Holographic model for nuclear matter.

Wayne de Paula

Instituto Tecnológico de Aeronáutica

PLB135339 (2020) 803

Colaborators: Chueng-Ji (North Caroline), JPBC de Melo (UNICSUL), T Frederico (ITA), O Lourenço (ITA).

wayne@ita.br

Outline

I. Holography - AdS/CFT.

II. 10d Type IIB Supergravity.

III. AdS/QCD models – Dynamical Soft Wall.

IV. Nuclear Matter and Holography.

V. Results: Energy density, Pressure and Incompressibility.

VI. Conclusions.

Holography - AdS/CFT

10 dimensions
Gravity Theory

4 dimensions
Quantum Field Theory

Type IIB String Theory on $AdS_5 \times S_5$ ⟺ *N*=4 Super Yang-Mills

Maldacena (1998)

Low-energy limit of String Theory is **Supergravity**.

For low-curvature regions, String action ~ classical action.

Weak coupling ⟺ Strong coupling

⇒ If one can extend to QCD, we would have an analytical tool to study the non-perturbative regime of strong interaction.

- Break SUSY
- Break Conf. Inv.
- Confinement

Holography - AdS/CFT

AdS_5 x S_5 $$ds^2 = \frac{R^2}{z^2}\left(\eta_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} - dz^2\right) + R^2 d\Omega_{S_5}{}^2$$

Holographic coordinate

Field/Operator correspondence Witten (1998)

Generating function of correlation function

$$\left\langle e^{i\int d^4x \varphi_0(\vec{x})\mathcal{O}(\vec{x})} \right\rangle_{CFT} = \mathcal{Z}_{Sugra}\left[\varphi_0(\vec{x})\right] = e^{-I_S(\varphi(\vec{x},z))|_{\varphi(\vec{x},0)=\varphi_0(\vec{x})}}$$

Partition function

field theory operators <=> classical fields

Mass of the field

$$\mathcal{O}(x) \Longleftrightarrow \varphi(\vec{x}, z)$$

Operator conformal dimension.

For Scalars

$$\varphi(z) \to z^{\Delta}$$

small z

$$M_5^2 = \Delta(\Delta - 4)$$

Twist dimension

For Spin S

$$\phi(z) \to z^{\tau}$$

small z

$$M_5^2 = (\Delta - S)(\Delta + S - 4) \qquad \tau = \Delta - S$$

10 dimensions
Gravity Theory

$AdS_5 \times S_5$

4 dimensions
Quantum Field Theory

N=4 SYM Conformal

Klebanov-Strassler
D3-branes on a Conifold

attempts to

N=1 SYM Non-conformal
Confinement

Maldacena-Nuñez

D5-branes wrapped
on a two sphere

Papadopoulos-Tseytlin ansatz

10d Type IIB Supergravity

Einstein Equation

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{2\phi}\partial_M\chi\partial_N\chi + \frac{1}{96}G_{MPQKL}G_N{}^{PQKL} + \frac{1}{4}e^{\phi}\hat{F}_{MPQ}\hat{F}_N{}^{PQ}$$
$$+\frac{1}{4}e^{-\phi}H_{MPQ}H_N{}^{PQ} - \frac{1}{48}g_{MN}[e^{\phi}\hat{F}_{LPQ}\hat{F}^{LPQ} + e^{-\phi}H_{LPQ}H^{LPQ}],$$

Field Equations

$$\nabla^2\phi = e^{2\phi}\partial_M\chi\partial^M\chi + \frac{1}{12}e^{\phi}\hat{F}_{LMN}\hat{F}^{LMN} - \frac{1}{12}e^{-\phi}H_{LMN}H^{LMN}$$

$$\nabla^M(e^{2\phi}\partial_M\chi) = -\frac{1}{6}e^{\phi}H_{LMN}\hat{F}^{LMN}$$

$$\nabla^M(e^{\phi}\hat{F}_{MNP}) = \frac{1}{6}G_{NPQRS}H^{QRS}$$

$$\nabla^M(e^{-\phi}H_{MNP} - e^{\phi}\chi\hat{F}_{MNP}) = -\frac{1}{6}G_{NPQRS}\hat{F}^{QRS}$$

