Constituent quark model for quarkyonic matter

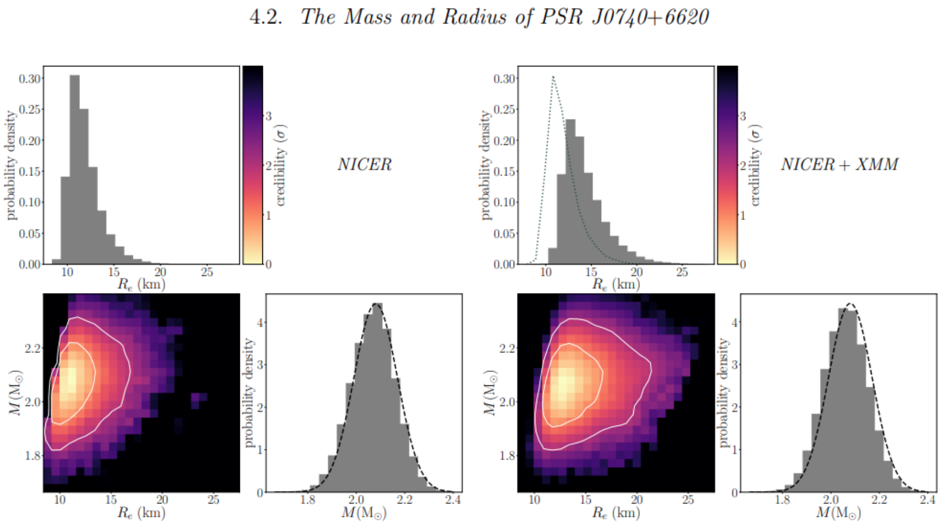
Aaron Park (Yonsei University)
collaborated with Kie Sang Jeong and Su Houng Lee

arXiv : 2105.12948

Contents

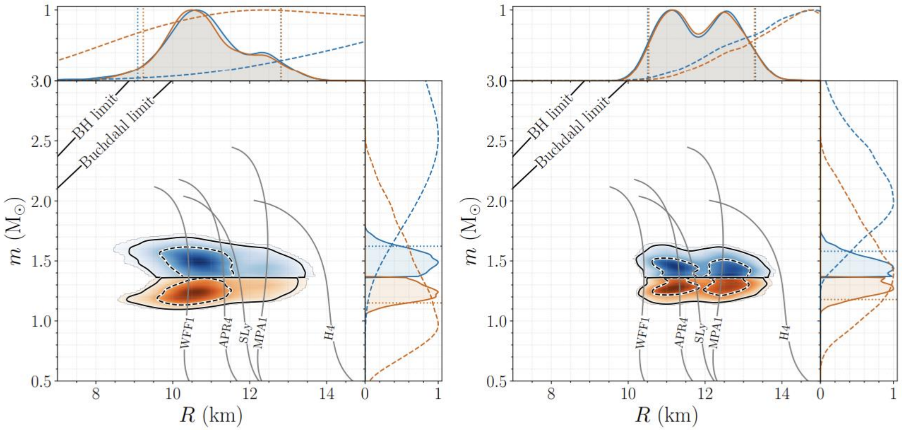
Introduction

Massive Neutron Star



M. C. Miller, et al. [arXiv:2105.06979 [astro-ph.HE]]

To support the massive neutron stars whose masses are larger than two times the solar mass, it was found that the equation of states for dense matter had to be sufficiently hard.

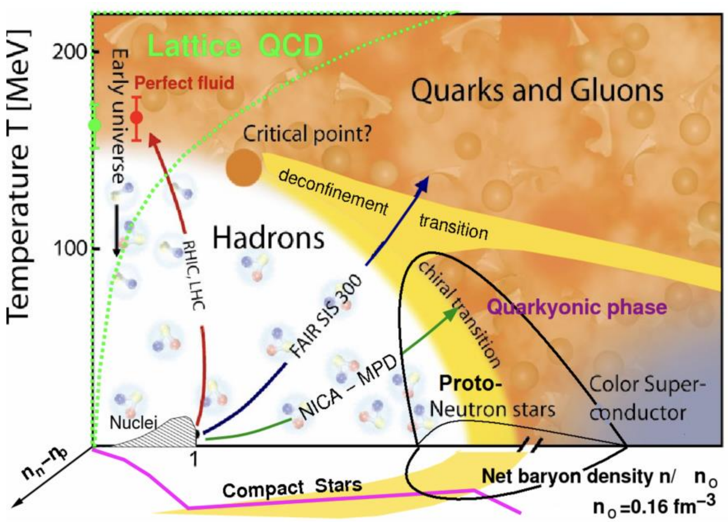


B. P. Abbott, et al. [LIGO Scientific and Virgo Collaborations], Phys. Rev. Lett. 121, no. 16, 161101 (2018)

The tidal deformability constrained via the GW170817 observation requires a relatively soft equation of states.

