July 16, 2021

Thermo-magnetic properties of QCD: Effective models constrained by lattice data

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APCTP Focus Program in Nuclear Physics 2021 Part I: Hadron properties in a nuclear medium from the quark and gluon degrees of freedom

Outline

- Motivation

- Thermo-magnetic effects on the coupling constants -> IMC

- The importance of implementing a proper regularization procedure in order to treat **thermo** and **magnetic** contributions within non renormalizable theories

- NJL -> Paramagnetic matter at high T

- Conclusions

Strong magnetic fields may be produced in off-central heavy ion collisions

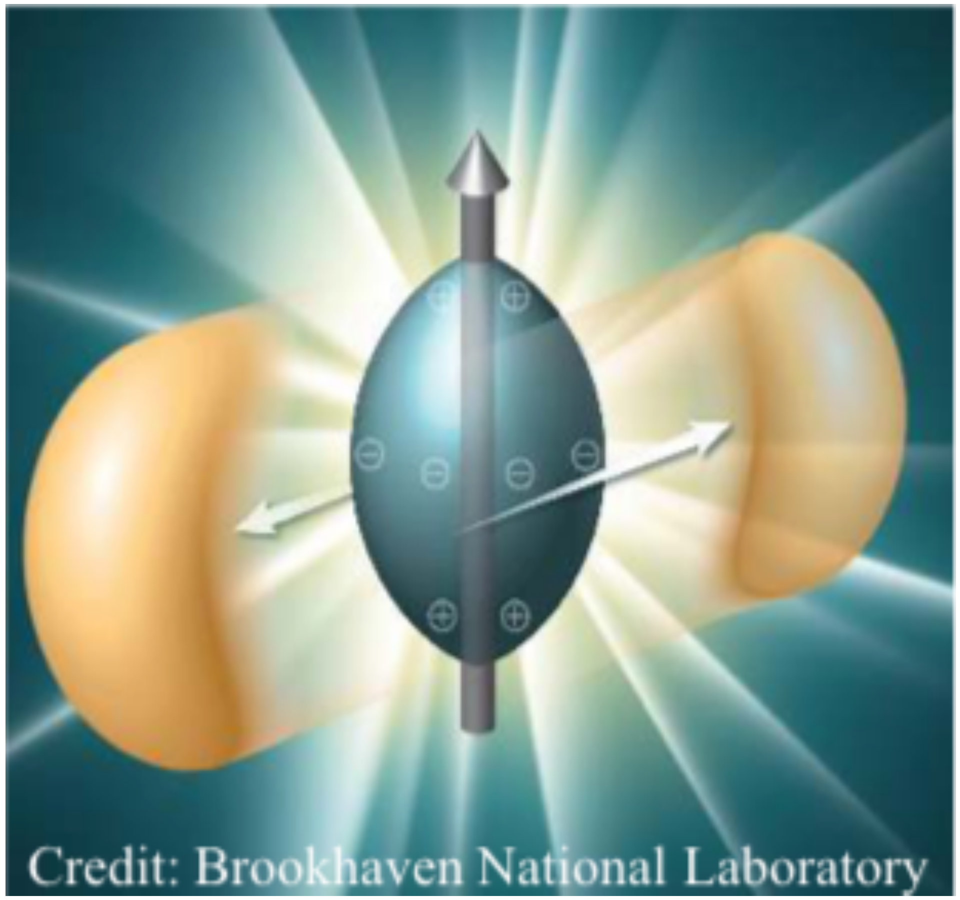
K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008). D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 80, 0304028 (2009). D. E. Kharzeev, Nucl. Phys. A 830, 543c (2009).

heavy-ion collisions:

temporarily $B \lesssim 10^{19}\,\mathrm{G}$

Skokov, Illarionov, Toneev, Int. J. Mod. Phys. A 24, 5925 (2009)

- Chiral magnetic effect
- Effects in EoS
- Anisotropies



Neutron Stars

- Strong magnetic fields are also present in magnetars:
  C. Kouveliotou et al., Nature 393, 235 (1998).

  magnetars:

  at surface $B \lesssim 10^{15}\,\mathrm{G}$

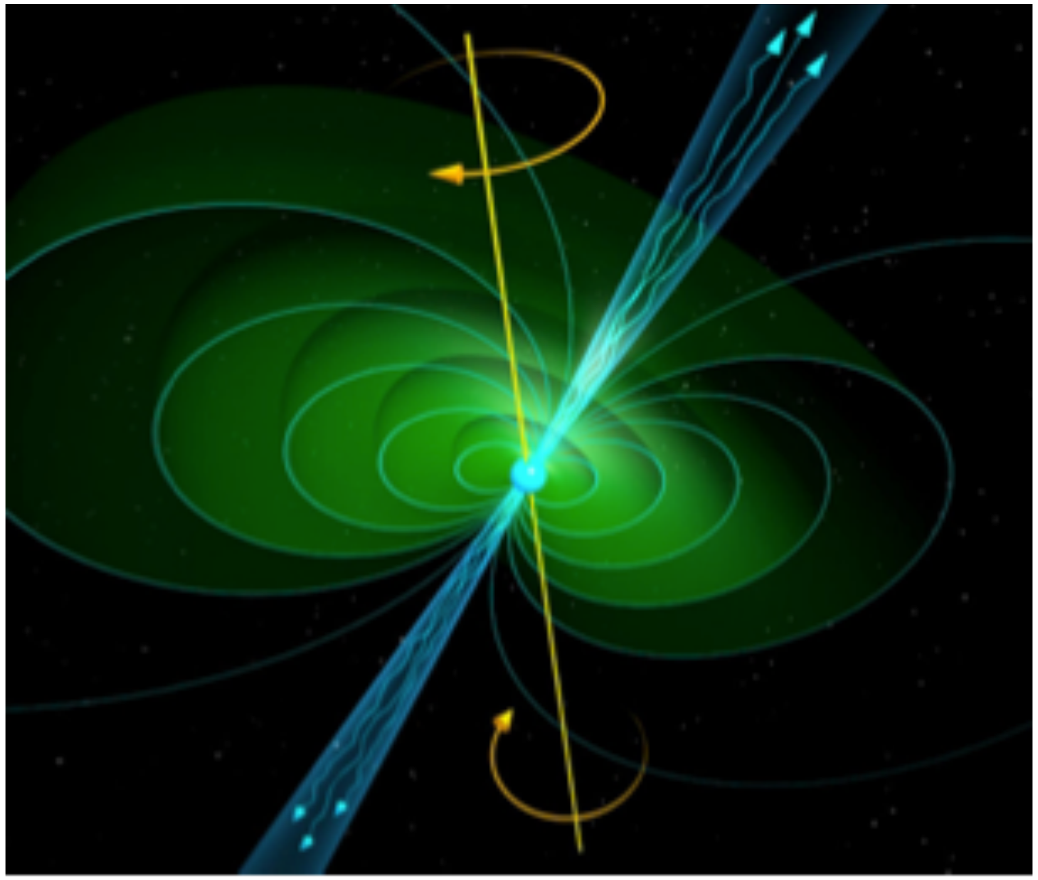
  Duncan, Thompson, Astrophys.J. 392, L9 (1992)

  larger in the interior,

  $B \sim 10^{18-20}\,\mathrm{G}$?

