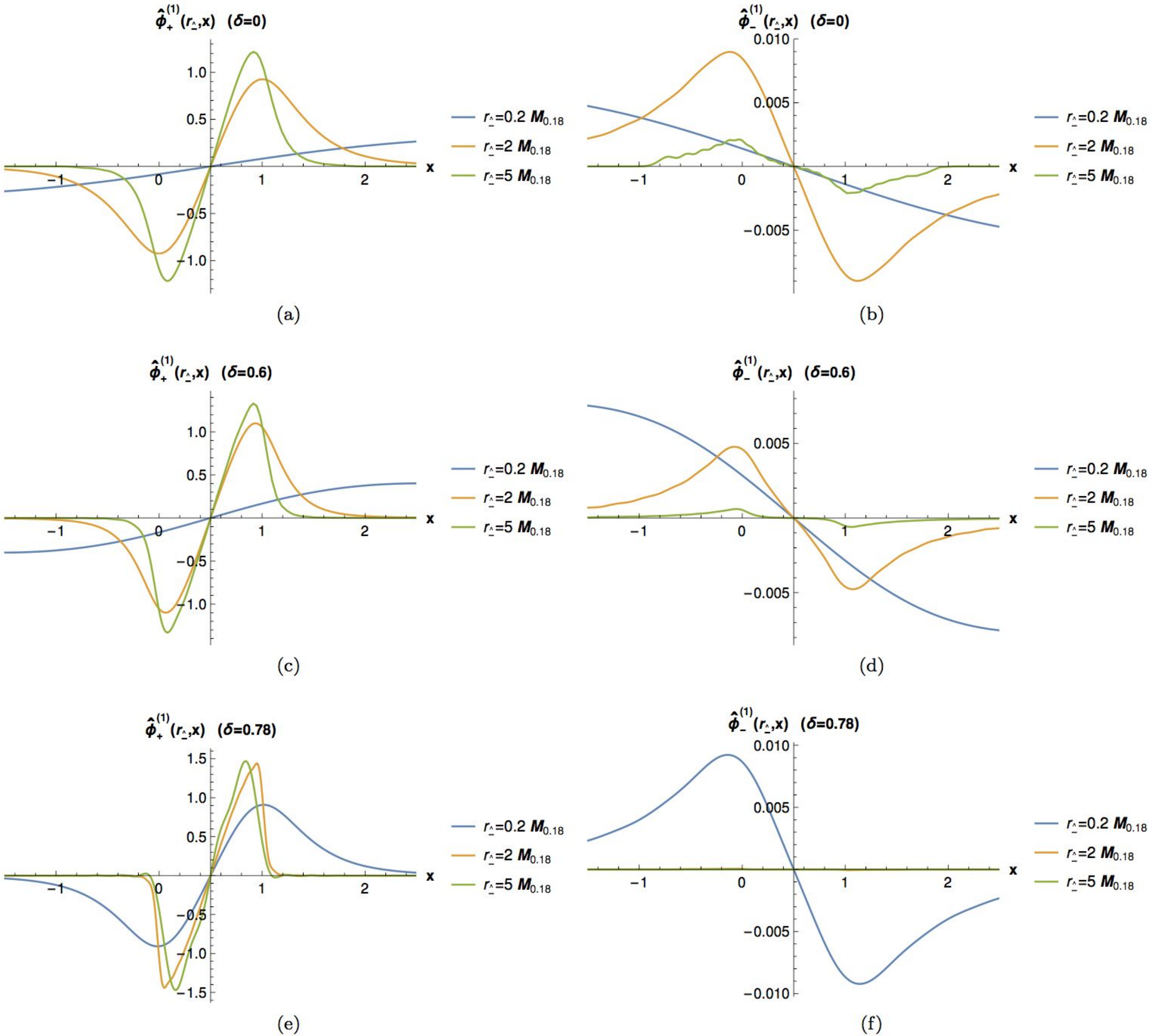
Meson Ground-state Wave-function for m=0 case

$$\hat{\phi}^{(0)}_{\pm}(r_{\hat{-}}, p_{\hat{-}}) =$$

$$\frac{1}{2}\left(\cos\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) - \theta(p_{\hat{-}})}{2} \pm \sin\frac{\theta(r_{\hat{-}} - p_{\hat{-}}) + \theta(p_{\hat{-}})}{2}\right)$$



