Delta baryon photoproduction with twisted photons

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Crim Dell

Talk based on work with Andrei Afanasev
plus related work with Asmita Mukherjee, Maria Solyanik-Gorgone, Christian Schmiegelow, Ferdinand Kaler-Schmidt, Jonas Schulz, Hao Wang

Bunhwangsa pagoda

Outline

- What are we talking about? Introduction to twisted photons.
- Do they really exist? Right now, yes, in atomic physics.
- Hadronic application: Photoproduction of $\Delta(1232)$ from nucleons.

Intro

- For plane wave photon states, we know that angular momentum along direction fo motion ("helicity") is just spin and just ± 1 (or $\pm\hbar$).
- Not true in general!
- Angular momentum along direction of motion can be any integer (times $\hbar$)
- Nor realized until 1992, Allen et al., PRA with 7600 citations (Google Scholar, July 17) and photon modes called Laguerre-Gaussian

Intro 2

- Could have been realized earlier
- Just for beginning, think of “scalar photons” (no polarization)
- 1987 JOSA-A Durnin: ∃ non-diffractive waves that are not plane waves.
- “Non-diffractive” means waves moving in some direction (e.g., z-direction) w/o spreading in the transverse direction.

Intro 3

- Field satisfies wave equation. Say monochromatic (time dep. $e^{-i\omega t}$) and propagating in *z*-direction (*z*-dependence $e^{i\beta z}$).

$$0 = \left(\frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \psi(t, \overrightarrow{x}) = \left(-\frac{\omega^2}{c^2} + \beta^2 - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \psi(t, \overrightarrow{x})$$

- Solvable: $\psi(t, \overrightarrow{x}) = e^{i\beta z - \omega t} J_0(\alpha\rho)$ $\quad [\alpha^2 = \frac{\omega^2}{c^2} - \beta^2 \, , \;\; \rho^2 = x^2 + y^2 \,]$

- Bessel function wavefront does not spread.

Intro 4

- Later (past '92) realized one could spin up the result.

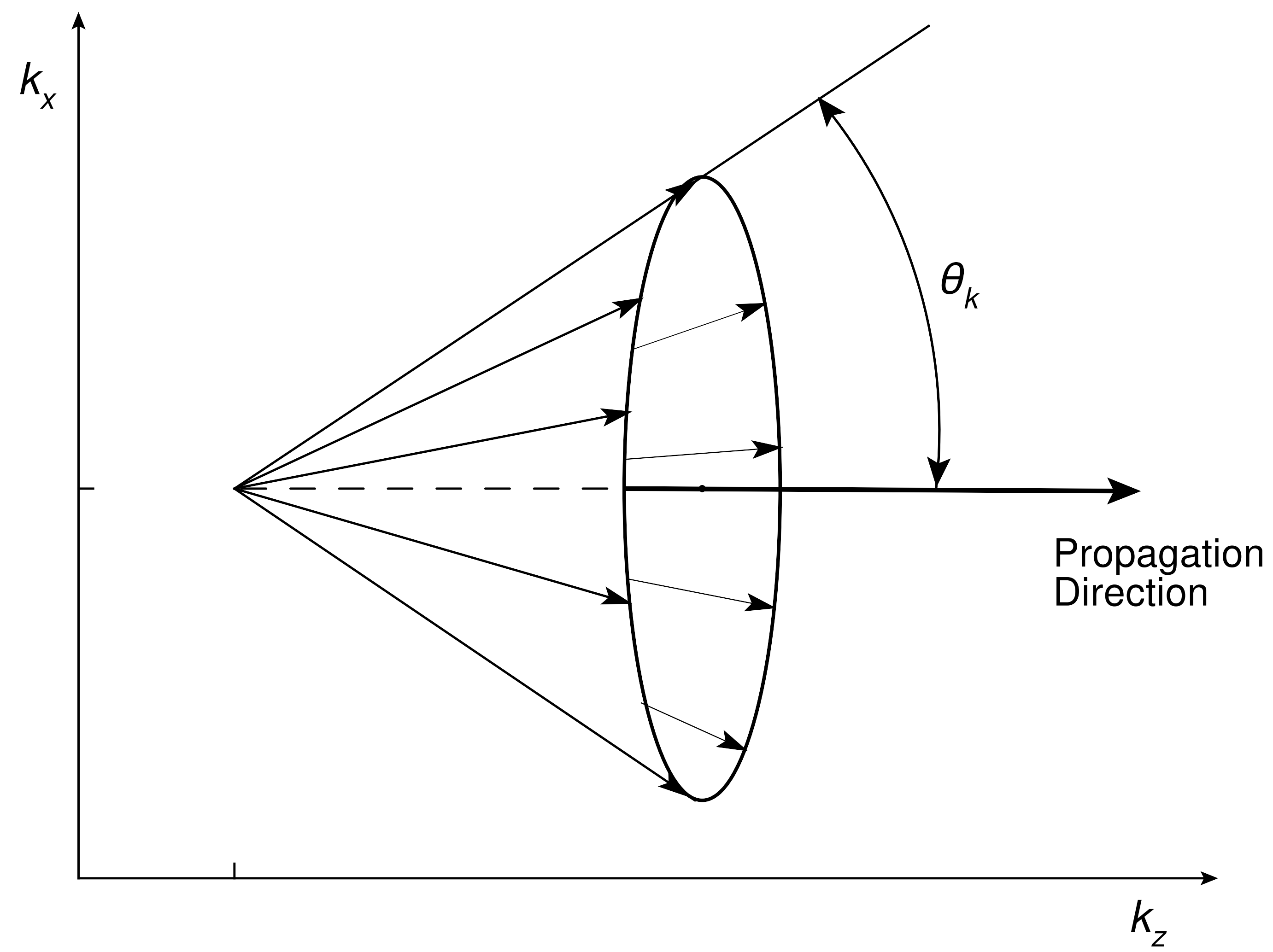
$$\psi(t, \overrightarrow{x}) = e^{i\beta z - \omega t} e^{im\phi} J_m(\alpha\rho)$$

- Orbital angular momentum operator $L_z = -i\hbar\partial/\partial\phi$ and

$$L_z \psi(t, \overrightarrow{x}) = m\hbar\, \psi(t, \overrightarrow{x}) \qquad [\, m = \text{any integer} \,]$$

- Fourier transform is simple $[\, \kappa = |\, \overrightarrow{k}_\perp | \,]$

$$\tilde{\psi}(t, \overrightarrow{k}) = \int d^3x\, e^{i\overrightarrow{k}\cdot\overrightarrow{x}}\, \psi(t, \overrightarrow{x}) = \ldots\ \delta(k_z - \beta)\, \delta(\kappa - \alpha)\, i^{-m}\, e^{im\phi_k}$$



- The allowed wave vectors $\overrightarrow{k}$ form a cone
  (Opening angle θ_k is "pitch angle")

Intro 5

- Also, Hilbert space representation. Characterize state by κ, m, k_z :

$$|\,\kappa, m, k_z\rangle = A_0 \int \frac{d\phi_k}{2\pi}\; i^{-m}\, e^{im\phi_k} |\,\overrightarrow{k}\,\rangle$$

(last is plane wave state)

- As exercise, can recover c.s. wave function using field operator $\hat{\psi}(x)$,

$$\psi_{\kappa m k_z}(t, \overrightarrow{x}) = \langle 0|\;\hat{\psi}(x)\;|\,\kappa, m, k_z\rangle = A_0 e^{i(k_z z - \omega t)} e^{im\phi} J_m(\kappa\rho)$$

- (Used $\langle 0|\;\hat{\psi}(x)\;|\,\overrightarrow{k}\,\rangle = e^{-ikx}$)

Intro 6

- Real photon has polarization. Easy to include in momentum space,

$$|\gamma(\kappa, m, k_z, \Lambda)\rangle = A_0 \int \frac{d\phi_k}{2\pi}\, i^{-m}\, e^{im\phi_k} |\gamma(\vec{k}, \Lambda)\rangle \qquad [\, \Lambda = \text{helicity} \,]$$

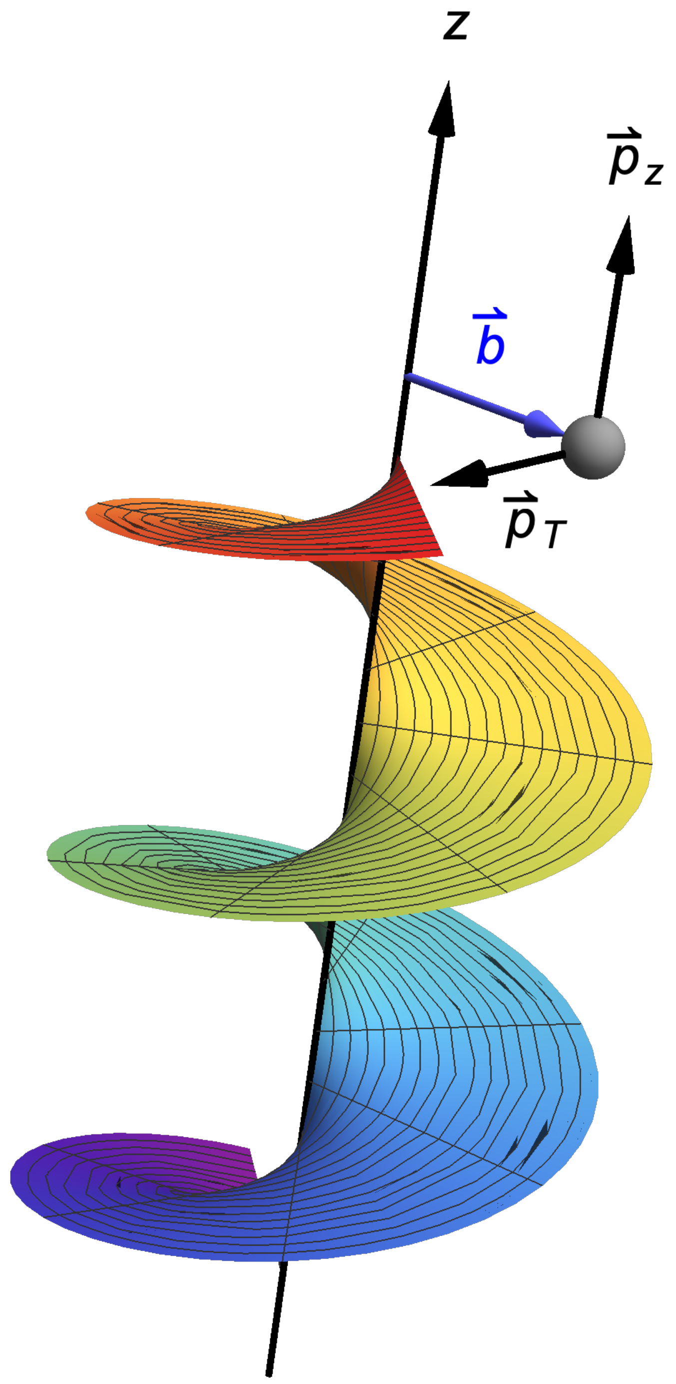
- Can get coordinate space potential from

$$A^{\mu}_{\kappa m k_z \Lambda}(x) = \langle 0 | \hat{A}^{\mu}(x) | \gamma(\kappa, m, k_z, \Lambda)\rangle$$