  Lai, Shapiro, Astrophys.J. 383, 745 (1991)

  E. J. Ferrer *et al.*, PRC 82, 065802 (2010)

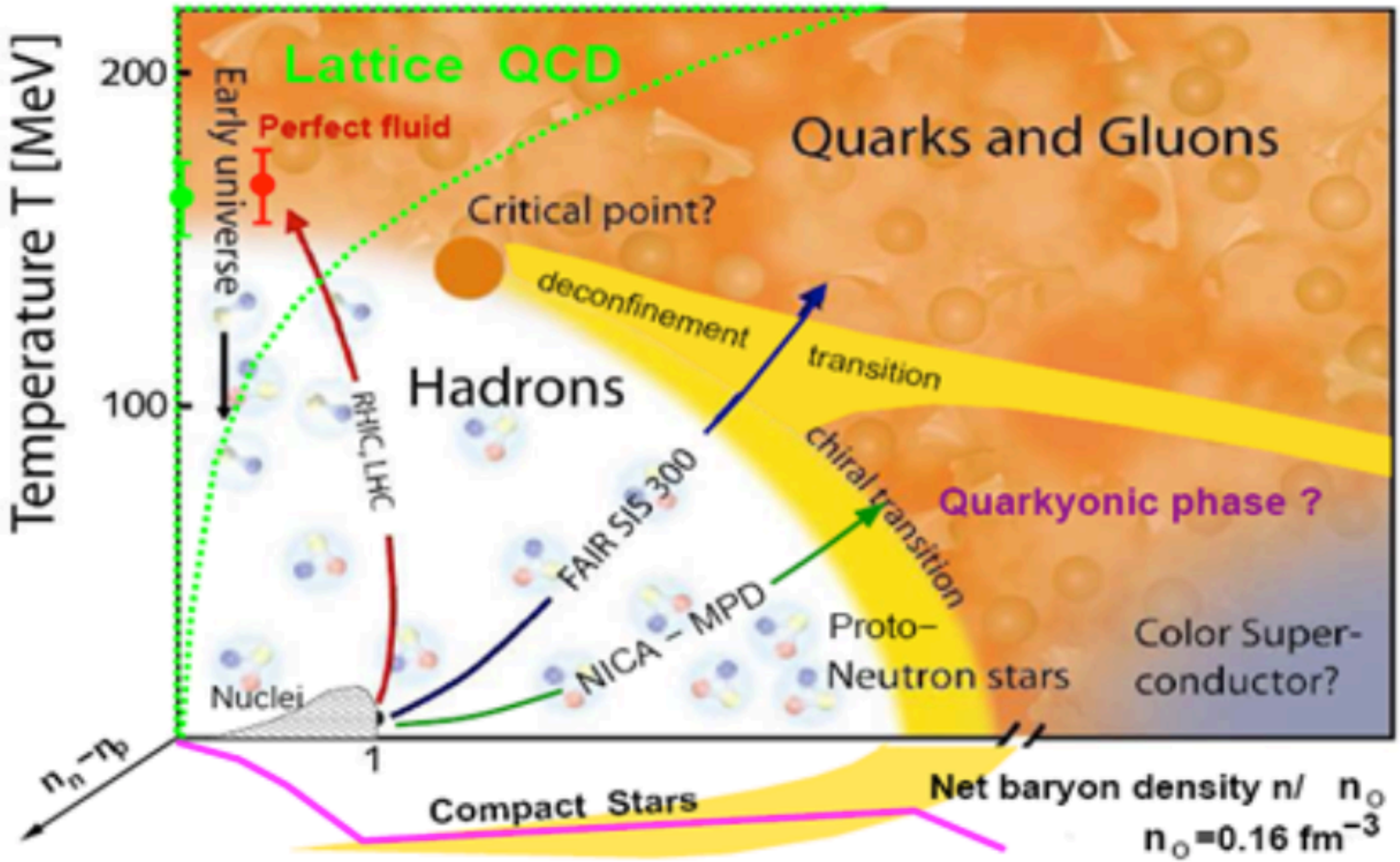


A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

- and might have played an important role in the physics of the early universe. T. Vaschapati, Phys. Lett. B 265, 258 (1991).

D. Grasso and H.R. Rubinstein, Phys. Rep. 348, 163 (2001).

QCD Phase Diagram



NICA - Nuclotron-based Ion Collider fAcility

FAIR - Facility for Antiproton and Ion Research

Lattice Results: Sign Problem

- fermion determinant is complex

$$[\det\ M(\mu)]^* = \det\ M(-\mu^*) \in \mathbb{C}$$

- no positive weight in path integral

$$Z = \int \mathcal{D}U\, e^{-S_{YM}}\ \det\ M(\mu)$$

- standard lattice methods base on importance sampling cannot be used!

To make progress

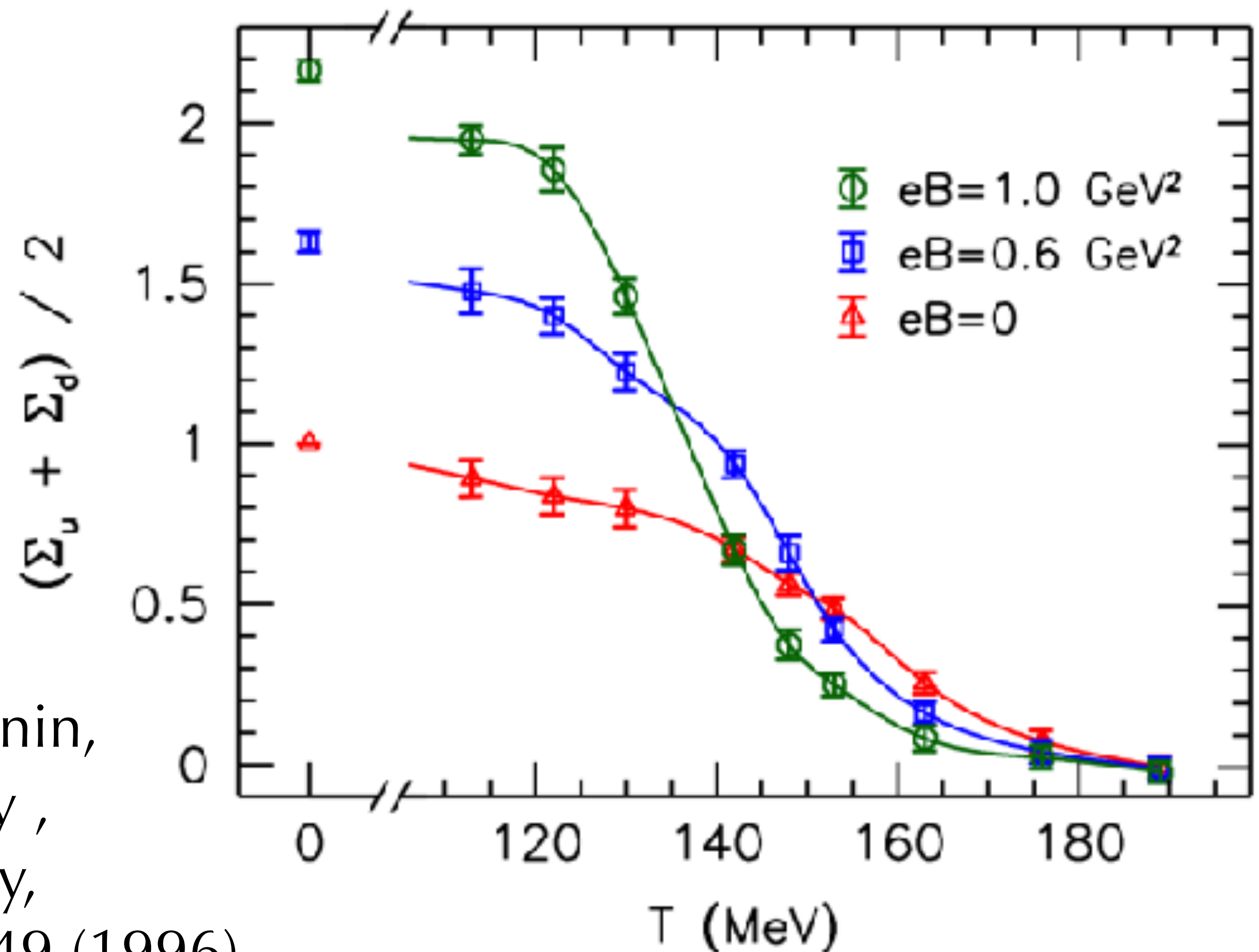
- We use Quantum Field Theory (in medium)
- DSE (beyond RL truncation…)
- Holographic models
- EOS + susceptibilities calculated on the lattice + Taylor series (limitations…)
- Effective models (just a few degrees of freedom): NJL/PNJL, Linear σ model, MIT,…

QCD at finite T and B : NO Sign Problem!

