Mechanical properties of particles

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- Interaction of the nucleon with gravity, gravitational ffs
- Three fundamental gravitational "charges": mass, spin & D (related to shear and pressure, *Druck-term*)
- Effective chiral theory in presence of gravity
- Normal and tangential forces inside large Nc nucleon. Stability conditions.
- First experimental results on gravitational form factors
- Forces between quark and gluon subsystems inside the nucleon
- Conclusion and outlook.

based on:
MVP, PLB555 (2003)
HD Son, MVP, JHEP09 (2018)
Alharazin, Djukanovic, Gegelia, MVP, PRD102 (2020)
Panteleeva, MVP, 2102.10902
for a review see: P. Schweitzer, MVP 1805.06596

p

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ∗∗∗∗

p MASS (atomic mass units u)

The mass is known much more precisely in u (atomic mass units) than in MeV. See the next data block.

VALUE (u)	*DOCUMENT ID*		*TECN*	*COMMENT*
1.00727646662 ±0.00000000009	**OUR AVERAGE**			Error includes scale factor of 3.1.
1.007276466583±0.000000000032	[1] HEISSE	17	SPEC	Penning trap
1.007276466879±0.000000000091	MOHR	16	RVUE	2014 CODATA value

Our intuitive perception of the mass is related to gravity (e.g. weighting experiment)

Spin can be also related to gravity, e.g. measurement of Earth angular velocity (spin) with help of Foucault pendulum

Interaction of a hadron with gravity

$$S=-\frac{1}{16\pi G}\int d^4x\sqrt{-g}\;R+\int d^4x\sqrt{-g}L_M$$ /Hilbert '1915/

Let us change the metric in long wave, static way

$$T^{\mu\nu}(x)=\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}(x)}$$

$$g^{\mu\nu}(x)=\eta^{\mu\nu}+\delta g^{\mu\nu}(\vec r)\qquad \lambda_{\rm grav}\gg\frac{1}{M_N}$$

Than the response of the nucleon to the static change of the space-time metric can be characterised by e. g. static EMT (Breit frame):

MVP, PLB555 (2003)

$$T_{\mu\nu}(\vec r)=\int\frac{d^3\Delta}{(2\pi)^3 2E}\;e^{-i\vec r\vec\Delta}\langle p'|T_{\mu\nu}(0)|p\rangle,$$

A location $\vec r$ is uncertain by $\delta r\sim\frac{\hbar}{Mc}$ Compton wave length or amplitude of Zitterbewegung

At distances smaller than the Compton length pairs can be created: one particle description is not adequate. At large distances the "relativistic corrections" behave $\sim e^{-2M_N r}$
The relative corrections to the distributions (via normalisation) $\sim 1/(2M_N R_N)\sim 0.2$

The relative correction to the radii $\sim 1/(2M_N R_N)^2\sim 0.05$

The problem discussed since1950th /Yennie et al. 1957/, the most recent discussion see e.g. /Jaffe '20/

It's a bad idea to use 3D Fourier tranforms for the proton.

Not really valid, but has correct non-relativistic limit.

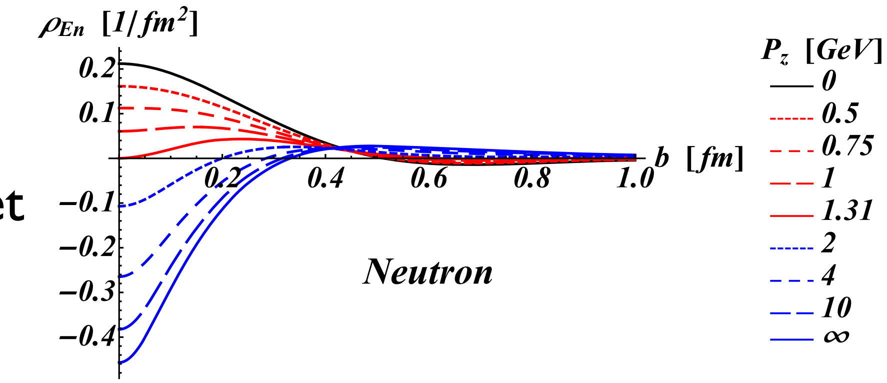
the Breit frame Fourier transform does not give a spatial density.

Instead this is a general problem that afflicts attempts to extract spatial distributions of local properties of any system that is not much larger than its Compton wave-length. The problem is quite fundamental, since it originates in the interplay between the uncertainty principle and relativity.

Different views of force distributions

- For e.m. ffs fully relativistic and model independent interpretation in terms of charge distributions can be given within a phase space approach /Lorce, PRL125 (2020), H-Ch Kim, J-Y Kim 2106.10986/. Analogous interpretation of EMT ffs is possible.

Everything can be reformulated using the phase space Wigner picture
The distortions appearing in the relativistic distributions for a moving target are entirely due to relativistic kinematical effects associated with spin.



- One can define static EMT in IMF= light cone pressure and shear forces. for charge distributions see /M. Burkardt '00, G.A. Miller '07/ for pressures /Lorce et al. '18, Freese, Miller '21/

- We showed recently /Panteleeva, MVP '21/ that 3D Breit frame and 2D light front force distribution are equivalent as they are related to each other by invertible Abel transformation. Any result in Breit frame (stability conditions, experimental data, model calculations, etc.) can be unambiguously transformed into corresponding result for light-front force distributions (and vice versa)

Here we prefer to work with Breit frame force distributions for which we can use our "rest frame intuition". Any statement about the Breit frame distributions can be unambiguously translated to the language of light-front distributions (and vice versa)

Different ways of the interpreting the form factors can be regarded as different choices of schemes. To my taste 3D Breit force distributions are more intuitive and physics appealing.

EMT form factors for the nucleon

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4}\,g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

Kobzarev, Okun '1962 , Pagels '1966

$P = (p' + p)/2,\ \Delta = p' - p.$

The name **"D-term"** is rather technical, it can be traced back to more or less accidental notations chosen in /Weiss, MVP '99/. Nowadays, given more clear physics meaning of this quantity, we might call this term as ***"Druck-term"*** derived from German word for pressure

Compare with electromagnetic FFs

$$\langle p'|J^{\rm em}_\mu(0)|p\rangle = \bar{u}'\left[F_1(t)\frac{P_\mu}{M_N} + G_M(t)\frac{i\sigma_{\mu\rho}\Delta^\rho}{2M_N}\right]u$$

Interaction of the nucleon with gravity

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

$a = g, Q$ (gluon or quark parts)

- $A^a(t)$ ↑ δg^{00} ↓ Mass: $\sum_a A^a(0) = 1$
- $J^a(t)$ ↑ δg^{0i} ↓ Spin: $\sum_a J^a(0) = \frac{1}{2}$
- $D^a(t)$ ↑ δg^{ij} ↓ deformation of space = elastic properties of N
- $\bar{c}^a(t)$ ↑ non – conservation of EMT pieces ↓ $\sum_a \bar{c}^a(t) = 0$

$$F_1(0) = 1 \qquad G_M(0) = 1 + \kappa$$

for e.m. FFs we have anomalous magnetic moment
NO anomalous gravitomagnetic moment /Kobzarev, Okun '1962/

Elasticity stress tensor (Landau & Lifshitz vol. 7)

Shear forces distribution (pressure anisotropy)

Pressure distribution

MVP '2003

$$(dF_i = T_{ij}dS_j)$$

$$T^a_{ij}(\vec{r}) = \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right)s^a(r) + \delta_{ij}\,p^a(r)$$

$$s^a(r) = -\frac{1}{4M_N}\,r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}^a(r), \quad p^a(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D}^a(r) - M_N\int\frac{d^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\vec{r}}\,\bar{c}^a(-\vec{\Delta}^2).$$

$$\widetilde{D}^a(r) = \int\frac{d^3\Delta}{(2\pi)^3}\,e^{-i\vec{\Delta}\vec{r}}\,D^a(-\vec{\Delta}^2)$$

O. Teryaev called it nucleon "cosmological term" $\delta p^a = -\delta\rho^a_E$

a) related to forces between quark and gluon subsystems /HDSon,MVP'18/
b) contribute to "gluon" and "quark" parts of energy density (mass decomposition) /Lorce '18/
c) instanton contribution to nucleon $\bar{c}^q(0) \approx +1.4\;10^{-2}$ /HDSon,MVP'18/
d) $\bar{c}^q(0) = 0$ for Goldstone bosons /Schweitzer, MVP '19/

Three global fundamental mechanical "charges" of nucleon: M, J, D

MVP '2003

Two the most important particle properties:

$$M = \int d^3\mathbf{r} \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{00}(\vec{r})}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

$T^{00}(\vec{r})$

$$J^i = \epsilon^{ikl} \int d^3\mathbf{r}\ r^k \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{0l}(\vec{r})}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

M, J, D are independent of way we localise hadron

Variations of (00) and (0l) components of metric can be such that the Riemann tensor =0, e.g. just going to non-inertial reference frame
That is why we can measure the mass by the shape of particle track in external em field, or Earth angular velocity with help of Foucault pendulum

Unexplored property:

$$D = -\frac{M}{2} \int d^3\mathbf{r} \left(r^i r^k - \frac{1}{3} r^2 \delta^{ik}\right) \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{ik}(\mathbf{r})}\Big|_{g_{\mu\nu}=\eta_{\mu\nu}}$$

Variation of (ik) components necessarily must lead to **non-zero Riemann tensor** ="true gravity"
Probably that is why D-term, being as fundamental as M & J, escaped attention of the community

MVP '2003

D-term is a global and fundamental quantity related to the distribution of strong forces (pressure and shear) inside a hadron

$$D = M \int d^3r\ r^2\ p(r) = -\frac{4M}{15} \int d^3r\ r^2\ s(r)$$

The Druck-term

last global ~~unknown~~ unexplored: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\text{em}} = 0$	$\langle N'\lvert J^\mu_{\text{em}}\rvert N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \ldots$
weak:	PCAC	$\langle N'\lvert J^\mu_{\text{weak}}\rvert N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \ldots$
gravity:	$\partial_\mu T^{\mu\nu}_{\text{grav}} = 0$	$\langle N'\lvert T^{\mu\nu}_{\text{grav}}\rvert N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \ldots$

global properties:

$$Q_{\text{prot}} = 1.602176487(40) \times 10^{-19}\text{C}$$
$$\mu_{\text{prot}} = 2.792847356(23)\mu_N$$
$$g_A = 1.2694(28)$$
$$g_p = 8.06(0.55)$$
$$M = 938.272013(23)\,\text{MeV}$$
$$J = \tfrac{1}{2}$$
$$D = \ ??$$

and more:
t-dependence ... ...
parton structure, etc ...

$\hookrightarrow$ $D =$ *"last" global ~~unknown~~* unexplored

which value does it have?

what does it mean?

Effective chiral action for nucleon and pions in external grav. field

based on /Alharazin, Djukanovic, Gegelia, MVP '20/

c_1	c_2	c_3
-1.22(2)(2)	3.58(3)(6)	-6.04(2)(9)

Contains known LECs /UG Meissner et al. '00/

$$S_{\pi N} = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\,\bar\Psi\, i e^\mu_a\gamma^a\nabla_\mu\Psi - \frac{1}{2}\,\nabla_\mu\bar\Psi\, i e^\mu_a\gamma^a\Psi - m\bar\Psi\Psi + \frac{g_A}{2}\,\bar\Psi e^\mu_a\gamma^a\gamma_5 u_\mu\Psi\right.$$

$$+\; c_1\langle\chi_+\rangle\bar\Psi\Psi - \frac{c_2}{8m^2}g^{\mu\alpha}g^{\nu\beta}\langle u_\mu u_\nu\rangle\left(\bar\Psi\{\nabla_\alpha,\nabla_\beta\}\Psi + \{\nabla_\alpha,\nabla_\beta\}\bar\Psi\Psi\right) + \frac{c_3}{2}g^{\mu\nu}\langle u_\mu u_\nu\rangle\bar\Psi\Psi$$

$$+\; \frac{ic_4}{4}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\,[u_\mu,u_\nu]\,\Psi + c_5\bar\Psi\hat\chi_+\Psi + \frac{c_6}{8m}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}F^+_{\mu\nu}\Psi + \frac{c_7}{8m}\bar\Psi\, e^\mu_a e^\nu_b\sigma^{ab}\langle F^+_{\mu\nu}\rangle\Psi$$

$$\left.+\; \frac{c_8}{8}R\bar\Psi\Psi + \frac{ic_9}{m}R^{\mu\nu}\left(\bar\Psi e^a_\mu\gamma_a\nabla_\nu\Psi - \nabla_\nu\bar\Psi e^a_\mu\gamma_a\Psi\right)\right\}$$

D-term is related to interaction with space-time curvature

New LECs: interaction with curvature /Alharazin, Djukanovic, Gegelia, MVP '20/

$$\frac{D(0)}{m_N} = c_8 + \frac{g_A^2}{16\pi F^2}M_\pi + \frac{-3g_A^2/m_N + 2\left(-4c_1 + c_2 + 2c_3\right)}{8\pi^2F^2}M_\pi^2\ln\left(\frac{M_\pi}{m_N}\right) + \mathcal{O}(M_\pi^3)$$

Chiral expansion of D-term /Alharazin, Djukanovic, Gegelia, MVP '20/

Computed previously /Belitsky, XD Ji '02/

Computed previously /Diehl, Manashov, Schafer '06/ we corrected their mistake. New result might be important to revisit chiral extrapolation of lattice data!

