APCTP Focus Program in Nuclear Physics 2021: Part I

Hadron properties in a nuclear medium from the quark and gluon degrees of freedom

14-16 July 2021

THE UNIVERSITY of ADELAIDE

Strong Interactions of Quarks and Gluons

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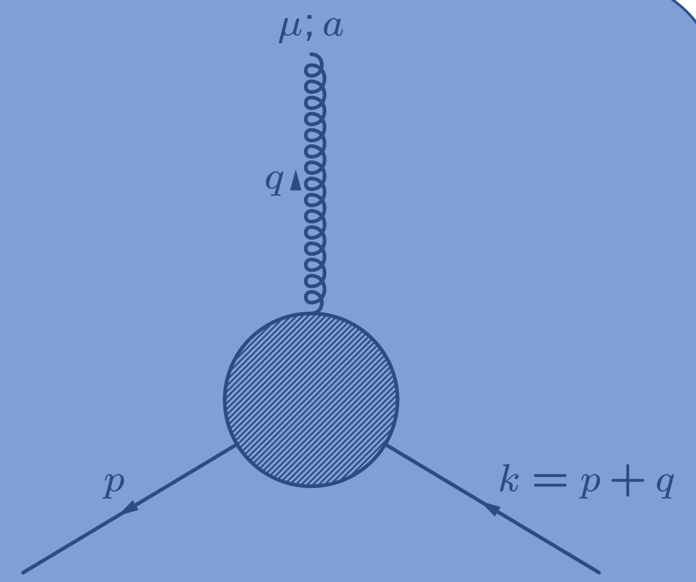
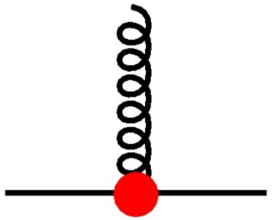
SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER



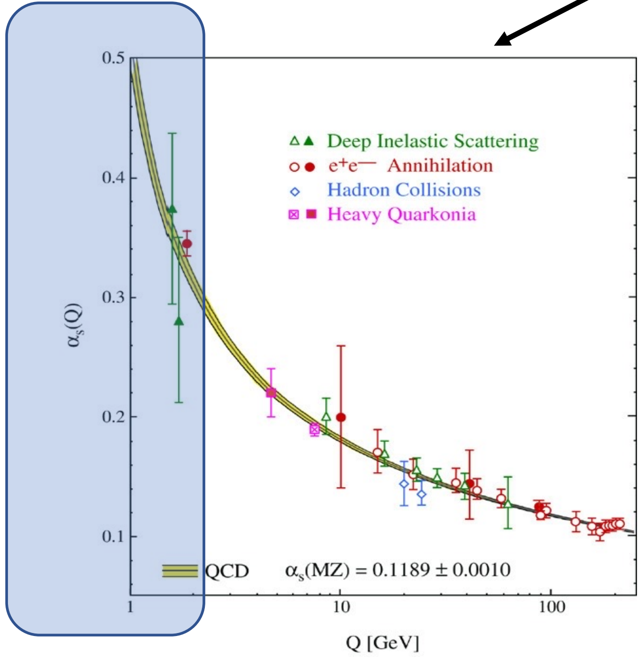
We study the quark-gluon vertex in the limit of vanishing gluon momentum using lattice QCD with two flavors for several lattice spacings and quark masses

QCD is the strong interaction sector in Standard Model

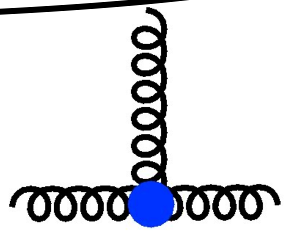
Quark-Gluon Vertex



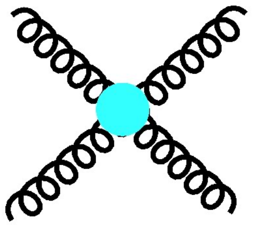
$$S_{QCD} = \int d^4x \left\{ -g\overline{\Psi}_i \gamma^\mu A^a_\mu T^a_{ij} \Psi_j + \overline{\Psi}_i (i \not{\partial} - m)\Psi - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right\}$$



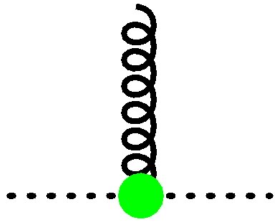
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$



3-Gluon Vertex



4-Gluon Vertex



Ghost-Gluon Vertex

IR behaviour of QCD is exciting

Non-Perturbative Methods: SDE, Lattice, Effective Field Theories ....

Lattice QCD Calculations *vs* *Schwinger-Dyson Calculations*

BOTH starts from first principles(Partition Function):

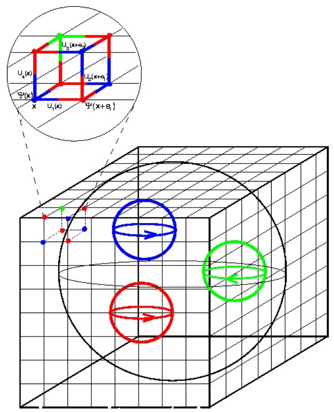
$$Z = \int DA_\mu D\Psi D\overline{\Psi}\; e^{-S_{QCD}}$$

Extrapolation to continuum is needed

Control:

Finite volume effects

Lattice discretization effects



COMPLEMENTARY METHODS

$$\int \mathcal{D}\Phi \frac{\delta}{\delta\Phi} \equiv 0$$

+

Gauge fixing

↓

Gauge independence of physical quantities

↓

Truncation of the infinite system

↓

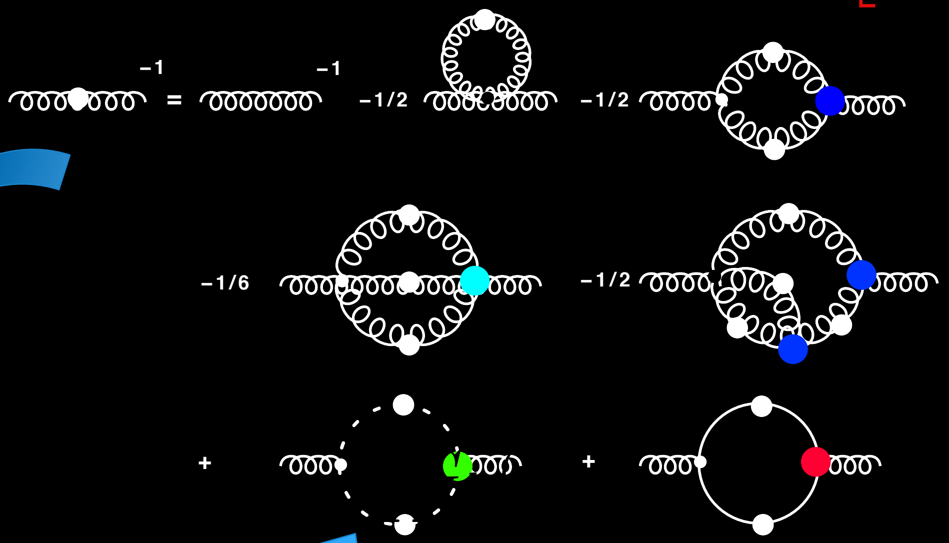
Danger of reintroducing the gauge dependence

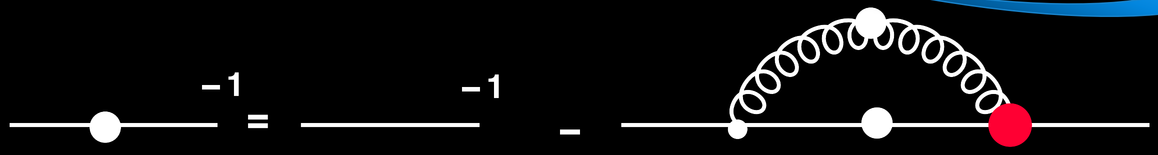
FULL QCD

Schwinger-Dyson Equations

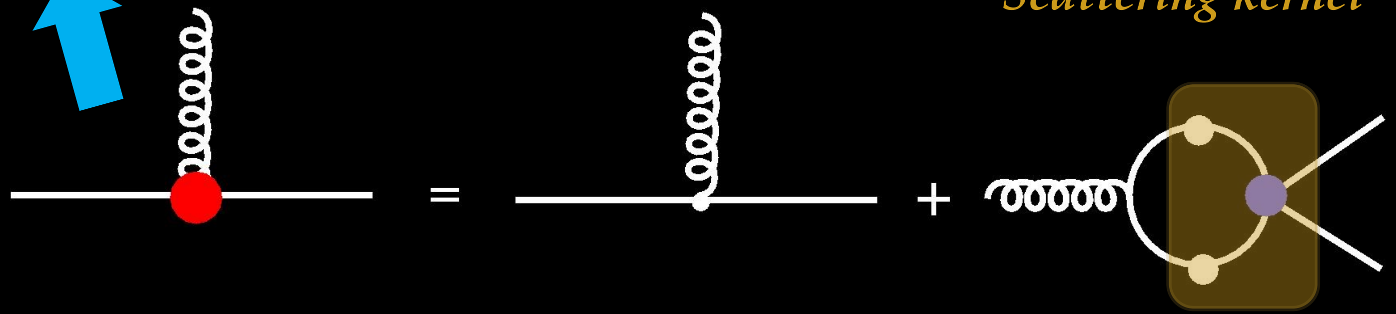
$$D_{\mu\nu}(q) = \frac{-1}{q^2}\left[G_l(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \xi \frac{q_\mu q_\nu}{q^2}\right]$$

$$D_G(q) = -\frac{G_h(q)}{q^2}$$





Fermion-Anti-fermion Scattering kernel

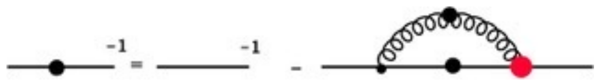


$$S_F(p) = \frac{F(p^2)}{\not{p} - M(p^2)} = \frac{1}{A(p^2)\ \not{p} - B(p^2)}$$

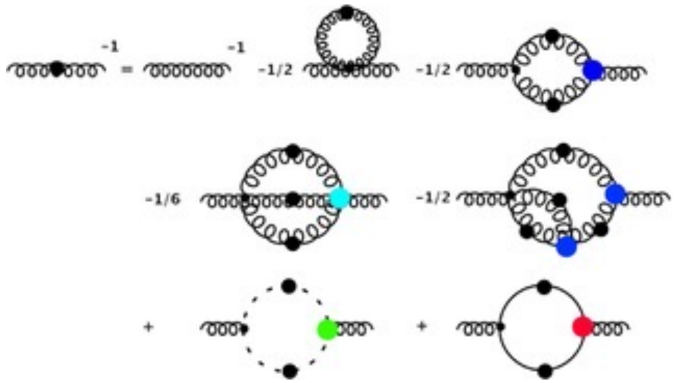
Non-Perturbative QCD

- **Confinement**
- **Dynamical Mass Generation**
- **Hadron Phenomenology**

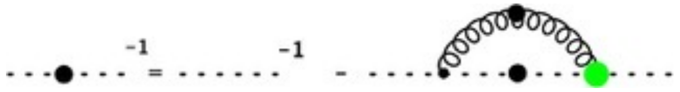
Perturbative QCD



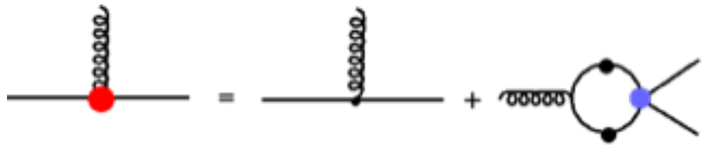
$$S_F(p) = \frac{F(p^2)}{\not p - M(p^2)} = \frac{1}{A(p^2)\,\not p - B(p^2)}$$