There is hope that lattice simulations of QCD with B can be used as a benchmark platform for comparing different effective models used in the literature.

B Effects on QCD phase transitions?

$$\Lambda_{QCD}^2 \sim (200\,\mathrm{MeV})^2 \sim 2 \times 10^{18}\,\mathrm{G}$$

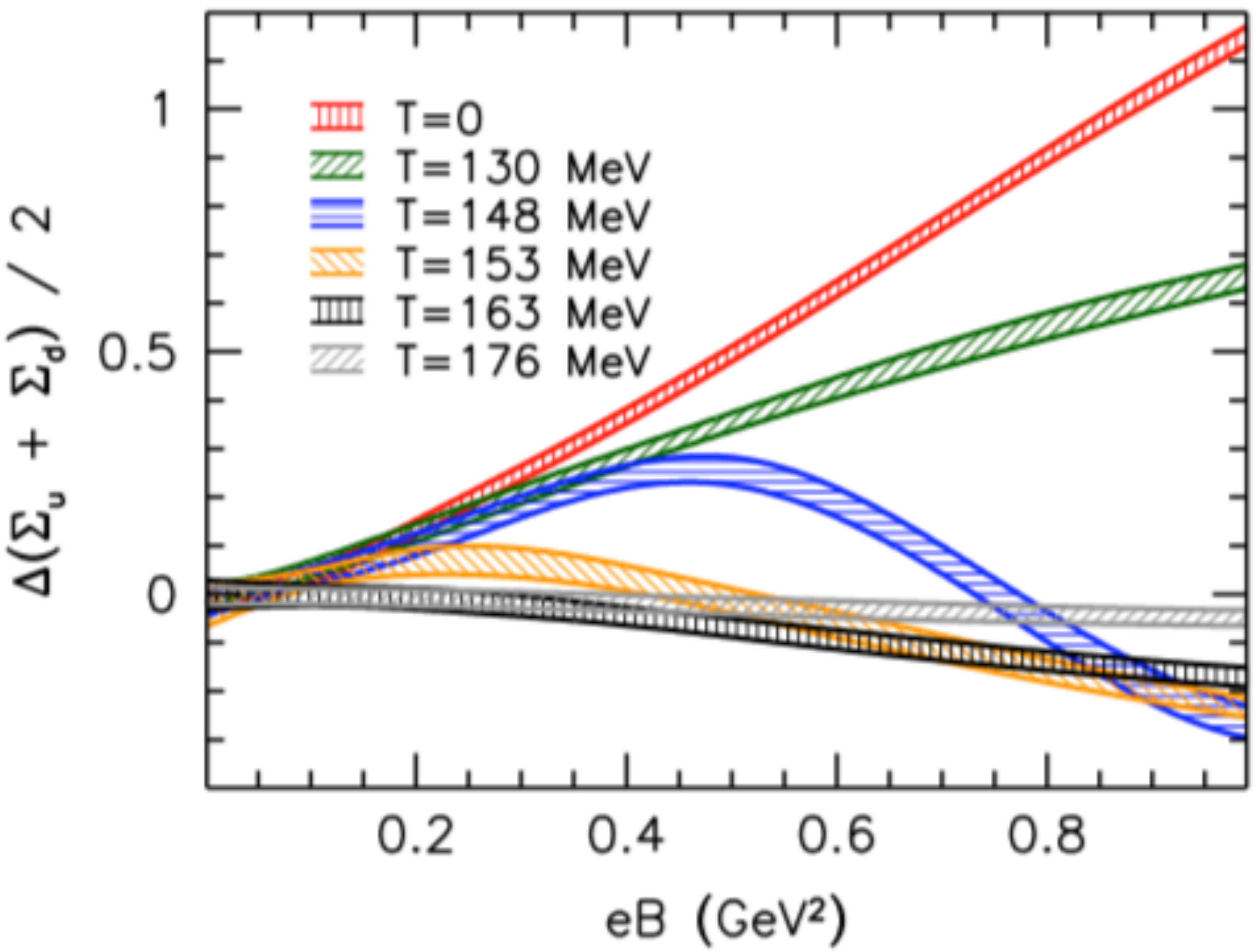


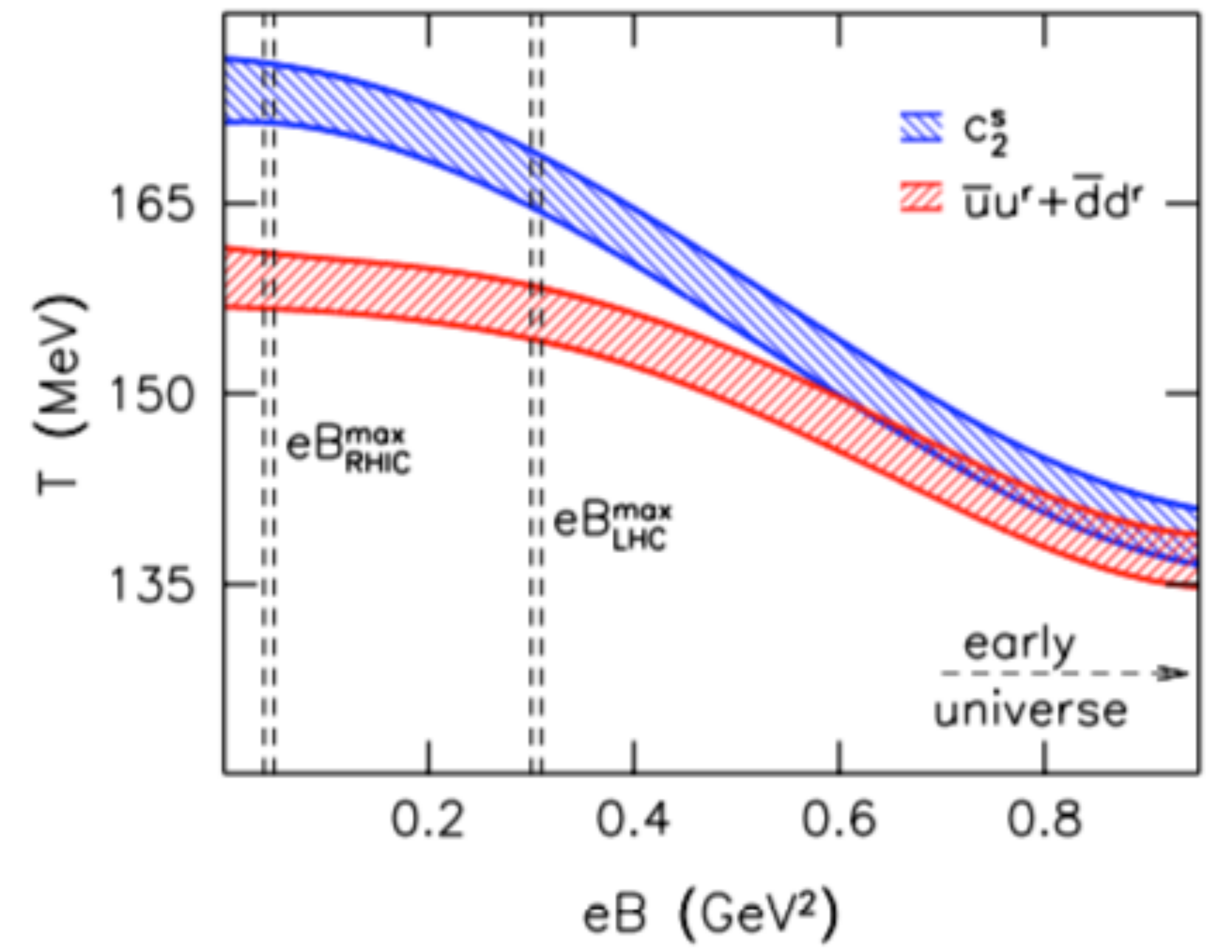
IMC: Bali, Bruckmann, Endrodi, Fodor, Katz et al. *JHEP* 02 (2012) 044 *Phys.Rev.D* 86 (2012) 071502

MC: V.P. Gusynin, V.A. Miransky , I.A. Shovkovy, *Nucl. Phys. B* **462** 249 (1996)

B Effects on QCD phase transitions?