PT ansatz: Isometries

Lie Derivative

$$\mathcal{L}_\xi(g_{MN}) = 0.$$

→ Killing Vector → Isometries

$$\nabla_M \xi_N + \nabla_N \xi_M = 0.$$

Killing Equations

→ $SU(2) \times \widetilde{SU(2)}$ **algebra**

Which is not expected from a N=1 SYM

M Bianchi and WP, JHEP 1004 (2010) 113

Maldacena-Nunez solution: Vector Fluctuations

$$ds^2 = e^{\frac{\phi}{2}}\left[dx^2 + N_5\left\{du^2 + e^{2g}\left({e_1}^2 + {e_2}^2\right) + \frac{1}{4}\left({\tilde{\omega}_1}^2 + {\tilde{\omega}_2}^2 + {\tilde{\omega}_3}^2\right)\right\}\right]$$

Sturm-Liouville equation

$$-\mathcal{Y}'' + p^2\mathcal{Y} + \left(\phi'' + g'' + (\phi' + g')^2\right)\mathcal{Y} = 0.$$

Effective Potential

$$V_{eff} = \left(\phi'' + g'' + (\phi' + g')^2\right)$$

IR limit $(z \to \infty)$ → goes to a constant

⇓

No mass gap

M Bianchi and WP, JHEP 1004 (2010) 113

From 10d to 5d perspective.

10 dimensions | **5 dimensions**

Sturm-Liouville equation for MN do not depend on the internal space. → Effective model independ of the internal space.

$$-\mathcal{Y}'' + p^2\mathcal{Y} + \left(\phi'' + g'' + (\phi' + g')^2\right)\mathcal{Y} = 0.$$

AdS/QCD Models

Hard Wall Model

- **QCD Scale** introduced by a **boundary condition**
- Metric is a **Slice of AdS**
- The metric **has Confinement** by the Wilson loops area law.
- **Does not** have **Regge Trajectories** ($m^2 \sim n^2$)

Polchinski, Strassler (2002)

Scalar glueball spectrum
Boschi, Braga (2003)

Soft Wall Model

- **QCD Scale** introduced by a **dilaton** field
- The background (**AdS + Dilaton**) **is not** a solution of Einstein Eq.
- The metric **does not** has **Confinement** by the Wilson loops area law .
- **Has Regge Trajectories** ($m^2 \sim n$)

Karch, Katz, Son, Stephanov (2006)

Dynamical Soft Wall

WP, T Frederico, H Forkel and M Beyer, PRD 79 (2009) 075019

Solve **Einstein's equations** coupled to a dilaton field.

The AdS metric is deformed in the IR limit.

- UV, $z \rightarrow 0$ scaling behavior
- IR, $z \rightarrow$ "large" (confinement)

Confining Metric

- AdS space with a IR deformation.

Regge Trajectories will

determine the IR deformation.

Background Field

- Scalar Field (dilaton)

5d Einstein Equations

Dilaton potential

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{|g|}\left(-R + \frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)\right)$$

Dilaton field

$$g_{MN} = e^{-2A(z)}\eta_{MN}$$

$$\begin{aligned} 6A'^2 - \frac{1}{2}\Phi'^2 + e^{-2A}V(\Phi) &= 0, \\ 3A'' - 3A'^2 - \frac{1}{2}\Phi'^2 - e^{-2A}V(\Phi) &= 0, \\ \Phi'' - 3A'\Phi' - e^{-2A}\frac{dV}{d\Phi} &= 0 \end{aligned}$$

Einstein's Equations

Dilaton Equation

Also discussed by Csaki and Reece (2007);
Gursoy, Kiristsis, Nitti (2008).

Hadronic Resonances

- Holographic Dual model:

- Hadrons in QCD (4D) correspond to the normalizable modes of 5D fields. These normalizable modes satisfy the linearized equation of motion in the background 5D-geometry.
- The eigenvalue corresponding to a normalizable

meson mode is its square mass.

$$\bar{q}\gamma_{\{\mu_1}\partial_{\mu_2}....\partial_{\mu_s\}}q$$

QCD Operator

For Spin S= 1, 2, 3, ...

$$I = \frac{1}{2}\int d^5x\,\sqrt{g}\,e^{-\Phi}\left\{\nabla_N\phi_{M_1...M_S}\nabla^N\phi^{M_1...M_S} + M^2(z)\phi_{M_1...M_S}\phi^{M_1...M_S} + \ldots\right\}$$