Effective chiral action for nucleon and pions in external grav. field

based on /Alharazin, Djukanovic, Gegelia, MVP '20/

Im part of D(t). Is useful for dispersion relations analysis

$$
\begin{aligned}
\mathrm{Im}D(t+i0) &= -\frac{3g_A^2 m_N\left(t-2M_\pi^2\right)\left(t+4M_\pi^2\right)}{64\pi F^2 t^{3/2}}\,\mathrm{arctg}\left(2m_N\frac{\sqrt{t-4M_\pi^2}}{t-2M_\pi^2}\right)\\
&+ \frac{m_N\sqrt{t-4M_\pi^2}}{40\pi F^2 t^{3/2}}\left[20c_1 M_\pi^2(t+2M_\pi^2)+c_2(t^2-3M_\pi^2 t-4M_\pi^4)+5c_3(t^2-4M_\pi^4)\right.\\
&+ \left.\frac{5}{16}\frac{g_A^2}{m_N}(t^2+20M_\pi^2 t-12M_\pi^4)\right].
\end{aligned}
$$

E. g. strong forces in nucleon periphery (chiral limit example):

$$
p(r) = -\frac{3g_A^2}{64\pi^2 F^2}\frac{1}{r^6}+\frac{(5g_A^2/m_N+4(c_2+5c_3))}{16\pi^3 F^2}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right),
$$

$$
s(r) = \frac{9g_A^2}{64\pi^2 F^2}\frac{1}{r^6}-\frac{21(5g_A^2/m_N+4(c_2+5c_3))}{128\pi^3 F^2}\frac{1}{r^7}+O\left(\frac{1}{r^8}\right).
$$

Useful for derivation general stability conditions and inequalities for strong forces.
Also it is important to describe large distance interaction of quarkonia with the nucleon.Important for hadrocha-charmonium picture of LHCb pentas.

/Eides, Petrov, MVP '15/

Note that at nucleon periphery

$$
\frac{dF_r}{dS_r} = \frac{2}{3}s(r)+p(r)\geq 0
$$

Many other applications:

- Soft hadron reactions in grav. field, e.g. pion gravitoproduction
- Maybe relevant for physics of LIGO mergers
- Low energy gravitoproduction can be probed in a lab, e.g non-diagonal DVCS (ongoing analysis at CLAS12, plans for EIC)

……………...

Total p(r) and s(r), normal and tangential forces, stability conditions

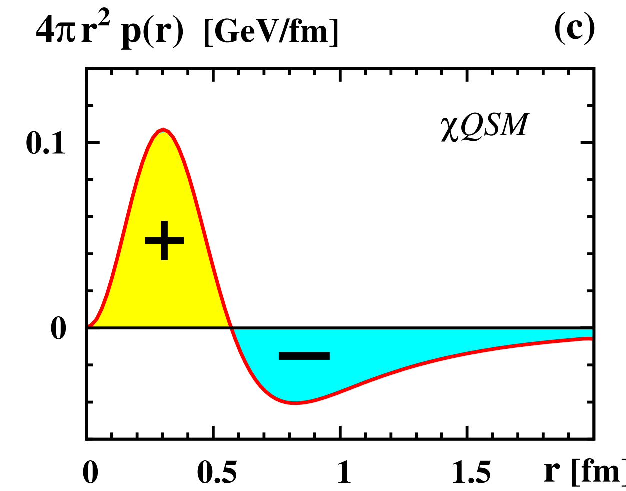
The force acting on the area element $d\vec{S} = dS_r\vec{e}_r + dS_\theta\vec{e}_\theta + dS_\phi\vec{e}_\phi$ $\quad (dF_i = T_{ij}dS_j)$

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r), \qquad \frac{dF_\theta}{dS_\theta} = \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r).$$

Eigenvalues of stress tensor

Normal forces

Tangential forces



Goeke et al. '2007

Stability condition
(spatial trace of EMT does not contribute to the mass)

$$\int d^3r\ p(r) = 0$$ von Laue '1911

Local stability condition
(Conjecture /Perevalova, Schweitzer, MVP' 17/
similar conjectures in astrophysics /Zeldovich, Novikov' 62, Herrera' 98/)

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

D-term $D(0) = -\frac{4m}{15}\int d^3r\ r^2\ s(r) = m\int d^3r\ r^2\ p(r) \leq 0$

All calculations of the D-term in various approaches give negative value for it.

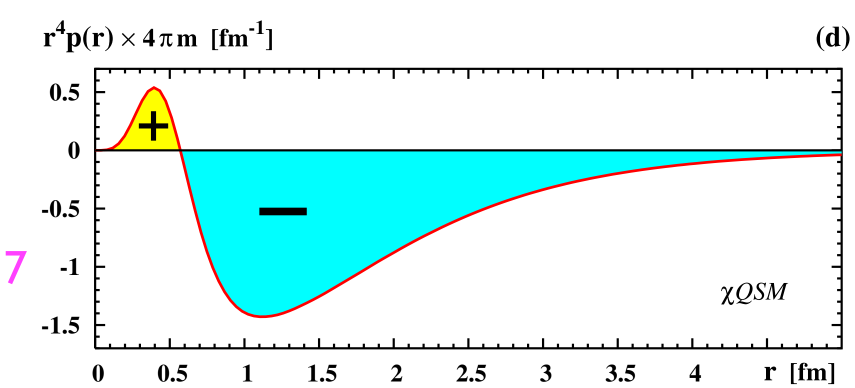
For some systems the D-term is fixed by general principles:

$D(0) = -1$ Goldstone bosons (pions etc.) Novikov, Shifman '1980, Voloshin, Zakharov' 1980

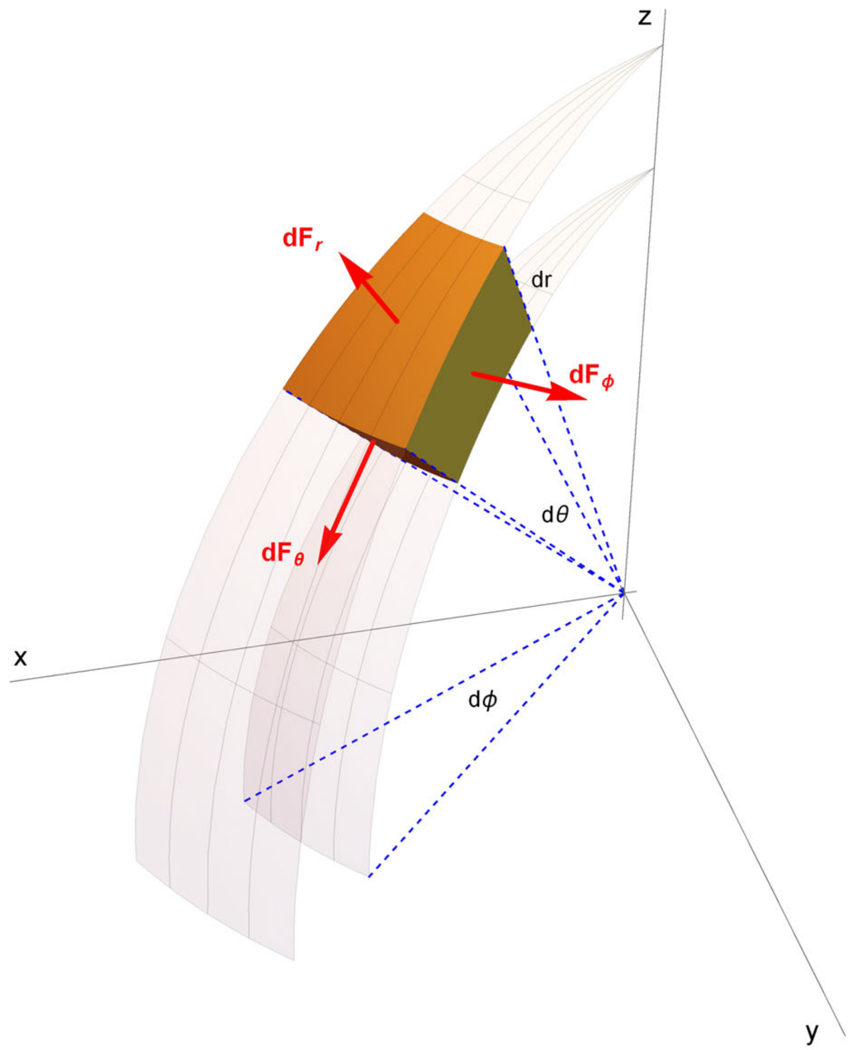
$D(0) = 0$ Free fermions Donoghue et al.' 02, Hudson, Schweitzer '17

$D(0) = -\frac{4\pi}{3}\gamma\ M_A\ R_A^4 \sim -A^{7/3}$ Heavy nuclei MVP '2003

Surface tension coeff in Weizsäcker mass formula ~ 1 MeV/fm^2

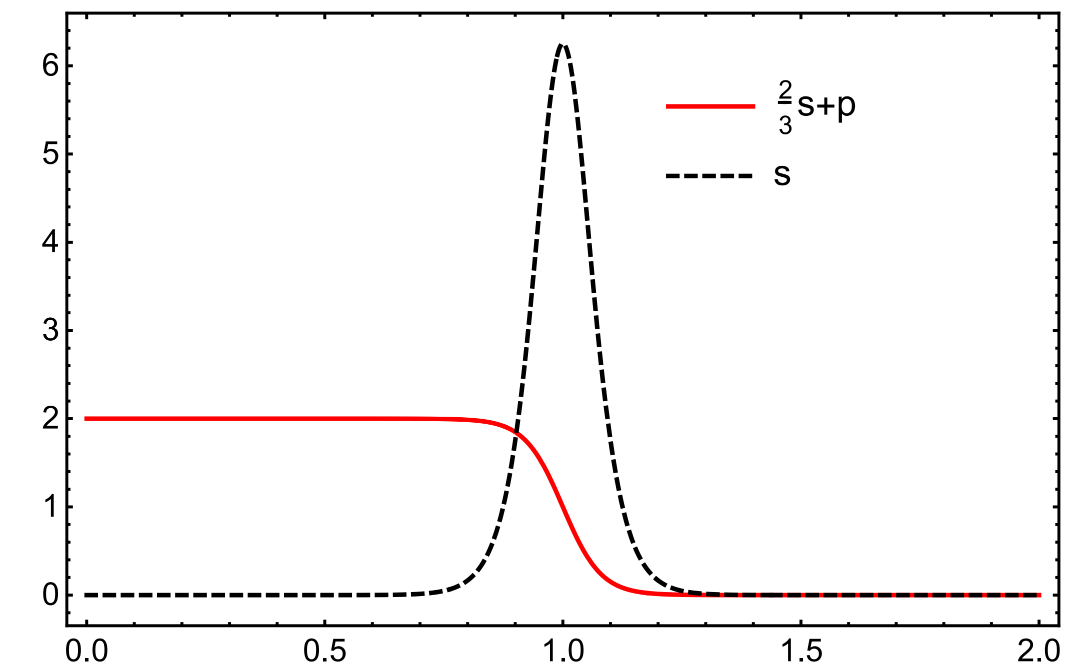


$(dF_i = T_{ij} dS_j)$ (Landau & Lifshitz vol. 7)



$$dF_r = \left(\frac{2}{3}s(r) + p(r)\right)\ dS_r$$

$$dF_\phi = \left(-\frac{1}{3}s(r) + p(r)\right)\ dS_\phi$$

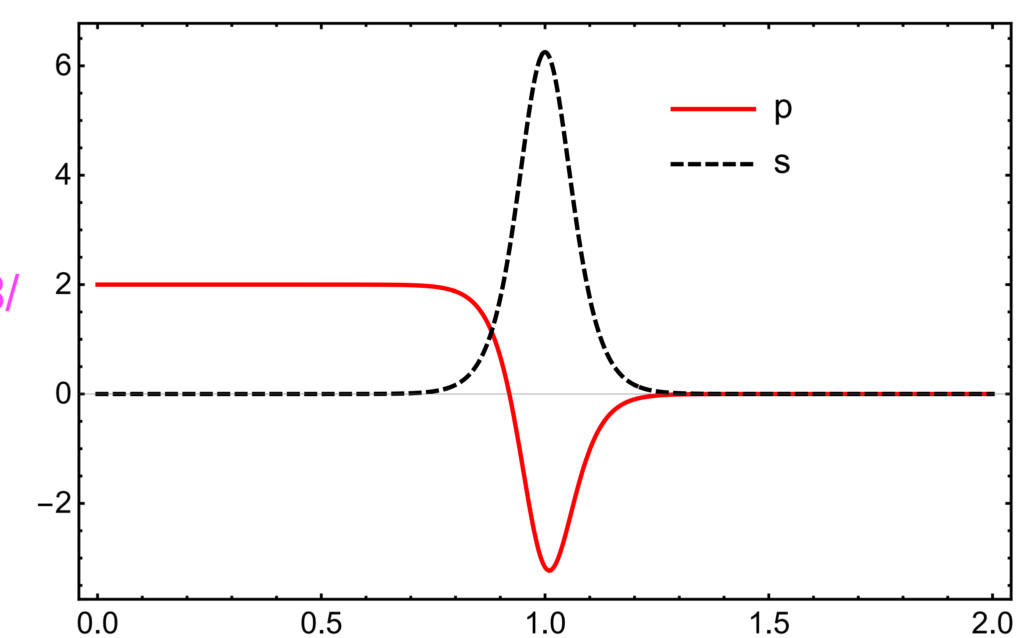


For a liquid drop

$$p(r) = p_0\theta(r - R) - \frac{p_0 R}{3}\delta(r - R), \quad s(r) = \gamma\delta(r - R),$$