MC and IMC



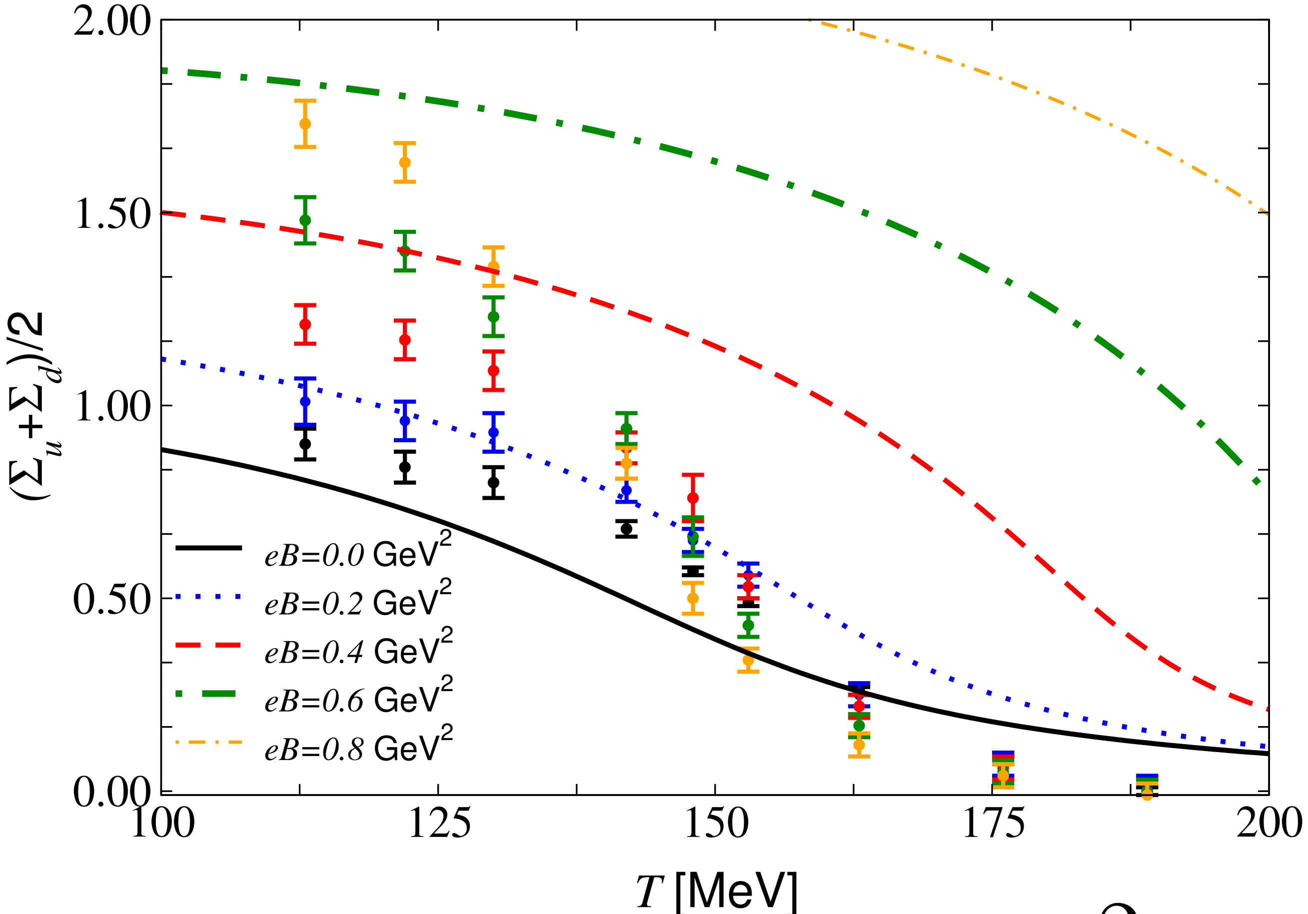


Failure of ALL effective models in providing inverse magnetic catalysis!

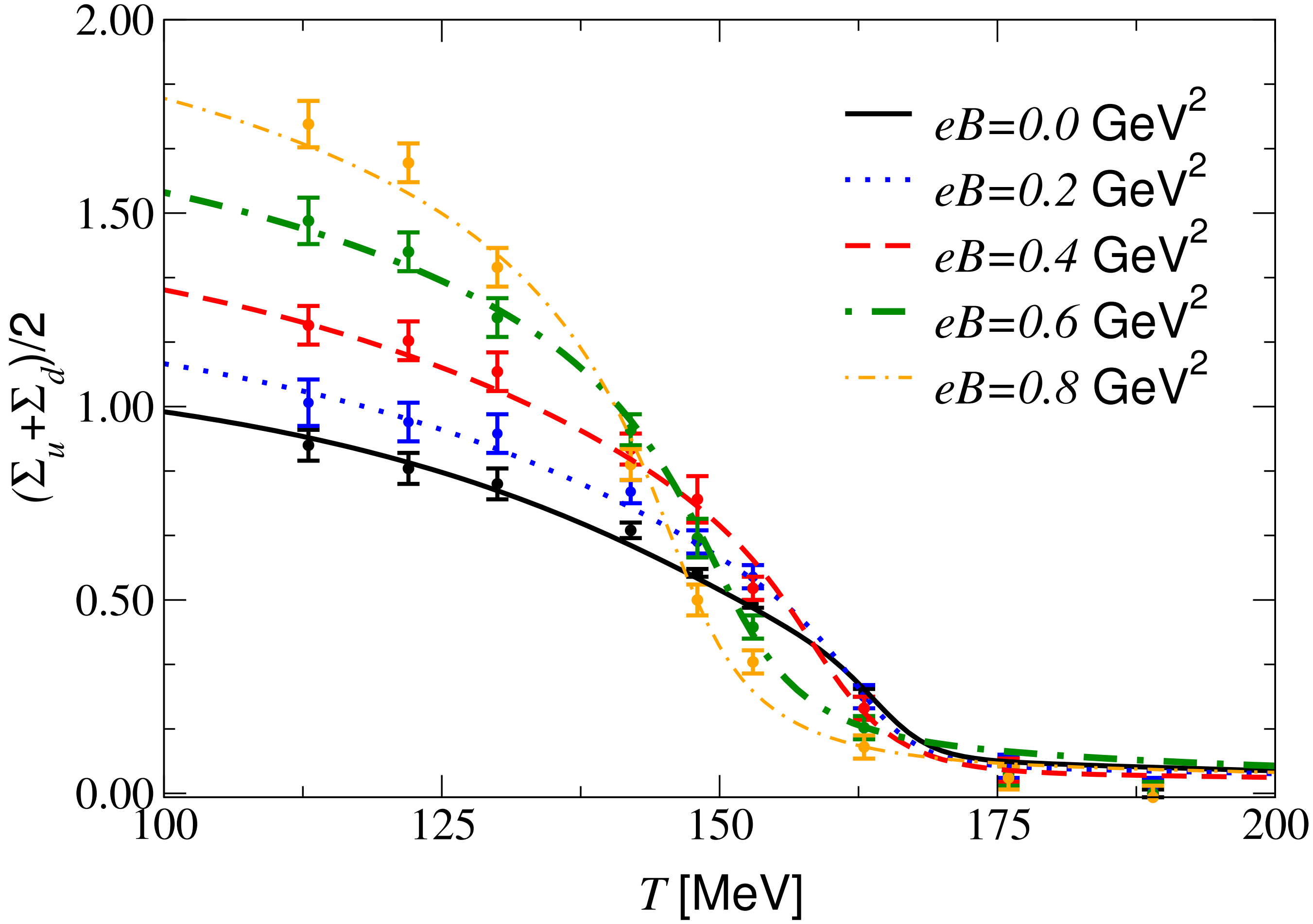
G. S. Bali et al., JHEP **1202**, 044 (2012)

SU(2) NJL + Thermo-Magnetic effects G(B,T)

G(0,0)



G(B,T)



$$\Sigma_f(B,T) = \frac{2m_f}{m_\pi^2 f_\pi^2}\left[\langle\bar{\psi}_f\psi_f\rangle - \langle\bar{\psi}_f\psi_f\rangle_0\right] + 1$$

RLSF, K.P. Gomes, M.B.Pinto, G. Krein, Phys. Rev. C **90**, 025203 (2014).