Quarkyonic Matter

The quarkyonic matter is a hypothetical new phase matter occurring at high baryon density and temperature $T < \Lambda_{QCD}$.



Larry MaLerran, “FAQS ABOUT QUARKYONIC MATTER”

If the quark Fermi momentum is large enough($k_F \gg \Lambda_{QCD}$), the quark distributed around Fermi surface will be confined into the baryon-like state.

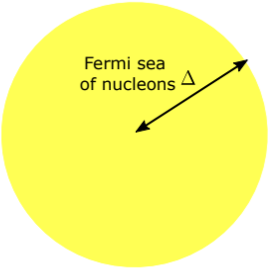
Then, the baryon momentum phase space distribution has a shell-like distribution due to the Pauli blocking effect from the quasi-free quarks occupying the lower distributions.

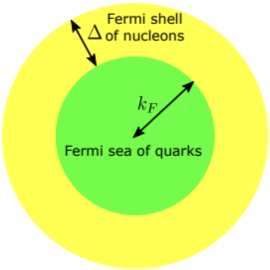
Quarkyonic Matter

At low baryon density quarkyonic matter resembles nuclear matter.

At high density the fermi distribution function of quarkyonic matter is different from purely hadronic or quark matter.

Within the quarkyonic matter framework, nucleon and quark degrees of freedom are described with a single fermi distribution function.



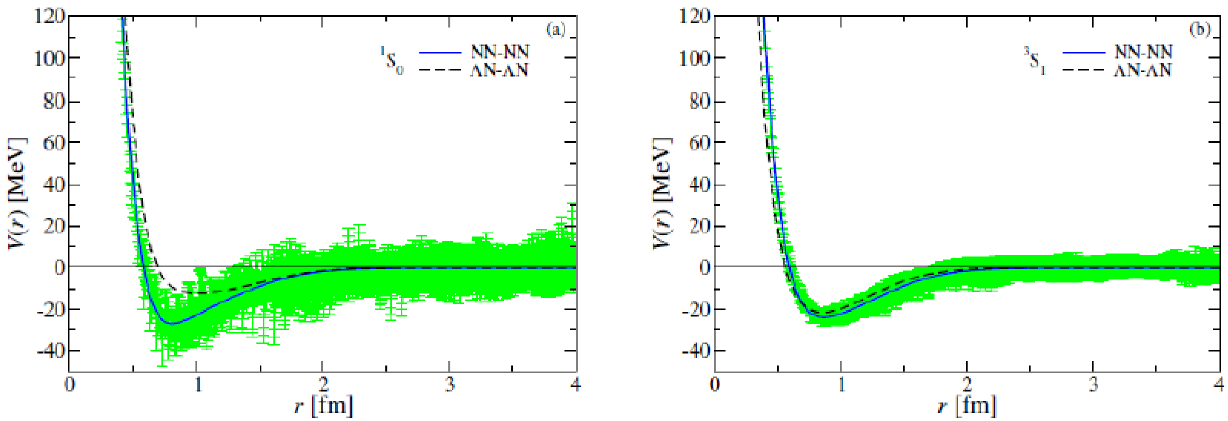


S. Sen and L. Sivertsen, arXiv:2011.04681

Excluded volume model

- To be more realistic

Lattice QCD study for hadron interaction (HALQCD T. Hatsuda et al.)