First Excited-state Meson Wave-functions for m=0 case



Parton Distribution Functions (PDFs)

$$q_n(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-}$$

$$\times \langle P_n^-, P^+ | \bar{\psi}(\xi^-) \gamma^+ \mathcal{W}[\xi^-, 0] \psi(0) | P_n^-, P^+ \rangle_C,$$

$$\mathcal{W}[\xi^-, 0] = \mathcal{P}\left[\exp\left(-ig_s \int_0^{\xi^-} d\eta^- A^+(\eta^-)\right)\right]$$

A^+=0 Gauge in LFD

Quasi-PDFs

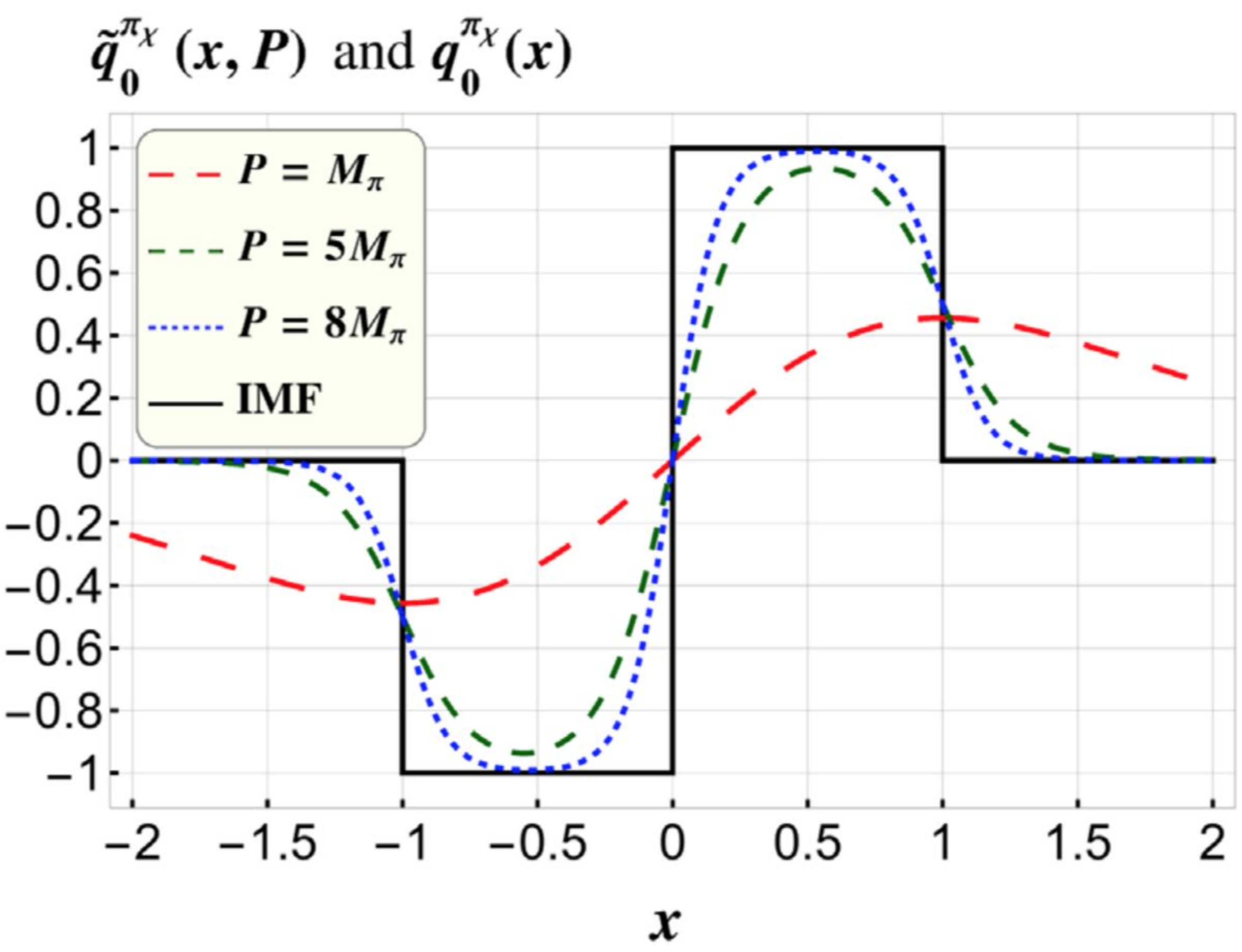
$$\tilde{q}_{(n)}(r_{\hat{-}}, x) = \int_{-\infty}^{+\infty} \frac{dx^{\hat{-}}}{4\pi} \mathrm{e}^{ix^{\hat{-}} r_{\hat{-}}}$$