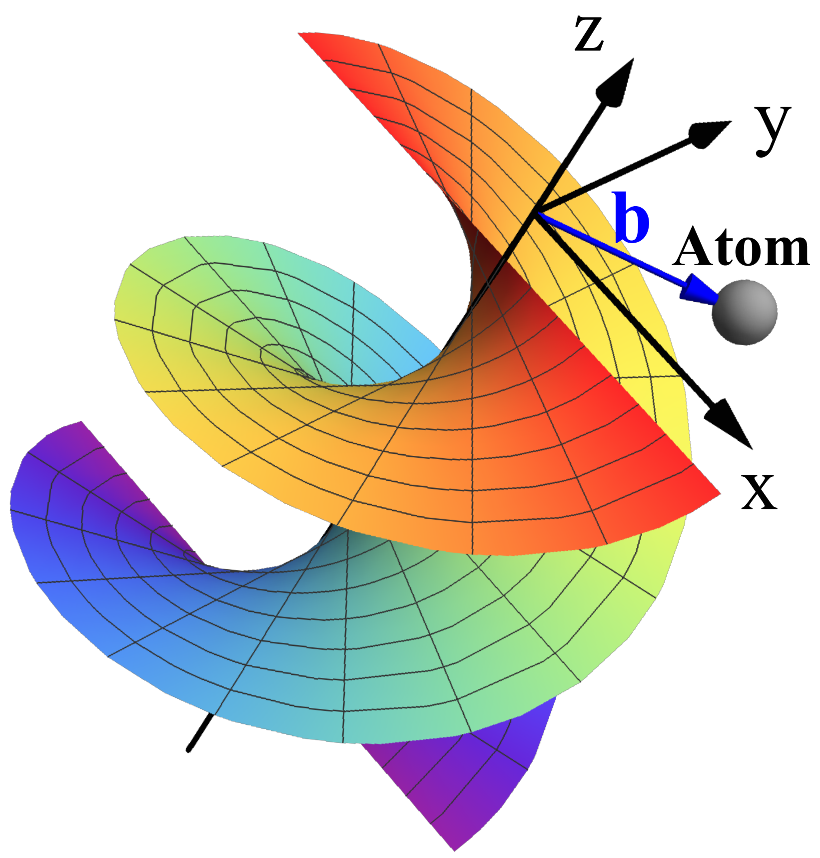
with $\langle 0 | \hat{A}^{\mu}(x) | \gamma(\vec{k}, \Lambda)\rangle = \epsilon^{\mu}_{\vec{k}\Lambda}\; e^{-ikx}$

- (Expressions complicated by polarization vectors depending on ϕ_k , but analytically given in terms of Bessel functions.)