- Hard core nature can be embodied by semi-classical size v_0

$$V_{ex} = V\ (1 - n/n_0)$$

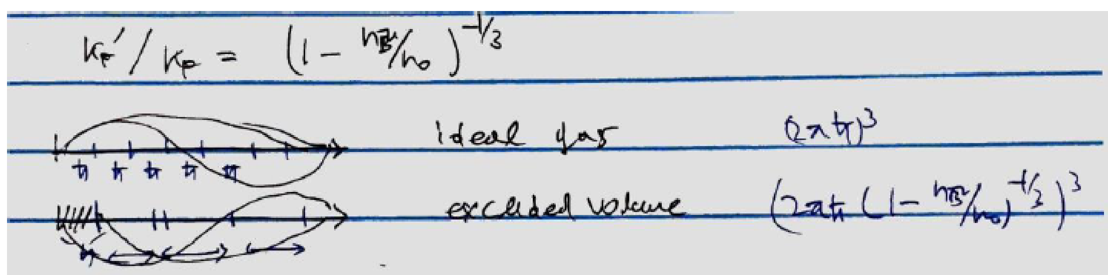
$$n_0 = 1/v_0$$

$$n_b^{ex} = \frac{n_b}{1 - n_b/n_0}$$

$$= \frac{2}{(2\pi)^3} \int_0^{K_F^b} d^3k$$

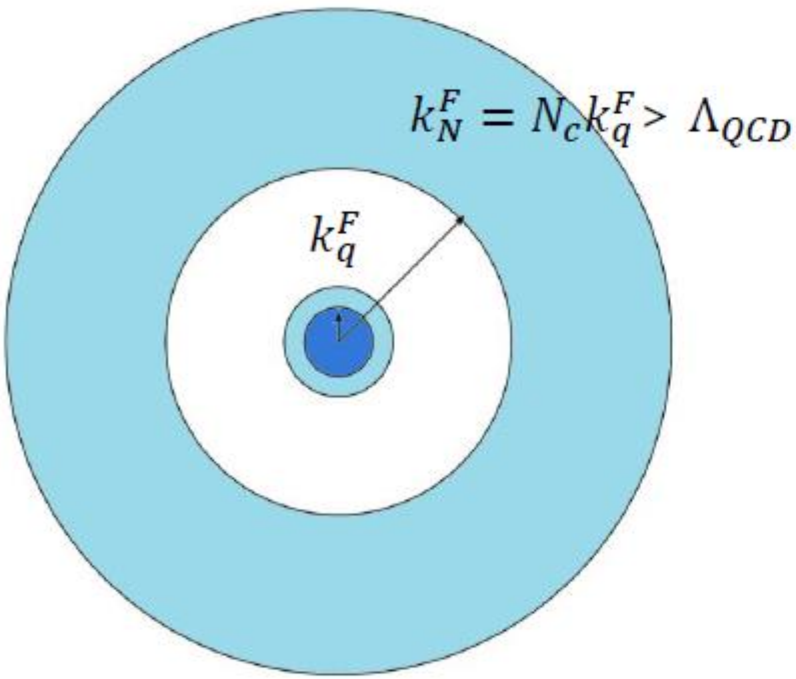
Hardcore effective size and excluded volume

→ reduced available space (fast nucleons)

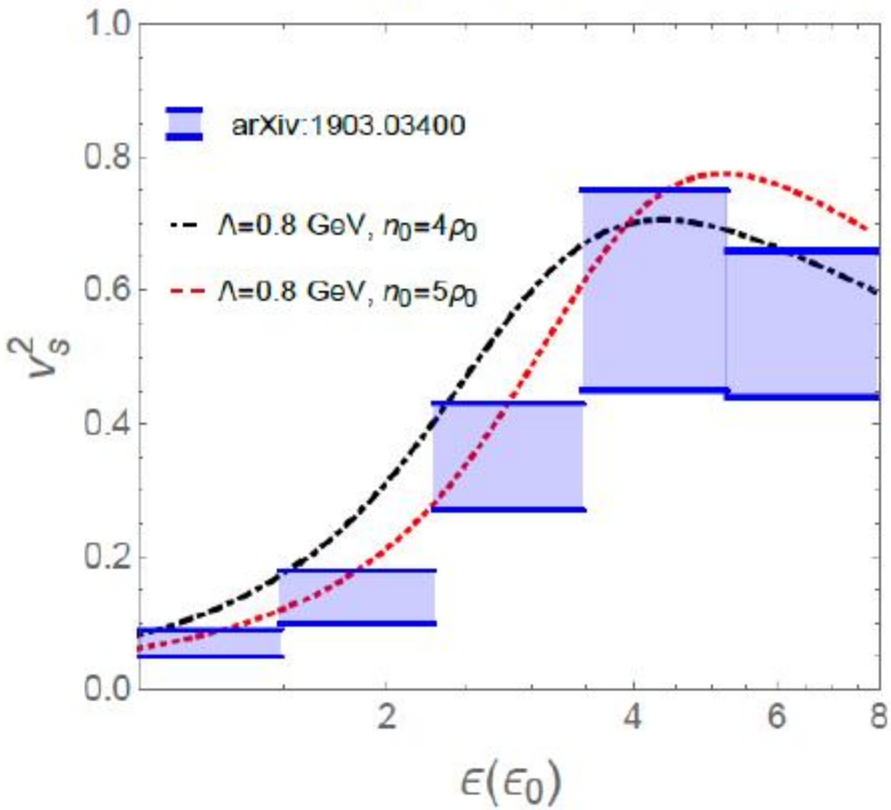


Quarkyonic-like baryon shell structure

$$\tilde{\epsilon} = 4\left(1 - \frac{n_N^N}{n_0}\right)\int_{k_F}^{k_F+\Delta}\frac{d^3k}{(2\pi)^3}\left((N_c m_Q)^2 + k^2\right)^{\frac{1}{2}} + \frac{2Nc}{\pi^2}\int_0^{k_F/N_c} dkk\left(\Lambda^2 + k^2\right)^{\frac{1}{2}}\left(m_Q^2 + k^2\right)^{\frac{1}{2}}$$



(PRC101 (2020) 035201 K.S.J., L.M., S.S.)