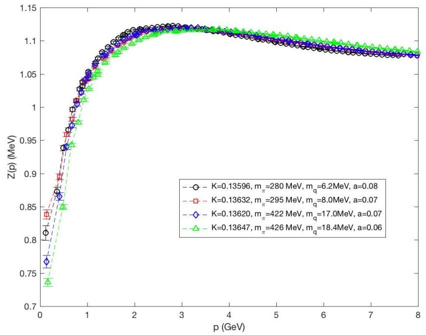


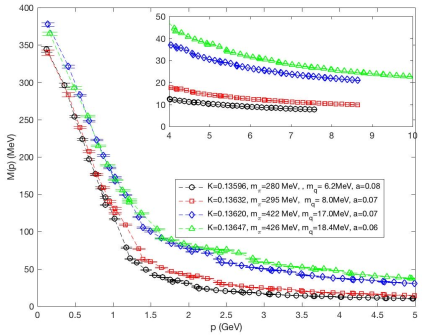
$$D_{\mu\nu}(q) = \frac{-1}{q^2}\left[\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \xi\frac{q_\mu q_\nu}{q^2}\right]$$

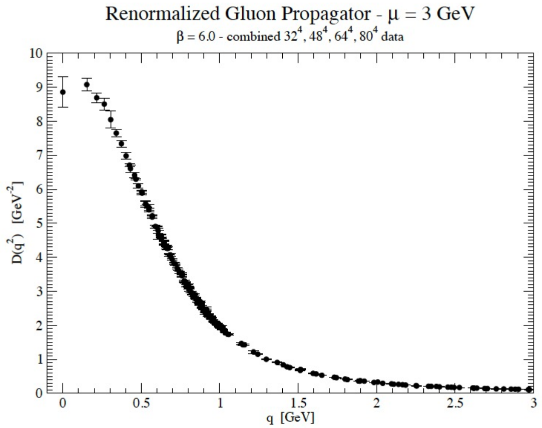


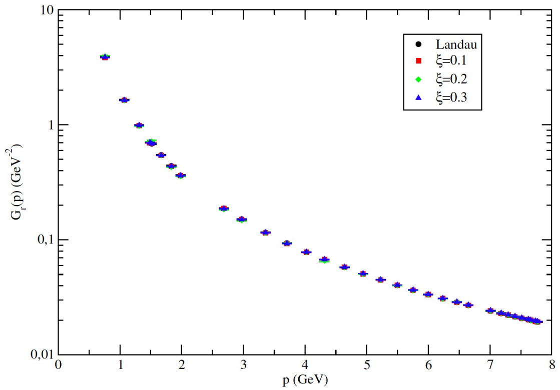
$$D_G(q) = -\frac{G_p}{q^2}$$



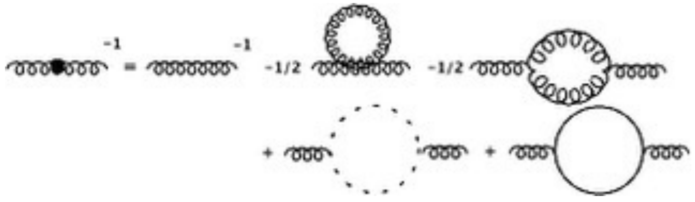




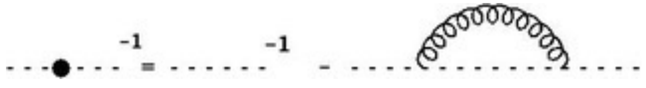




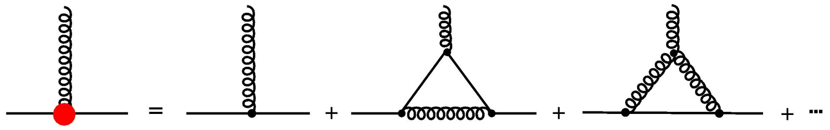
$$S_0(p) = \frac{1}{\not p - m_0}$$



$$D_{\mu\nu}(q) = \frac{-1}{q^2}\left[g_{\mu\nu} - (\xi - 1)\frac{q_\mu q_\nu}{q^2}\right]$$

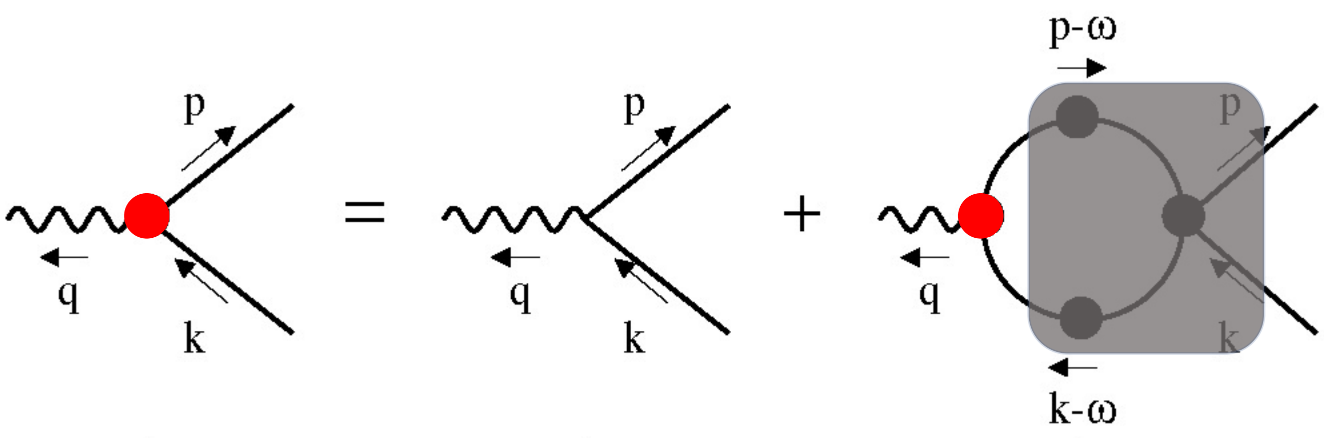


$$D_G(q) = -\frac{1}{q^2}$$



Non-Perturbative Quark-Gluon Vertex:

$(\Lambda_\mu)_{\beta\rho} =$



$$\Lambda_F^\mu(p,k,q) = \sum_{i=1}^{4} \lambda^i(p^2,k^2,q^2,\xi,m)\, L_i^\mu(p,k) + \sum_{i=1}^{8} \tau^i(p^2,k^2,q^2,\xi,m)\, T_i^\mu(p,k)$$

Non-Transverse Part

Transverse Part

$$q \cdot T_i(p,k,q) = 0 \qquad \Lambda_T^\mu(p,p,0) = 0$$

$$q_\mu \Lambda^\mu(p,q,k) = q_\mu \Lambda_{NT}^\mu(p,q,k)$$

Ward-Takahashi Identity (QED)

$$q_\mu \Lambda^\mu(p,q,k) = S^{-1}(k) - S^{-1}(p)$$

Slavnov-Taylor Identity (QCD)

$$q_\mu \Lambda^\mu(p,q,k) = G_h(q^2)\left[\bar{H}(k,-p,-q)S^{-1}(k) - S^{-1}(p)H(-p,k,-q)\right]$$

Ghost-Quark Scattering Kernel

Non-Perturbative Vertex (SDE)

$$\Lambda_F^{\mu}(p,k,q)=\sum_{i=1}^{4}\lambda^{i}(p^2,k^2,q^2,\xi,m)\,L_i^{\mu}(p,k)+\sum_{i=1}^{8}\tau^{i}(p^2,k^2,q^2,\xi,m)\,T_i^{\mu}(p,k)$$

- Bare Vertex $(\Lambda_\mu)=\gamma_\mu$ + bare gluon propagator (Rainbow-Ladder app.). Very popular!

- Ball-Chiu Vertex (Non-Transverse Part)

(QED)

$$\lambda_1^M(p^2,k^2)=\frac{1}{2}\left[A(k^2)+A(p^2)\right],$$

$$\lambda_2^M(p^2,k^2)=\frac{1}{2(k^2-p^2)}\left[A(k^2)-A(p^2)\right],$$

$$\lambda_3^M(p^2,k^2)=\frac{-1}{k^2-p^2}\left[M(k^2)\,A(k^2)-M(p^2)\,A(p^2)\right],$$

$$\lambda_4^M(p^2,k^2)=0$$

(QCD)

$$\lambda_1(p^2,0,p^2)=G(0)\left[A(p^2)\chi_0(p^2,0,p^2)+B(p^2)\left(\chi_1(p^2,0,p^2)+\chi_2(p^2,0,p^2)\right)-2p^2A(p^2)\chi_3(p^2,0,p^2)\right]$$

Ghost-Quark Scattering Kernel form factors

- Curtis-Pennington Vertex (Transverse Part) : $\tau_6(p,k)$ (MR)

- Kizilersu-Penington Vertex (Transverse Part) : $[\tau_2,\tau_3,\tau_6,\tau_8](p,q,k)$

In Continuum…
Non-Perburbative (DSE)
Ball_Chiu, Curtis-Pennington, C.D. Roberts, P. Tandy, P. Maris, Haeri, H. Matevosyan,-
A. Thomas, A.Bashir, Kizilersu-Pennington, R. Williams, R. Alkofer et.al.,J. Pawlowski,
C. Aguilar, D. Binosi, D. Ibanez, J. Papavassiliou …

Lattice action

- Wilson gauge action
- ***(Sheikholeslami-Wohlert)*** clover fermions

$$S_{SW} = S_W - i\frac{a}{4} g_0 c_{SW} \sum_x \sum_{\mu\nu} \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

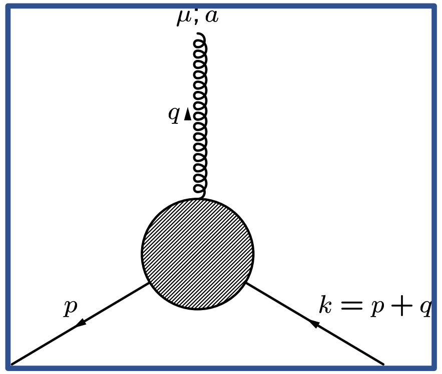
In Continuum

$$\overline{\psi}(x)\left(i \not{D} + m\right)\psi(x) + \overline{\psi}(x)D^2\psi(x) + \overline{\psi}(x)\sigma_{\mu\nu}F_{\mu\nu}(x)\psi(x)$$

Other fermion actions such as Staggered, overlap fermions are used in different studies…

However continuum limit must stay the same

Non-Perturbative Vertex (from LATTICE)



Quark-gluon vertex on the lattice :

$$\mathbf{\Lambda^{a,lat.}_{\mu}(p,q) = S_R(p)^{-1} V^a_{\nu}(p,q) S_R(p+q)^{-1} D(q)^{-1}_{\nu\mu}}$$

Unamputated vertex $V^a_\mu(p,q) =<< S_R(p;U) A^a_\mu(q) >>$

Transverse Projection

$D_{\mu\nu}^{-1}$ does not exist, so we will be looking at transverse projection

$$\tilde{\Lambda}^T_\mu(p,k,q) = P^T_{\mu\nu}(q)\Lambda_\nu = \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Lambda_\nu(p,k,q)$$

Some of the Form Factors in Special Kinematics in Landau gauge are calculated in quenched QCD

J.Skullerud, A.Kizilersu, JHEP09(2002)013

J. Skullerud,P. Bowman, A.Kizilersu, D.Leinweber,A.Williams, JHEP04(2003)047

Lattice Parameters of Gauge Ensembles in this Study (N_f=2)

Lattice action

- Wilson gauge action
- Wilson clover fermions
- $\mathcal{O}(\alpha)$ improved rotated propagator
- Landau gauge $(\xi = 1)$

Name	β	κ	a [fm]	V	m_π [MeV]	m_q [MeV]	N_{cfg}	N_{src}
L08	5.20	0.13596	0.081	$32^3 \times 64$	280	6.2	900	4
H07	5.29	0.13620	0.071	$32^3 \times 64$	422	17.0	900	4
L07	5.29	0.13632	0.071	$32^3 \times 64$	295	8.0	908	4
L07-64	5.29	0.13632	0.071	$64^3 \times 64$	290	8.0	750	2
H06	5.40	0.13647	0.060	$32^3 \times 64$	426	18.4	900	2
Q07	6.16	0.13400	0.071	$32^3 \times 64$	1000	130	998	4