$$\times < r^{\hat{+}}_{(n)}, r_{\hat{-}} \mid \bar{\psi}(x^{\hat{-}}) \, \gamma_{\hat{-}} \, \mathcal{W}[x^{\hat{-}}, 0] \, \psi(0) \mid r^{\hat{+}}_{(n)}, r_{\hat{-}} >_C,$$

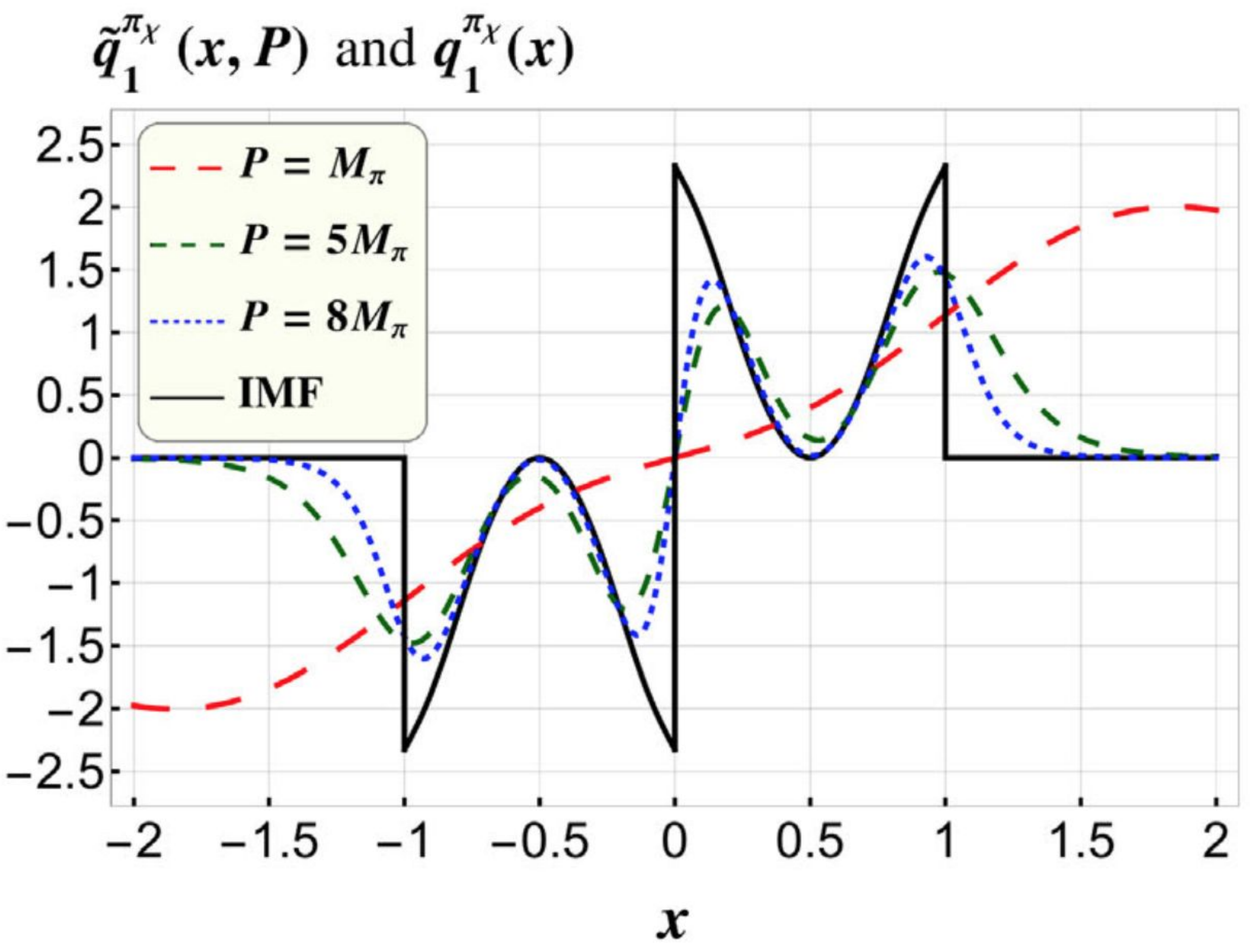
$$\mathcal{W}[x^{\hat{-}}, 0] = \mathcal{P}\left[\exp\left(-ig \int_0^{x^{\hat{-}}} dx'^{\hat{-}} A_{\hat{-}}(x'^{\hat{-}})\right)\right]$$