$k_Q < k_F/N_c$

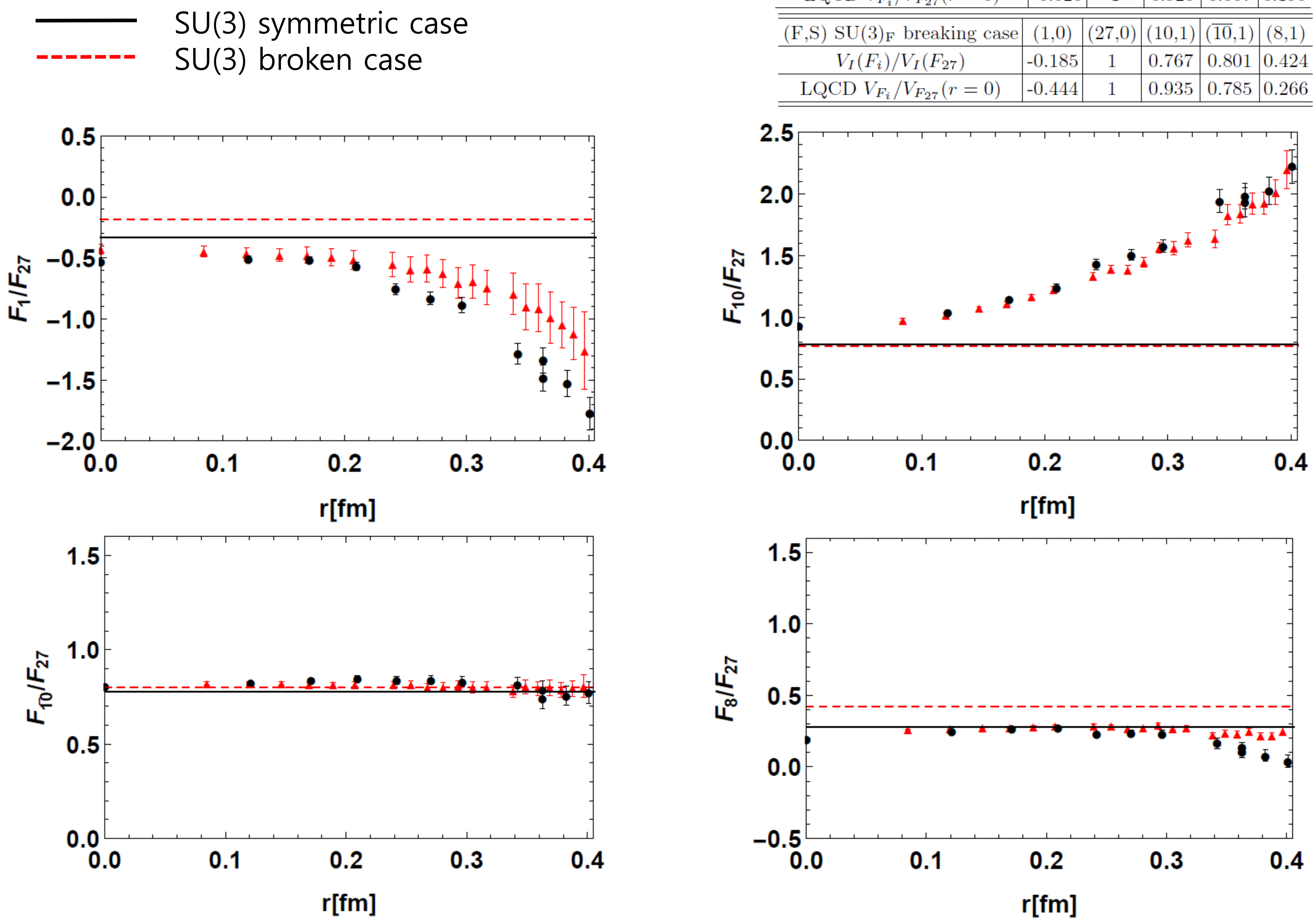
$k_F < k_N < k_F + \Delta$

Quark model approach

Comparison with Lattice QCD

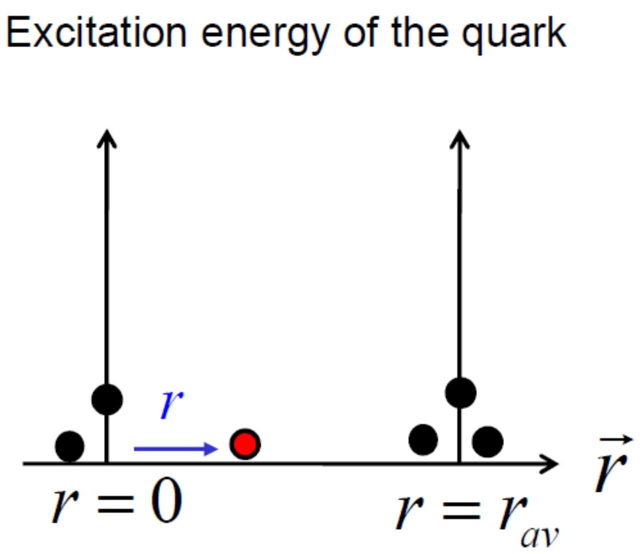
(F,S) $SU(3)_F$ symmetric case	(1,0)	(27,0)	(10,1)	($\overline{10}$,1)	(8,1)
$\Delta\mathcal{CS}(F_i)$	-8	24	$\frac{56}{3}$	$\frac{56}{3}$	$\frac{20}{3}$
$V_I(F_i)/V_I(F_{27})$	-0.333	1	0.778	0.778	0.278