$p_0 = 2\gamma/R$ Relation between pressure in t
he drop and the surface tension /Lord Kelvin '1858/

Hence for a liquid drop $\dfrac{dF_r}{dS_r} = \dfrac{2}{3}s(r) + p(r) = p_0\theta(r - R)$



Mechanical radius and surface tension

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

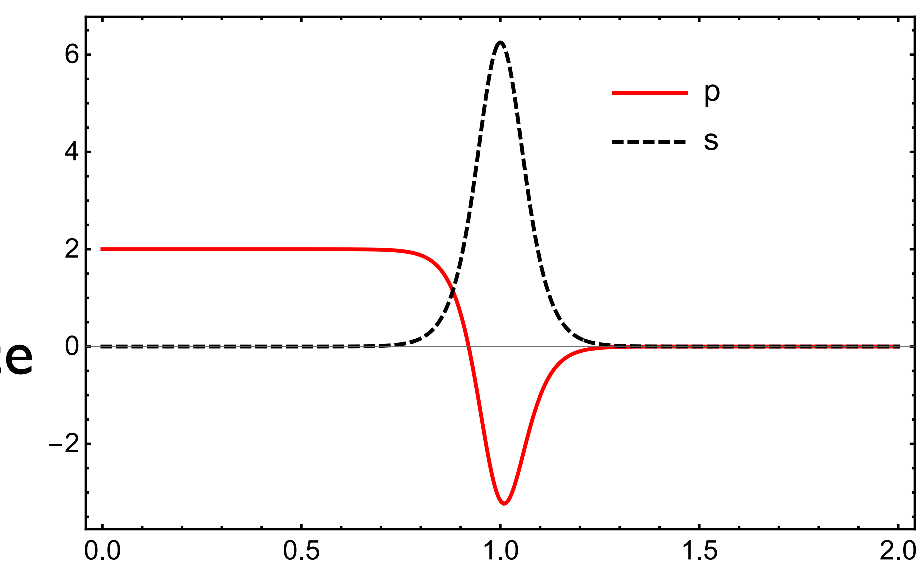
Positive quantity - allows to define the mechanical radius

$$\langle r^2 \rangle_{\rm mech} = \frac{\int d^3r\ r^2\ \left[\frac{2}{3}s(r) + p(r)\right]}{\int d^3r\ \left[\frac{2}{3}s(r) + p(r)\right]} = \frac{6D(0)}{\int_{-\infty}^{0} dt\ D(t)}$$

Note that mech radius is NOT the slope of D(t)

For a liquid drop

$$p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R), \quad s(r) = \gamma\delta(r-R),$$



$p_0 = 2\gamma/R$ Relation between pressure in the drop and the surface tension Lord Kelvin '1858

Hence for a liquid drop $\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) = p_0\theta(r-R)$

mechanical radius has the intuitive clear value

For general systems one can obtain the generalisation of the Kelvin relation $p(0) = \int_0^\infty dr\ \frac{2s(r)}{r}$

s(r) can be called surface tension for the system

Mechanical radius and surface tension

The surface tension energy $$\int d^3r\ s(r) = -\frac{3}{8m}\int_{-\infty}^{0} dt\ D(t)$$

This energy must be less than the total energy of the system $\int d^3r\ s(r) \leq m$ this implies

$\langle r^2\rangle_{\rm mech} \geq -9D/(4m^2)$ we checked that for stable systems (stable solitons) is always satisfied. Violated for unstable systems!

$\langle r^2\rangle_{\rm mech} \approx 0.75\ \langle r^2\rangle_{\rm charge}$ in chiral soliton picture of the nucleon

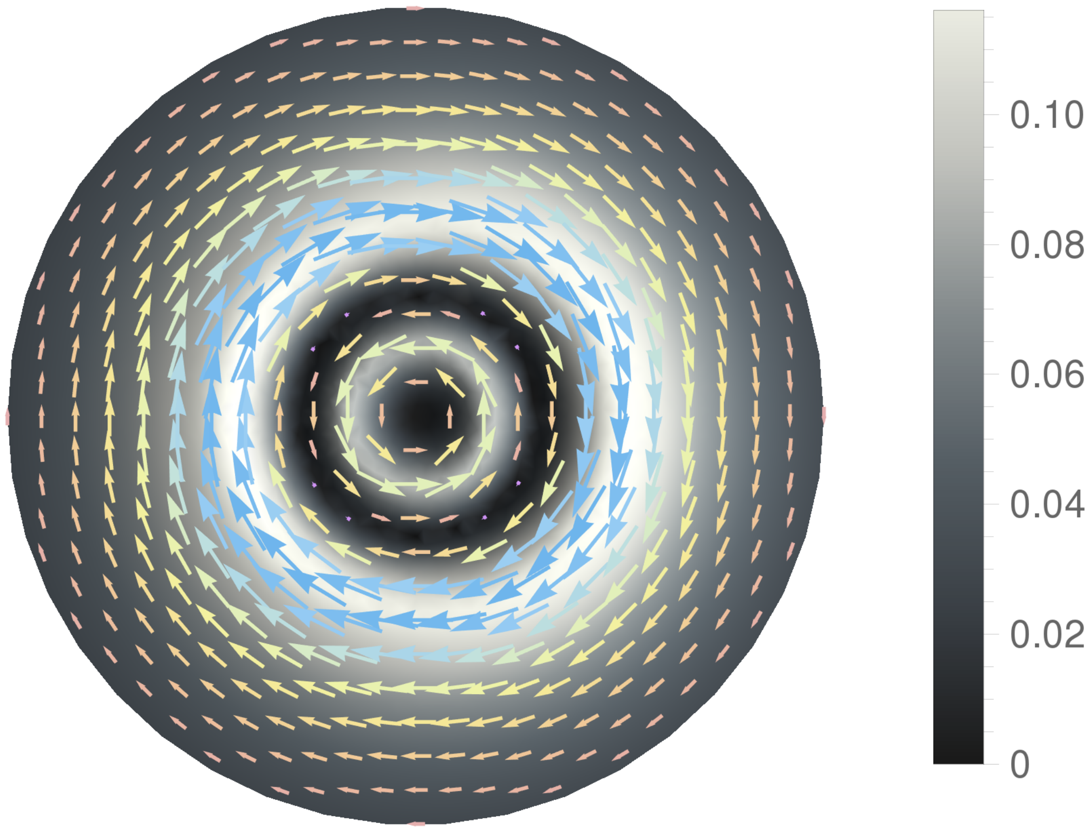
Shear forces distribution s(r) is important for forming the shape of the hadron.
For s(r)=0 the hadron corresponds to homogeneous, isotropic fluid. Hence has infinite mechanical radius. Non-zero s(r) is responsible for *hadron structure formation*!

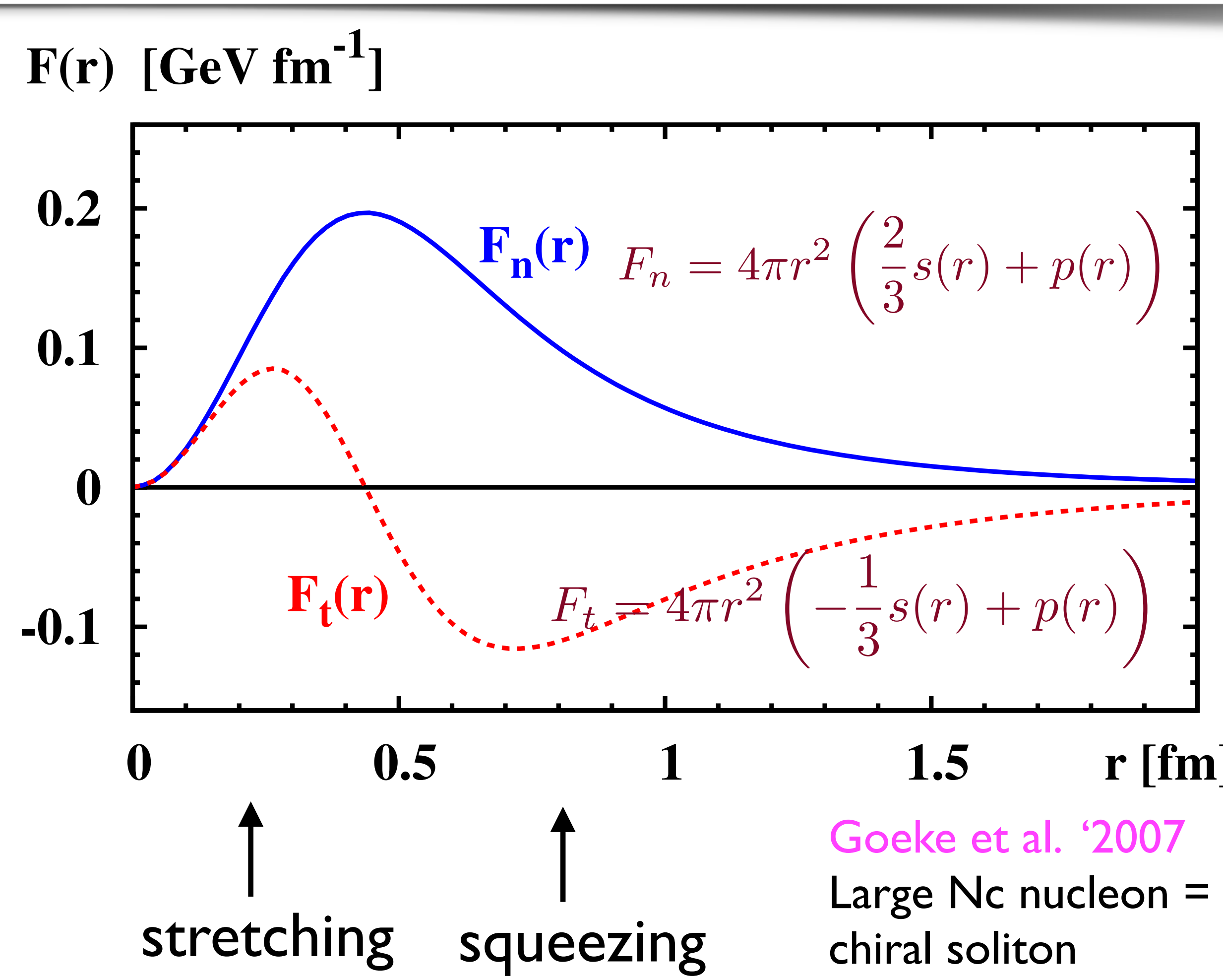
$$p'(r) + \frac{2}{3}\ s'(r) + \frac{2}{r}s(r) = 0$$

Equilibrium equation (conservation of EMT)

Interestingly the pressure anisotropy (shear forces distribution) plays an essential role in astrophysics, see the review [Herrera:1997plx] on the role of pressure asymmetry for self-gravitating systems in astrophysics and cosmology.

Size of the forces in the nucleon. Comparison with confinement forces





Goeke et al. '2007
Large Nc nucleon = chiral soliton

Compare with the linear potential force of ~1 GeV/fm !