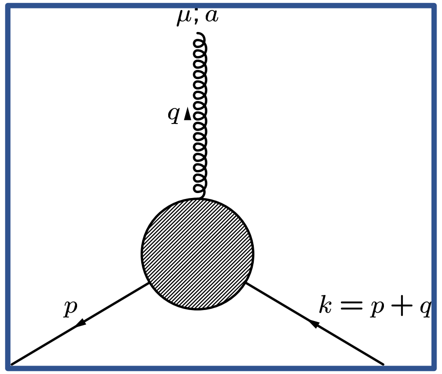
Acknowledgements

N_f = 2 congurations provided by RQCD collaboration (Regensburg), *S. Bali et all, Phys Rev D91, 054501 (2014)*

A. Kizilersu, O. Oliveira, P.J. Silva, J. Skullerud and A. Sternbeck, Phys.Rev.D103 (2021)114515

Form Factor Extraction

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

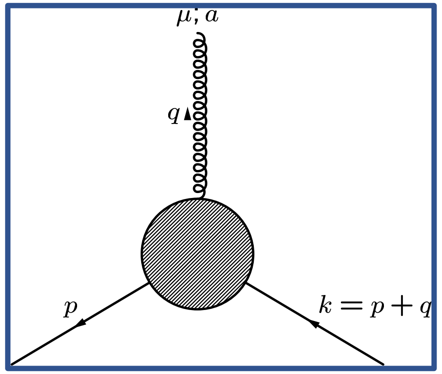


$$(\tilde{\Lambda}^a_\mu) = -ig_0\,(\lambda_1\,[\gamma_\mu] + \lambda_2\,[-4\,\not{p}p_\mu] + \lambda_3\,[-2ip_\mu]\,)$$

Covariant Form factors:

- $$\lambda_1 = \frac{1}{(-ig_0)}\left\{\frac{1}{3}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - \frac{p_\mu p_\nu}{p^2}\,\mathrm{Tr}_4(\gamma_\nu\tilde{\Lambda}_\mu)\right]\right\}$$
- $$\lambda_2 = \frac{1}{(-ig_0)}\left\{\frac{1}{3p^2}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - 4\frac{p_\mu p_\nu}{p^2}\,\mathrm{Tr}_4(\gamma_\nu\tilde{\Lambda}_\mu)\right]\right\}$$
- $$\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\,\frac{p_\mu}{p^2}\,\mathrm{Tr}_4(I\,\overline{\Lambda}_\mu)\right\}$$

Form Factor Extraction



Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

$$(\tilde{\Lambda}^a_\mu) = -ig_0\left(\lambda_1\,[\gamma_\mu] + \lambda_2\,[-4\,\not{p}p_\mu] + \lambda_3\,[-2ip_\mu]\right)$$

Covariant Form factors:

- $$\lambda_1 = \frac{1}{(-ig_0)}\left\{\frac{1}{3}\left[\mathrm{Tr}_4(\gamma_\mu\overline{\Lambda}_\mu) - \frac{p_\mu p_\nu}{p^2}\,\mathrm{Tr}_4(\gamma_\nu\tilde{\Lambda}_\mu)\right]\right\}$$
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- $$\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\,\frac{p_\mu}{p^2}\,\mathrm{Tr}_4(I\,\overline{\Lambda}_\mu)\right\}$$

Non-covariant Form factors:

- $$\lambda_1 = \frac{1}{(-ig_0)}\left\{\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu\\ p_\mu=0}}\right\}$$
- $$\lambda_2 = \frac{1}{(-ig_0)}\left\{-\frac{1}{4\,p^2}\,\frac{p_\alpha p_\mu}{p^2}\left[Tr_4(\gamma_\alpha\overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right]\right\}$$
- $$\lambda_3 = \frac{1}{(-ig_0)}\left\{\frac{i}{2}\,\frac{p_\mu}{p^2}Tr_4(I\,\overline{\Lambda}_\mu)\right\}$$

MOM Renormalisation: $\lambda_1^R(\mu^2, 0, \mu^2) = 1$ ⇒ $\Gamma^{\mathrm{lat}}_\mu(p,k,q) = Z_1\Gamma^R_\mu(p,k,q)$

Lattice form factors and Tree-Level Corrections

Continuum form factors

$$\lambda_1 = \frac{1}{(-ig_0)} \left\{ \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu \\ p_\mu=0}} \right\}$$

$$\lambda_2 = \frac{1}{(-ig_0)} \left\{ -\frac{1}{4\,p^2} \frac{p_\alpha p_\mu}{p^2} \left[Tr_4(\gamma_\alpha \overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right] \right\}$$

$$\lambda_3 = \frac{1}{(-ig_0)} \left\{ \frac{i}{2} \frac{p_\mu}{p^2} Tr_4(I\,\overline{\Lambda}_\mu) \right\}$$

Tree-level corrected, lattice equivalents of the form factors :

$$\lambda_1(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0} \left\{ \left[\mathrm{Tr}_4(\gamma_\alpha \overline{\Lambda}_\mu)\right]\Big|_{\substack{\alpha=\mu \\ p_\mu=0}} \right\} \Big/ \lambda_1^{(0)}$$

$$\lambda_2(p^2,0,p^2) = \frac{\mathrm{Im}}{g_0} \left\{ -\frac{1}{4\,K(p)^2} \frac{K_\alpha(p) K_\mu(p)}{K(p)^2} \left[\mathrm{Tr}_4(\gamma_\alpha \overline{\Lambda}_\mu)\Big|_{\alpha\neq\mu}\right] \right\} - \left(\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)}\right)$$

$$\lambda_3(p^2,0,p^2) = \frac{\mathrm{Re}}{(-g_0)} \left\{ \frac{1}{2} \frac{K_\mu(p)}{K^2(p)} \mathrm{Tr}_4(I\,\overline{\Lambda}_\mu) \right\} - \left(\lambda_3^{(0)} + \overline{\lambda}_{3(\mu)}^{(0)}\right)$$

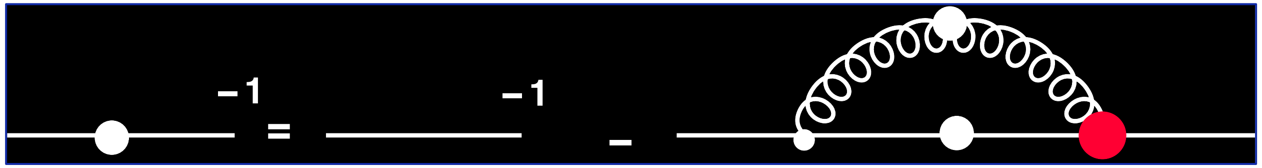
Lattice tree-level form factors

$$\lambda_1^{(0)} = F(p)\big(1 + c_q^2 a^2 K^2(p)\big)^2$$

$$\lambda_2^{(0)} + \overline{\lambda}_{2(\mu)}^{(0)} = a^2 F(p) \Big[-c_q \left(1 - c_q^2 a^2 K^2(p)\right) + 2 c_q^2 a\, C_\mu(p) \Big]$$

$$\lambda_3^{(0)} + \overline{\lambda}_{3,(\mu)}^{(0)} = \frac{a}{2} F(p) \Big[\big(1 - c_q^2 a^2 K^2(p)\big)^2 - 4 c_q^2 a^2 K^2(p) - 4 c_q \big(1 - c_q^2 a^2 K^2(p)\big) C_\mu(p) \Big]$$

Quark Propagator in Landau Gauge



In Continuum

$$S_F(p) = \frac{Z(p^2)}{i \not{p} + M(p^2)}$$

$$= \frac{1}{A(p^2) i \not{p} + B(p^2)}$$

In Discrete

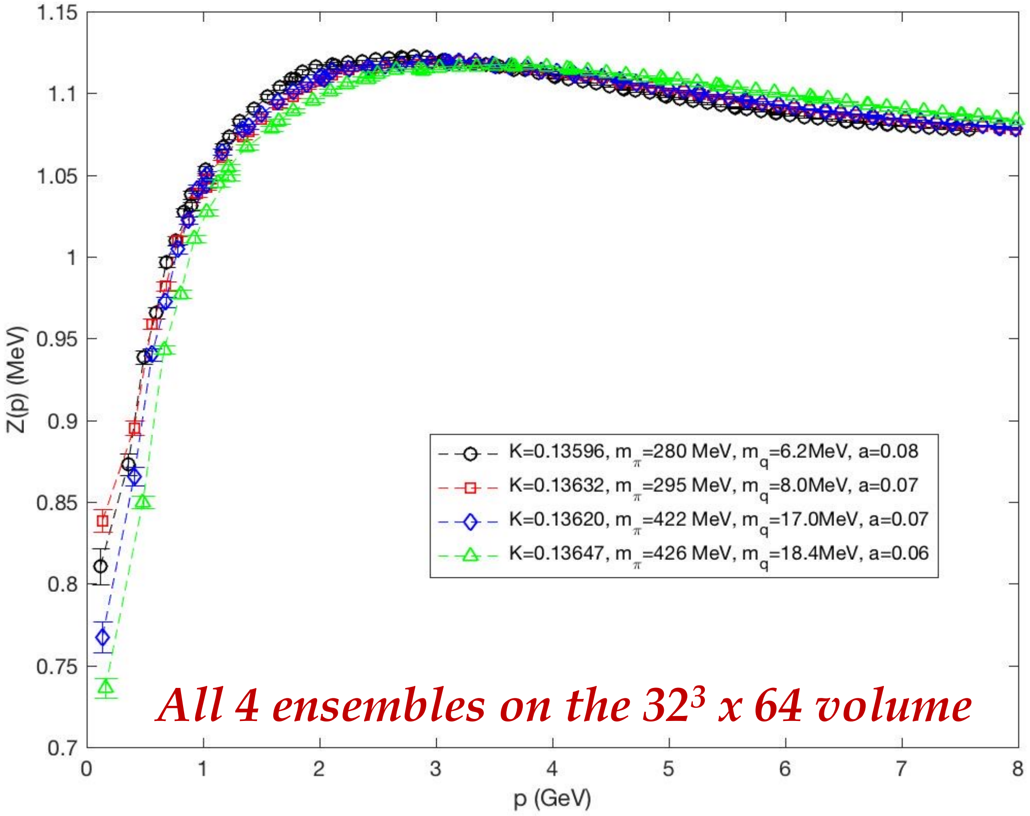
$$S^L(pa) = \frac{Z^L(pa)}{ia \not{K}(p) + aM^L(pa)}$$