LQCD $V_{F_i}/V_{F_{27}}(r=0)$	-0.528	1	0.928	0.807	0.195

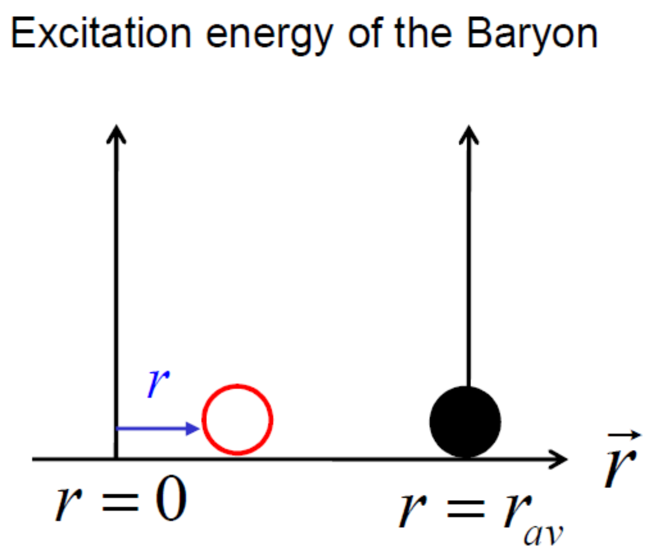
(F,S) $SU(3)_F$ breaking case	(1,0)	(27,0)	(10,1)	($\overline{10}$,1)	(8,1)
$V_I(F_i)/V_I(F_{27})$	-0.185	1	0.767	0.801	0.424
LQCD $V_{F_i}/V_{F_{27}}(r=0)$	-0.444	1	0.935	0.785	0.266



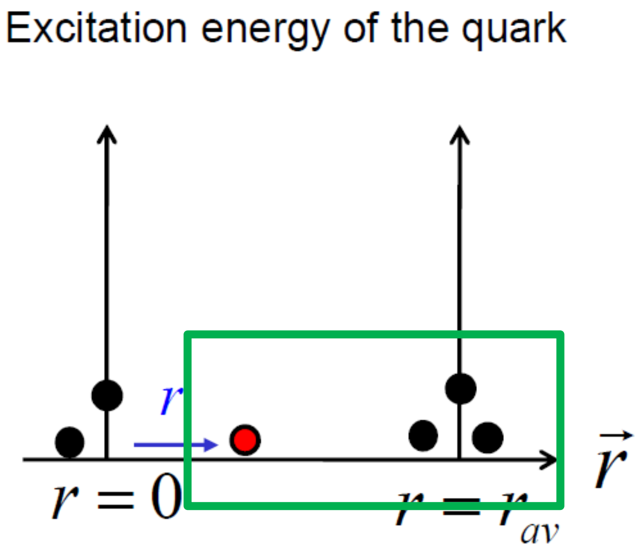
A. Park, S.H.Lee, T. Inoue, T. Hatsuda, Eur. Phys. J. A 56, 93(2020)