RLSF, V.S. Timoteo, S.S.Avancini,M.B.Pinto and G.Krein Eur. Phys. J. A (2017) **53**: 101

Thermo-Magnetic effects G(B,T)

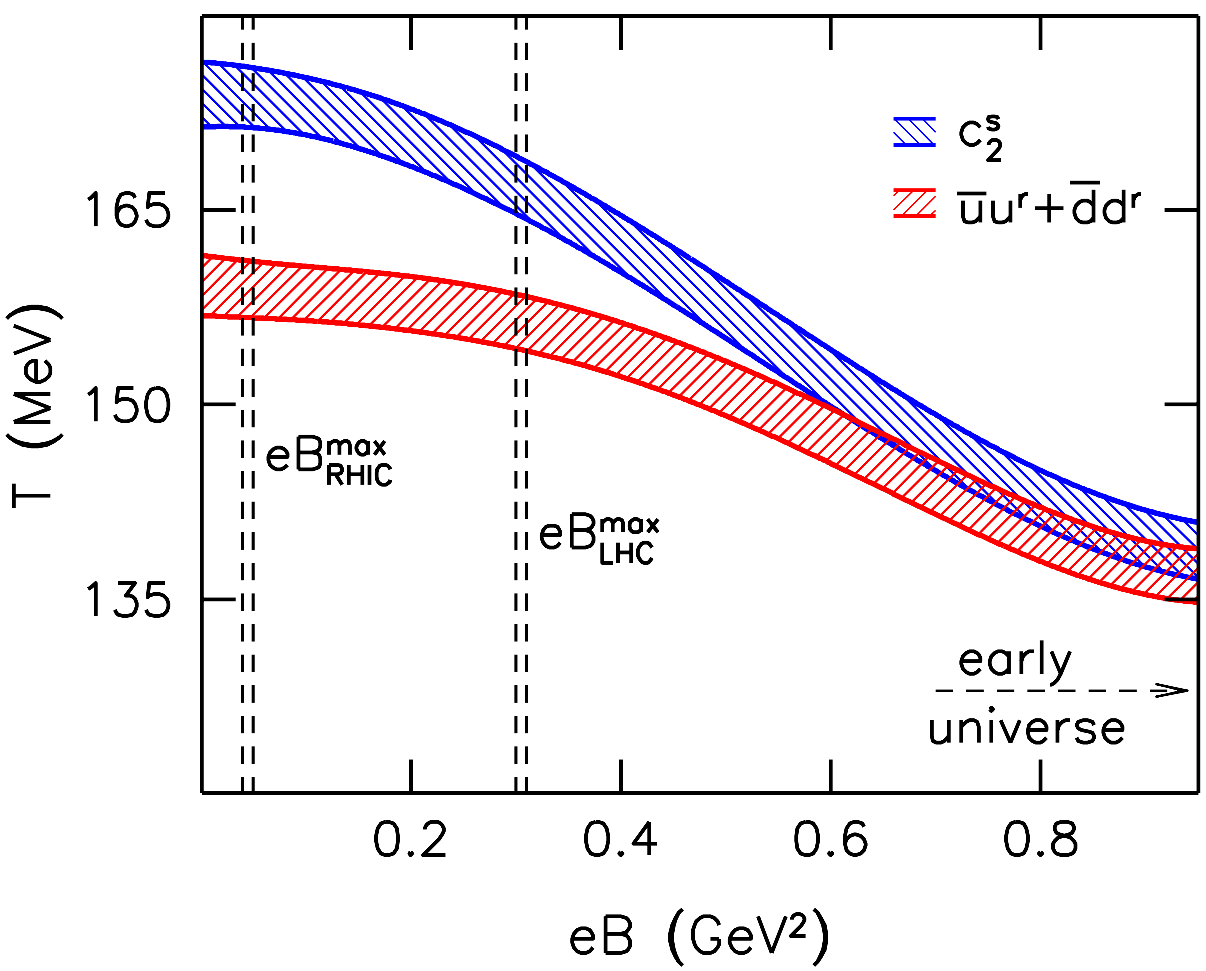
$$G(B,T) = c(B)\left[1 - \frac{1}{1 + e^{\beta(B)[T_a(B)-T]}}\right] + s(B)$$

eB [GeV^2]	c [GeV^{-2}]	T_a [MeV]	s [GeV^{-2}]	β [MeV^{-1}]
0.0	0.9000	168.000	3.73110	0.40000
0.2	1.2256	167.922	3.2621	0.34117
0.4	1.7693	169.176	2.2942	0.22988
0.6	0.7412	155.609	2.8638	0.14401
0.8	1.2887	157.816	1.8040	0.11506

RLSF, V.S. Timoteo, S.S.Avancini,M.B.Pinto and G.Krein Eur. Phys. J. A (2017) **53**: 101