$$K_\mu(p) \equiv \frac{1}{a} sin(p_\mu a)$$

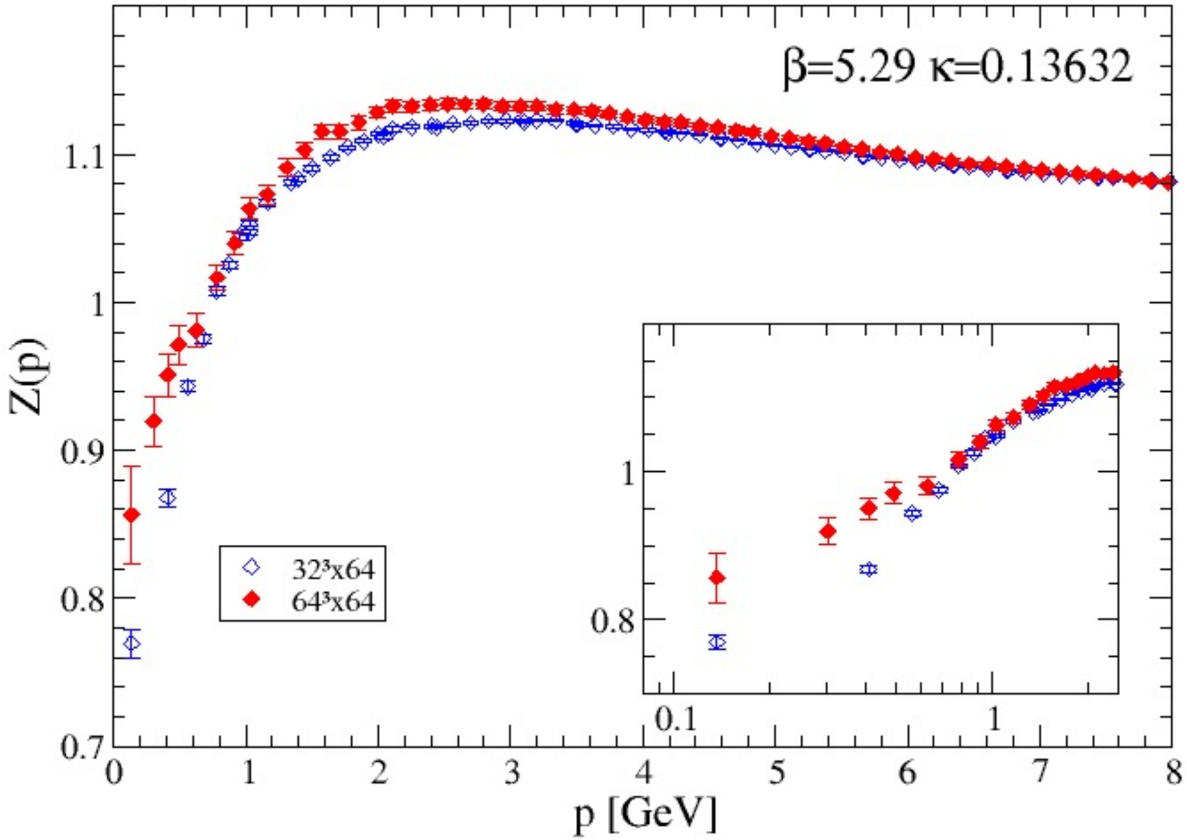
Quark wave function renormalization Z(p) in Landau Gauge (N_f=2), *tree-level corrected*

$$S_F(p) \quad = \quad \frac{Z(p^2)}{\not{p} - M(p^2)}$$

$Z(p^2)$ suppressed in IR



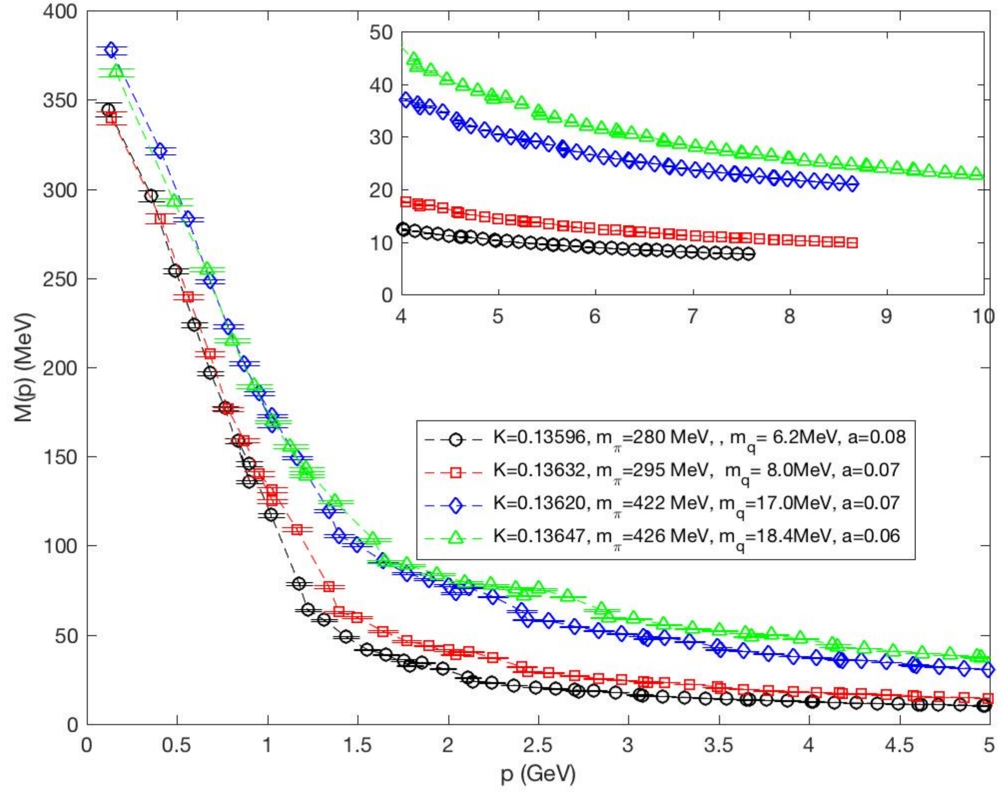
All 4 ensembles on the 32^3 x 64 volume

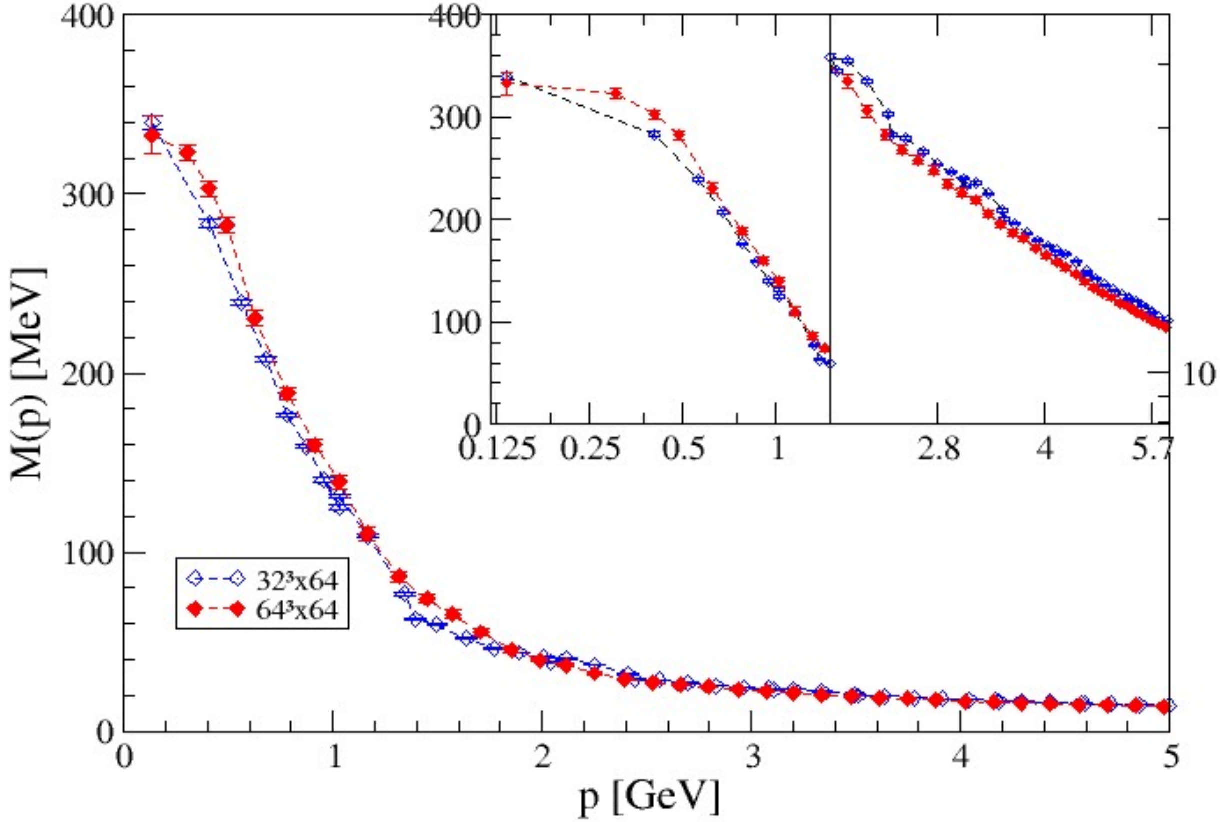


Quark Mass function M(p) in Landau Gauge (N_f=2) *, tree-level corrected*

$$S_F(p) = \frac{Z(p^2)}{\not{p} - M(p^2)}$$

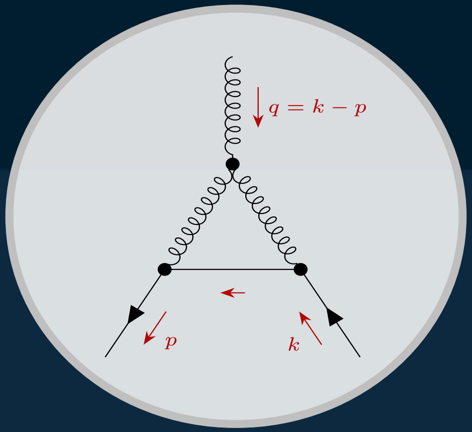
M = 349.7 ± 5.2 MeV @ p= 136 MeV for m_π=290 MeV