Quark model approach

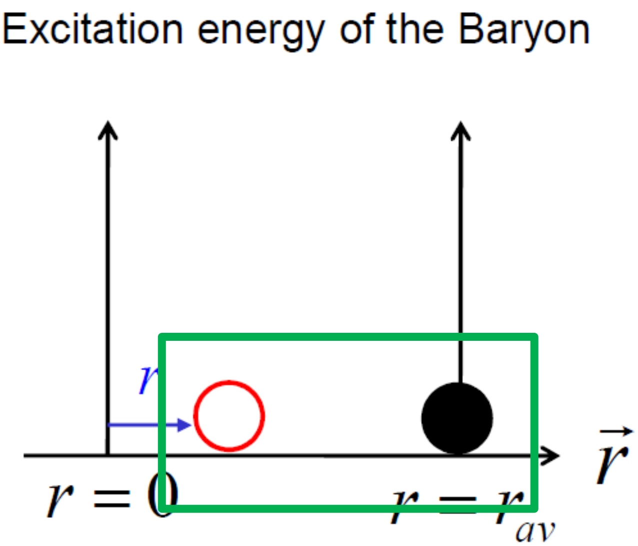




Quark model approach



4-quark configuration



dibaryon configuration

Hamiltonian

$$H = \sum_{i=1}^{N} (m_i + \frac{\mathbf{p}_i^2}{2m_i}) - \frac{3}{16} \sum_{i<j}^{N} (V_{ij}^C + V_{ij}^{CS}),$$

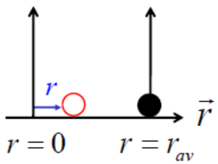
$$V_{ij}^C = -\lambda_i^c \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D \right),$$

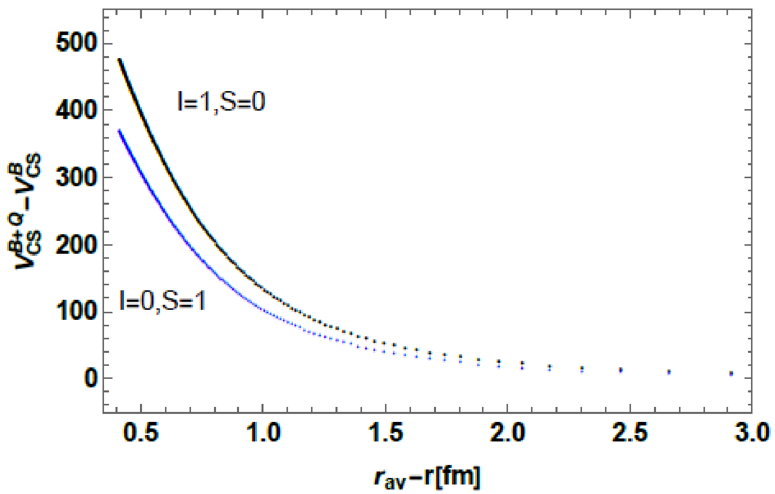
Confinement potential

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \, \sigma_i \cdot \sigma_j$$

Hyperfine potential

Baryon excitation





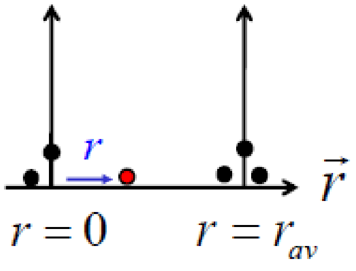
$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \; \sigma_i \cdot \sigma_j$$

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r - r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

m_B : nucleon mass
a_B = 0.218 GeV·fm
b_B = 1.474 fm

Quark excitation

Quark excitation

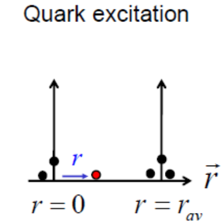


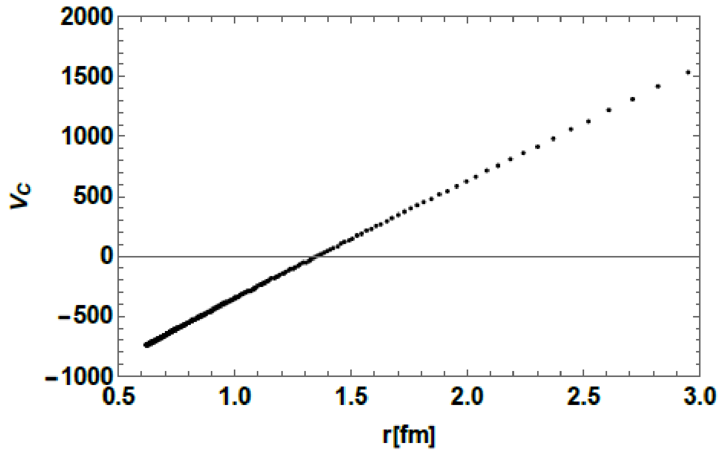
$$\mathcal{X} \equiv -\sum_{i<j}^{N} \langle \lambda_i^c \lambda_j^c \, \sigma_i \cdot \sigma_j \rangle$$
$$= N(N-10) + \frac{4}{3} S(S+1) + 4C_{\rm F} + 2C_{\rm C}.$$