Meson states in the Dilaton-Gravity Background

- Sturm-Liouville type eigenvalue problem for mesons

$$\left[-\partial_z^2 + \mathcal{V}_S(z)\right]\psi_{n,S} = m_{n,S}^2\psi_{n,S}$$

- Sturm-Liouville Potential

$$\mathcal{V}_S(z) = S^2 A'^2 + S\left(A'\sqrt{3A'^2 + 3A''} - A'^2 - A''\right) + A'^2 + \frac{5}{4}A'' - \sqrt{3}\,\frac{A''' + 4A'A'' + 2A'^3}{4\sqrt{A'^2 + A''}} + M^2 e^{-2A}$$

- Deformed AdS metric $A(z) = \ln z + C(z)$

For example $C_\lambda(z) = z^\lambda$

Confinement and Regge Trajectories

IR limit

$$\mathcal{V}_S(z) \stackrel{z\to\infty}{\longrightarrow} \frac{\lambda^2}{4}(2S+\sqrt{3}-1)^2\; z^{2\lambda-2}$$

$\lambda > 1$ ➡ Mass Gap

It is in agreement with the area law condition
Gursoy, Kiritsis and Nitti (2008)

$\lambda = 2$ ➡ $m^2_{n,S} \sim 2(2S - 1 + \sqrt{3})\; n$

Regge Trajectories

WP, T Frederico, H Forkel and M Beyer, PRD 79 (2009) 075019

Nuclear Matter and Holography

$$S_\psi = \int d^4x dz \, \sqrt{g} \, e^{-\varphi(z)} \, \overline{\Psi}(x,z) \left\{ i\Gamma^M \left(D_M + g_v \, \omega_M \right) - m_5^* \right\} \Psi(x,z)$$

Where $\Psi(x,z)$ is the fermion field, m^*_5 is the effective 5D mass, $\boldsymbol{\sigma}$ is the nuclear scalar field, $\boldsymbol{\omega}$ is the vector field and g_v is the coupling constant.

We propose that the five dimensional mass is modified by the mean scalar field.

$$\frac{d}{d\sigma} m_5^* = -\frac{m_5^*}{\sigma_0}$$

where the scale $\boldsymbol{\sigma}_0$ represents a typical nuclear matter value, which we will be fitted to the properties of the equation of state of the symmetric nuclear matter.

$$\frac{d}{d\sigma} m_5^* = -\frac{m_5^*}{\sigma_0} \quad \Rightarrow \quad m_5^* = m_5 \, e^{-\frac{\sigma}{\sigma_0}}$$

In the limit of small density, one has $m_5^* = m_5 - g_\sigma \sigma + ...$

where $g_{\boldsymbol{\sigma}} = m_5 / \boldsymbol{\sigma}_0$ is interpreted as the quark-sigma coupling constant.

Nuclear Matter and Holography

$$S_\psi = \int d^4x dz \sqrt{g}\, e^{-\varphi(z)}\, \overline{\Psi}(x,z)\left\{ i\Gamma^M \left(D_M + g_v\,\omega_M\right) - m_5^* \right\}\Psi(x,z)$$

The metric is a deformed 5d AdS

$$g_{\mu\nu} = e^{-2A(z)}\eta_{\mu\nu}$$

where the warp factor is

$$A(z) = -\log\left(\frac{R}{z} + \frac{\lambda^2 z}{R}\right)$$

In the UV, has conformal invariance.
In the IR, has confinement and Regge Trajectories for nucleon.

We redefine the fermionic field as $\Psi(x,z) = e^{\varphi(z)/2}\psi(x,z)$

We decompose the nucleon dual field into the left and right components:

$$\psi(x,z) = \left[\frac{1+\gamma^5}{2}F_+(z) + \frac{1-\gamma^5}{2}F_-(z)\right]\psi_4(x)$$

where

$$\left(i\gamma^i\partial_i - M^*\right)\psi_4(x) = 0.$$ 4D Dirac Equation

Changing variables $$F_\pm(z) = e^{2A(z)}f_\pm(z)$$

We obtain a Sturm-Liouville equation

$$-f''_\pm(z) + V^*_\psi f_\pm(z) = (E^* + g_v\,\omega)^2 f_\pm(z),$$

where the medium modified effective potential is

$$V^*_\psi(z) = m_5 R\, e^{-\frac{\sigma}{\sigma_0}}\frac{(m_5 R\, e^{-\frac{\sigma}{\sigma_0}} \mp 1)}{z^2} + \lambda^2\frac{m_5}{R}\, e^{-\frac{\sigma}{\sigma_0}}(2m_5 R\, e^{-\frac{\sigma}{\sigma_0}} \pm 1) + \lambda^4\frac{m_5^2}{R^2}\, e^{-2\frac{\sigma}{\sigma_0}} z^2\,,$$