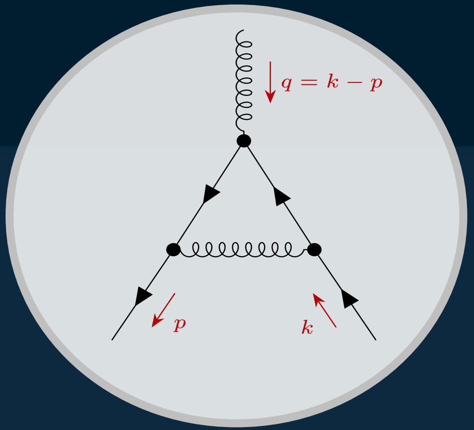


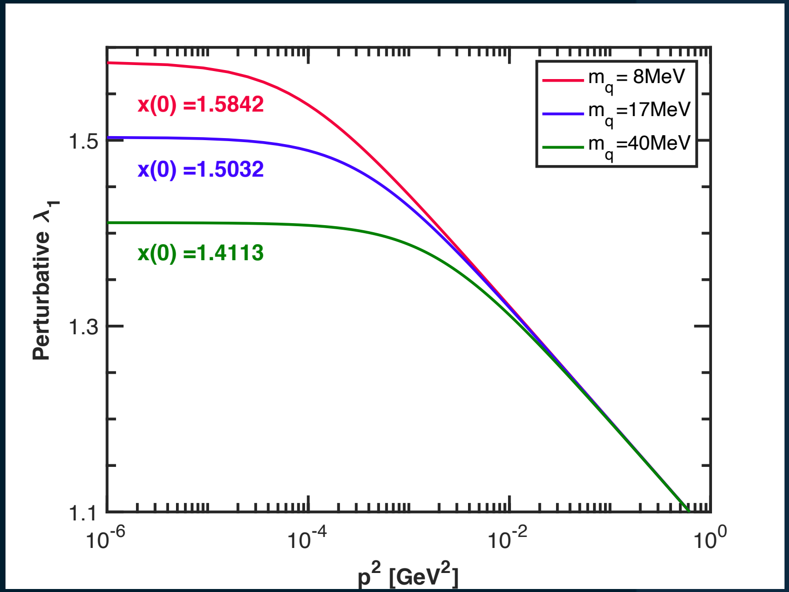


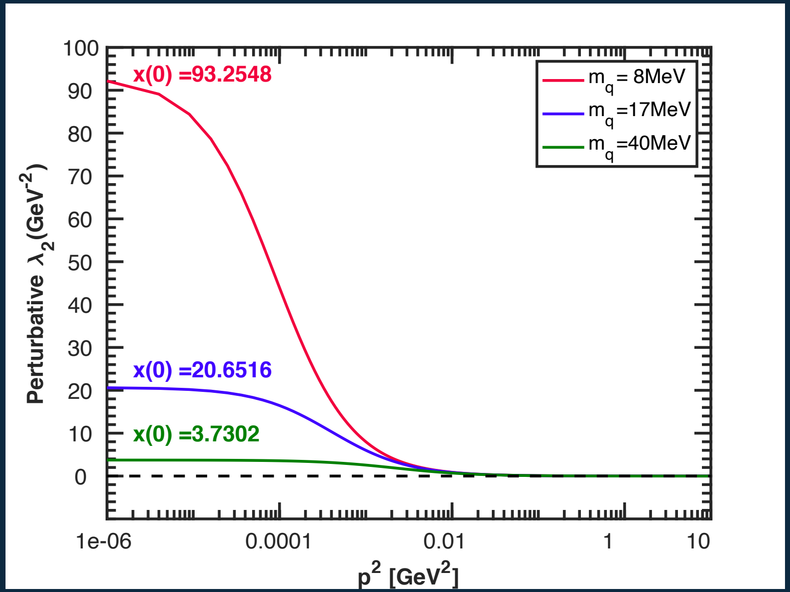
All 4 ensembles on the 32^3 x 64 volume

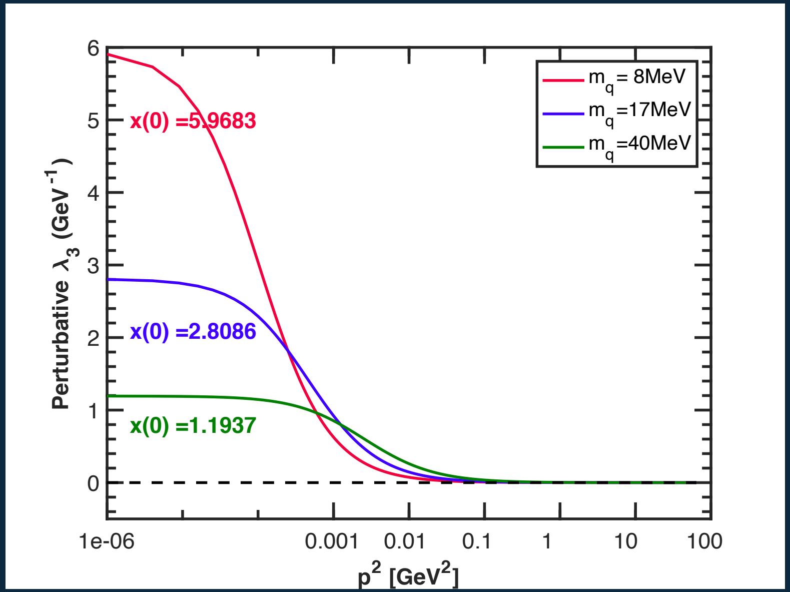
Perturbative One-Loop Form Factors in Soft Gluon Kinematics





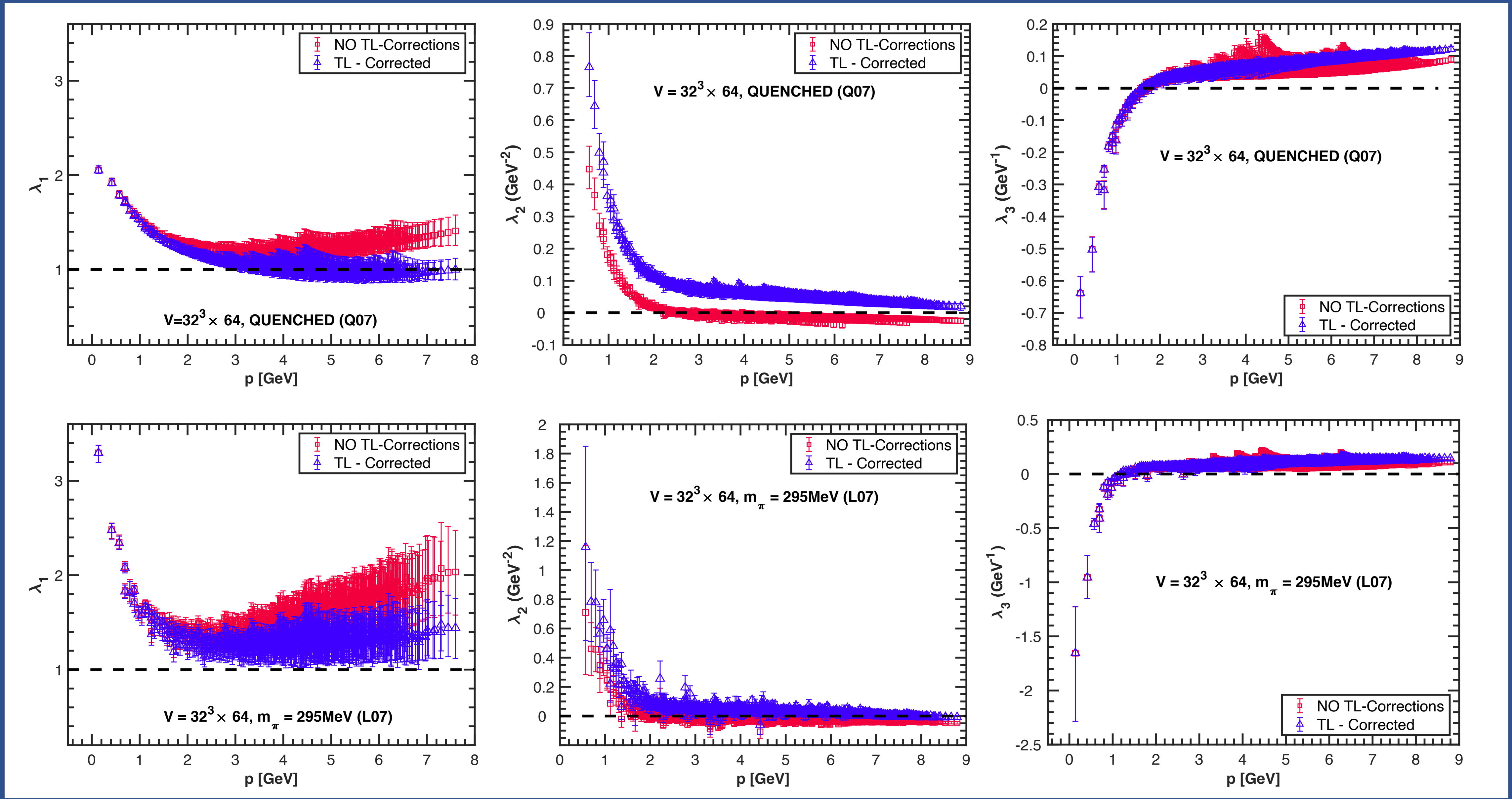






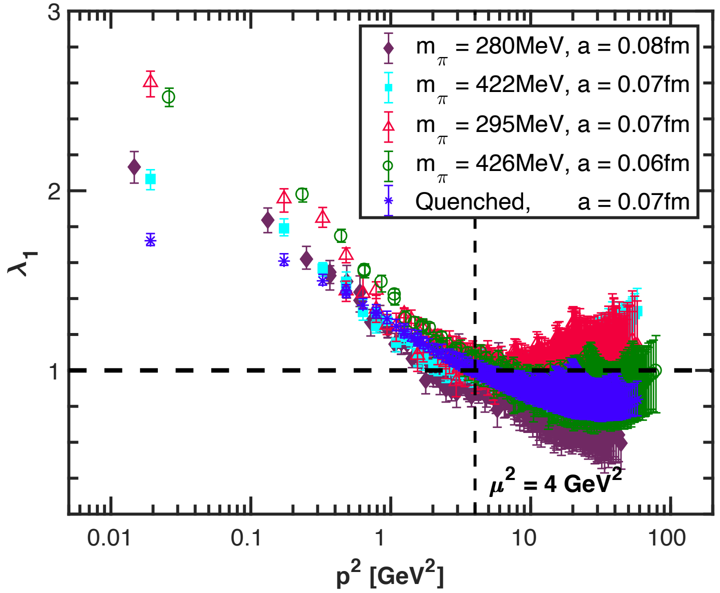
J.S.Ball and T.W. Chiu, Phys.Rev.D22, 2542(1980)
A.Kizilersu,M.Reenders and M.R.Pennington, Phys.Rev.D52,1242 (1995). (QED)
A.I.Davydychev, P.Osland and L.Saks, Phys.Rev.D63,014022 (2001) (QCD)
J.A.Gracey, Phys.Rev.D90,025014(2014) (QCD)

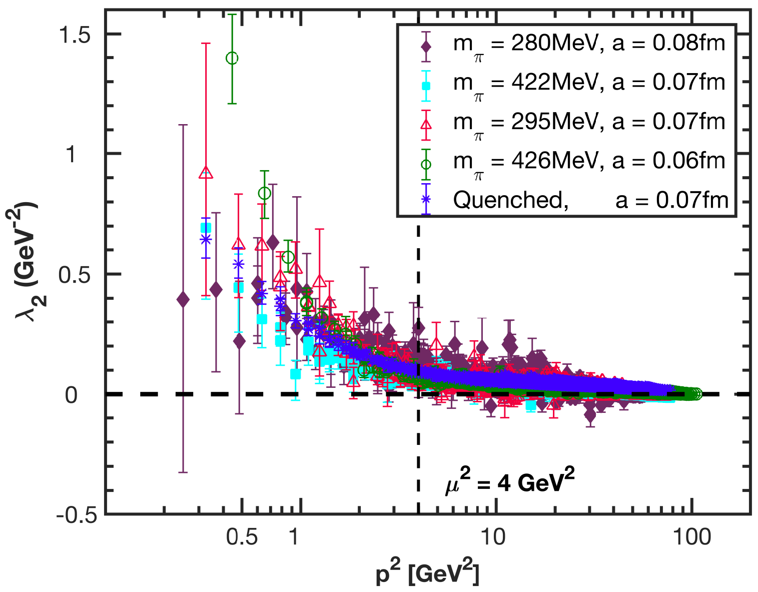
Tree Level Corrected vs Uncorrected Form Factors

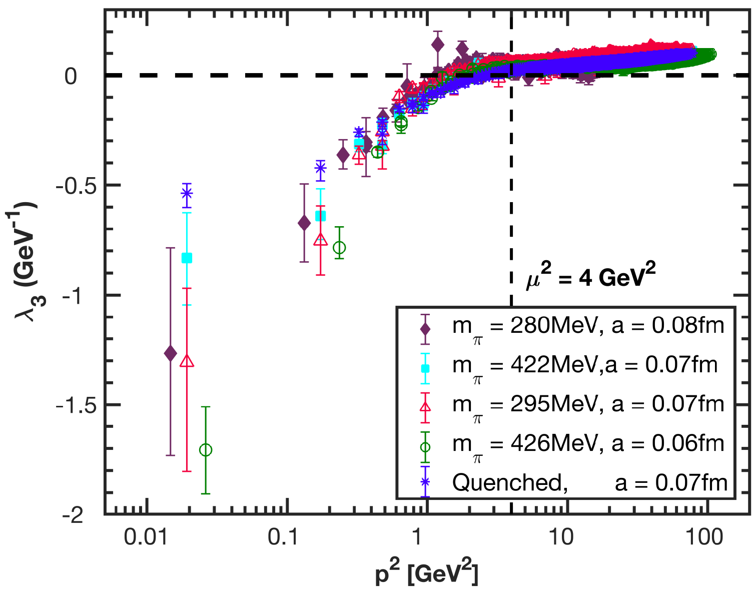


Tree Level Corrected vs Uncorrected Form Factors

Renormalised $\Gamma_\mu(p,q,k) = Z_1\,\Gamma^R_\mu(p,q,k)$ $\lambda^R_1(\mu^2,0,\mu^2) = 1$