Intro 7 - pictures

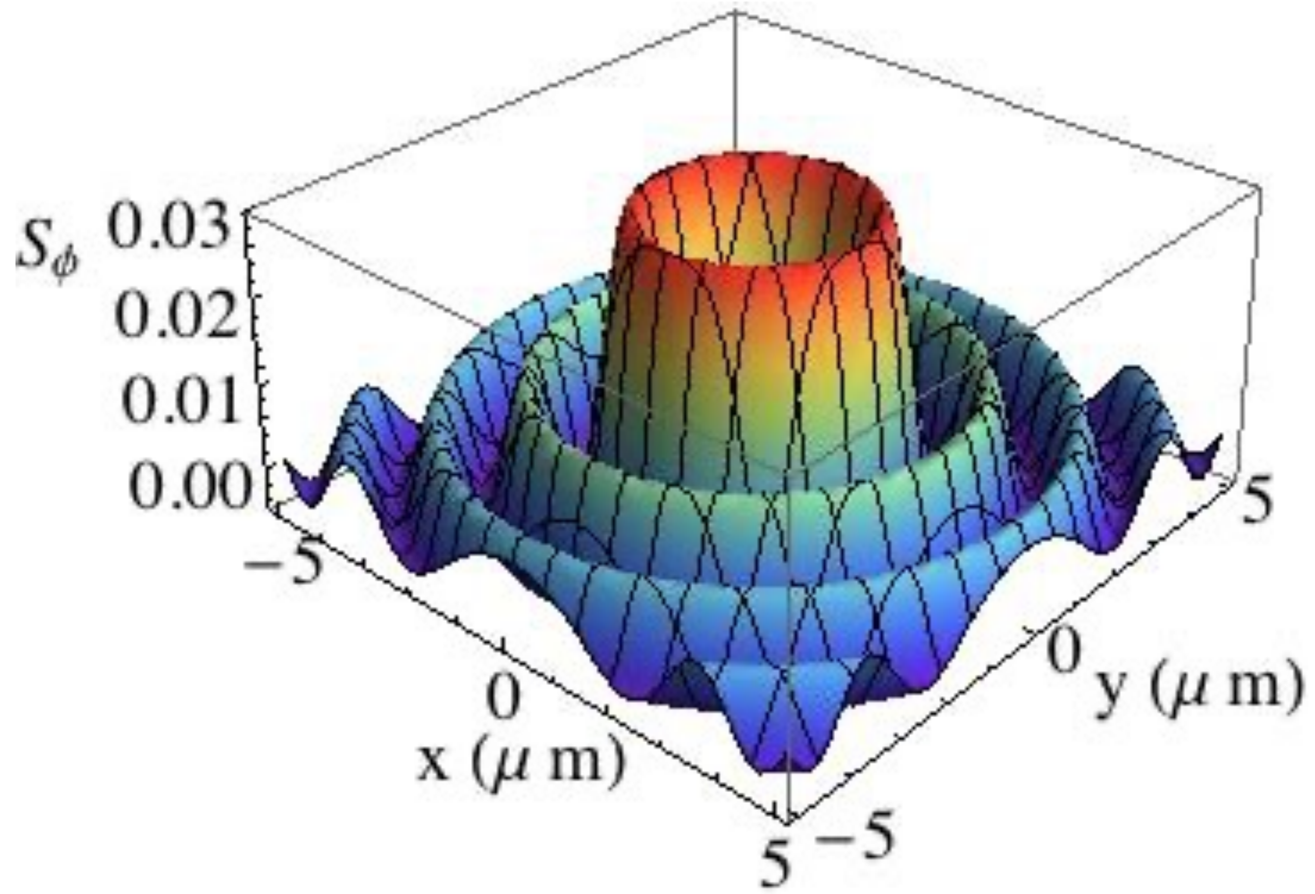


$m_\gamma - \Lambda = -1$

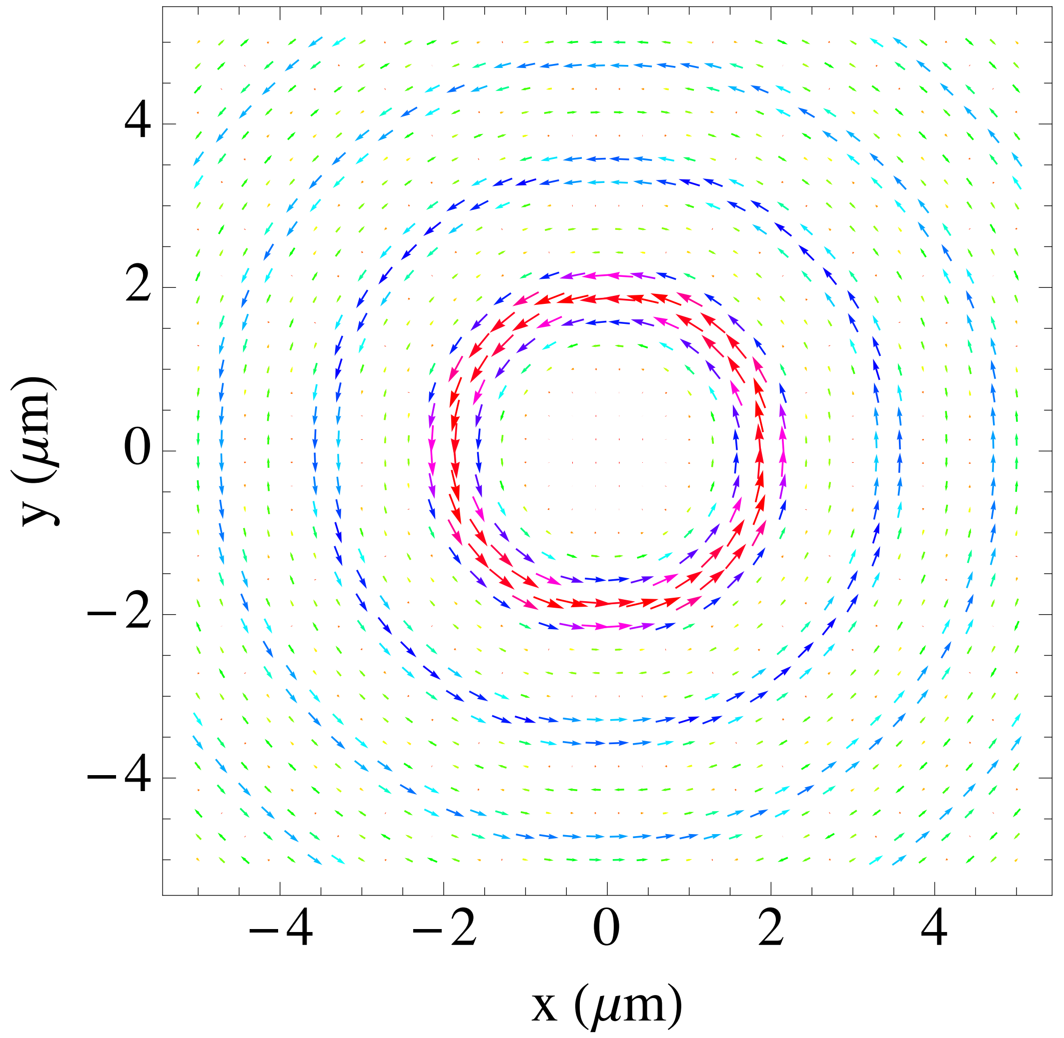


$m_\gamma - \Lambda = -2$

Surfaces of constant phase



Azimuthal component of Poynting vector magnitude



Azimuthal component of Poynting vector magnitude and direction

Names: Twisted photons, Vortex photons, Structured light

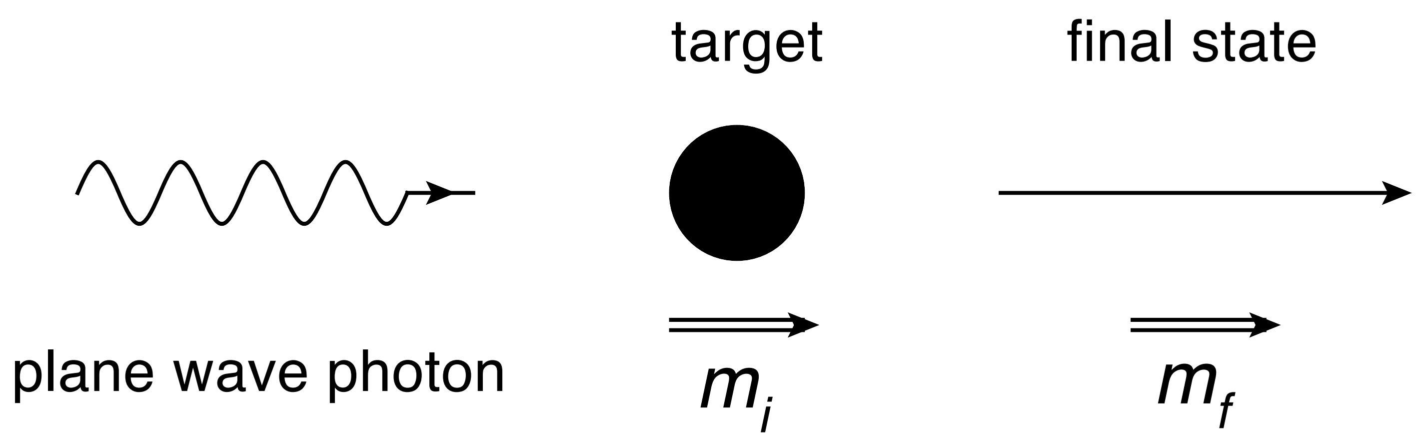
Intro 8

- Have photons with $m_\gamma \hbar$ angular momentum in direction of propagation

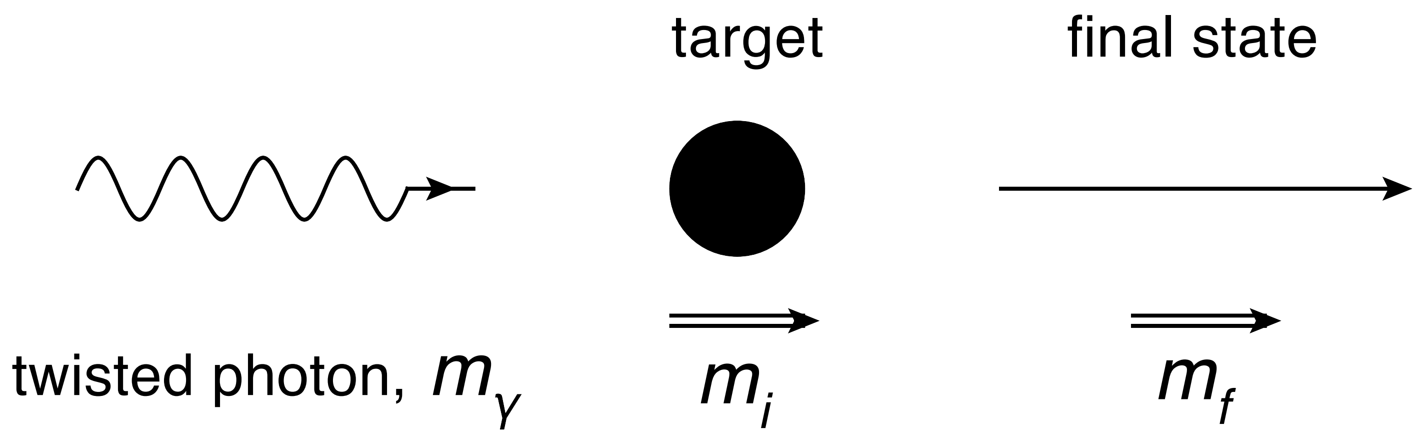
- Applications?

- Take advantage of selection rules.

  E.g., in photoproduction or photoexcitation



$m_f = m_i \pm 1$ only



$m_f = m_i + m_\gamma$ and m_γ can be any integer

Intro 9-more on selection rules

- Could selectively excite and study high angular momentum final states w/o background of low angular momentum states

- Thoughts about hitting targets off vortex axis:

  - Plane wave: there is no off-axis

  - Vortex states: Recall plots of Poynting vectors. There is local transverse momentum. Can give sideways kick and OAM to target.

    - On-axis: all angular momentum must go to internal excitation

    - Off-axis: angular momentum shared between internal excitation of target and OAM of center of mass of final state.

Atomic studies

- Can we do it?
- Learn from atomic physics.
- Can localize atoms in Paul trap to few 10’s of nanometers
- “Hole” in middle of vortex photons are few wavelengths

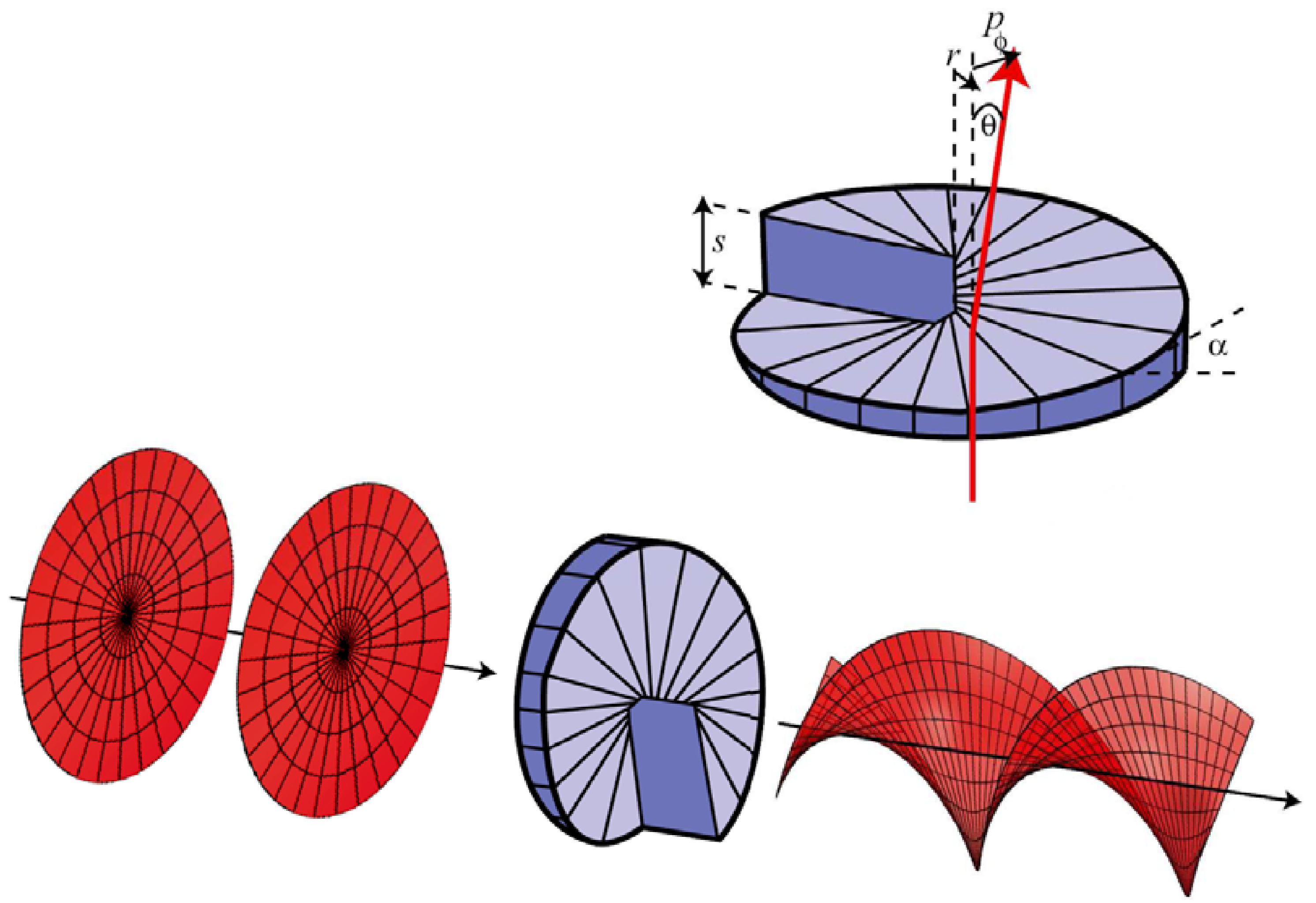
    i.e., few times several hundred nm.

- Hence localization very good.