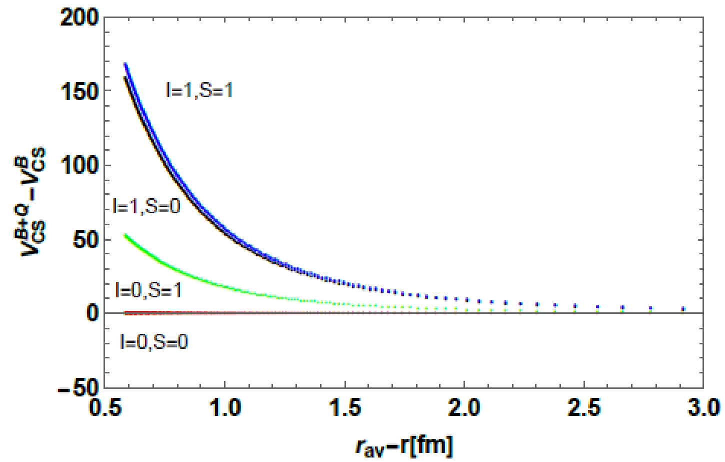
$\chi = -8$ for both nucleon and ud diquark.

→ The excitation of a d quark will not cost any color-spin energy as the attractive (ud) diquark remains intact.

Quark excitation







$$V_{ij}^{C} = -\lambda_i^c \lambda_j^c \left(-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0} - D \right),$$

$$V_{ij}^{CS} = \frac{\kappa'}{m_i m_j r_{0ij}^2} \frac{1}{r_{ij}} e^{-(r_{ij}/r_{0ij})^2} \lambda_i^c \lambda_j^c \, \sigma_i \cdot \sigma_j$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma \left(rH(r_{\max} - r) + r_{\max} H(r - r_{\max}) \right) + R_n \left(\frac{a_Q}{|r - r_{av}|} e^{-\frac{|r - r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}} \right)$$

m_D : diquark mass
m_Q = 0.343 GeV
σ = 0.962 GeV/fm
a_Q = 0.2 GeV·fm
b_Q = 0.745 fm

$H(r)$: Heaviside step function
R_n = 0.7

Results

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r - r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma \left(r H(r_{\max} - r) + r_{\max} H(r - r_{\max}) \right) + R_n \left(\frac{a_Q}{|r - r_{av}|} e^{-\frac{|r-r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}} \right)$$

$$\Delta E = E_d(k_F^d) + m_D - E_n(k_F^n = k_F^d).$$

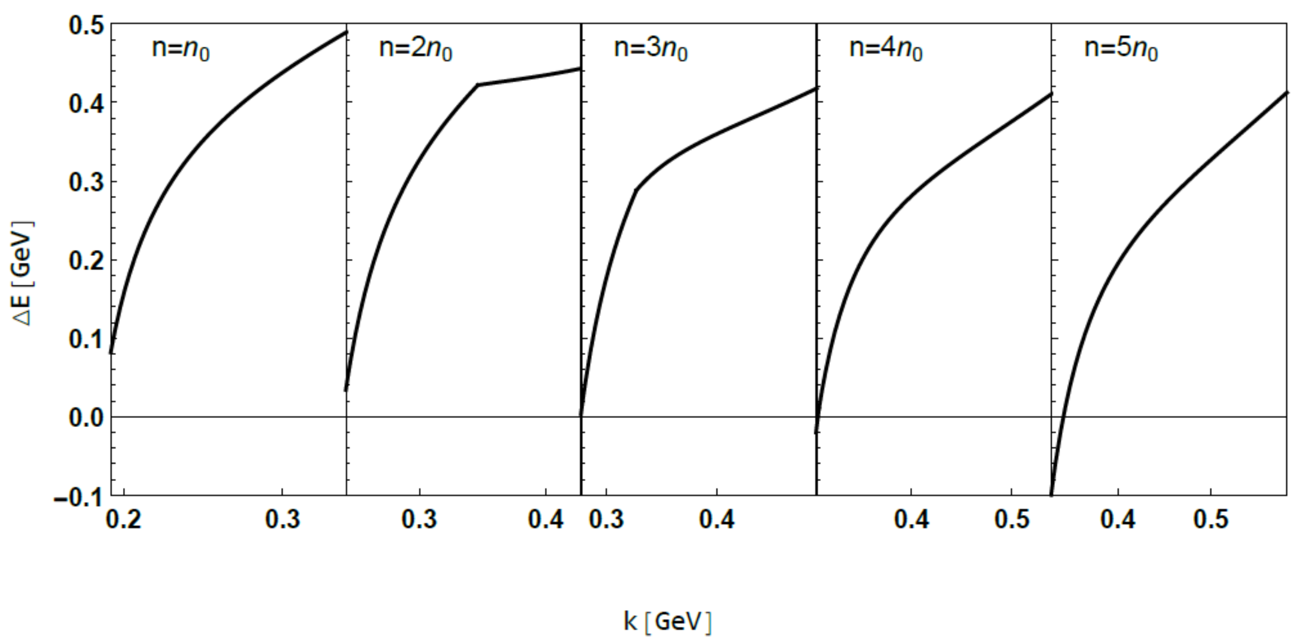


FIG. 5. ΔE where $R_n = 0.7$. Here, n_0 is a normal nuclear density.