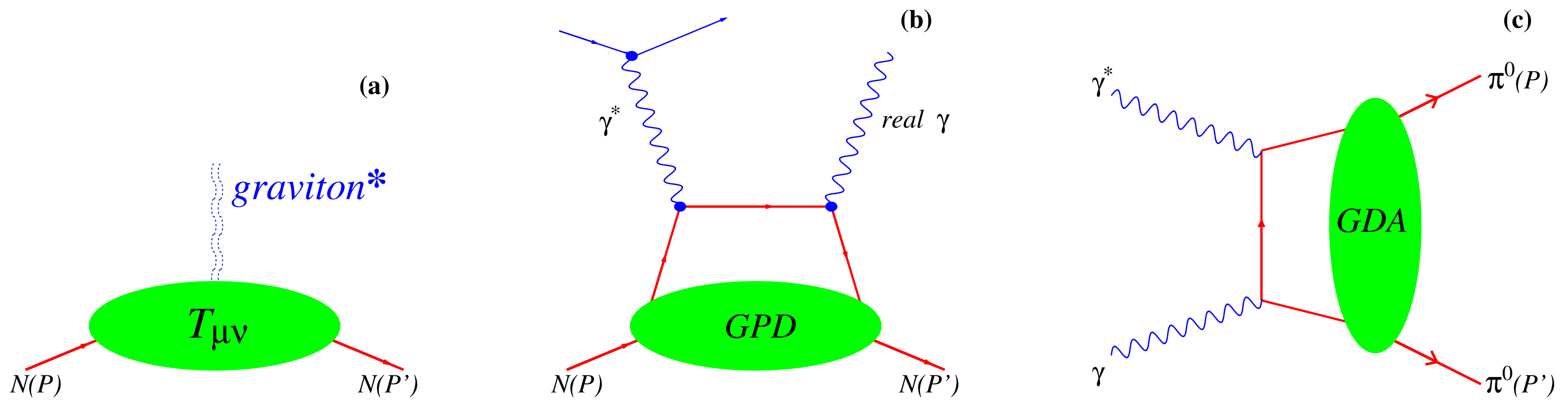
What does it imply for pictures of the confinement?

Values of D-term for the nucleon:

$-2 \geq D(0) \geq -4$ Chiral Quark Soliton model Boffi, Radici, Schweitzer '2001 Goeke et al. '2007

$D^Q(0) \approx -1.56$ at $\mu = 4$ GeV^2 Dispersion relations Pasquini, Vanderhaeghen, MVP '2014

Accessing D(t) in hard exclusive processes



$$\int_{-1}^{1} \mathrm{d}x\, x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)\,, \qquad \int_{-1}^{1} \mathrm{d}x\, x\, E^a(x,\xi,t) = 2J^a(t) - A^a(t) - \xi^2 D^a(t)\,.$$

X. D. Ji '96

Unfortunately the Mellin moments are not observable in model independent way. However, D(t) is related to subtraction constant in dispersion relations for amplitudes (observables!)

$$\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0}\right) H(x,\xi,t)$$ DVCS amplitude at LO (directly measurable!)

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\mathrm{vp}\int_0^1 d\xi'\ \mathrm{Im}\mathcal{H}(\xi',t)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

MVP '2003 (small-x DR)
Teryaev '2005
Anikin, Teryaev '2007
Diehl, Ivanov '2007

$$\Delta(t) = \frac{4}{5}\sum_q e_q^2\ D^q(t) + \sum_q e_q^2\ d_3^q(t) + \ldots$$

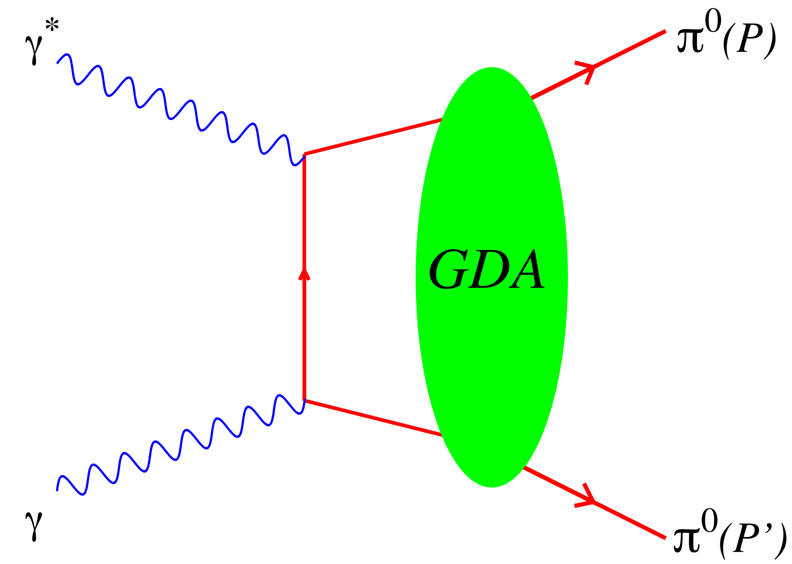
D(t) is more easy access than J(t). It is possible model independent extraction of D(t) in contrast to J(t)

Pion D-term

- **D-term of π^0**

access EMT form factors of unstable particles through generalized distribution amplitudes (analytic continuation of GPDs) via $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^-

Masuda et al (Belle), PRD 93, 032003 (2016)



best fit to Belle data $\to$ $D^Q_{\pi^0} \approx -0.7$
at $\langle Q^2 \rangle = 16.6$ GeV^2
compatible with soft pion theorem $D_{\pi^0} \approx -1$
(if gluons contribute the rest)

Kumano, Song, Teryaev, PRD97, 014020 (2018)

Slopes obtained:

$$\frac{1}{A^Q(0)}\frac{d}{dt}A^Q(0) = 1.33 \sim 2.02\ \mathrm{GeV}^{-2}, \quad \frac{1}{D^Q(0)}\frac{d}{dt}D^Q(0) = 8.92 \sim 10.35\ \mathrm{GeV}^{-2}.$$

Considerably larger than estimates in chiral effective theory! Why?

Schweitzer, MVP '2018

$$-D'(0) = \frac{N_c}{48\pi^2 f_\pi^2} + \frac{\ln\left(\mu^2/m_\pi^2\right)}{24\pi^2 f_\pi^2} = (0.73 + 1.66)\ \mathrm{GeV}^{-2} = 2.40\ \mathrm{GeV}^{-2}$$

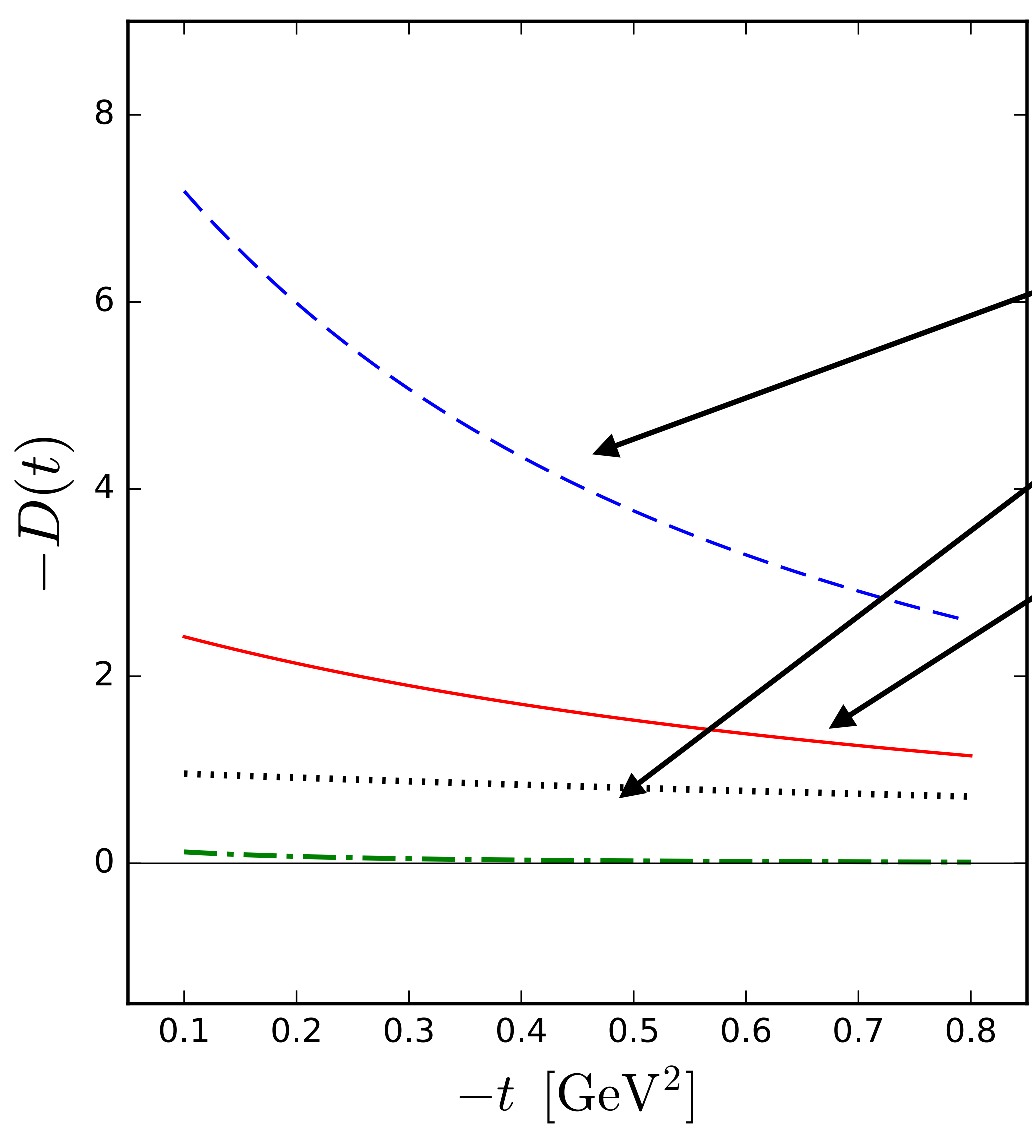
Accessing p(r) and s(r) in hard exclusive processes

Simplifying assumptions (for present state of art of the experiment):

1) d3(t), d5(t), … much smaller than D(t). It is so at large normalisation scale.
2) Flavour singlet D(t) is dominant. Justified in large Nc limit. Can be relaxed for more precise data.

Under these assumptions we obtain:

$$D^Q(t) = \frac{4}{5}\,\frac{1}{2(e_u^2+e_d^2)}\Delta(t) = \frac{18}{25}\,\Delta(t).$$



The first determination of D(t) from DVCS
Kumericki, Mueller Nucl. Phys. B841 (2010) 1

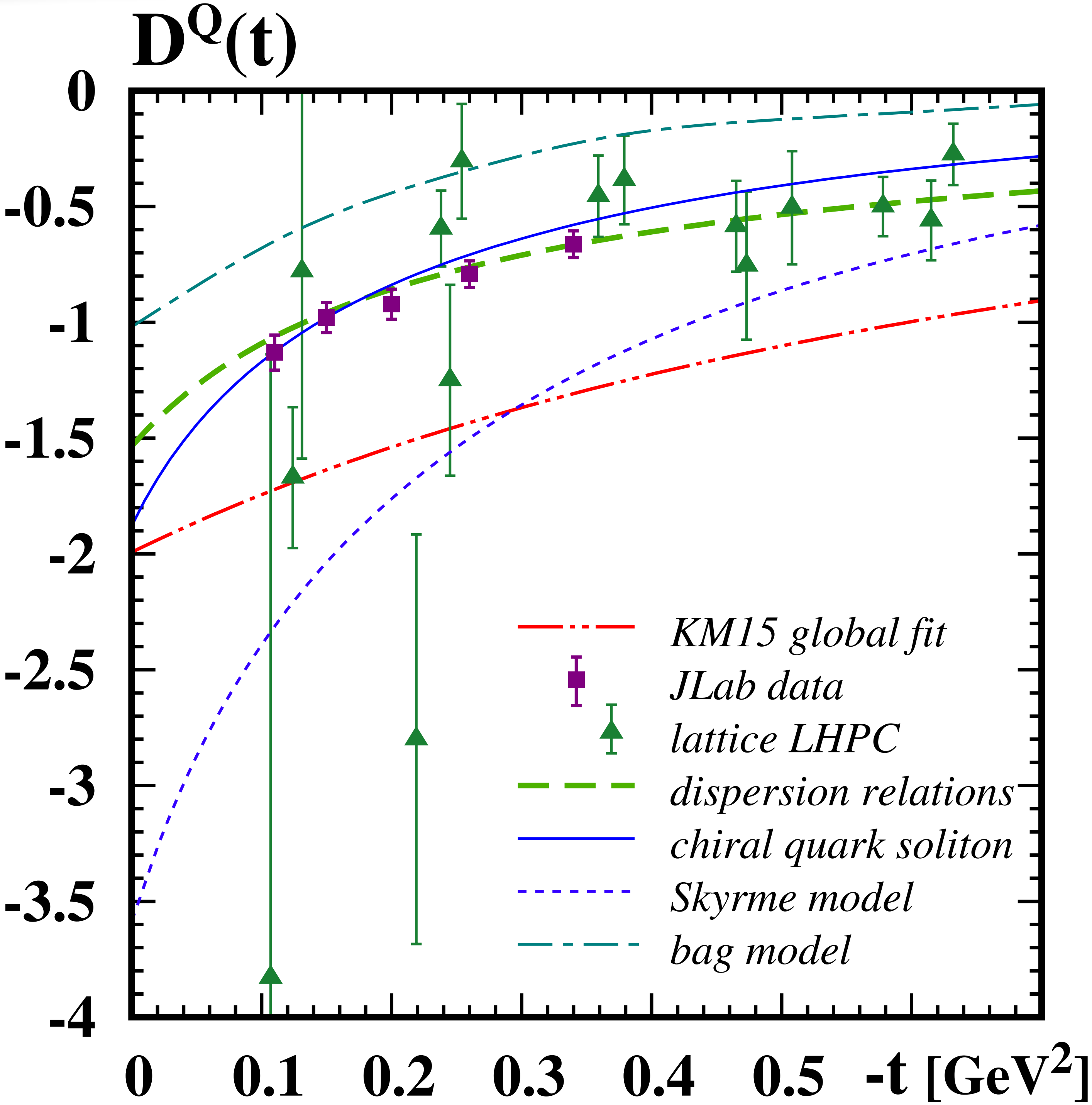
KM10, statistical accuracy 60%

KM12, statistical accuracy 50%

KM15, statistical accuracy 20%

The D-term is *negative,* statistical accuracy is increasing with new data added.