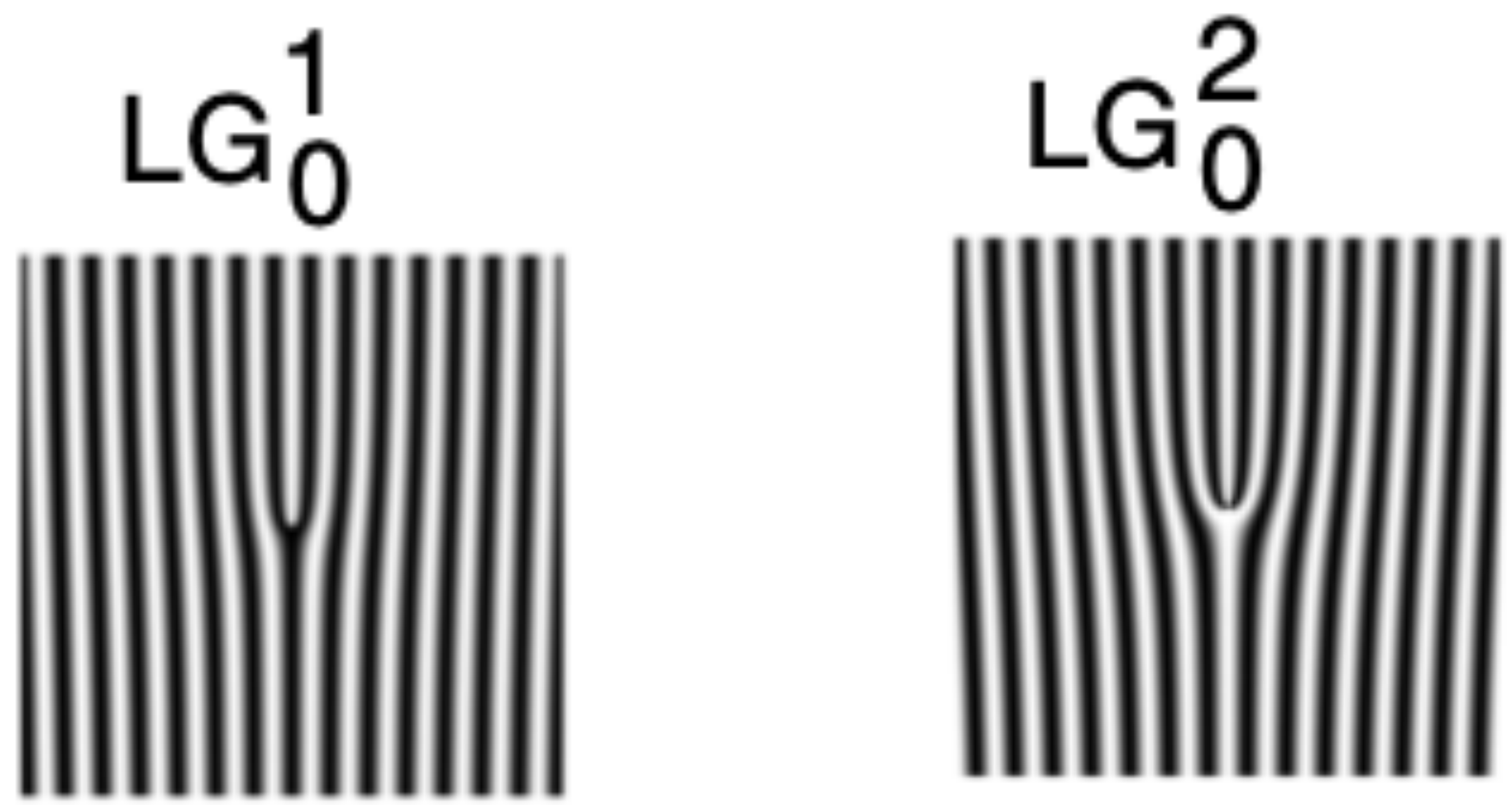
Atomic studies 2

- Acknowledgement: I learned a lot from the QUANTUM group (Ferdinand Schmidt-Kaler and co) at the JGU Mainz

- Get visible wavelength twisted light from laser with circular polarization passing through spiral wave plates or tuning fork diffraction grating.



A spiral phase plate can generate a helically phased beam from a Gaussian. In this case $\ell = 0 \to \ell = 2$.



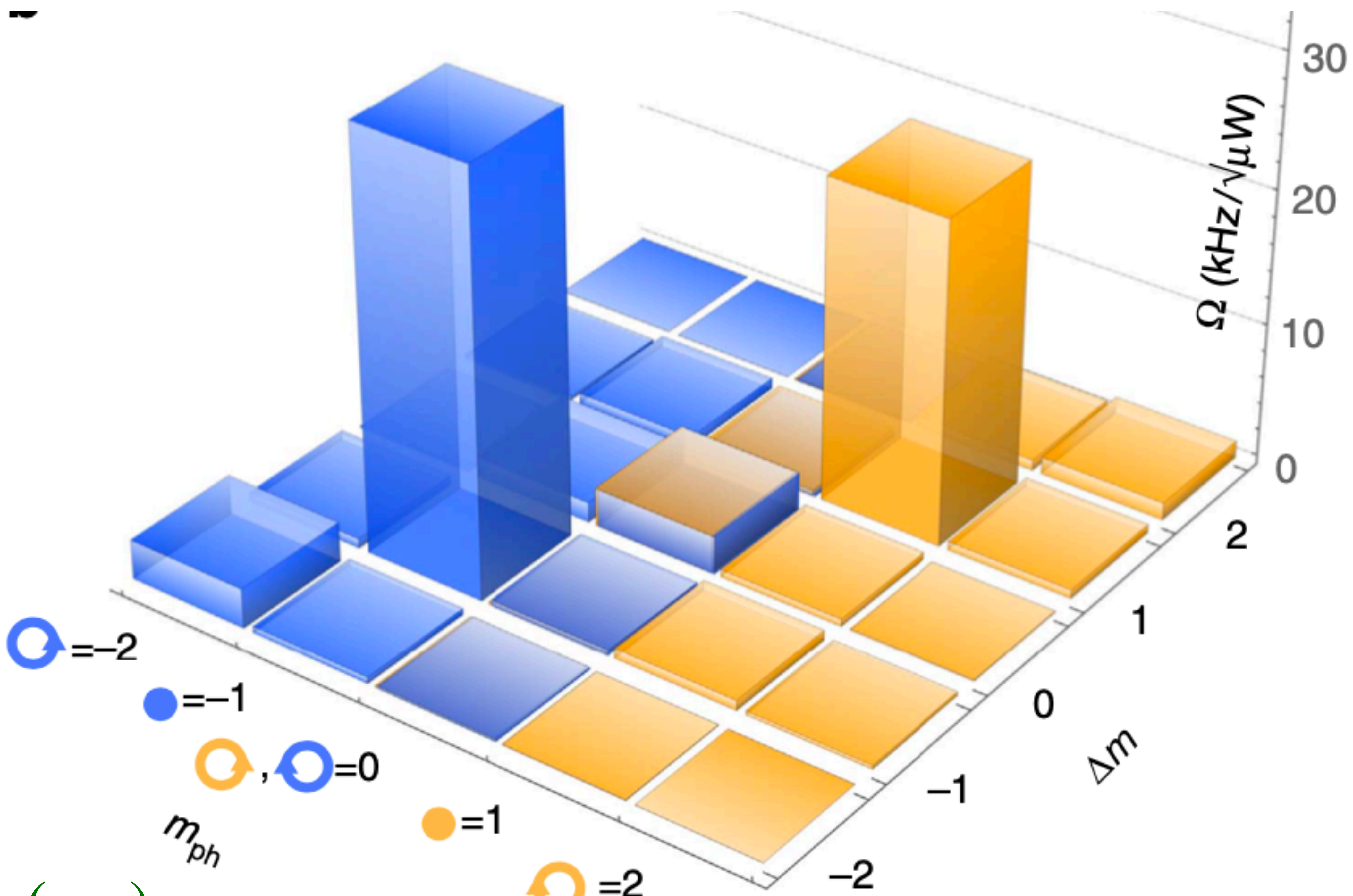
Atomic studies 3

- Excellent study ion is $^{40}\text{Ca}^+$
  - Ground state has $4s_{1/2}$ valence electron
  - Excite to $3d_{5/2}$ with 729 nm photons
  - $3d_{5/2}$ (of course) has $m_f = -5/2, -3/2, \ldots, 5/2$
- Zeeman split levels by magnetic field and determent m_f from small changes in γ-energy needed to excite them.

Atomic studies 4

- Hence $\Delta m = m_f - m_i$ measurable. m_γ known.

$\Delta m = m_\gamma$ testable



(m_γ)