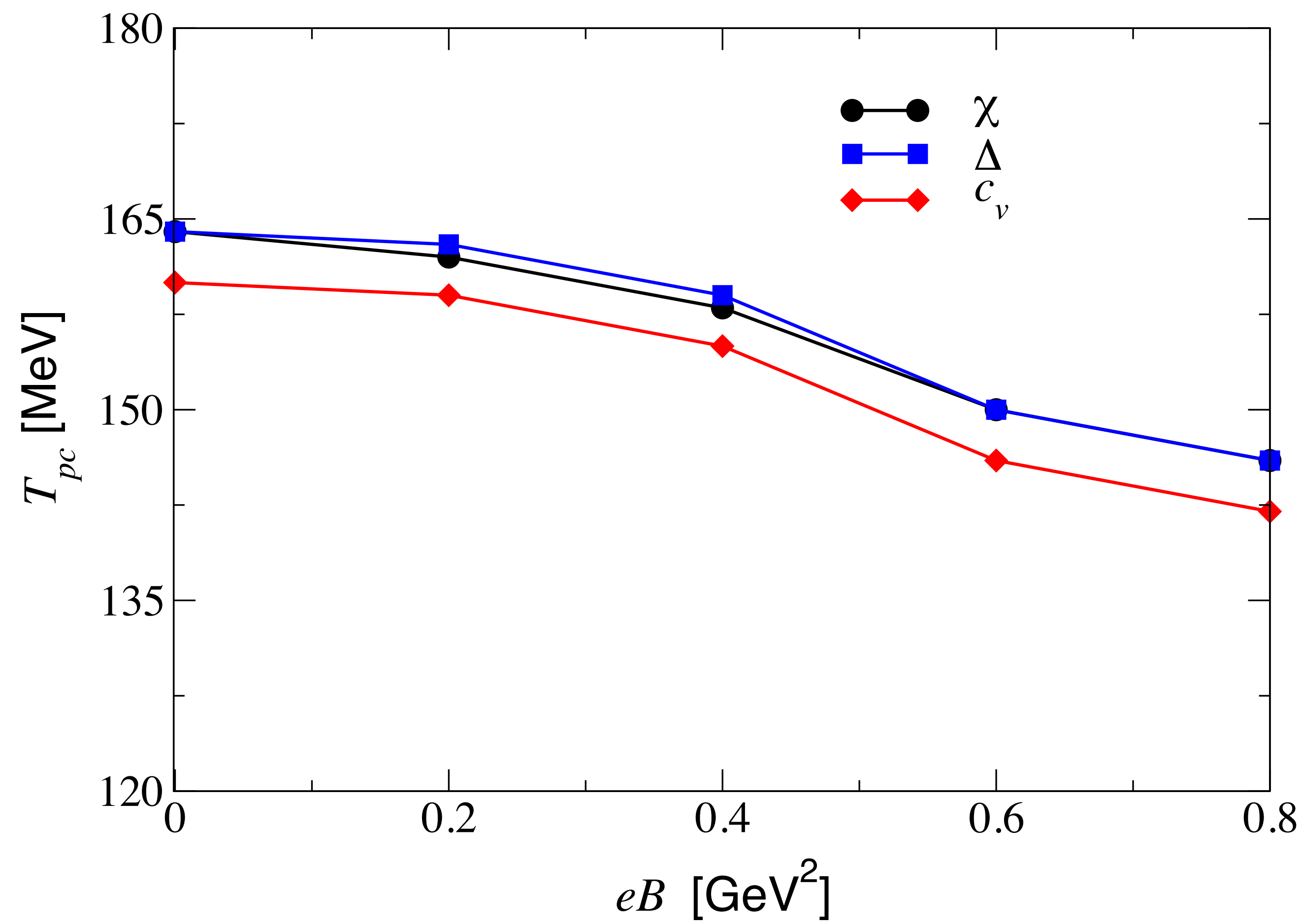
T_{pc} lattice X SU(2) NJL

Lattice



JHEP **1207** (2012) 056

G(B,T)



RLSF, V.S. Timoteo, S.S.Avancini,M.B.Pinto and G.Krein Eur. Phys. J. A (2017) **53**: 101

G(B,T) **Thermo-magnetic effects!**

B Effects on QCD phase transitions?

Inverse magnetic catalysis: how much do we know about?

A. Bandyopadhyay, R.L.S. Farias, *Eur. Phys. J. ST* 230 (2021) 3, 719-728,

B. e-Print: 2003.11054 [hep-ph]

Possible explanations for IMC:

- Competition of B effects on sea and valence quarks, F. Bruckmann, G. Endrodi, T. G. Kovacs, *JHEP* 04 (2013) 112

- Inclusion of plasma screening effects that capture the physics of collective, long-wave modes, and thus describe a prime property of plasmas near transition lines, namely, long distance correlations. e-Print: 2104.05854 [hep-ph], arXiv:2107.05521

IMC $\neq$ Tc decreasing with eB

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L}_{NJL} = \bar{\psi}\left(\not{D} - m\right)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

$$D^\mu = (i\partial^\mu - QA^\mu)$$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

Q=diag(q_u= $2e/3$, q_d=-$e/3$)

good **chiral** physics, pions,…
BUT no confinement

✓ **strong magnetic field background that is constant and homogeneous!**

G, Λ and m_c → m_π, f_π and $\langle\bar{\psi}\psi\rangle$

natural units: $1\text{GeV}^2 \approx 5.34 \times 10^{19}\,\text{G}$ and $e = \sqrt{\frac{4\pi}{137}}$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

NJL at finite B

At B=0

$$\mathcal{F} = \frac{(M-m_c)^2}{4G} - N_c \sum_{f,s} \int \frac{d^4p}{(2\pi)^4} \ln\left[p^2 + M^2\right]$$

By using the replacement $\vec{p}^{\,2} \to p_3^2 + 2k|q_f|B$

$$\int \frac{d^4p}{(2\pi)^4} \to \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \sum_{k=0}^{\infty} \alpha_k$$

$$\alpha_k = 2 - \delta_{k0}$$

$$\begin{aligned} \mathcal{F} &= \frac{(M-m_c)^2}{4G} \\ &- N_c \sum_f \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \ln\left[p_4^2 + p_3^2 + 2k|q_f|B + M^2\right] \end{aligned}$$

And the gap equation: $\partial\mathcal{F}/\partial M = 0 \longrightarrow \infty$

We need a regularization procedure!

Which procedure/method is more appropriate?

Is there any criteria?

MFIR - Magnetic Field Independent Regularization

- ✓ D. Ebert and K.G. Klimenko, Nucl. Phys. **A728**, 203 (2003).
- ✓ D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providência, Phys. Rev. C **79**, 035807 (2009).
- ✓ P. G. Allen, A. G. Grunfeld, and N. N. Scoccola, Phys. Rev. D **92**, 074041 (2015).
- ✓ D.C.Duarte, P.G.Allen, R.L.S.Farias, P.H.A.Manso,R.O.Ramos,and N. N. Scoccola, Phys. Rev. D **93**, 025017 (2016).
- ✓ S. S. Avancini, W. R. Tavares, and M. B. Pinto, Phys. Rev. D **93**, 014010 (2016).
- ✓...

Noncovariant Regularizations

Form factors: $$\sum_{k=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi} \to \sum_{k=0}^{\infty}\int_{-\infty}^{\infty}\frac{dp_3}{2\pi}\, U_\Lambda(p_3^2 + 2k|q_f|B)$$

✓ Lorenztian: $$U_\Lambda^{(Lor N)}(x) = \left[1 + \left(\frac{x}{\Lambda^2}\right)^N\right]^{-1}$$

✓ Wood-Saxon: $$U_\Lambda^{(WS\alpha)}(x) = \left[1 + \exp\left(\frac{x/\Lambda^2 - 1}{\alpha}\right)\right]^{-1}$$

✓ Fermi-Dirac: $$U_\Lambda^{\mathrm{FD}}(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{\frac{x}{\Lambda} - 1}{\alpha}\right)\right]$$

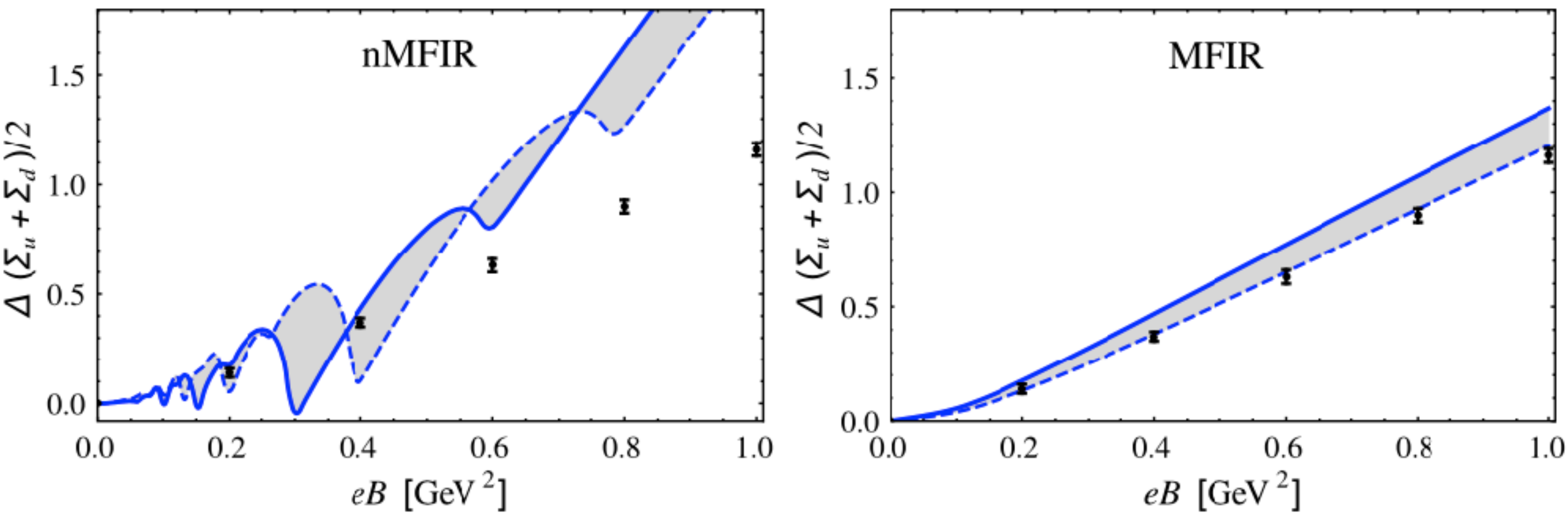
✓ 3D sharp cutoff

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Fermi-Dirac Form Factor

$$U_\Lambda^{\mathrm{FD}}(x) = \frac{1}{2}\left[1 - \tanh\left(\frac{\frac{x}{\Lambda} - 1}{\alpha}\right)\right]$$

$$245\,\mathrm{MeV} < -\bar{\Phi}_0^{1/3} < 260\,\mathrm{MeV}$$



Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Covariant Regularizations:

- ✓ **4D sharp cutoff**
- ✓ **Proper time**
- ✓ **Pauli-Villars**

RLSF, S.S. Avancini, N. Scoccola, W.R. Tavares, PRD **99**, 116002 (2019).

Magnetized baryons and the QCD phase diagram: NJL model meets the lattice

jhep.pdf (página 1 de 17)

G. Endrődi[a] and G. Markó[b]

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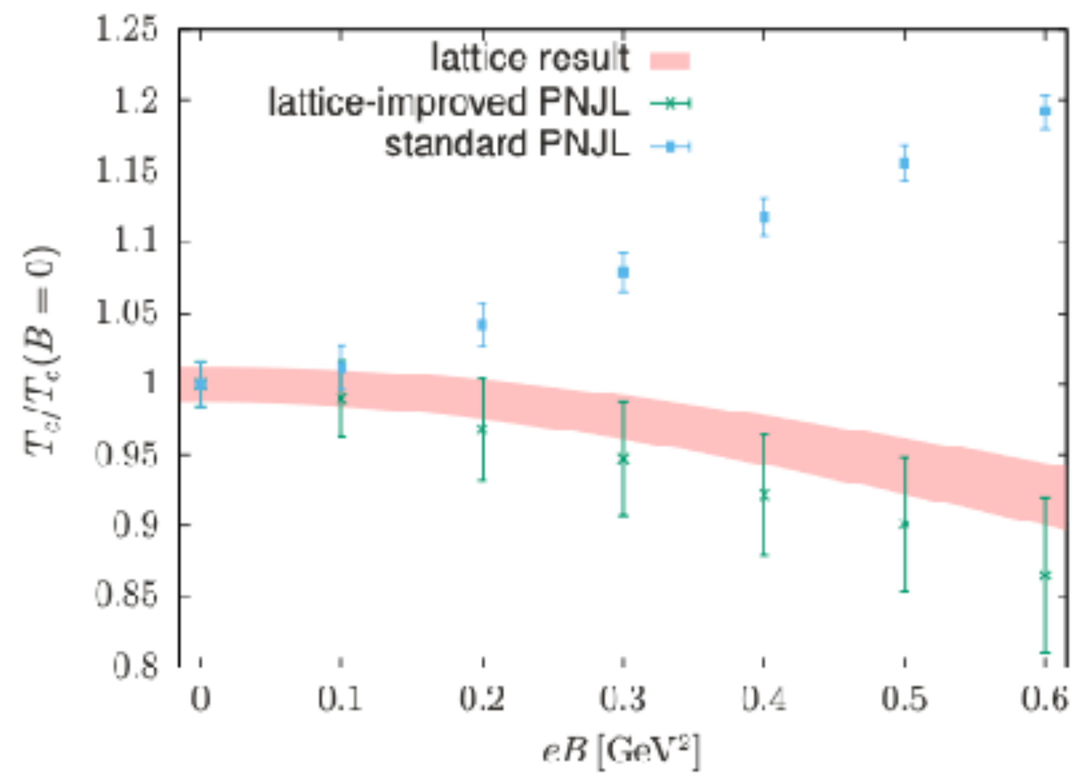
[b] *Eötvös University, Department of Theoretical Physics, Pázmány P. s. 1/A, H-1117 Budapest, Hungary*



G(eB) in PNJL SU(2) -> IMC

Their assumption is that the baryon masses can be obtained by merely summing the masses of their constituents + TRPT

G. Endrodi and G. Marko, JHEP **08**, 036 (2019).

TRPT and SPT frameworks

$$\Omega_{TRPT}(M,\Phi,T,B) = \mathcal{U}(\Phi,T) + \frac{(M-m_c)^2}{4G} + \frac{N_c}{8\pi^2}\sum_{f=u,d}(|q_f|B)^2\int_{\frac{|q_f|B}{\Lambda^2}}^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)$$

$$+\frac{1}{8\pi^2}\sum_{f=u,d}(|q_f|B)^2\int_{\frac{|q_f|B}{\Lambda^2}}^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)\left\{2\sum_{n=1}^{\infty}e^{-\frac{|q_f|Bn^2}{4sT^2}}(-1)^n\left[2\cos\left(n\cos^{-1}\frac{3\Phi-1}{2}\right)+1\right]\right\}$$

In SPT: $\dfrac{|q_f|B}{\Lambda^2} \to 0$

Entangled vacuum and magnetic contributions

G. Endrodi and G. Marko, JHEP **08**, 036 (2019).

VMR and MFIR frameworks

vacuum magnetic regularization

$$\frac{N_c}{8\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\int_{\frac{|q_f|B}{\Lambda^2}}^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)$$

$$=\frac{N_c N_f}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{ds}{s^3}e^{-M^2 s}$$

$$+\frac{N_c}{24\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\left[\ln\left(\frac{\Lambda^2}{2|q_f|B}\right)+1-\gamma_E\right]$$

usually neglected

$$-N_c\sum_{f=u,d}\frac{(|q_f|B)^2}{2\pi^2}\left[\zeta'(-1,x_f)-\left[x_f^2-x_f\right]\frac{\ln x_f}{2}+\frac{x_f^2}{4}\right]$$

where $x_f=\frac{M^2}{2|q_f|B}$, and ζ represents the Hurwitz-Riemann zeta function

RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

VMR and MFIR frameworks

$$\Omega_{VMR}(M,\Phi,T,B) = \mathcal{U}\left(\Phi,T\right) + \frac{(M-m_c)^2}{4G} + \frac{N_c N_f}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{ds}{s^3}e^{-M^2 s}$$

$$+\frac{N_c}{24\pi^2}\sum_{q_f=u,d}(|q_f|B)^2\left[\ln\left(\frac{\Lambda^2}{2|q_f|B}\right)+1-\gamma_E\right]$$

$$-N_c\sum_{f=u,d}\frac{(|q_f|B)^2}{2\pi^2}\left[\zeta'\left(-1,x_f\right)-\frac{1}{2}\left[x_f^2-x_f\right]\ln x_f+\frac{x_f^2}{4}\right]$$