Accessing p(r) and s(r) in hard exclusive processes



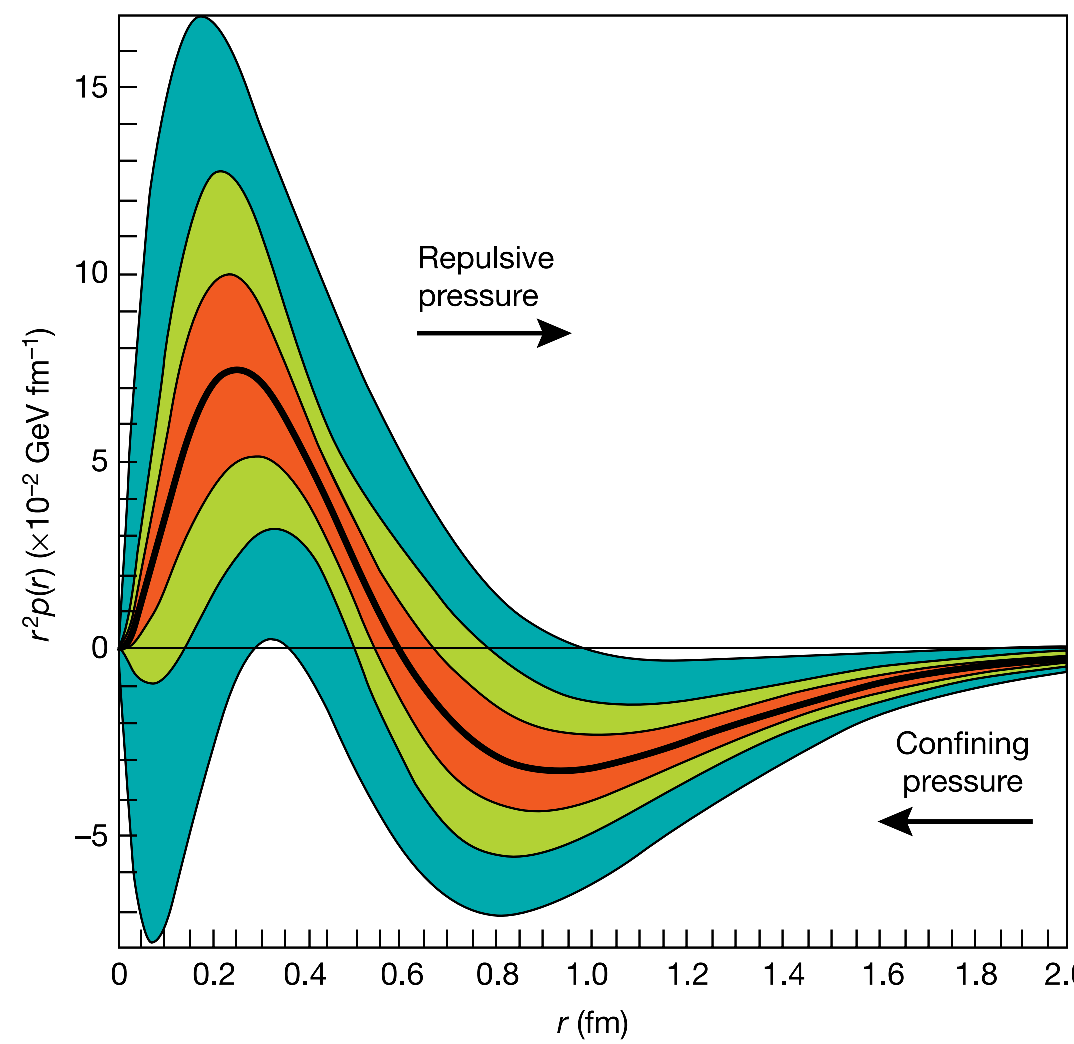
The systematic uncertainty needs more detailed estimate!

Details in [K. Kumericki, Nature 570 (2019)] and in [Dutrieux, Lorce et al. (2021)]

Recent analysis of CLAS data

Burkert, Elouadrhiri, Girod, **Nature 557, 396 (2018)**

1) D-term negative and sizeable
2) Agrees with chiral quark soliton model DR calculations



Nucleon "cosmological term".
Interaction of the gluon and quark subsystems inside the nucleon

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t) g_{\mu\nu}\right]u$$

$$\Delta^\beta M_N\ \bar{c}^Q(t)\ \bar{u}'u = \langle p'|ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p\rangle$$

In QCD: $\partial_\mu T^Q_{\mu\nu} = -g\ \bar{\psi}G_{\mu\nu}\gamma_\mu\psi$ $\qquad \partial_\mu T^g_{\mu\nu} = -\frac{1}{2}\ \mathrm{tr}\left(G_{\nu\alpha}\left[\mathcal{D}^\sigma, G_{\sigma\alpha}\right]\right)$

$\partial_\mu(T^Q_{\mu\nu} + T^g_{\mu\nu}) = 0$ due to EOM $\qquad [\mathcal{D}^\sigma, G_{\sigma\alpha}] = j^a_\alpha t^a \quad \text{with} \quad j^a_\alpha = -g\ \bar{\psi}\gamma_\alpha t^a\psi$

$\partial_\mu T^Q_{\mu\nu} = G^a_{\mu\nu}\ j^a_\mu$ ⟵ Expression for the Lorentz force experienced by a quark in external gluon field. We may expect that Cbar(t) is related to forces between quark and gluon subsystems.

Interaction of the gluon and quark subsystems inside the nucleon

$$\frac{\partial T_{ij}^{Q}(\boldsymbol{r})}{\partial r_j} + f_i(\boldsymbol{r}) = 0. \qquad (5.5)$$

Landau, Lifshitz, vol. 7

This equation can be interpreted (see e.g §2 of [28]) as equilibrium equation for quark internal stress and external force (per unit of the volume) $f_i(\boldsymbol{r})$ from the side of the gluons. This gluon force can be computed in terms of EMT form factor $\bar{c}^Q(t)$ as:

$$f_i(\boldsymbol{r}) = M_N \frac{\partial}{\partial r_i} \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta r}}\, \bar{c}^Q(-\boldsymbol{\Delta}^2) \qquad (5.6)$$

H.-D. Son, MVP '2018

For $\bar{c}^Q(0) > 0)$ the corresponding force is directed towards the nucleon centre, therefore we call it squeezing (compression) force. For opposite sign the corresponding force is stretching.

The total squeezing gluon force acting on quarks in the nucleon is equal to

$$F_{\text{total}} = \frac{2M_N}{\pi} \int_{-\infty}^{0} \frac{dt}{\sqrt{-t}}\, \bar{c}^Q(t).$$

Cbar(t) FF important to know what are (compressing or stretching) forces experienced by quarks from side of gluons inside the nucleon. Size of this forces?

The nucleon's "cosmological term" from instantons.

Instantons form a dilute liquid in the QCD vacuum. They provide a mechanism of spontaneous breakdown of chiral symmetry in QCD.

Shuryak '1982
Diakonov, Petrov '1983

$$\Delta^\beta M_N\ \bar{c}^Q(t)\ \bar{u}'u = \langle p'|ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p\rangle$$

Computed in QCD vacuum using the method of Diakonov, Weiss, MVP '1996; Balla, Weiss, MVP '1997

We found a strong suppression by the instanton packing fraction

$$\bar{c}^Q(t) = \frac{\bar{c}_{\rm quark}}{(1-t/\Lambda^2)^2} \qquad \bar{c}_{\rm quark} \sim \frac{1}{6}\frac{\bar{\rho}^4}{\bar{R}^4}\ \ln\left(\frac{\bar{R}}{\bar{\rho}}\right) \qquad \bar{c}^Q(0) = \bar{c}_{\rm quark} \simeq 1.4\cdot 10^{-2}.$$

H.-D. Son, MVP '2018

We obtained small and *positive* value at a low normalisation point of ~0.5 GeV^2.
This corresponds to rather *small* compression forces experienced by quarks!

$$F_{\rm total} = \bar{c}_{\rm quark}\ M_N\Lambda \simeq 5.9\cdot 10^{-2}\frac{\rm GeV}{\rm fm}$$

it looks like the two systems almost decouple.
Justification of Teryaev's equipartition conjecture ?

We estimate that the contribution of $\bar{c}^Q(t)$ to the pressure distribution inside the nucleon is in the range of $1-20\%$ relative to the contribution of the quark D-term.

↖ negative

No Goldstone boson "cosmological term"

Schweitzer, MVP: 1812.06143

EMT ffs for Goldstone boson (pion in the chiral limit):

$$\langle p'|\Theta_a^{\mu\nu}(0)|p\rangle = 2P^\mu P^\nu A^a(t) + \frac{1}{2}\left(\Delta^\mu\Delta^\nu - \eta^{\mu\nu}\Delta^2\right) D^a(t) + 2f_\pi^2\eta^{\mu\nu}\,\bar{c}^a(t)$$

Soft Goldstone (pion in the chiral limit) theorem:

$$\lim_{p'^\mu\to 0}\langle p'|\Theta_Q^{\mu\nu}(x)|p\rangle = 0.$$

$$0 = \frac{1}{2}p^\mu p^\nu\left(A^Q(0) + D^Q(0)\right) + 2f_\pi^2\,\eta^{\mu\nu}\bar{c}^Q(0).$$

This equation is satisfied if the EMT form factors of massless Goldstone boson are related other by:

$$D^Q(0) = -A^Q(0), \quad \bar{c}^Q(0) = 0.$$

Conclusions

- gravitational D-form factor is related to "elastic properties" of the nucleon, and gives access to details of strong forces inside the nucleon.

- We showed that 3D Breit frame and 2D light front force distribution are equivalent as they are related to each other by invertible Abel transformation. Any result in Breit frame (stability conditions, experimental data, model calculations, etc.) can be unambiguously transformed into corresponding result for light-front force distributions (and vice versa)

- Different ways of the interpreting the form factors can be regarded as different choices of schemes. To my taste 3D Breit force distributions are more intuitive and physics appealing.

- D(0) (the D-term) is the last unexplored global (in the same sense as mass and spin) property of the nucleon

- First experimental results for D(t) of the nucleon and of the pion are obtained. It is negative, as expected from stability conditions.

- Cbar(t) ("nucleon cosmological term") FF is important to understand forces between quark and gluon subsystems inside hadrons. Instanton picture of QCD vacuum predicts small positive value of the FF. That corresponds to compression forces experienced by quark subsystem (at variance with lattice results)

Outlook

- knowledge of the D-term can be important to understand hadron interaction in gravitational field relevant to BH or NS mergers (LIGO events). Gegelia, MVP in preparation

- the pressure distribution inside hadrons important to understand the physics of quarkonia interaction with the nucleon and the physics of hadro-charmonia (LHCb pentaquarks, tetraquarks with hidden charm) Eides, Petrov, MVP '2016, Perevalova, Schweitzer, MVP '2017, Panteleeva, Perevalova, Schweitzer, MVP '2018

- several theoretical issues - relation between pressure and energy density (elastic waves in hadrons?), analogies with cosmology, hadrons as projection of higher dimensional objects, relations to exactly solvable 2D models, etc.

Thank you!