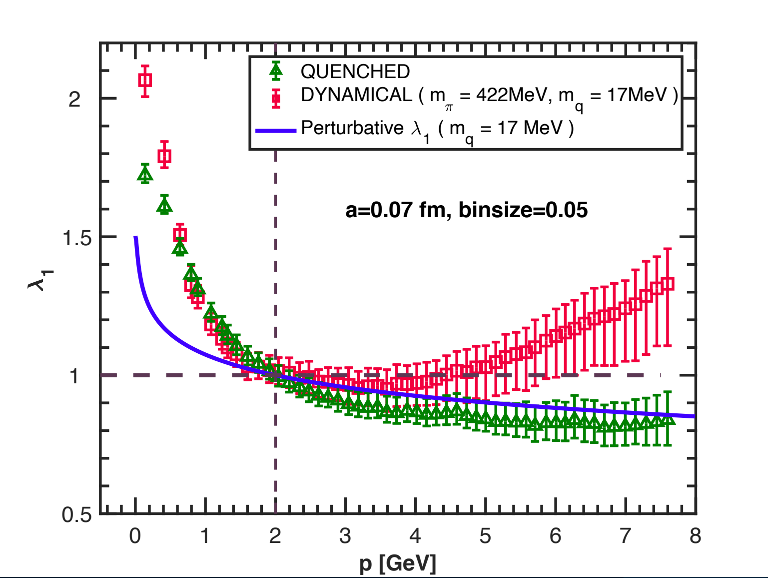


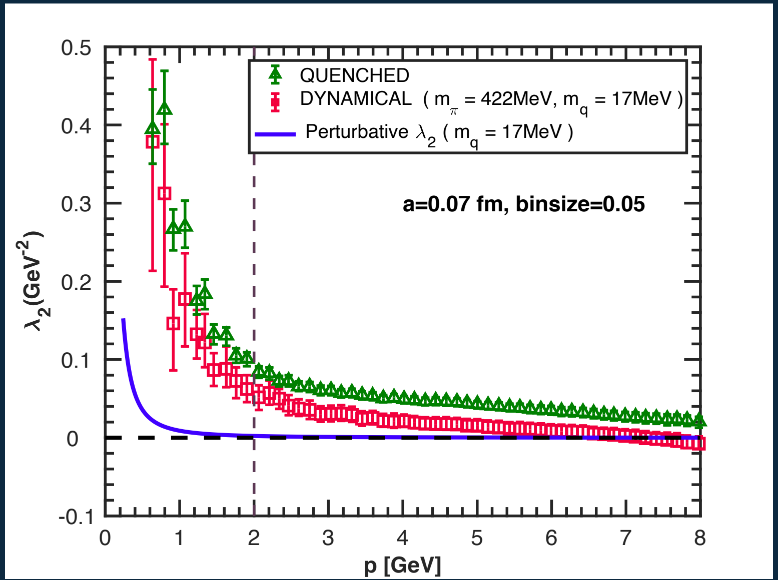


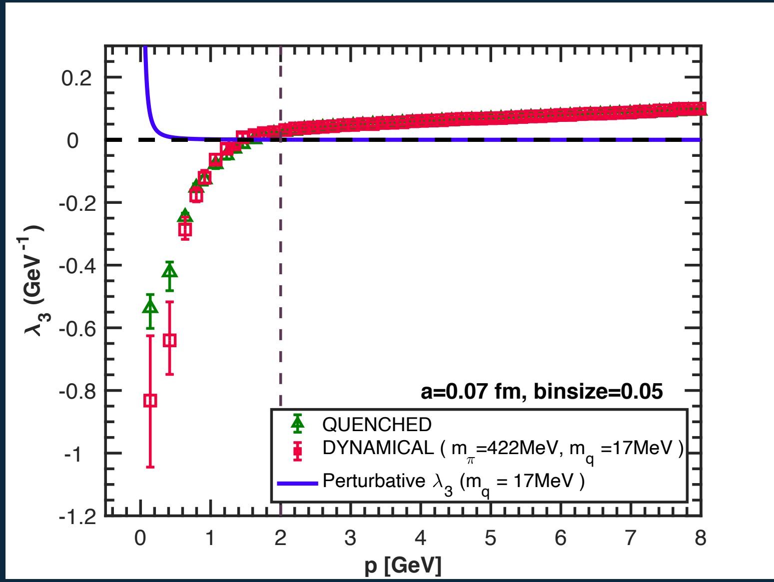


Quenched vs Dynamical

$N_f = 0$ vs. $N_f = 2$ (a=0.07 fm)



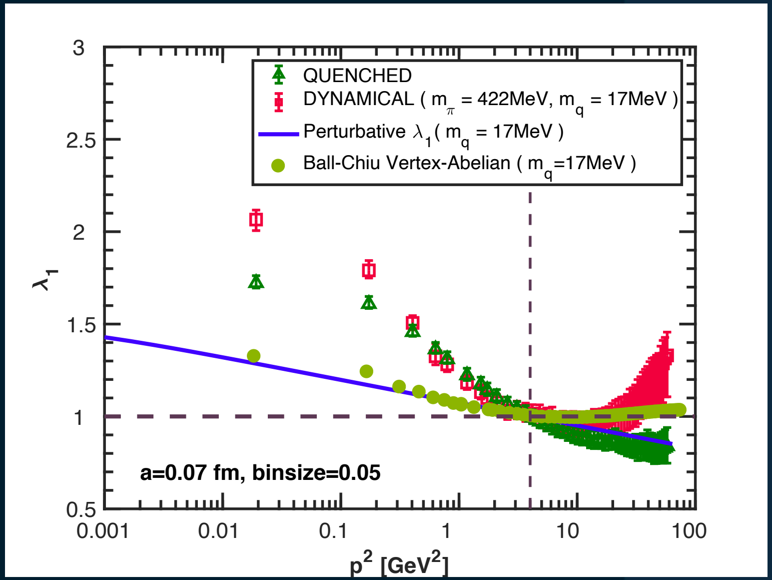


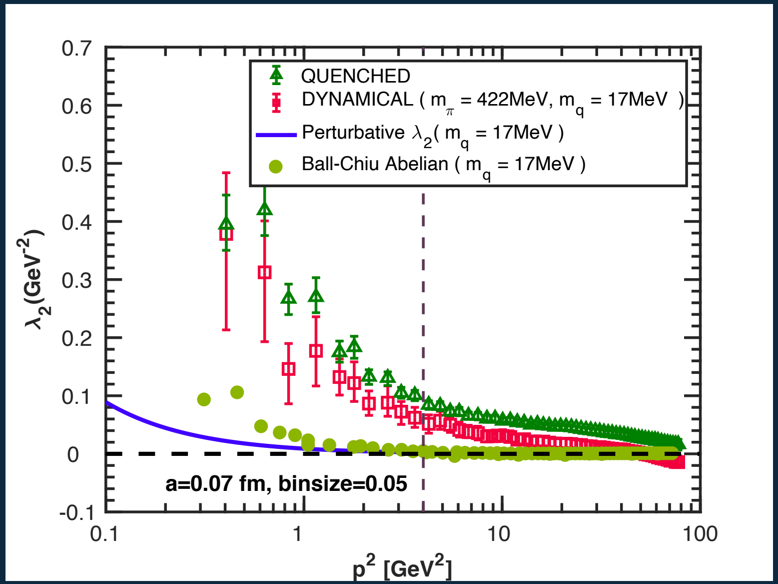


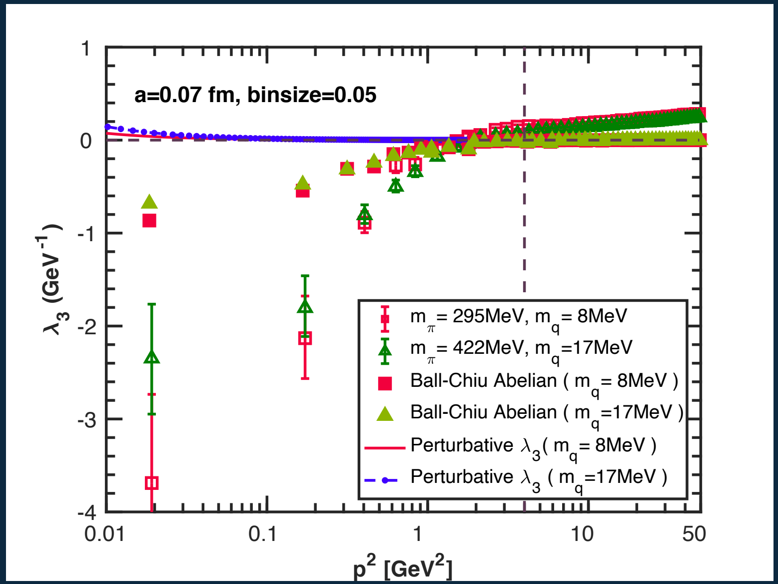
Quenched vs Dynamical

$N_f = 0$ vs. $N_f = 2$ (a=0.07 fm)

$$S_F(p) = \frac{F(p^2)}{\not{p} - M(p^2)} = \frac{1}{A(p^2)\,\not{p} - B(p^2)}$$





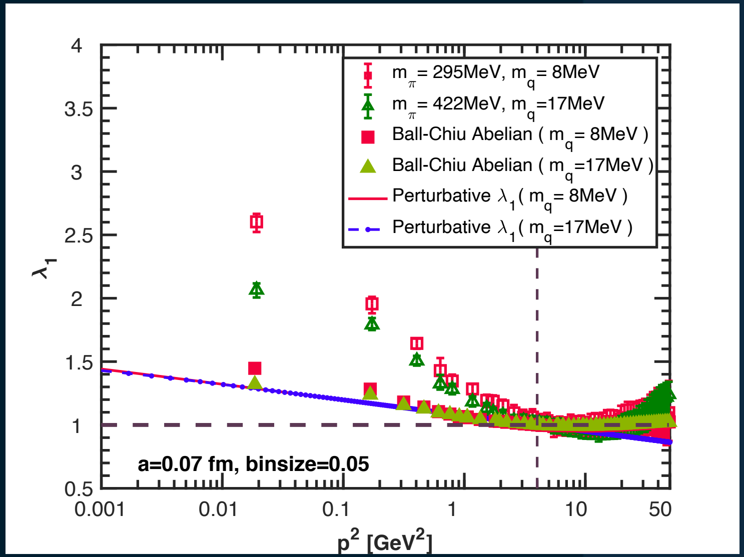


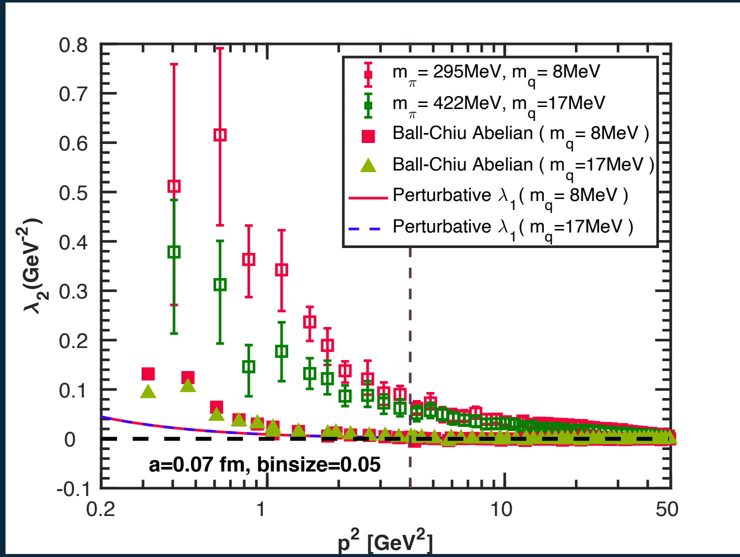
$$\lambda_1^{BC} = A(p^2)$$

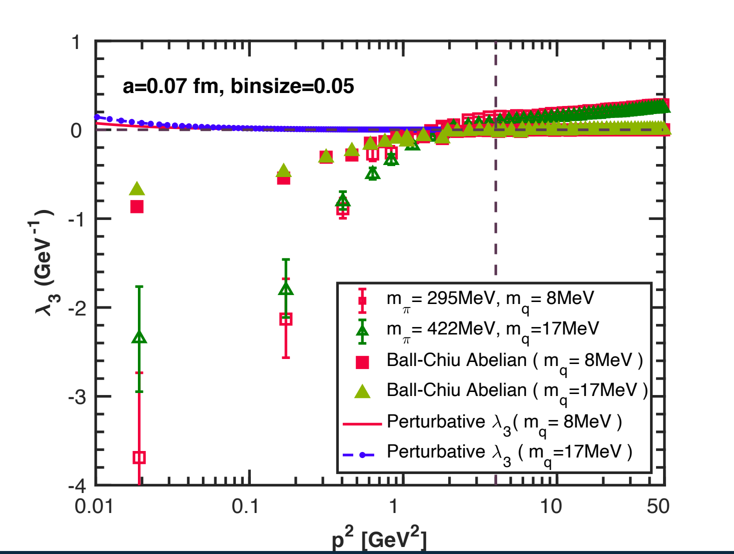
$$\lambda_2^{BC} = -\frac{1}{2}\frac{dA(p^2)}{dp^2}$$