Excitation amplitude

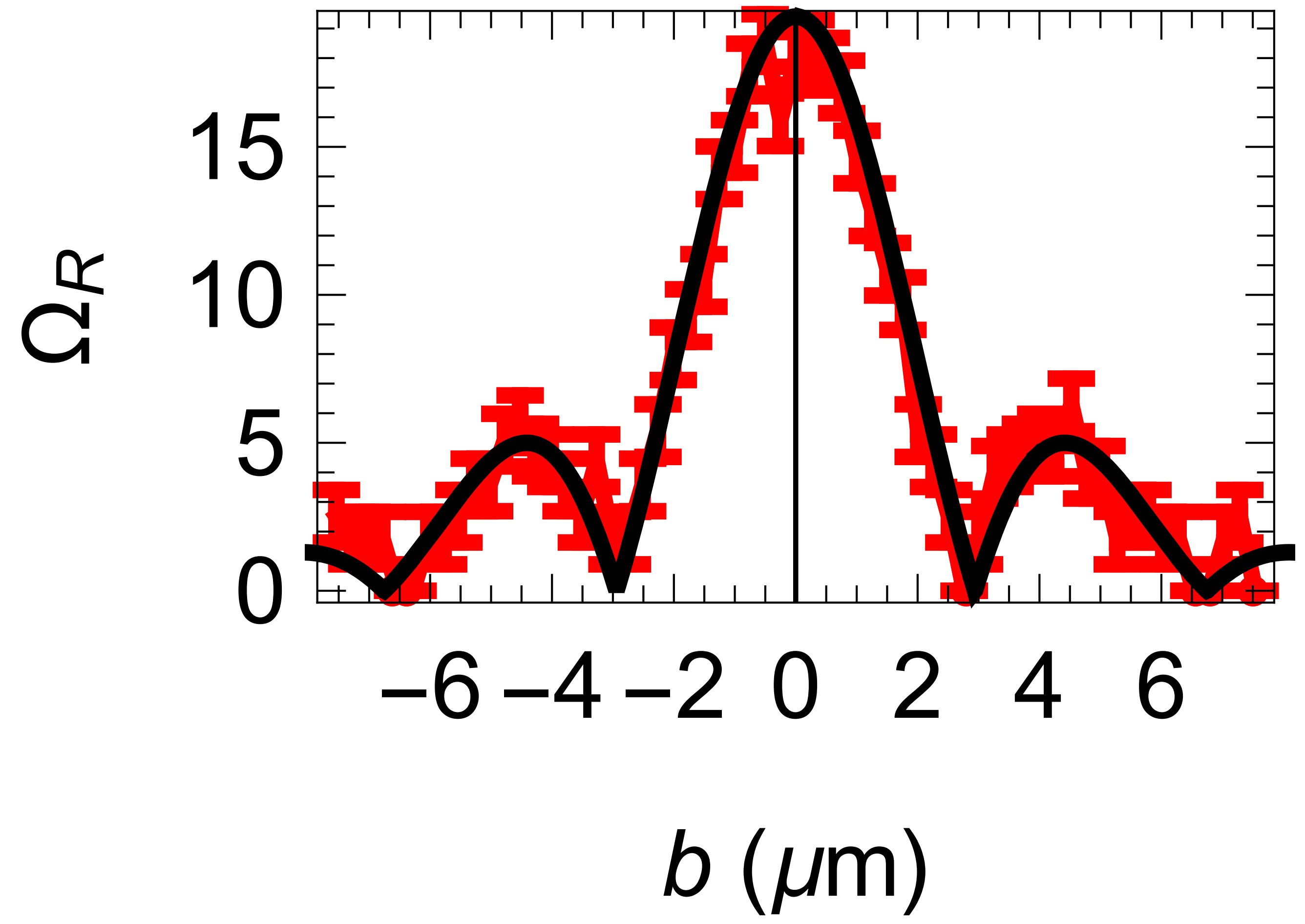
Fig. from Schmiegelow et al., Nat. Comm. (2016)

Atomic studies 5

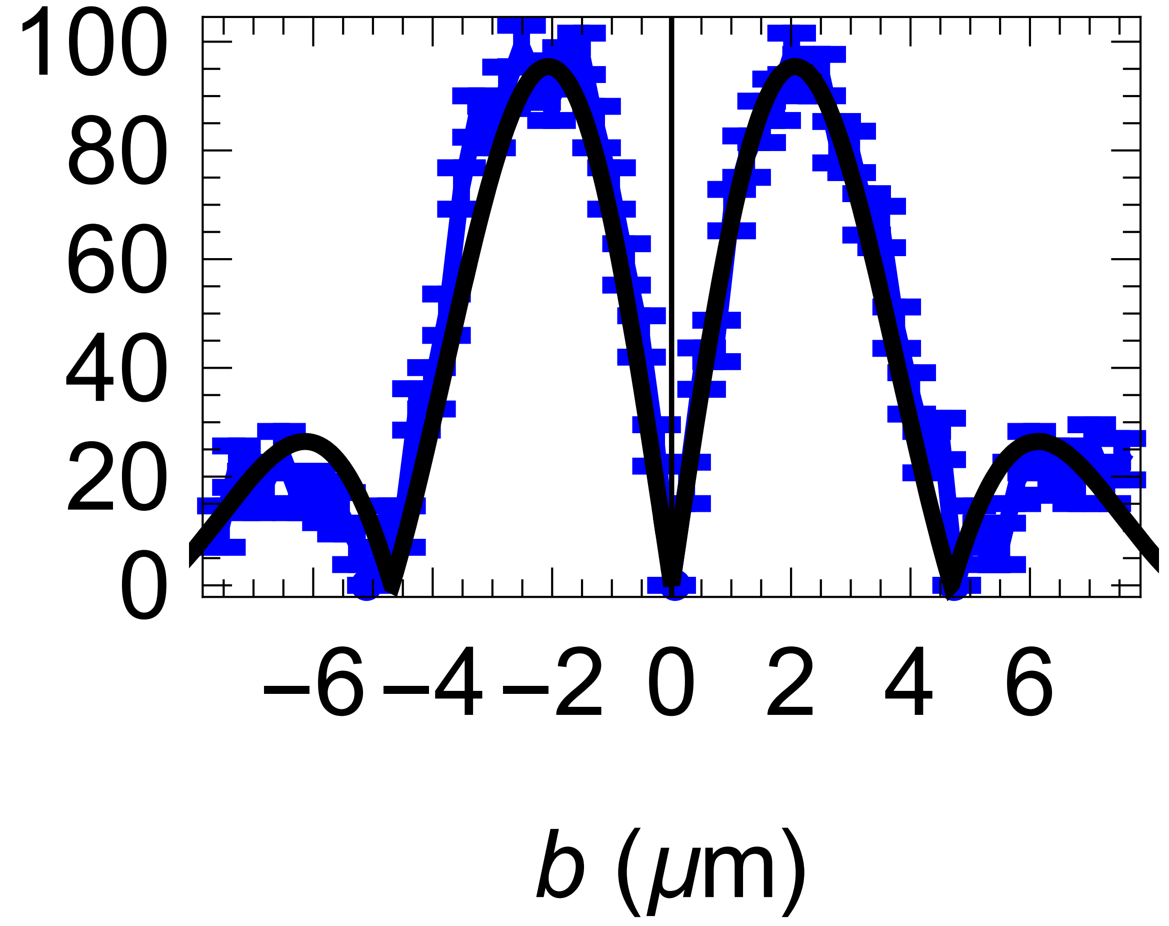
- In addition there is off-axis data for many m_γ, m_f, m_i

- Have $m_\gamma = \pm 2, \pm 1, 0$ (two versions), $m_f = 5$ values (one missing), $m_i = \pm 1/2$.
  60 plots, among them for $m_\gamma = -2,\ \Lambda = -1,\ m_i = -1/2$

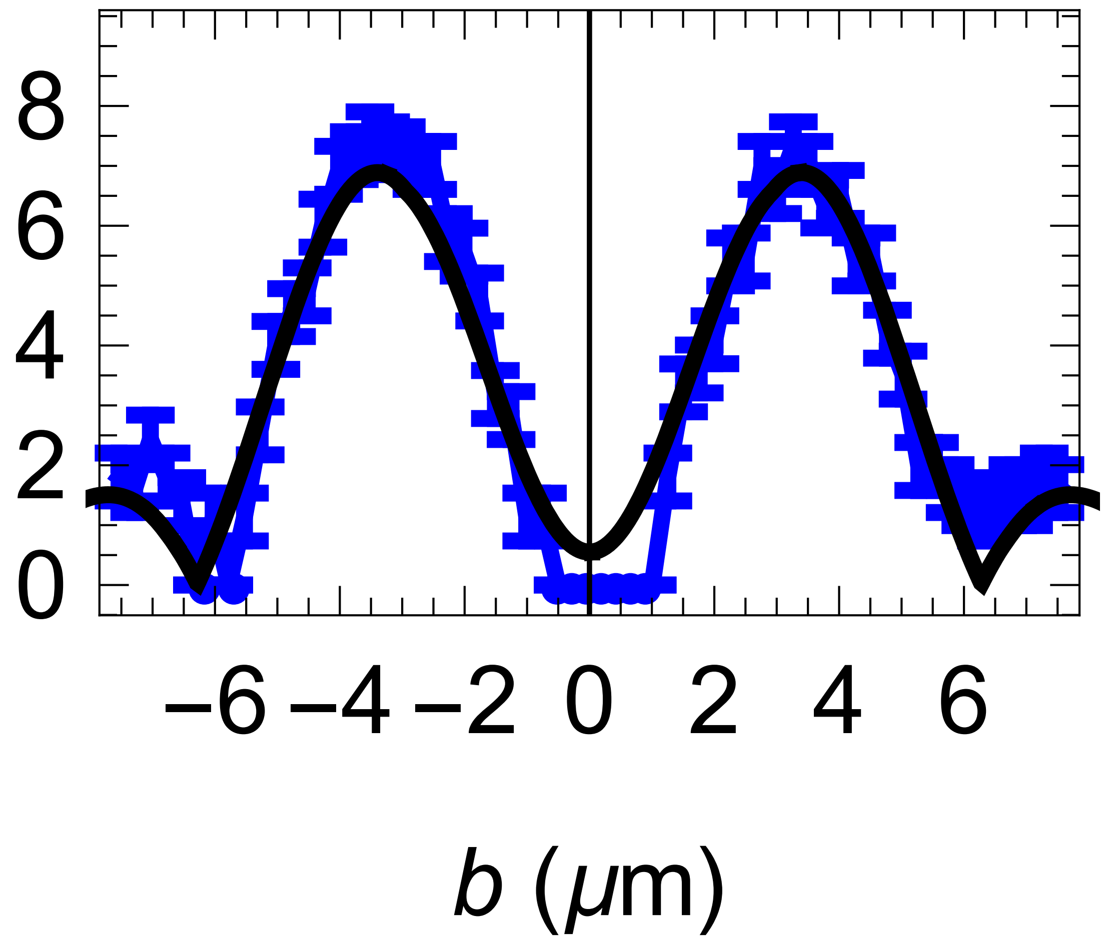
$m_f = -5/2$



$m_f = -3/2$



$m_f = -1/2$



$\theta_k \approx 0.1$

impact parameter

Plots from Afanasev et al., NJP 2018 and Solyanik-Gorgone et al, JOSA-B 2019

Hadronic physics

- Generic: selective excitation of high spin baryon states
- Specific possibility: in $\gamma + N \to \Delta$, isolate E2 transitions from dominant M1
- Notes: N is $J^P = (1/2)^+$; Δ is $J^P = (3/2)^+$
- Hence $N \to \Delta$ photoproduction requires M1 or E2 transitions
  - M1 means angular momentum change $L = 1$, parity $(-1)^{L+1} = +$
  - E2 means angular momentum change $L = 2$, parity $(-1)^{L} = +$

Hadronic physics 2

- M1 amplitude often order $(ka)^2 \sim (a/\lambda)^2$ compared to typical E1 scale
- Atomic: λ (wavelength) >> a (linear scale of target), and M1 small
- Nuclear and particle physics: λ and a scales comparable
- For $N \rightarrow \Delta$, both mostly orbital S-states; M1 only needs spin flip and dominates
- The E2 need two units of orbital angular momentum, involving the (small) D-state of the N or Δ, hence small. But important.