Equivalence of 3D Breit and light-front force distributions

/Panteleeva, MVP '21/

Light front pressure and shear force distributions /Lorce'18, Freese, Miller '21/:

$$\widetilde{D}(x_\perp) = \frac{1}{4P^+} \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} D(-\boldsymbol{\Delta}_\perp^2) e^{-i\boldsymbol{\Delta}_\perp \cdot \mathbf{x}_\perp},$$

$$p^{(2D)}(x_\perp) = \frac{1}{2x_\perp} \frac{d}{dx_\perp} \left(x_\perp \frac{d}{dx_\perp} \widetilde{D}(x_\perp) \right), \qquad s^{(2D)}(x_\perp) = -x_\perp \frac{d}{dx_\perp} \left(\frac{1}{x_\perp} \frac{d}{dx_\perp} \widetilde{D}(x_\perp) \right)$$

Rescaling with the Lorentz factor:

$$\mathcal{S}(x_\perp) = \frac{P^+}{2m} s^{(2D)}(x_\perp), \qquad \mathcal{P}(x_\perp) = \frac{P^+}{2m} p^{(2D)}(x_\perp)$$

One-to-one mapping (Abel transformation) between Breit and LF force distributions /Panteleeva, MVP'21/

$$\mathcal{S}(x_\perp) = \int_{x_\perp}^{\infty} \frac{dr}{r} s(r) \frac{x_\perp^2}{\sqrt{r^2 - x_\perp^2}}, \qquad \frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) = \int_{x_\perp}^{\infty} \frac{dr}{r} s(r) \sqrt{r^2 - x_\perp^2}$$

$$s(r) = -\frac{2}{\pi} r^2 \int_r^{\infty} dx_\perp \frac{d}{dx_\perp} \left(\frac{\mathcal{S}(x_\perp)}{x_\perp^2} \right) \frac{1}{\sqrt{x_\perp^2 - r^2}}, \qquad \frac{2}{3} s(r) + p(r) = \frac{4}{\pi} \int_r^{\infty} \frac{dx_\perp}{x_\perp} \mathcal{S}(x_\perp) \frac{1}{\sqrt{x_\perp^2 - r^2}}.$$

Abel transformation is used in tomography of spherically symmetric systems (spin-0 and 1/2 hadrons)
For non-spherical objects (spin>1/2) the Radon transformation should be used.

Equivalence of 3D Breit and light-front force distributions

/Panteleeva, MVP '21/

Von Laue stability conditions in 3D and 2D

$$\int d^3\mathbf{r}\; p(r) = 0 \quad \Longleftrightarrow \quad \int d^2\mathbf{x}_\perp \mathcal{P}(x_\perp) = 0$$

Local stability conditions in 3D and 2D are equivalent to each other
(positivity property of Abel transformation)

$$\frac{2}{3} s(r) + p(r) > 0 \quad \Longleftrightarrow \quad \frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) > 0$$

Relations between 2D and 3D mechanical radii
(as usually 3D and 2D radii are related by the geometric factor 2/3)

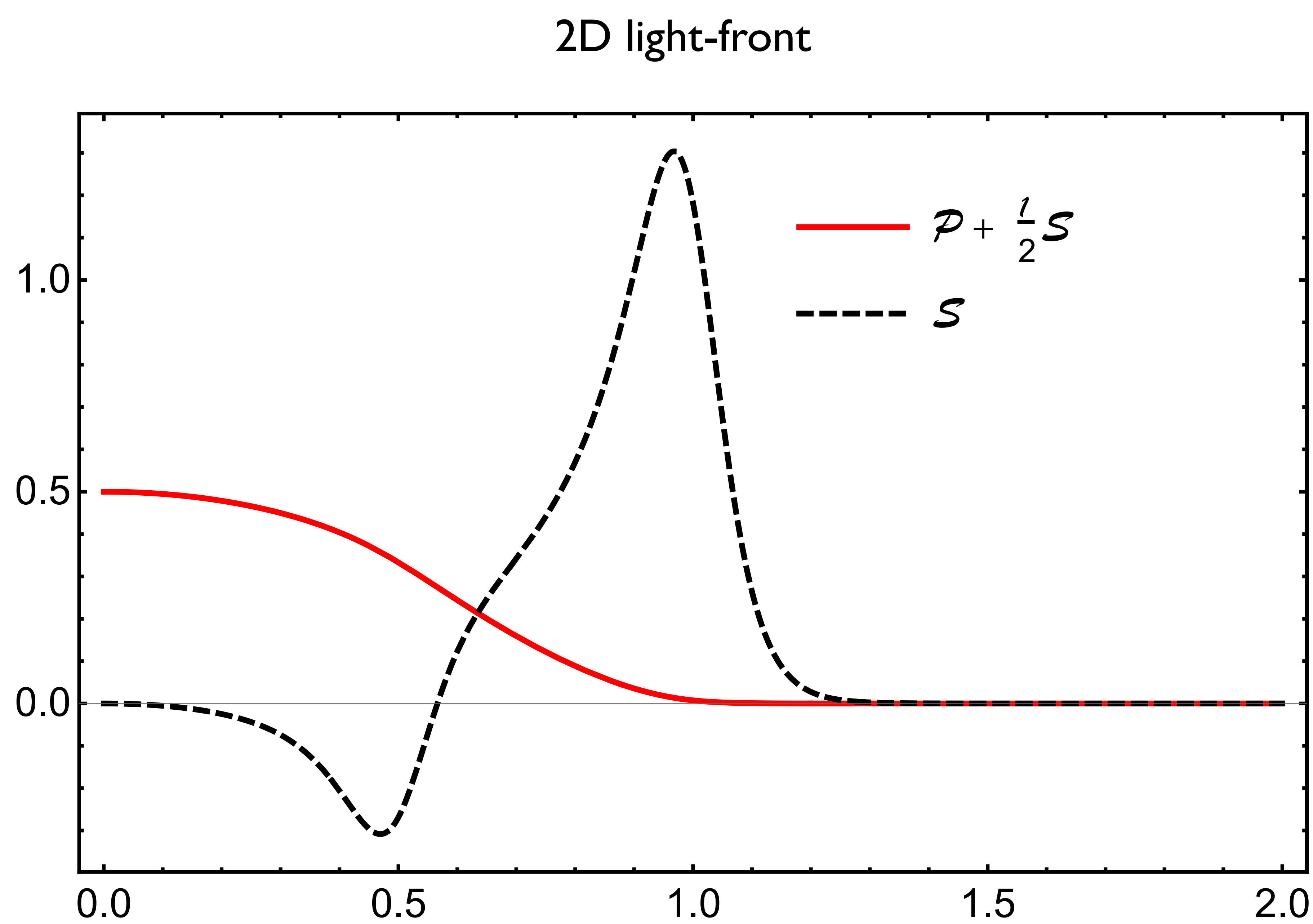
$$\langle x_\perp^2\rangle_{\rm mech} = \frac{\int d^2x_\perp x_\perp^2 \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)}{\int d^2x_\perp \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)} = \frac{4D(0)}{\int\limits_{-\infty}^{0} dt D(t)} = \frac{2}{3}\langle r^2\rangle_{\rm mech}$$

Druck term via 3D Breit and 2D LF force distributions

$$D(0) = -\frac{4m}{15}\int d^3\mathbf{r}\; r^2 s(r) = m\int d^3\mathbf{r}\; r^2 p(r) \Longleftrightarrow D(0) = -m\int d^2x_\perp x_\perp^2 \mathcal{S}(x_\perp) = 4m\int d^2x_\perp x_\perp^2 \mathcal{P}(x_\perp)$$

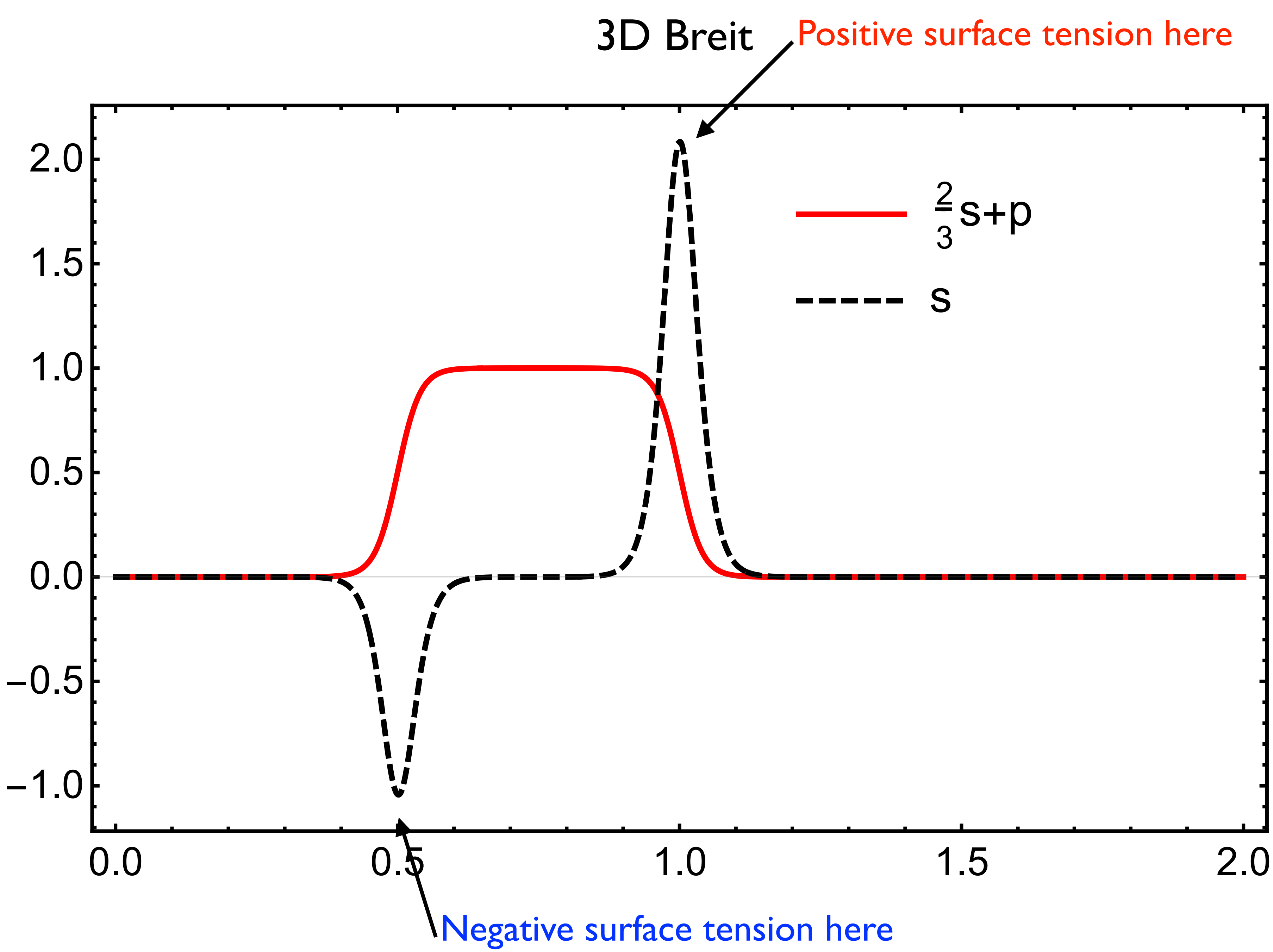
Small physics quiz

Q: Which physics system has the light-front force distributions as below?



Small physics quiz

A: It is a spherical shell with positive surface tension on outer edge, and with negative ST on inner edge. It is clearly seen from Breit frame force distributions!



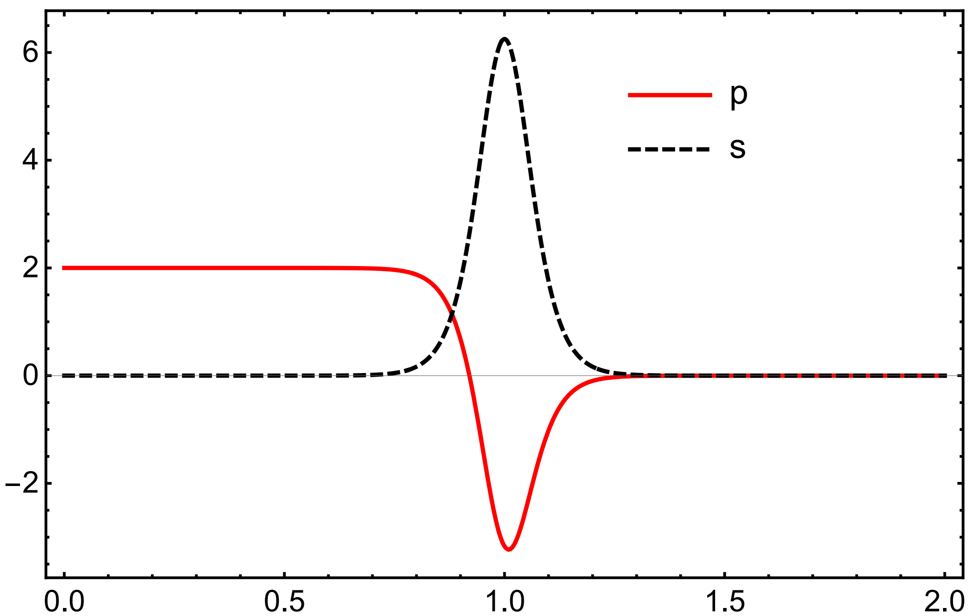
Equivalence of 3D Breit and light-front force distributions