Interpolating dynamics

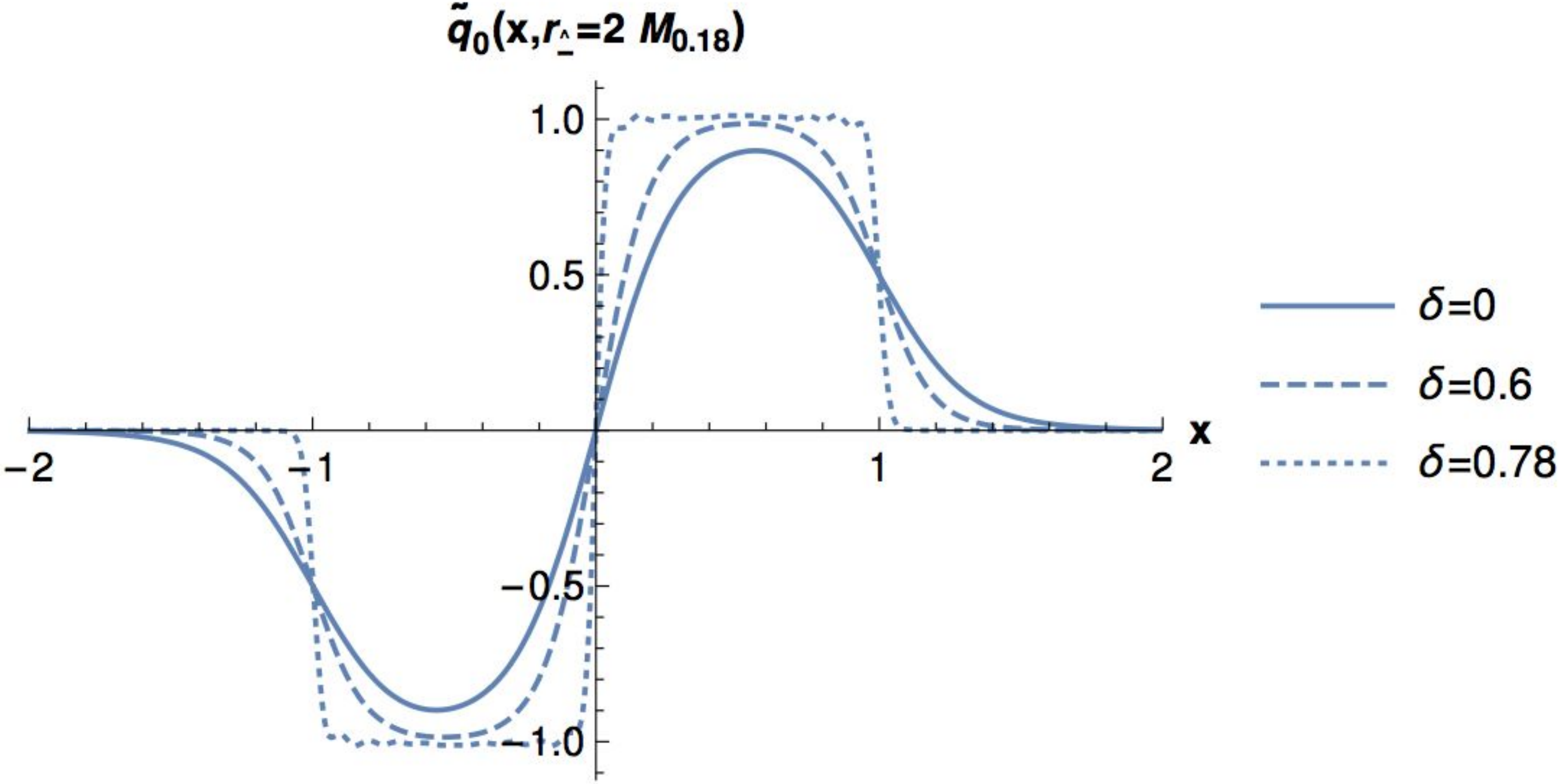
- Y. Jia, et al., PRD98, 054011('18)
  - IFD (quasi-PDFs)