Hadronic physics 3

- How actually to calculate?
- Will start with atomic-like treatment (ignoring recoil), and then correct for recoil (in archived paper if not in talk).
- Will work to relate twisted photon amplitudes to plane wave helicity amplitudes, which in turn are related to M1 and E2 amplitudes

Hadronic physics 4

- Put target at origin with twisted photon with vortex line passing through $\overrightarrow{b} = b_\perp$ in the x-y plane,

$$|\gamma(\kappa m_\gamma k_z \Lambda \overrightarrow{b})\rangle = A_0 \int \frac{d\phi_k}{2\pi} (-i)^{m_\gamma} e^{i m_\gamma \phi_k - i \overrightarrow{k} \cdot \overrightarrow{b}} |\gamma(\overrightarrow{k}, \Lambda)\rangle$$

- Want amplitude

$$\mathscr{M} = \langle \Delta(m_f) | \mathscr{H}(0) | N(m_i); \gamma(\kappa m_\gamma k_z \Lambda \overrightarrow{b})\rangle$$

- $\mathscr{H}$ is interaction Hamiltonian; nucleon is at rest; m_i & m_f are spin projections in z-direction.

Hadronic physics 5

- Plane wave states obtained by rotating states w/momentum in z-direction,

$$|\gamma(\overrightarrow{k},\Lambda)\rangle = R(\phi_k,\theta_k,0)\,|\gamma(k\hat{z},\Lambda)\rangle = R_z(\phi_k)\,R_y(\theta_k)\,|\gamma(k\hat{z},\Lambda)\rangle$$

  using Wick (1962) phase convention.

- Rewrite the previous amplitude, since $\mathscr{H}$ is rotation invariant, in terms of photon states in z-direction and rotated N and Δ states.

- Since the nucleon is at rest, rotations are given in terms of Wigner functions

$$R^\dagger(\phi_k,\theta_k,0)\,|N(m_i)\rangle = e^{im_i\phi_k}\sum_{m_i'} d^{1/2}_{m_i,m_i'}(\theta_k)\,|N(m_i')\rangle$$

  (all spins projected along z-axis).

Hadronic physics 6

- Can do the same for Δ, if we neglect recoil

$$\langle \Delta(m_f) \,|\, R(\phi_k, \theta_k, 0) = \sum_{m_f'} \langle \Delta(m_f') \,|\, e^{-i m_f \phi_k} d^{3/2}_{m_f, m_f'}(\theta_k)$$

- Put together. Do ϕ_k integral to obtain Bessel function. Result is

$$\boxed{\mathcal{M} = A_0 (-i)^{m_f - m_i} e^{i(m_\gamma + m_i - m_f)\phi_b} J_{m_f - m_i - m_\gamma}(\kappa b) \sum_{m_i'} d^{3/2}_{m_f, m_i' + \Lambda}(\theta_k)\, d^{1/2}_{m_i, m_i'}(\theta_k)\, \mathcal{M}^{(\mathrm{pw})}_{m_i', \Lambda}}$$

- The plane wave amplitude comes from

$$\langle \Delta(m_f') \,|\, \mathcal{H}(0) \,|\, N(m_i');\, \gamma(k\hat{z}, \Lambda) \rangle = \mathcal{M}^{(\mathrm{pw})}_{m_i', \Lambda}\, \delta_{m_f',\, m_i' + \Lambda}$$

Hadronic physics 7

- There are two independent plane wave amplitudes (and others connected by parity):

$$\mathcal{M}^{(\text{pw})}_{1/2,1} = -\frac{3eE_\gamma}{2}\sqrt{\frac{2}{3}}\left(G^*_M + G^*_E\right)$$

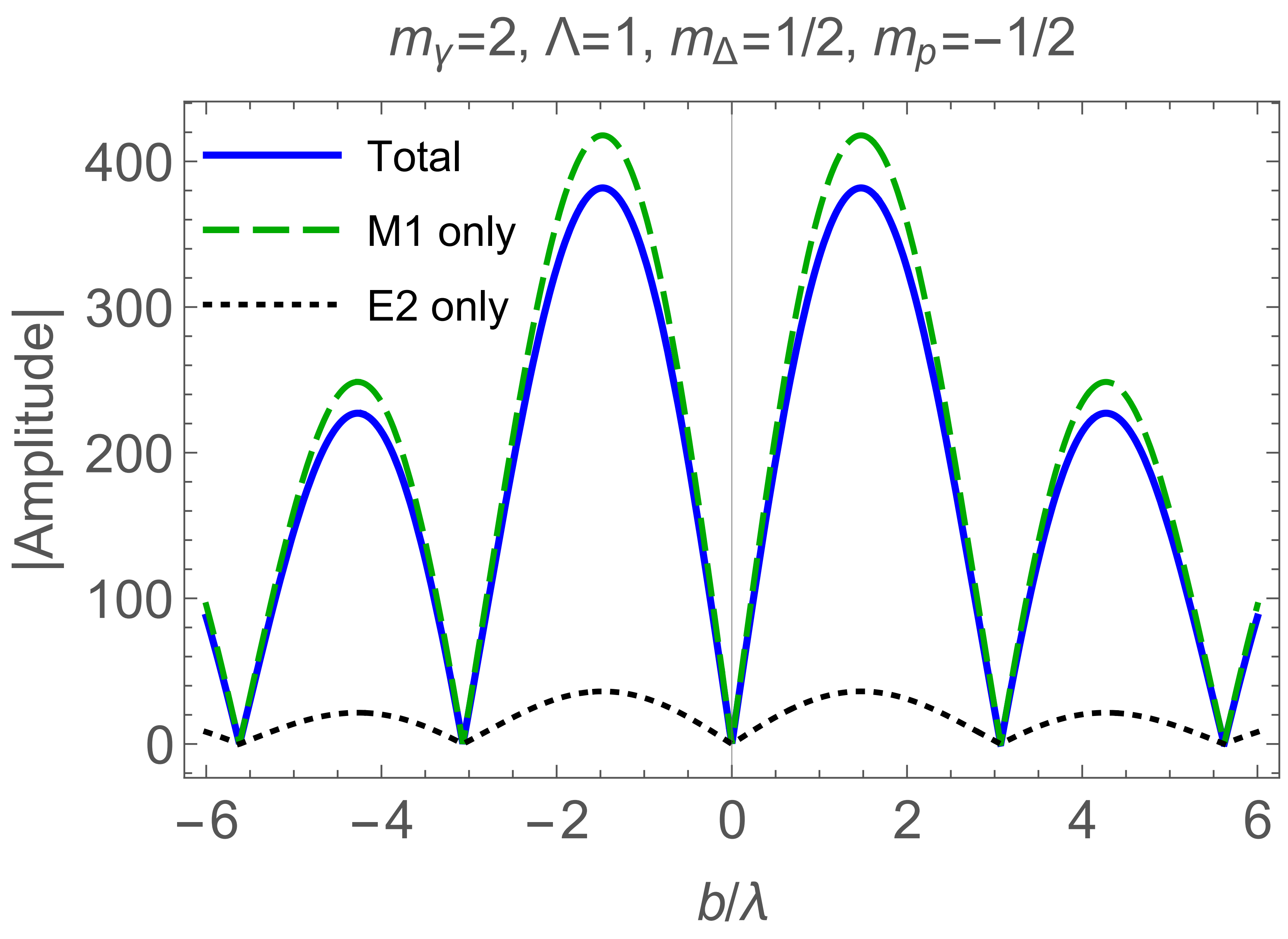
$$\mathcal{M}^{(\text{pw})}_{-1/2,1} = -\frac{\sqrt{3}\,eE_\gamma}{2}\sqrt{\frac{2}{3}}\left(G^*_M - 3G^*_E\right)$$

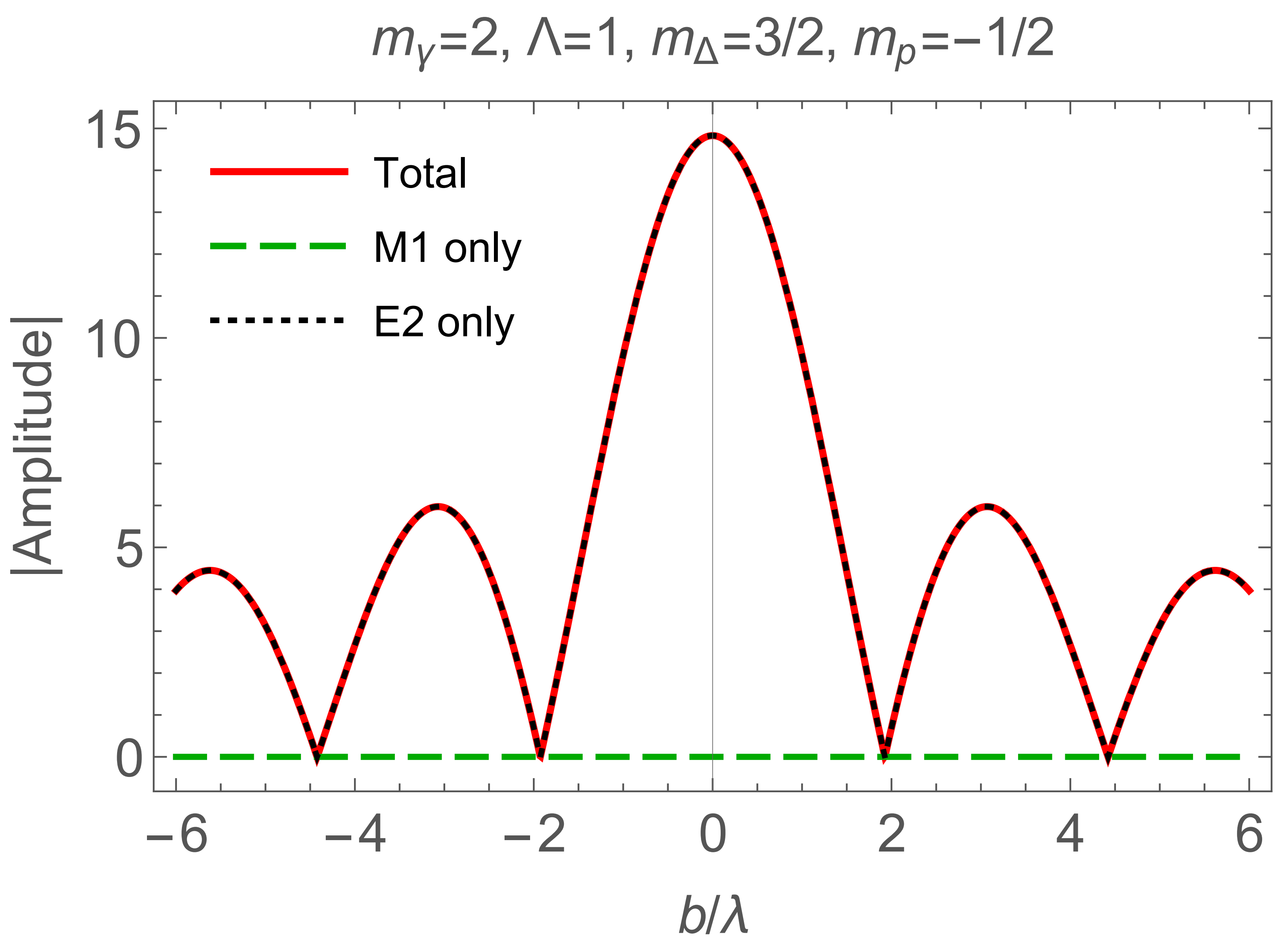
- The M1 and E2 amplitudes represented by Jones-Scadron form factors G^*_M and G^*_E , resp., and