$$\lambda_3^{BC} = \frac{dB(p^2)}{dp^2}$$

Quark Mass Dependence

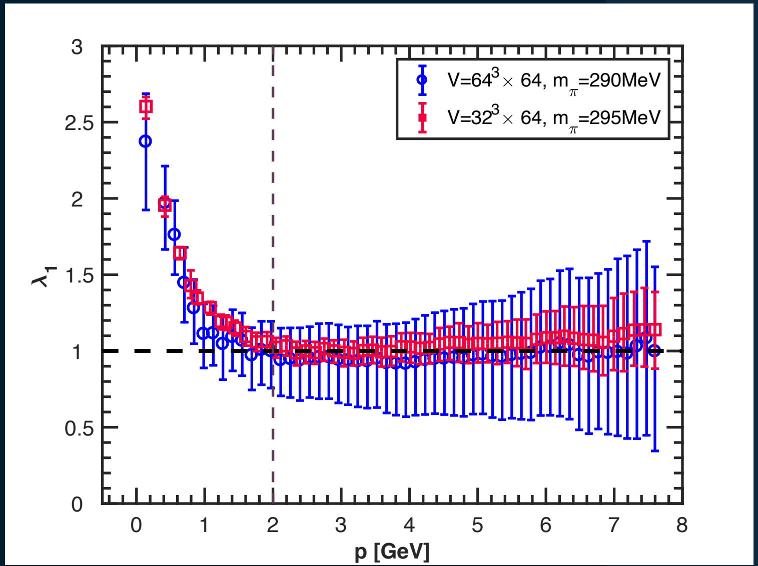


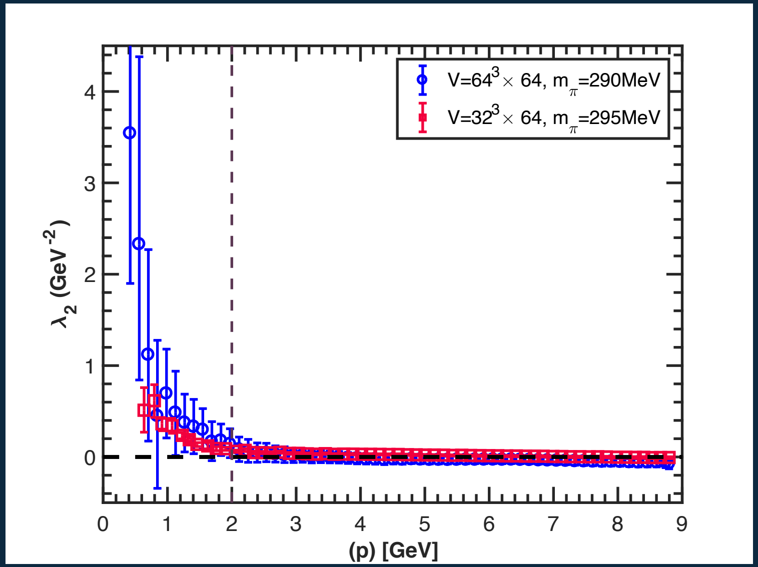


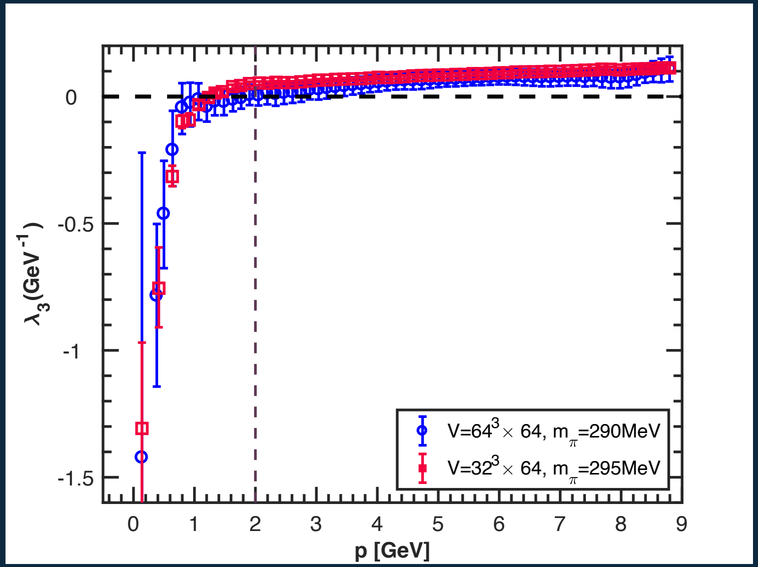


discret./Volume Dependence

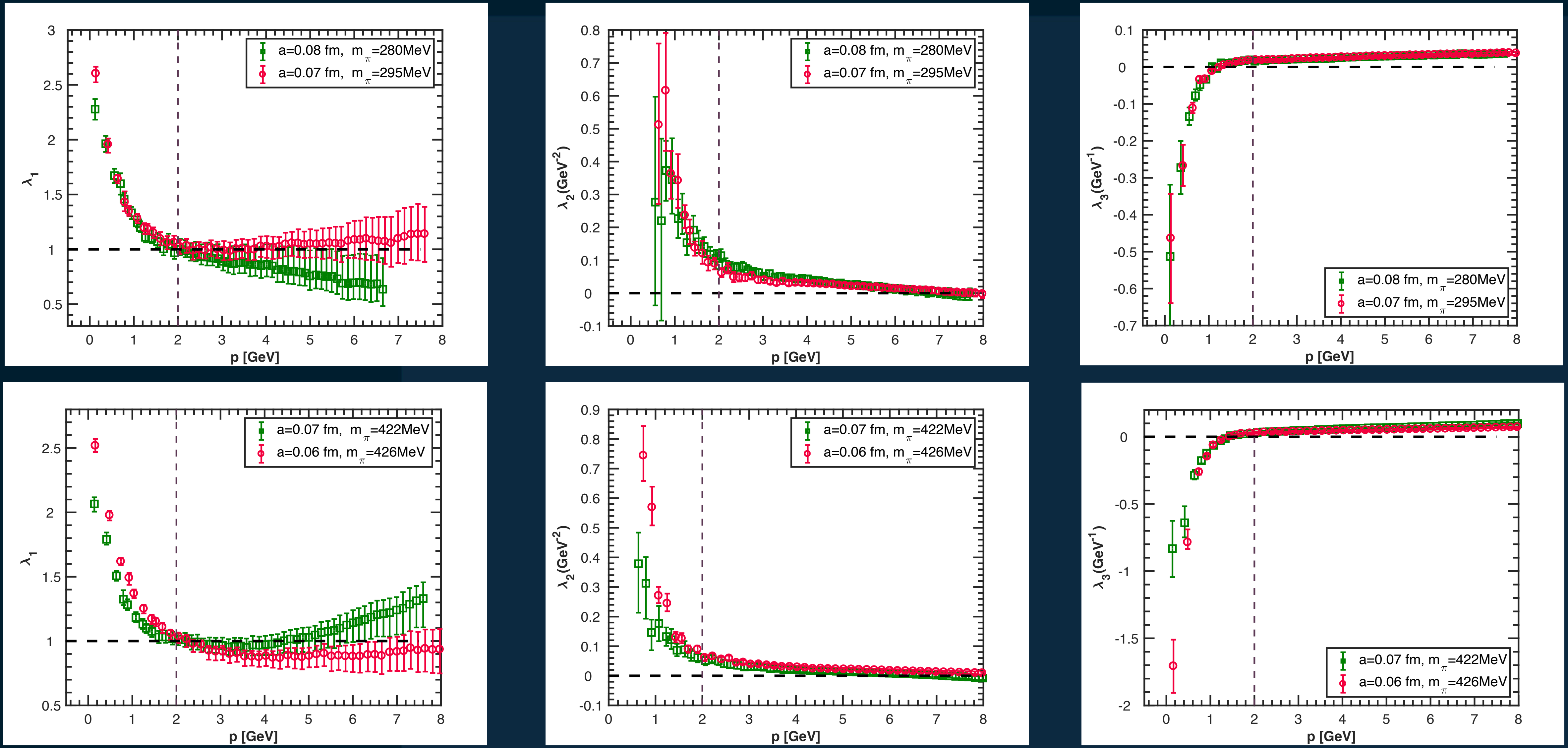
(a=0.07 fm)