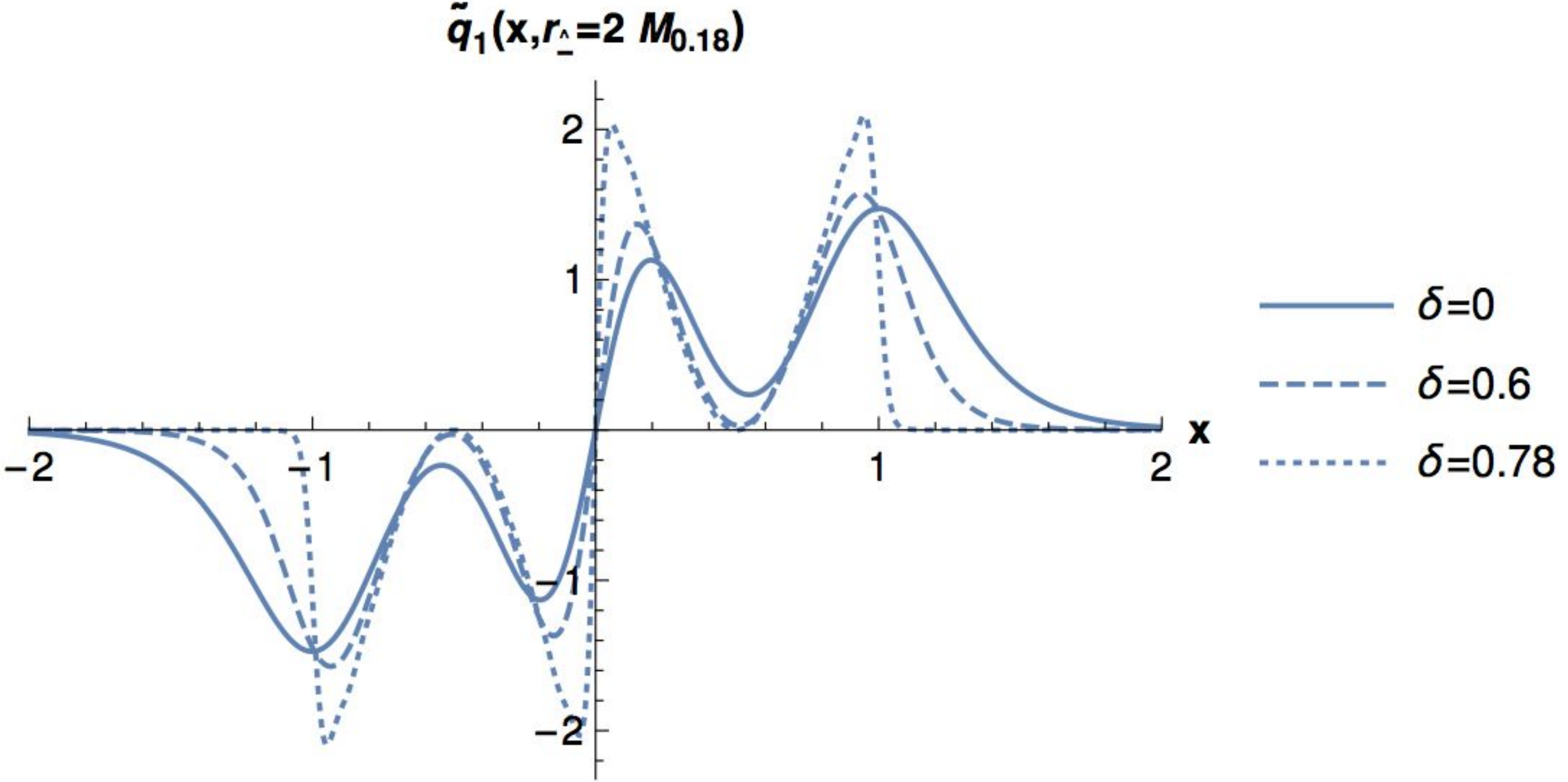
- Y. Jia, et al., PRD98, 054011('18)
  - IFD (quasi-PDFs)



- B.Ma&C.Ji, arXiv:2105.09388v1[hep-ph]



- B.Ma&C.Ji, arXiv:2105.09388v1[hep-ph]



Extended Wick Rotation

$$p^0 \rightarrow \tilde{P}^0 = ip^0 \quad (\delta = 0)$$

$For \quad 0 < \delta < \pi / 4,$

$$p^{\hat{+}} / \sqrt{C} \rightarrow \tilde{P}^{\hat{+}} / \sqrt{C} = ip^{\hat{+}} / \sqrt{C}.$$

Correspondence to Euclidean Space

$$p'^2_{\underline{\wedge}} = p^2_{\underline{\wedge}} / C \leftrightarrow -\tilde{P}^2$$

Conclusion and Outlook

- Whole landscape between IFD and LFD has been revealed in QED tree-level with interpolating spinors,gauge bosons,their propagators.
- Maximal stability group of LFD saves significant dynamic efforts.
- Interpolating quantum field theory appears useful in resolution of theoretical issues, e.g. LFZM.

- **QCD(1+1) in large Nc “tHooft model’ is interpolated between IFD and LFD and solved for its mass gap to find interpolation angle independent energy function including the wavefunction renormalization.**
- **Chiral condensate, meson mass spectra bearing the feature of Regge trajectories and GOR relation are found independent of interpolation angle indicating the persistence of nontrivial vacuum even in LFD.**

- **Applying to quasi-PDFs in the interpolating formulation, we note a possibility of utilizing not only the reference frame dependence but also the interpolation angle dependence to get an alternative effective approach to the LFD's PDFs.**