Results

$$E_B^{I=1,S=0} = m_B + \frac{k^2}{2m_B} + \frac{a_B}{|r-r_{av}|} e^{-\frac{|r-r_{av}|^2}{b_B^2}} - \frac{a_B}{|r_{av}|} e^{-\frac{|r_{av}|^2}{b_B^2}}$$

$$E_Q = m_B - m_D + \frac{k^2}{2m_Q} + \sigma\left(rH(r_{\max}-r) + r_{\max}H(r-r_{\max})\right) + R_n\left(\frac{a_Q}{|r-r_{av}|} e^{-\frac{|r-r_{av}|}{b_Q}} - \frac{a_Q}{|r_{av}|} e^{-\frac{|r_{av}|}{b_Q}}\right)$$

$$\Delta E = E_d(k_F^d) + m_D - E_n(k_F^n = k_F^d).$$

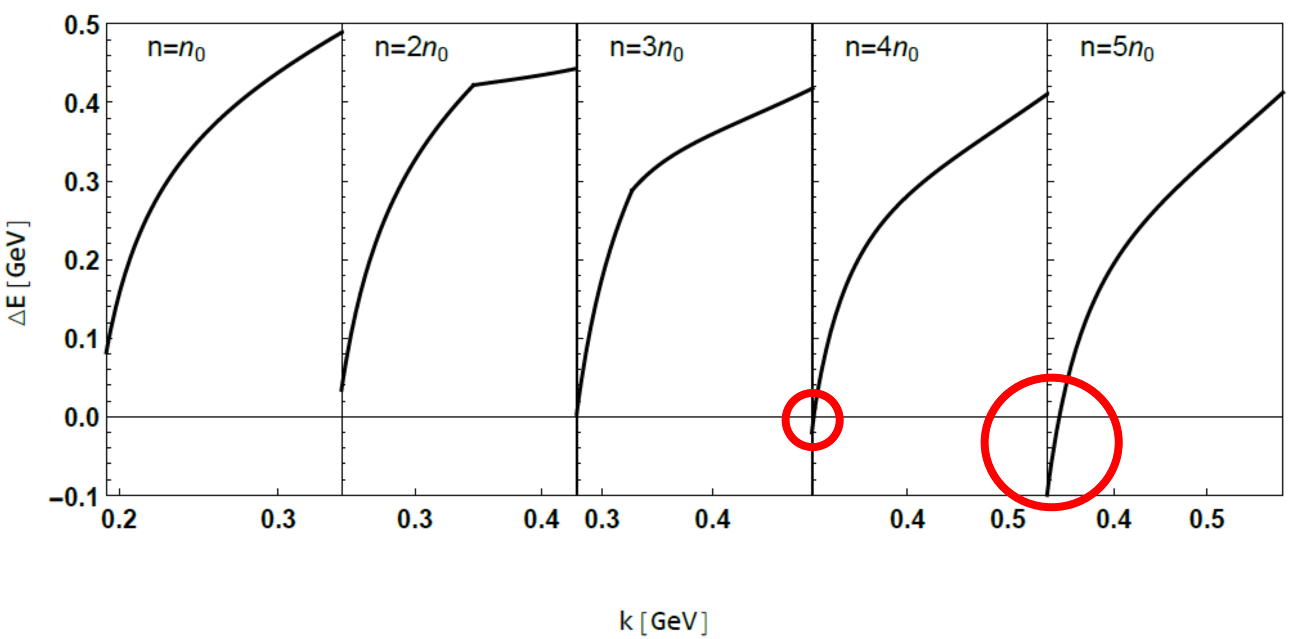


FIG. 5. ΔE where $R_n = 0.7$. Here, n_0 is a normal nuclear density.

Summary

1. Based on the fact that the constituent quark model reproduces the recent lattice result on baryon-baryon interaction at short distance, we analyzed to what extent the quarkyonic modes appear as one increases the density.

2. We analyzed the excitation modes of the baryon and quarks in the presence of a neighbouring nucleon.

3. We found that the initial excitation may involve the d-quark from a neutron, which will leave the most attractive (ud) diquark intact.