Nuclear Matter and Holography

The mass spectrum for the nucleon and its excited states is analytical for the given metric and its solution of the eigenvalue equation is:

$$(E_n^* + g_v\,\omega)^2 = 4\frac{\lambda^2}{R^2}\,m_5\,R\,\exp\left(-\sigma/\sigma_0\right)\left(n + m_5\,R\,\exp\left(-\sigma/\sigma_0\right) + \frac{1}{2}\right)\,,$$

where the slope of the Regge Trajectory diminishes with the increase of the background field.

The nucleon is the ground state and its energy is given by

$$E_0^* \equiv E_N^* = -g_v\,\omega + M_N^*(\sigma)$$

where the effective mass is given by

$$M_N^* = 2\frac{\lambda}{R}\sqrt{m_5\,R\,e^{-\frac{\sigma}{\sigma_0}}\left(m_5\,R\,e^{-\frac{\sigma}{\sigma_0}} + \frac{1}{2}\right)}$$

Medium-modified holographic-hadron dynamics (MHD)

We find that AdS/QCD provides a different scenario for the nuclear dynamics when compared to both QHD (Quantum Hadrodynamics) and QMC (Quark-Meson-Coupling) models.

The MHD provides the non-linear **σ**-dependent nucleon mass:

$$M_N^* = 2\frac{\lambda}{R}\sqrt{m_5\,R\,e^{-\frac{\sigma}{\sigma_0}}\left(m_5\,R\,e^{-\frac{\sigma}{\sigma_0}} + \frac{1}{2}\right)}$$

Scalar coupling is given by:

$$g_\sigma(\sigma) = -\frac{\partial}{\partial\sigma}M_N^* = \frac{\lambda m_5}{2\sigma_0}\,\frac{e^{-\frac{\sigma}{\sigma_0}} + 4m_5 R\,e^{-2\frac{\sigma}{\sigma_0}}}{\sqrt{m_5 R\,e^{-\frac{\sigma}{\sigma_0}}\left(m_5 R\,e^{-\frac{\sigma}{\sigma_0}} + \frac{1}{2}\right)}}$$

The back reaction to the nuclear medium of the nucleons is a depletion of g_σ

Medium-modified holographic-hadron dynamics (MHD)

The energy density and pressure are given by

$$\mathcal{E} = \frac{G_\omega^2}{2}\rho^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\gamma}{2\pi^2}\int_0^{k_F} dk\, k^2 (k^2 + M_N^{*2})^{1/2}$$

$$P = \frac{G_\omega^2}{2}\rho^2 - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\gamma}{6\pi^2}\int_0^{k_F} dk \frac{k^4}{(k^2 + M_N^{*2})^{1/2}}$$

where k_f is the Fermi momentum, γ is the degeneracy factor (γ=4 for symmetric nuclear matter).

We define the coupling constant $G_\omega^2 = g_\omega^2/m_\omega^2$

Medium-modified holographic-hadron dynamics (MHD)

The nuclear matter stability condition $\frac{\partial \mathcal{E}}{\partial \sigma} = 0$ gives the mean field equation

$$\sigma = \frac{g_\sigma(\sigma)}{m_\sigma^2}\rho_s,$$

where the scalar density $\boldsymbol{\rho}_s$ is given by

$$\rho_s = \frac{\gamma M_N^*}{2\pi^2}\int_0^{k_f} dk\, \frac{k^2}{\sqrt{k^2 + M_N^{*2}}}$$

Scalar coupling:

$$g_\sigma(\sigma) = -\frac{\partial}{\partial \sigma}M_N^* = \frac{\lambda m_5}{2\sigma_0}\frac{e^{-\frac{\sigma}{\sigma_0}} + 4m_5 R\, e^{-2\frac{\sigma}{\sigma_0}}}{\sqrt{m_5 R\, e^{-\frac{\sigma}{\sigma_0}}\left(m_5 R\, e^{-\frac{\sigma}{\sigma_0}} + \frac{1}{2}\right)}}$$

Medium-modified holographic-hadron dynamics (MHD)