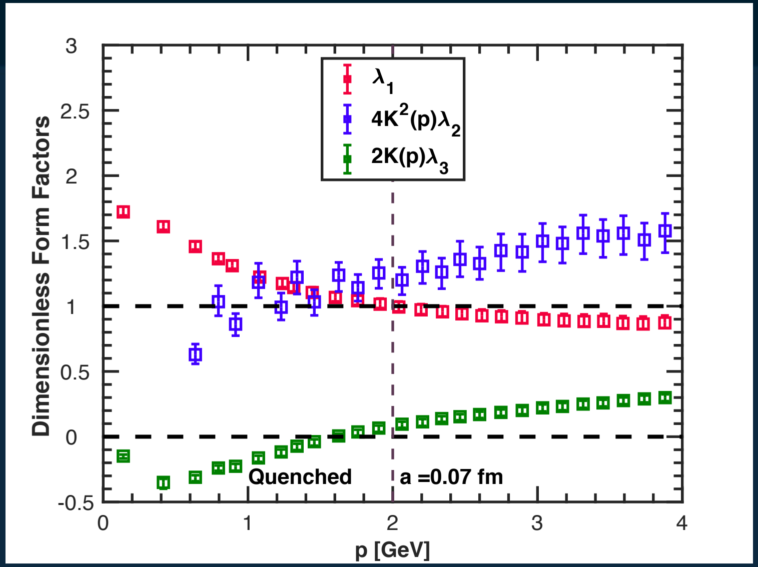


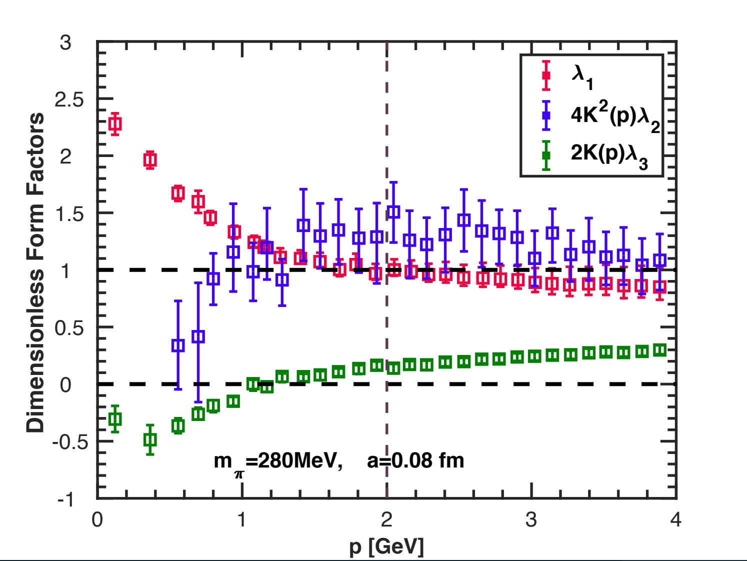
Lattice Spacing

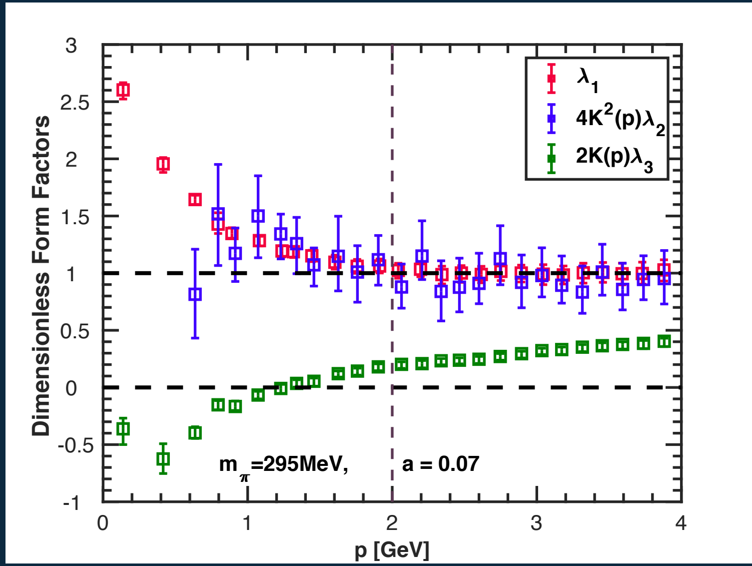


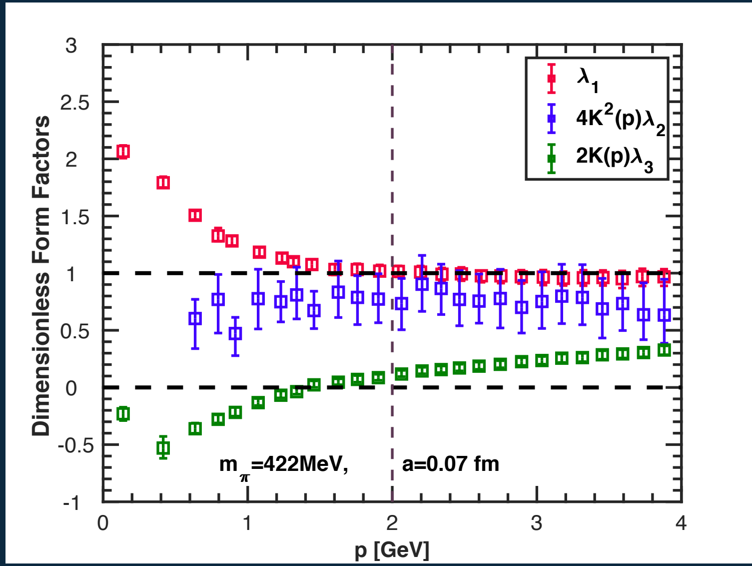
Dimensionless Form Factors

$$K_\mu(p) = \frac{1}{a}\,sin(p_\mu a)$$



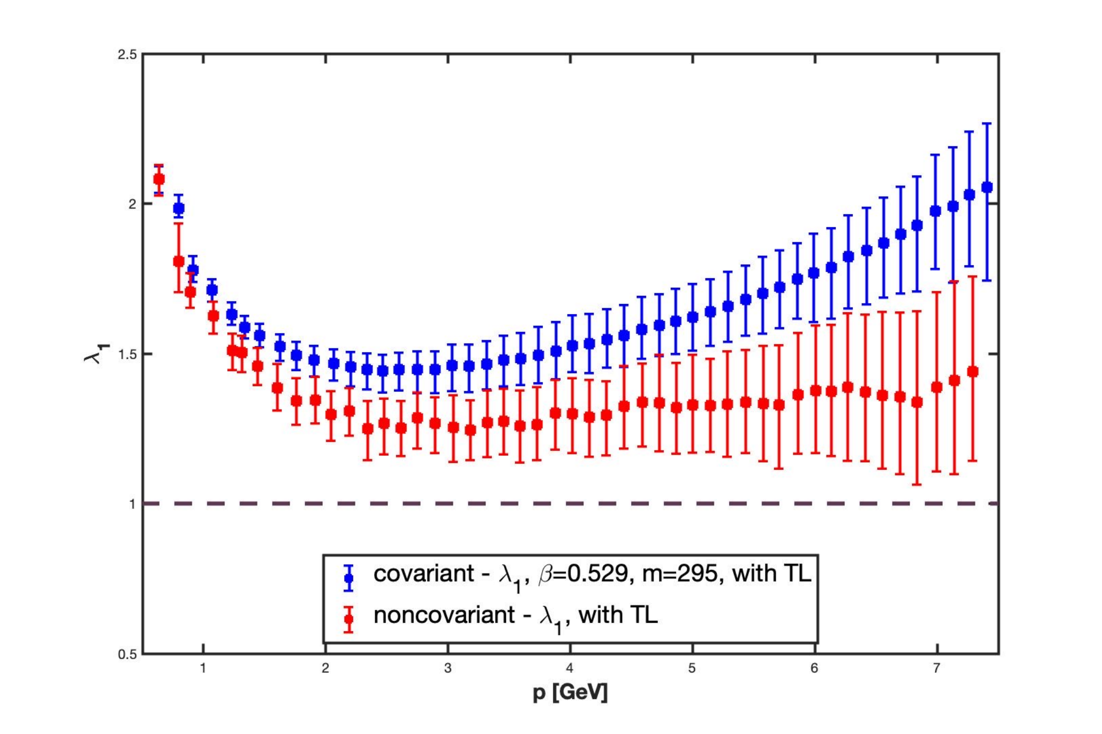


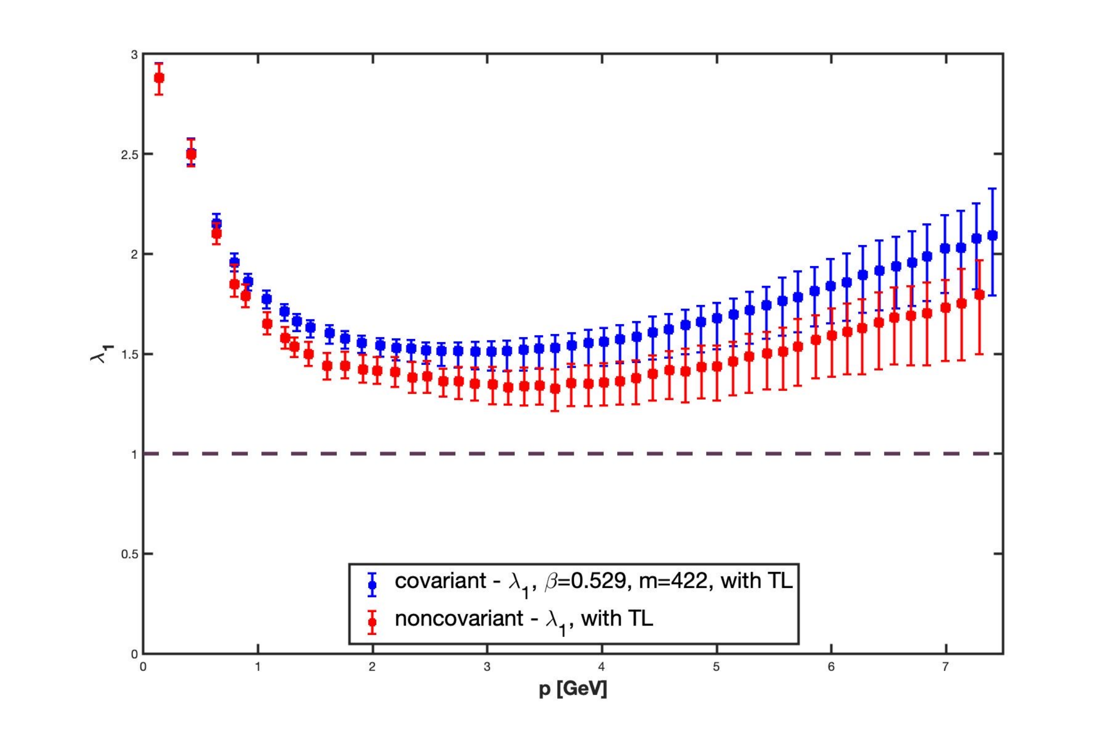




Covariant Form Factor

PRELIMINARY





CONCLUSION

First ever study of Quark-Gluon Vertex in Soft Gluon Kinematics for Landau Gauge w

N_f=2 dynamical fermions

Soft Gluon Kinematics : $(q_\mu = 0, k_\mu = p_\mu)$

$$\boldsymbol{\lambda_1, \lambda_2, \lambda_3, \lambda_4 = 0}$$

λ_1 is significantly enhanced in the IR this enhancement is stronger than in the quenched approximation and increases as the chiral limit is approached

λ_2 exhibits an infrared strength smaller than λ_1, the enhancement increases as the continuum, infinite-volume, and chiral limits are approached

λ_3 shows considerably infrared strength larger than in the quenched approximation, increases as the continuum limit is approach

Orthogonal Kinematics : $q \cdot P = 0$ $k^2 = p^2$ $\boldsymbol{\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \tau_6, \tau_5, \tau_7}$

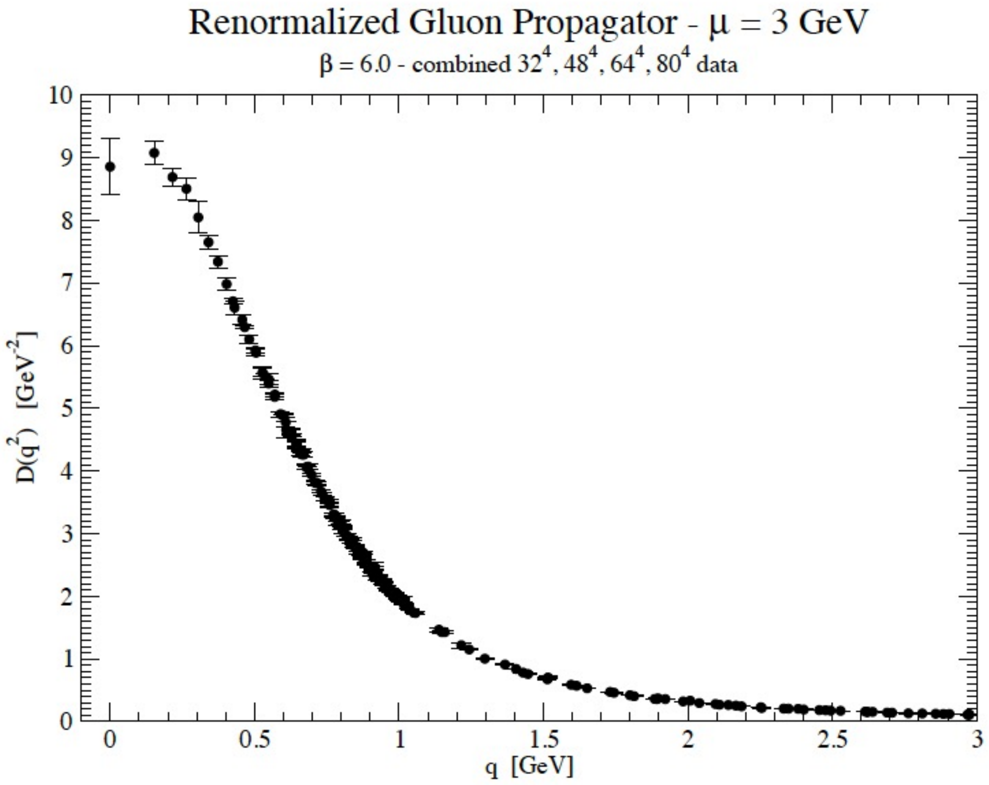
SUB CRUCE LUMEN

THE UNIVERSITY of ADELAIDE

Landau gauge

GLUON PROPAGATOR

O. Oliveira, P. Bicudo, J Phys G38, 045003 (2011)



GHOST PROPAGATOR

Attilio Cucchieri, David Dudal, Tereza Mendes,a Orlando Oliveira, Martin Roelfs and Paulo J. Silva, POS Lattice 2018, 252