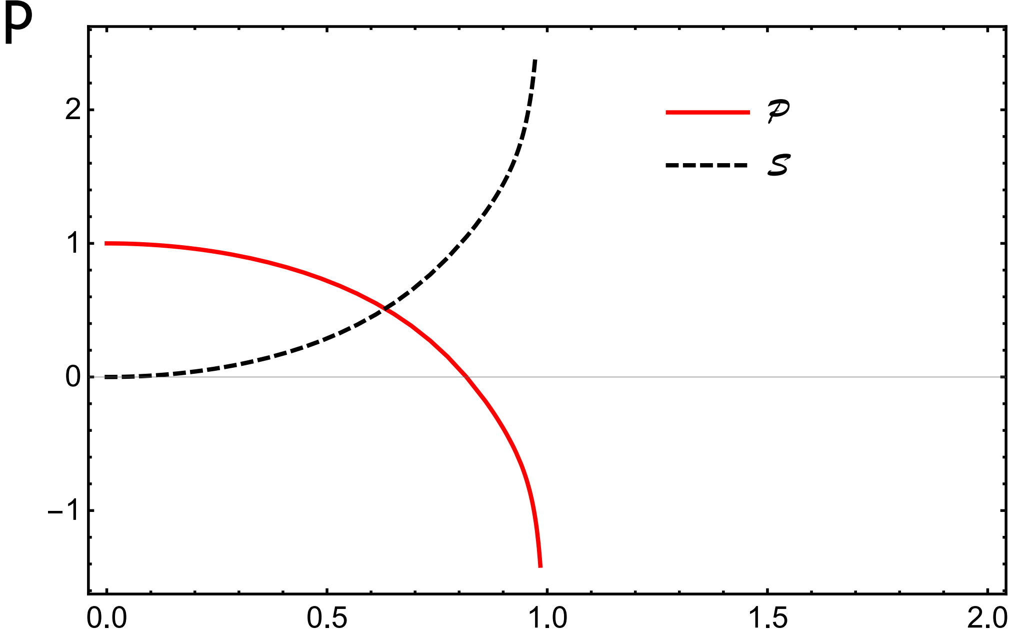
/Panteleeva, MVP '21/

3D Breit

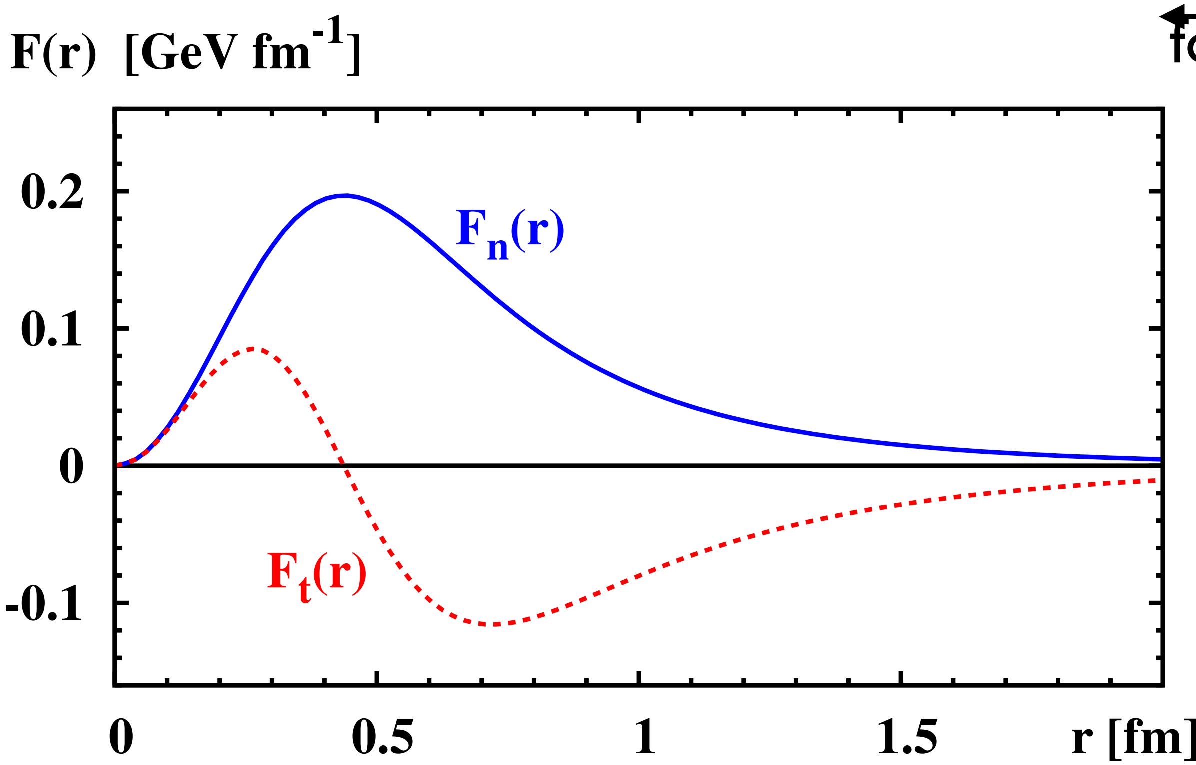
Abel transform
for a liquid drop

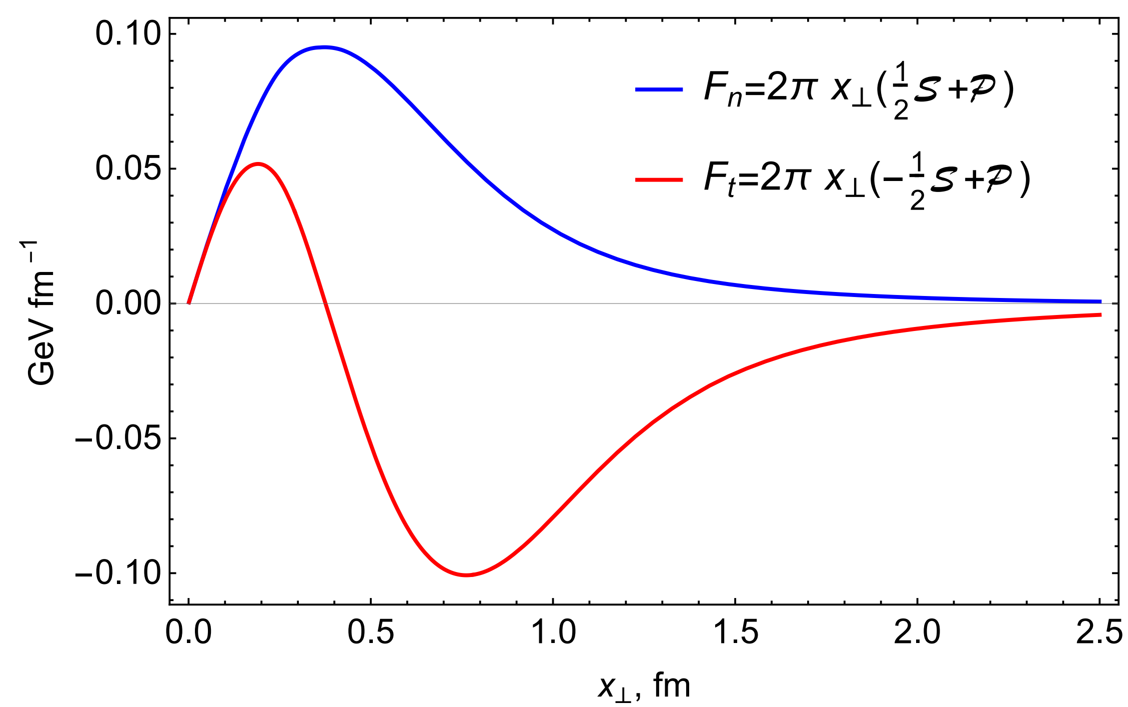
2D light-front





Abel transform
for chiral soliton





Goeke et al. '2007

Large Nc nucleon =chiral soliton

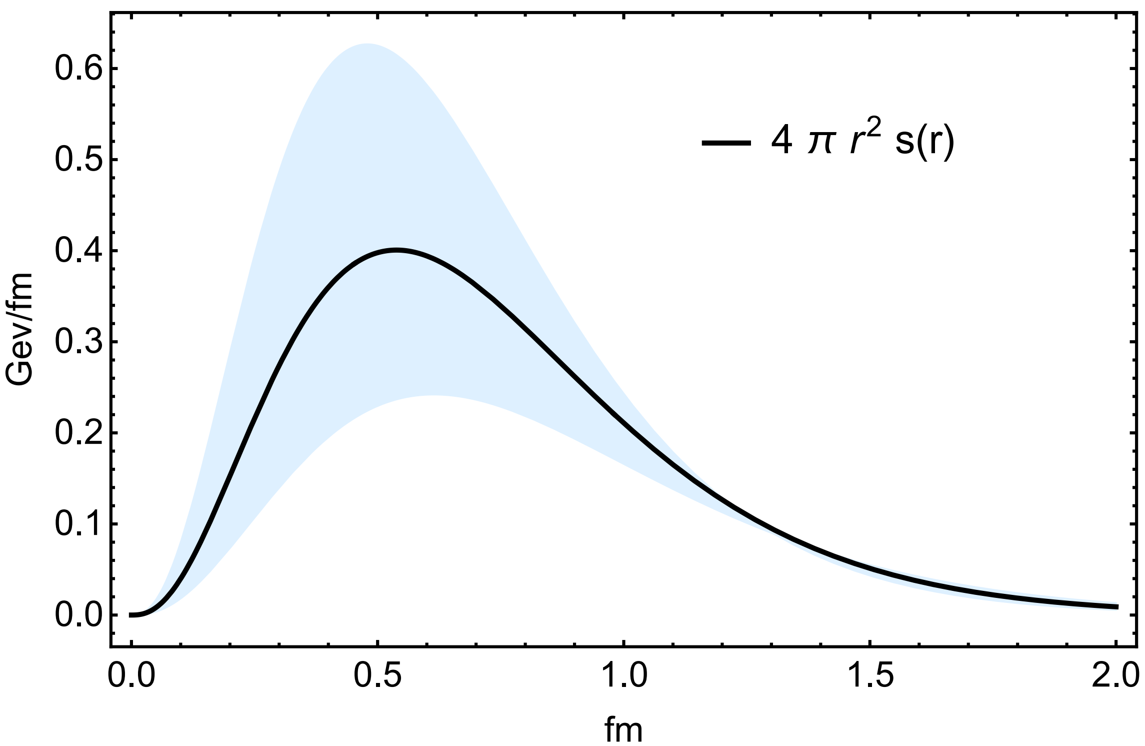
Equivalence of 3D Breit and light-front force distributions

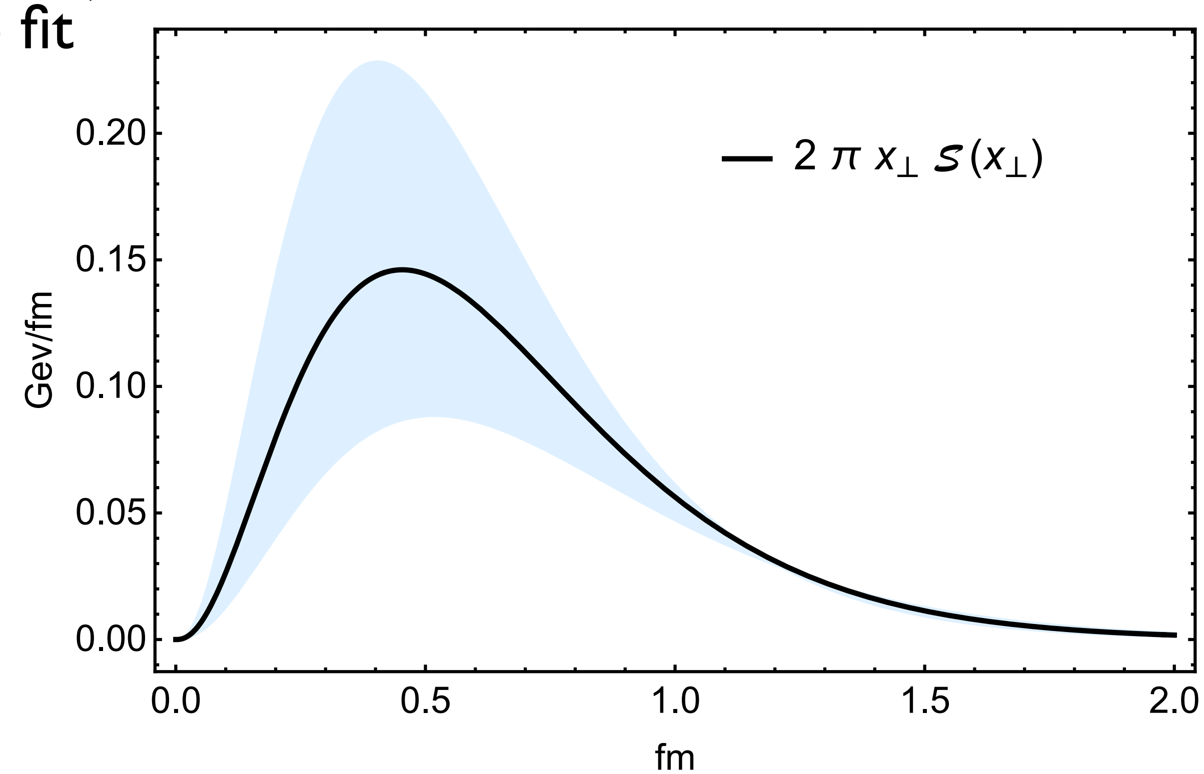
Results from the fit to DVCS by /Burkert, Elouadrhiri, Girod 2104.02031/