$$E_\gamma = (m^2_\Delta - m^2_N)/(2m_N) = 2\pi/\lambda$$

Hadronic physics 8

- Can calculate. With $m_\gamma = \pm 2, \pm 1, 0$ (two versions), $m_f = 4$ values, and $m_i = \pm 1/2$, could make 48 plots. Show 2. Done for $G_E^*/G_M^* = 3\,\%$.

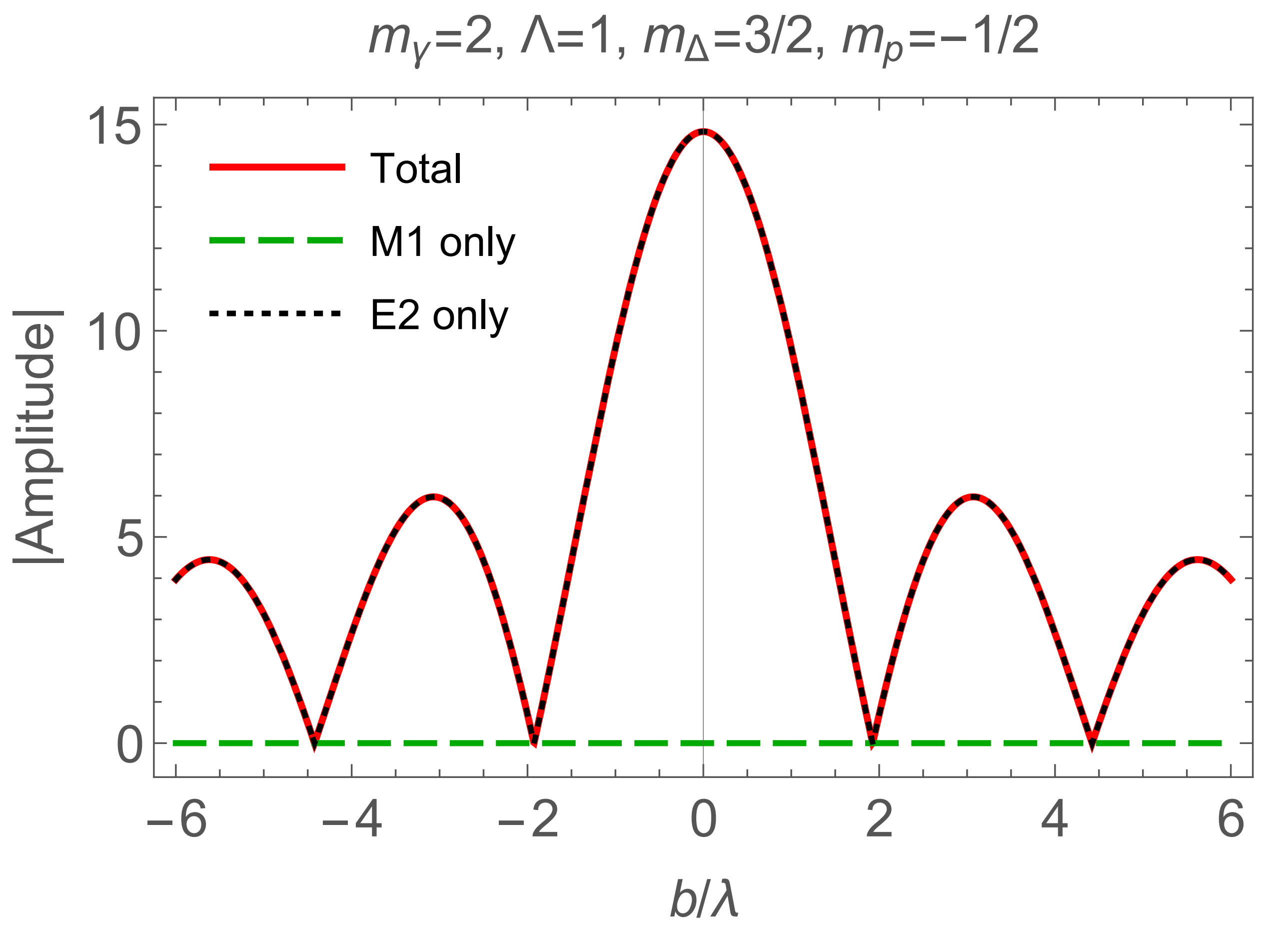




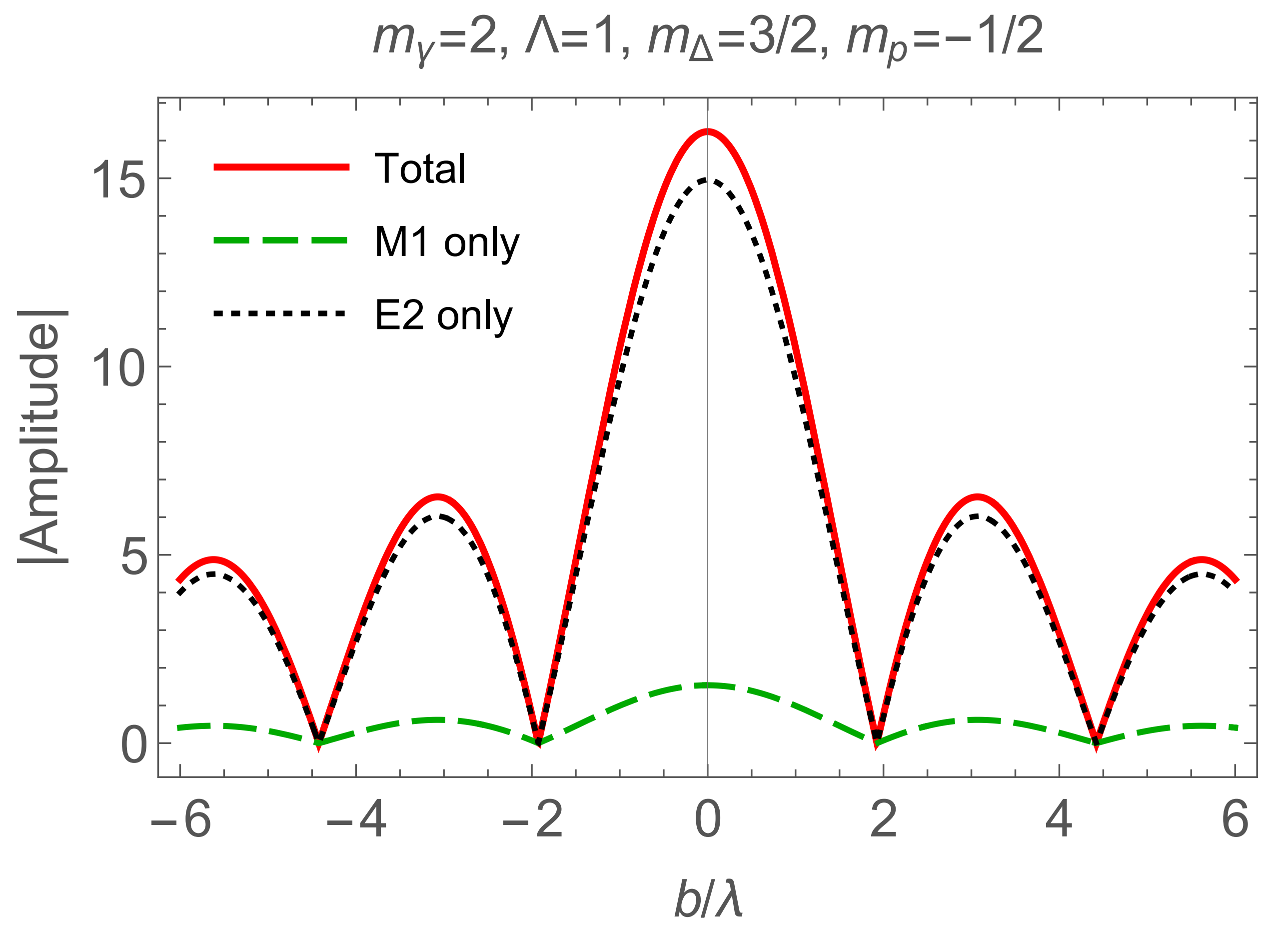
- No M1 contribution for $\Delta m = 2$ case (right hand plot)

Hadronic physics 9

- Comments (only) about recoil. Corrections worked out in archived work. Size depends on small components of Δ wave function squared, nominally $(E_\gamma/2M_\Delta)^2 \approx 1.9\,\%$. Not so serious.



becomes



Last slide

- One attitude: stick to plane wave photons (momentum eigenstate photons) so that $m_\gamma = \pm 1$ only. Very standard.
- Explore what can be done with an extra degree of freedom.
    - Communications.
    - Atomic physics. Can reverse an use final QN and off-axis behavior as diagnostic of structured photon state.
    - Hadronic physics
        - Higher spin states
        - Isolate small amplitudes, as E2 in Δ photoproduction.
- Beam requirements not trivial and not currently possible. But there is a future.