We choose the following quantities and observables in order to obtain the free parameters to build the EOS of symmetric nuclear matter:

Nucleon rest mass – M_N = 939 MeV
Sigma meson mass - m_σ = 550 MeV
Saturation density - $\boldsymbol{\rho}_0$ = 0.15 fm^{-3}
Binding energy – B_0 = -15.6 MeV

For a given density, we solve

$$\sigma = \frac{g_\sigma(\sigma)}{m_\sigma^2}\rho_s, \qquad \rho_s = \frac{\gamma M_N^*}{2\pi^2}\int_0^{k_f} dk \, \frac{k^2}{\sqrt{k^2 + M_N^{*2}}}$$

From the solution $\boldsymbol{\sigma}$, we calculate the energy dentity and pressure

$$\mathcal{E} = \frac{G_\omega^2}{2}\rho^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\gamma}{2\pi^2}\int_0^{k_F} dk\, k^2 (k^2 + M_N^{*2})^{1/2}$$

$$P = \frac{G_\omega^2}{2}\rho^2 - \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\gamma}{6\pi^2}\int_0^{k_F} dk \, \frac{k^4}{(k^2 + M_N^{*2})^{1/2}}$$

Numerical Results

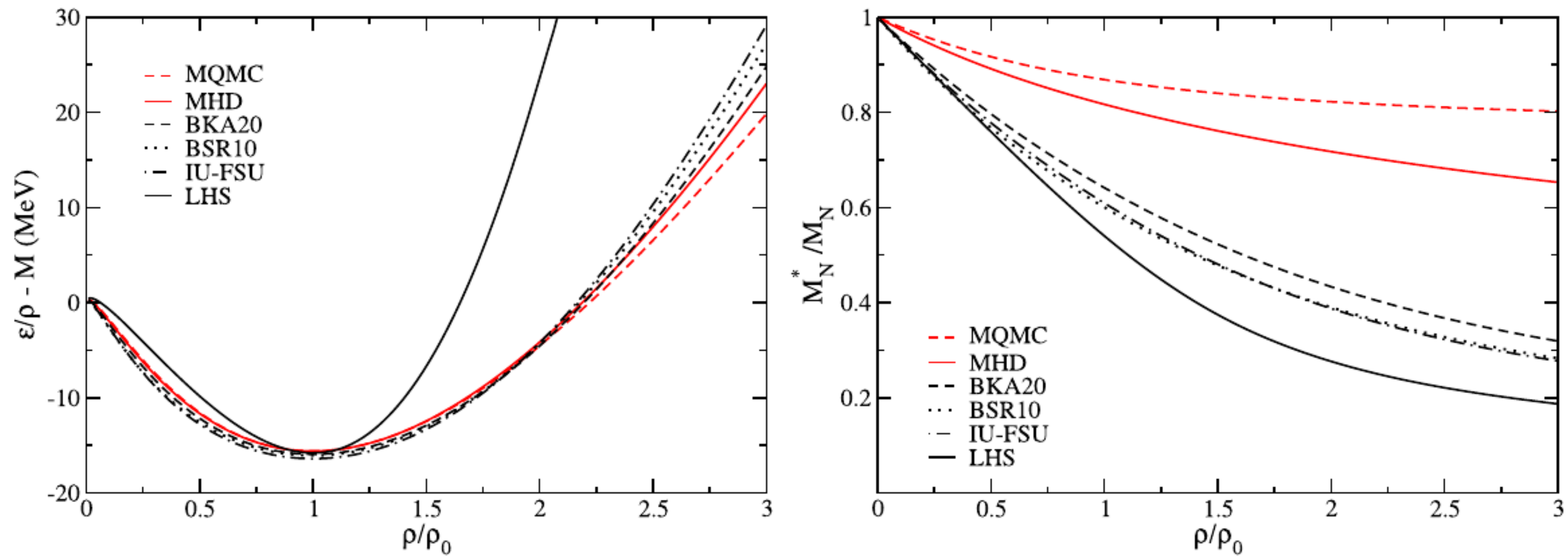

Fig. 1. Energy per nucleon (left panel) and M_N^*/M_N (right panel) as a function of ρ/ρ_0 (SNM) for the MHD model in comparison with some relativistic models, namely, BKA20 [35], BSR10 [36], and IU-FSU [37]. Also displayed a Walecka model, namely, LHS [31] and the MQMC one [5].