3D Breit

Abel transform ⟷ for a exp fit

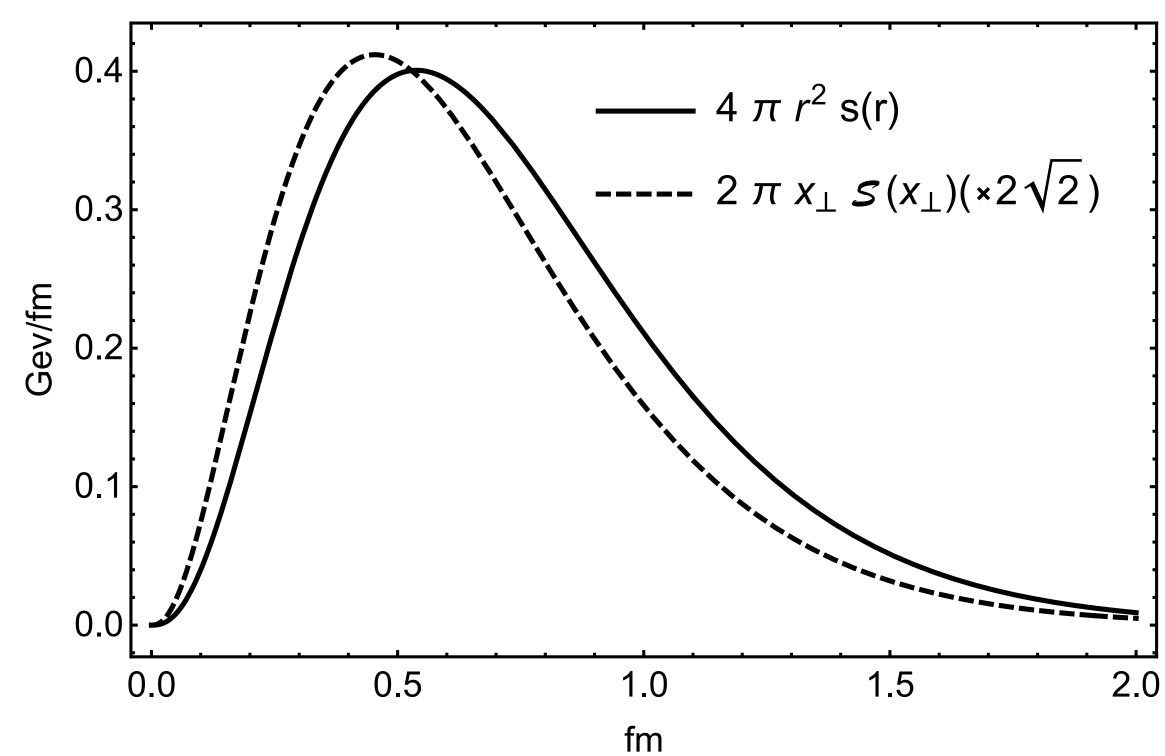
2D light-front





The systematic uncertainty needs more detailed estimate!

Details in [K. Kumericki, Nature 570 (2019)] and in [Dutrieux, Lorce et al. (2021)]



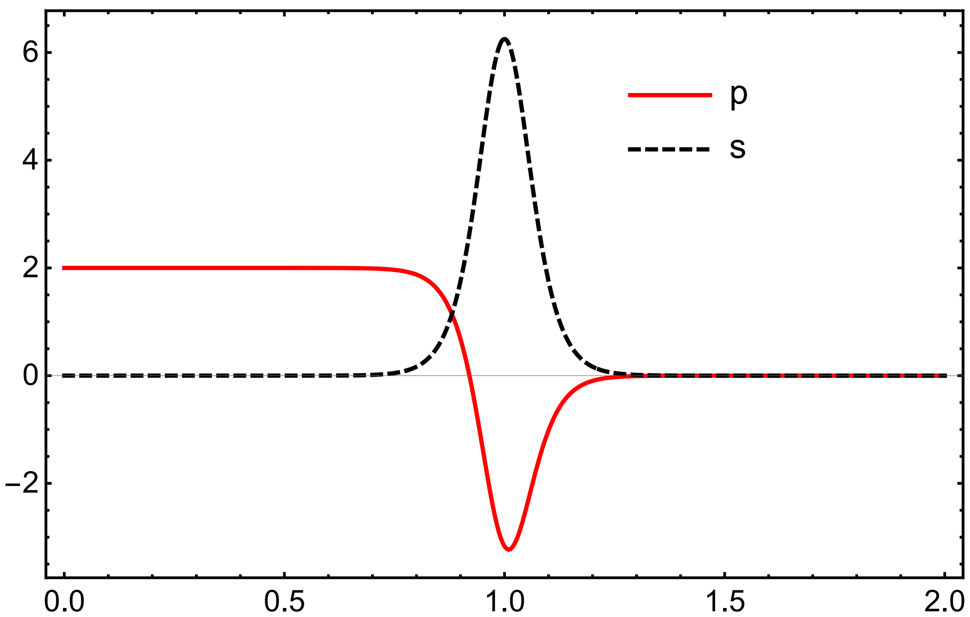
Equivalence of 3D Breit and light-front force distributions

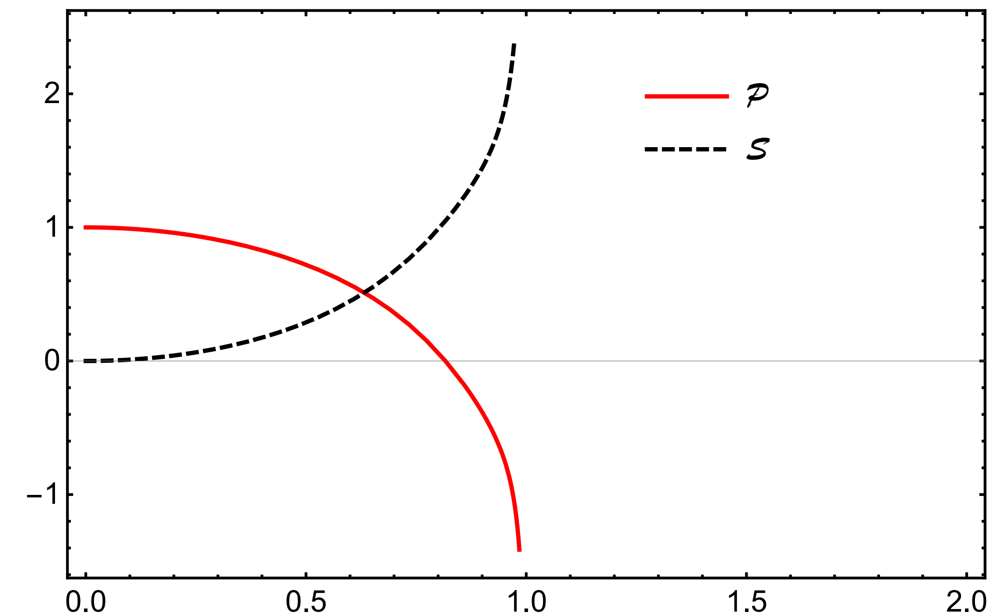
/Panteleeva, MVP '21/

3D Breit

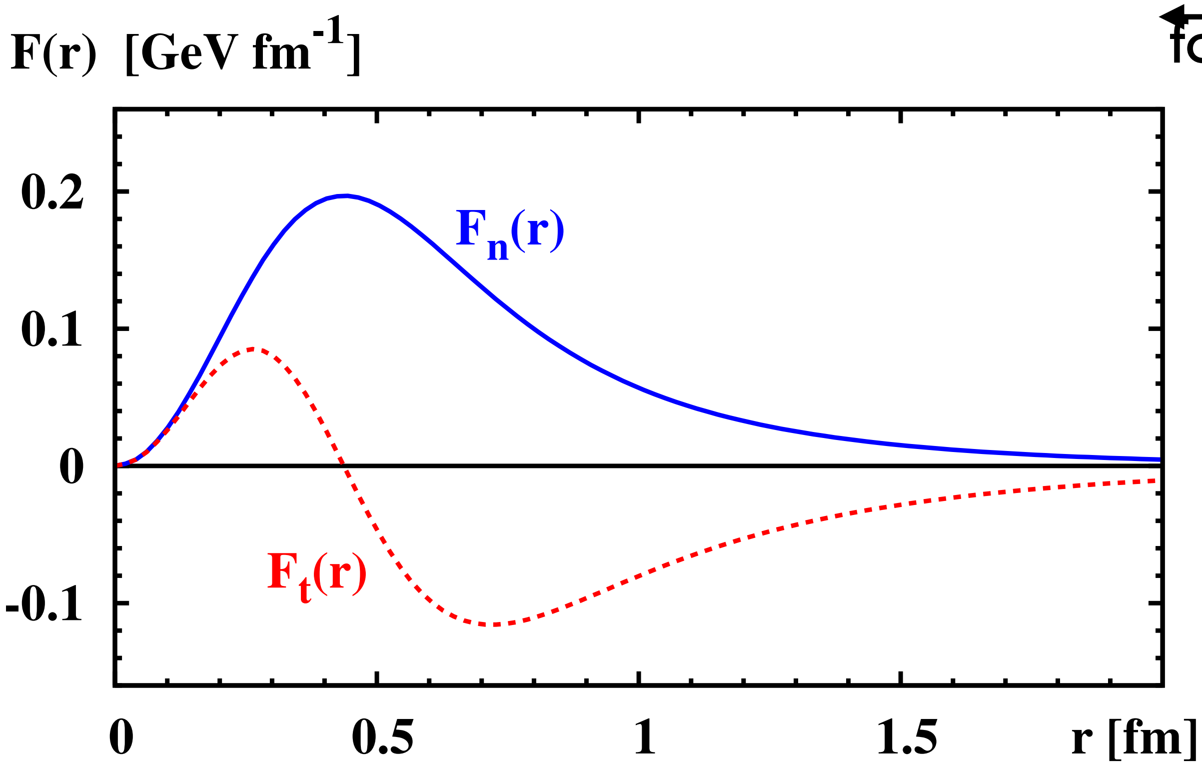
Abel transform ⟷ for a liquid drop

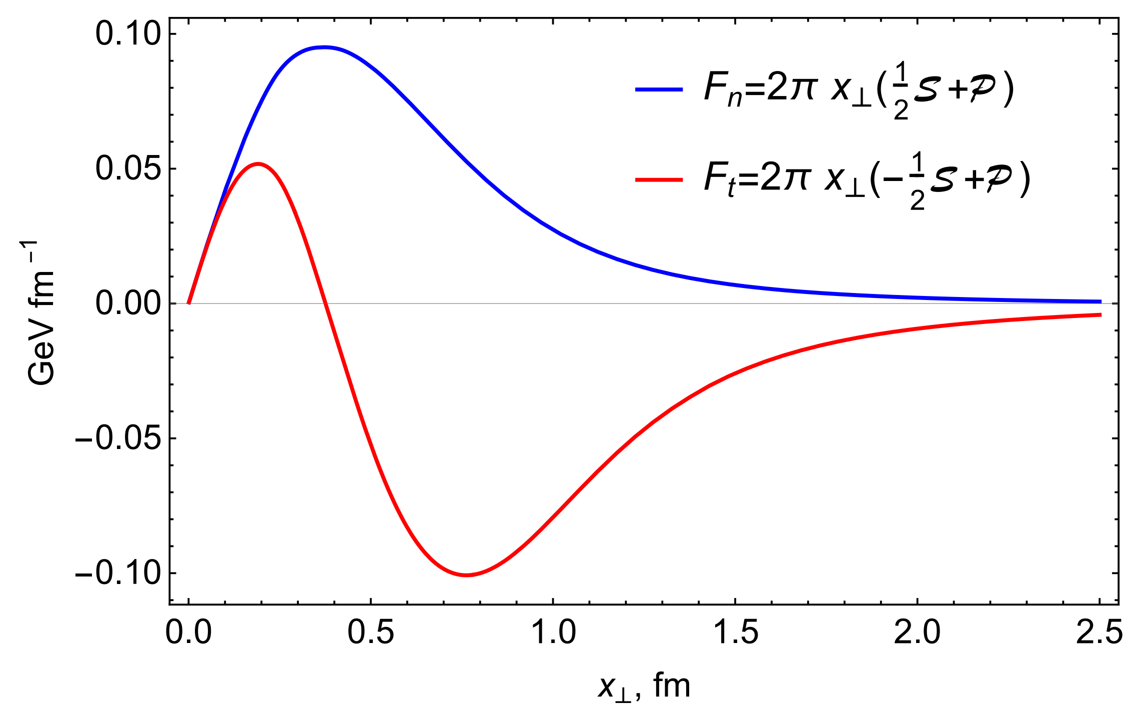
2D light-front





Abel transform ⟷ for chiral soliton





Goeke et al. '2007

Large Nc nucleon =chiral soliton