Extra

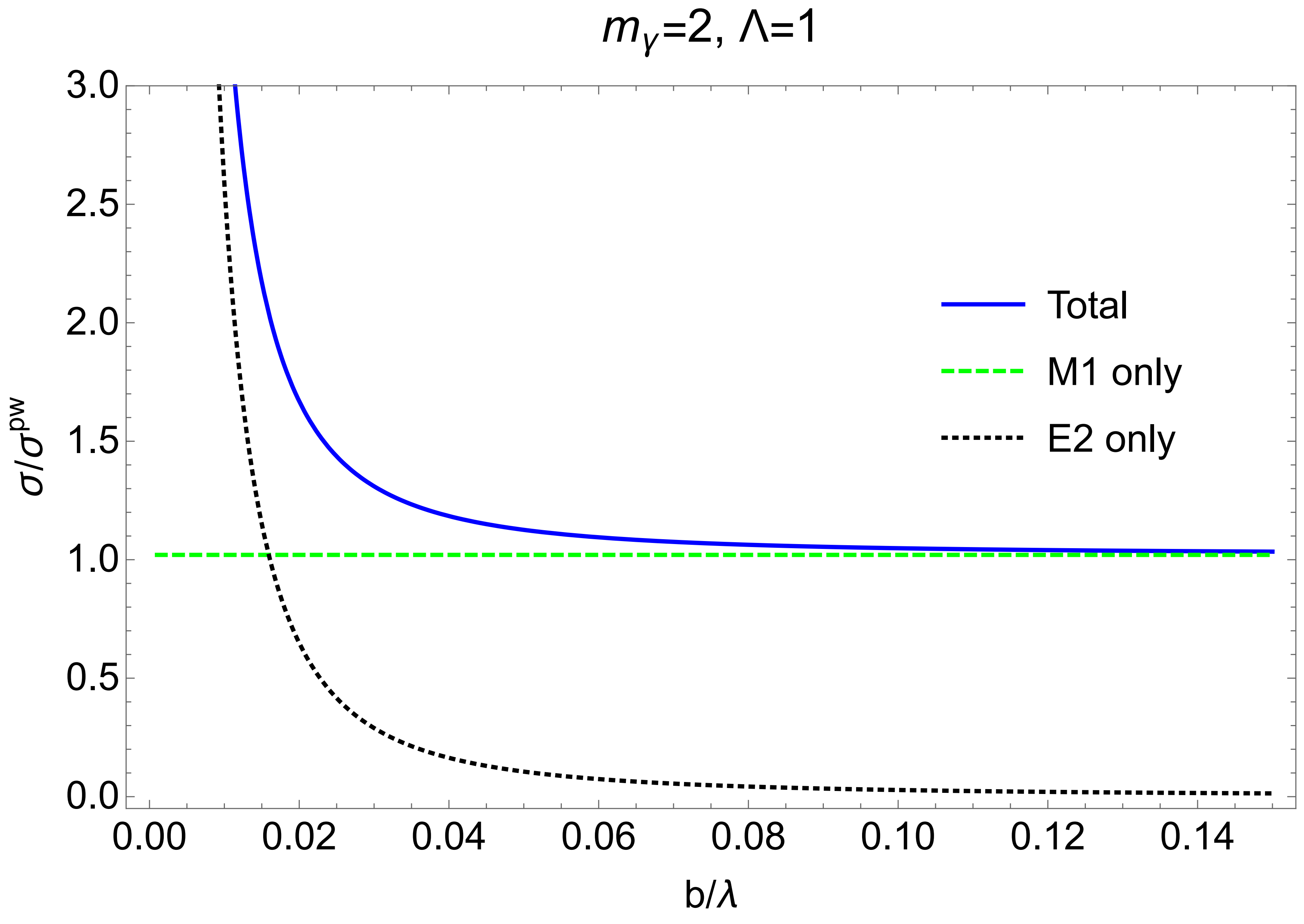
Target localization

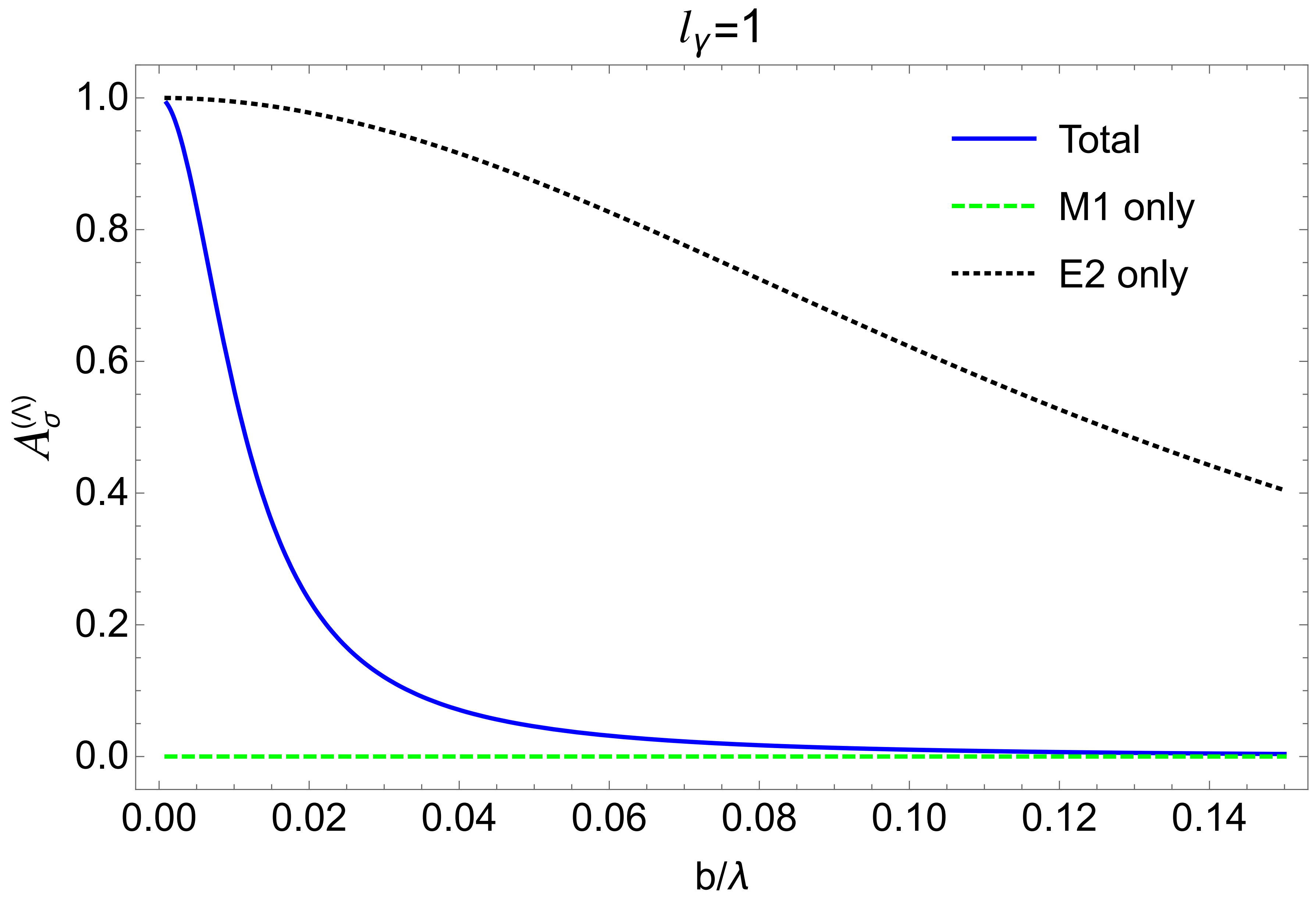
- Want target at rest and stationary. But we know quantum mechanics. Best we can do is

$$\Delta x\,\Delta p \geq \frac{1}{2}\hbar \approx \frac{1}{2}200\ \text{MeV fm}$$

- A minimal possibility: $\Delta x \approx 3$ fm and $\Delta p \approx 30$ MeV. Could be o.k.
- For Δ kinetic energy, $\Delta E_\Delta \approx 2E_\gamma \Delta p/(2M_\Delta) \approx 7$ MeV , small compared to Δ width.
- Amplitude plots have minima a few or several λ, and $\lambda = 3.65$ fm. So 3 fm for Δx acceptable, and can adjust. See also Zheludev et al on super-resolution ideas

Unpolarized results





$$A_\sigma^{(\Lambda)} = \frac{\sigma_{\Lambda=1} - \sigma_{\Lambda=-1}}{\sigma_{\Lambda=1} + \sigma_{\Lambda=-1}}$$

Twisted vector potential

- Components for $(\kappa, m_\gamma, k_z, \Lambda)$ and $b = 0$, Coulomb gauge.
- $A_\rho = i\frac{A_0}{\sqrt{2}}\, e^{i(k_z z - \omega t + m_\gamma \phi)} \left[\cos^2\frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa\rho) + \sin^2\frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa\rho)\right]$
- $A_\phi = -\Lambda\,\frac{A_0}{\sqrt{2}}\, e^{i(k_z z - \omega t + m_\gamma \phi)} \left[\cos^2\frac{\theta_k}{2} J_{m_\gamma - \Lambda}(\kappa\rho) - \sin^2\frac{\theta_k}{2} J_{m_\gamma + \Lambda}(\kappa\rho)\right]$
- $A_z = \Lambda\,\frac{A_0}{\sqrt{2}}\, e^{i(k_z z - \omega t + m_\gamma \phi)} \sin\theta_k\, J_{m_\gamma}(\kappa\rho)$