- LHS: parametrization of the Walecka model
- BKA20, BSR10, IU-FSU: parametrizations of the RMF model with mesons interaction.
- MQMC: relativistic hadronic model with quarks inside nucleons.
- MHD: our model. Very similar to the sophisticated hadronic models.

Numerical Results

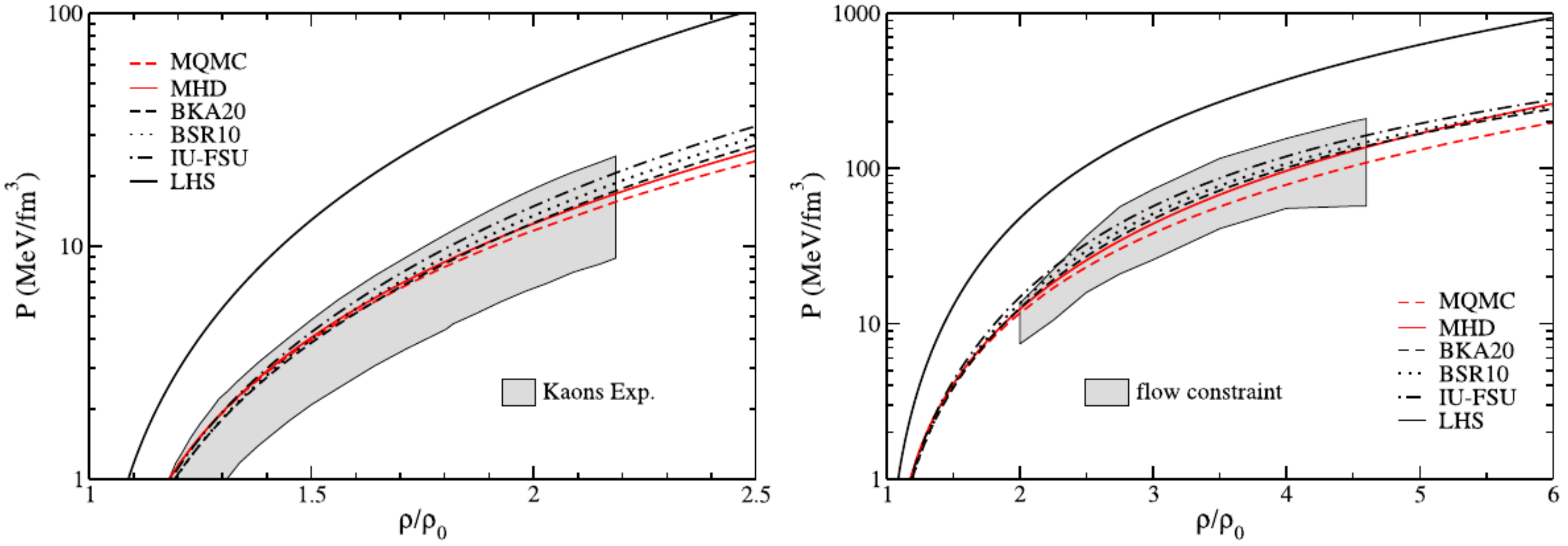

Fig. 2. $P \times \rho/\rho_0$ for SNM described by the MHD model in comparison with LHS, BKA20, BSR10, and IU-FSU parametrizations, along with the MQMC model. Bands: data extracted from Refs. [12] (left panel) and [13] (right panel).

In the left, the band is constructed from the experiment of Kaon condensation in heavy ion collisions.
In the right, the constrain is obtained from the experimental data of the motion of ejected matter in the energetic nucleus-nucleus collisions.

- Furthermore, we calculate the incompressibility at the saturation point:

- Walecka model: $K_0 \sim 500$ MeV.
- MHD model: $K_0 = 254$ MeV.
- Literature: 220 MeV $< K_0 <$ 260 MeV.

Stone, Stone, Moszkowski, PRC 89 (2014) 044316.

Summary

We discussed some aspects of AdS/CFT duality.

We presented AdS/QCD models, in particular the Dynamical Soft Wall, which has confinement and Regge Trajectories.

We proposed a Holographic model for Nuclear Matter.

For symmetric nuclear matter, the energy density and pressure are compatible to the one obtained by sophisticated RMF models and MQMC.

At the saturation point, we obtain an incompressibility of 254 MeV, which is consistent with the range found in literature.

We intend to generalize the model for asymmetric matter and apply for compact stars.