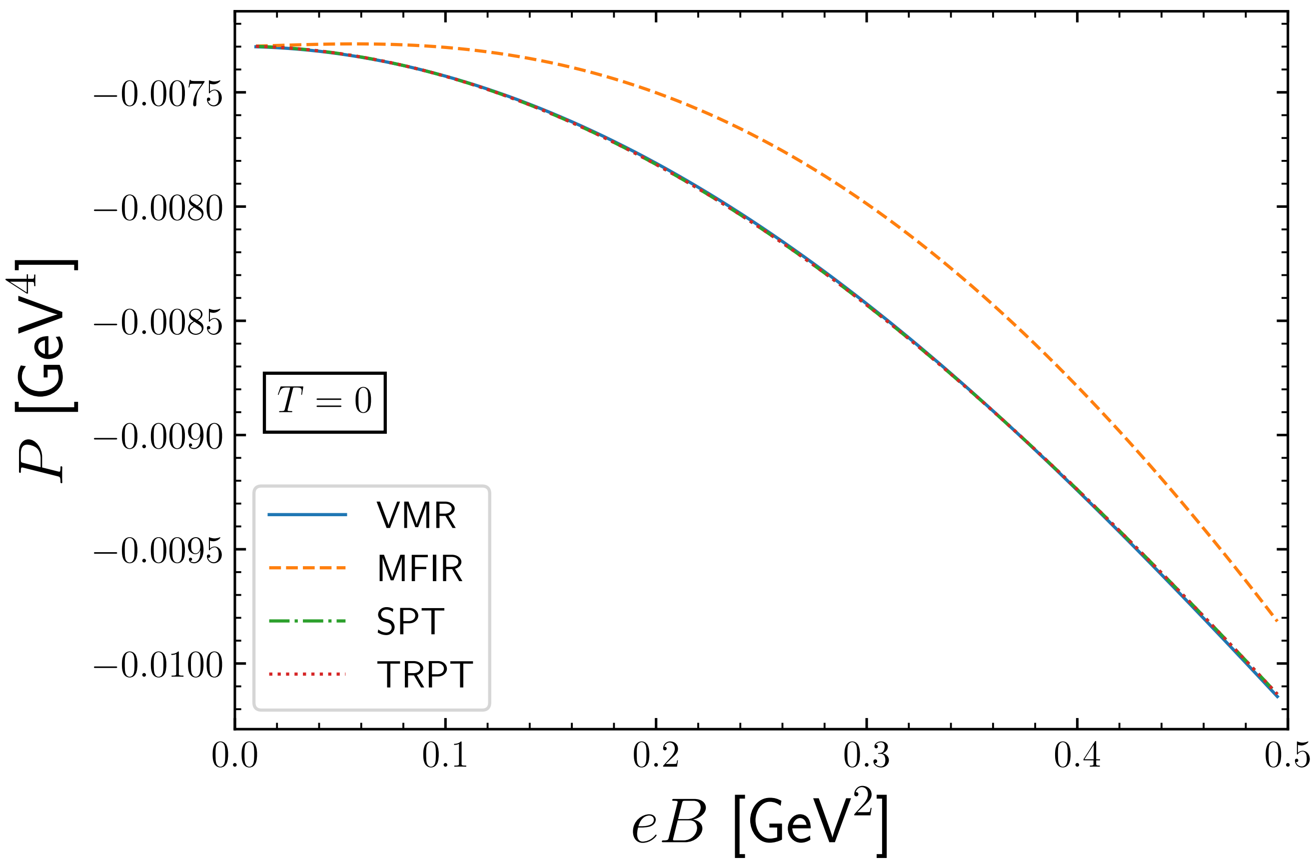
$$+\frac{1}{8\pi^2}\sum_{f=u,d}(|q_f|B)^2\int_0^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)\left\{2\sum_{n=1}^{\infty}e^{-\frac{|q_f|Bn^2}{4sT^2}}(-1)^n\left[2\cos\left(n\cos^{-1}\frac{3\Phi-1}{2}\right)+1\right]\right\}$$

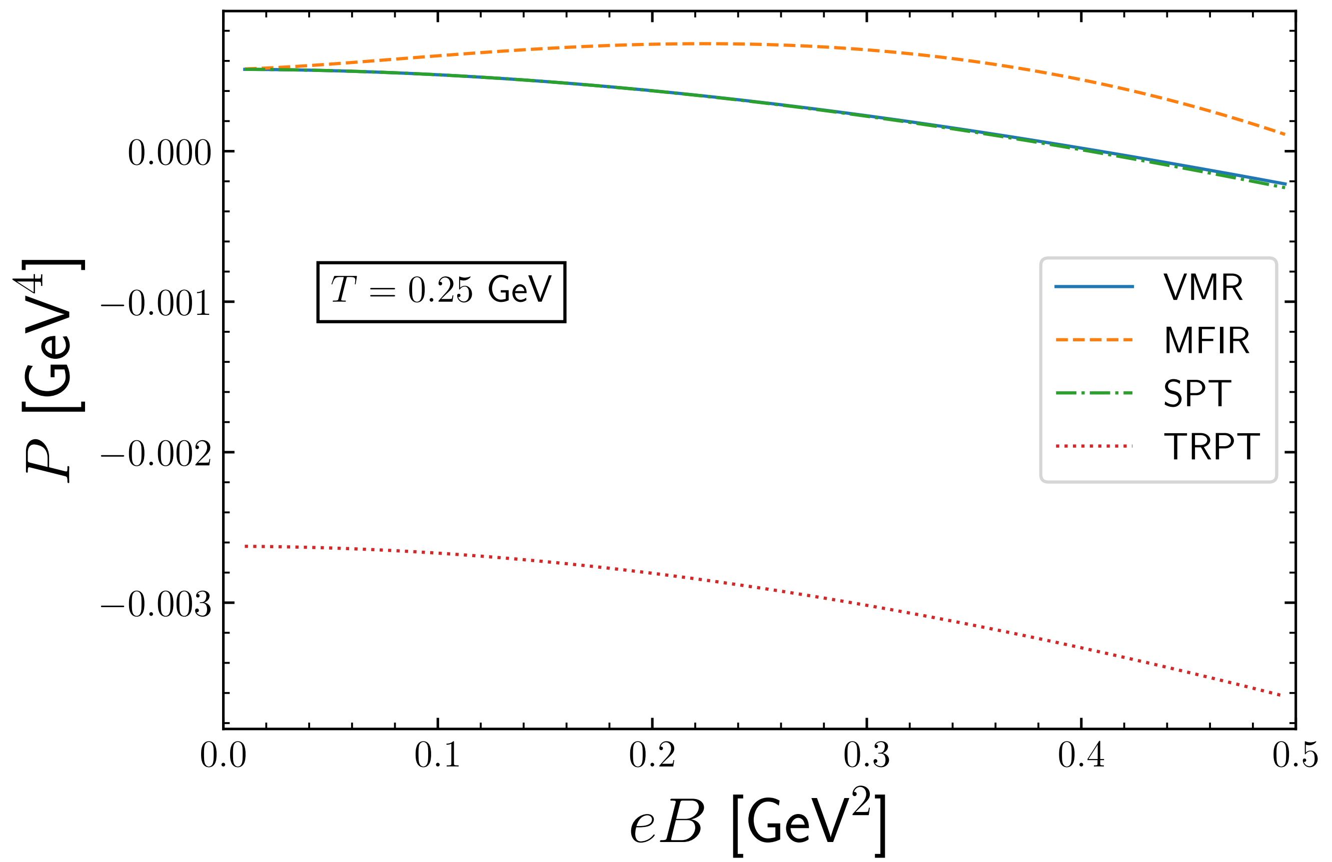
$$\Omega_{MFIR}(M,\Phi,T,B) = \mathcal{U}\left(\Phi,T\right) + \frac{(M-m_c)^2}{4G} + \frac{N_c N_f}{8\pi^2}\int_{\frac{1}{\Lambda^2}}^{\infty}\frac{ds}{s^3}e^{-M^2 s}$$

$$-N_c\sum_{f=u,d}\frac{(|q_f|B)^2}{2\pi^2}\left[\zeta'\left(-1,x_f\right)-\frac{1}{2}\left[x_f^2-x_f\right]\ln x_f+\frac{x_f^2}{4}\right]$$

$$+\frac{1}{8\pi^2}\sum_{f=u,d}(|q_f|B)^2\int_0^{\infty}\frac{ds}{s^2}e^{-\frac{M^2 s}{|q_f|B}}\coth(s)\left\{2\sum_{n=1}^{\infty}e^{-\frac{|q_f|Bn^2}{4sT^2}}(-1)^n\left[2\cos\left(n\cos^{-1}\frac{3\Phi-1}{2}\right)+1\right]\right\}$$

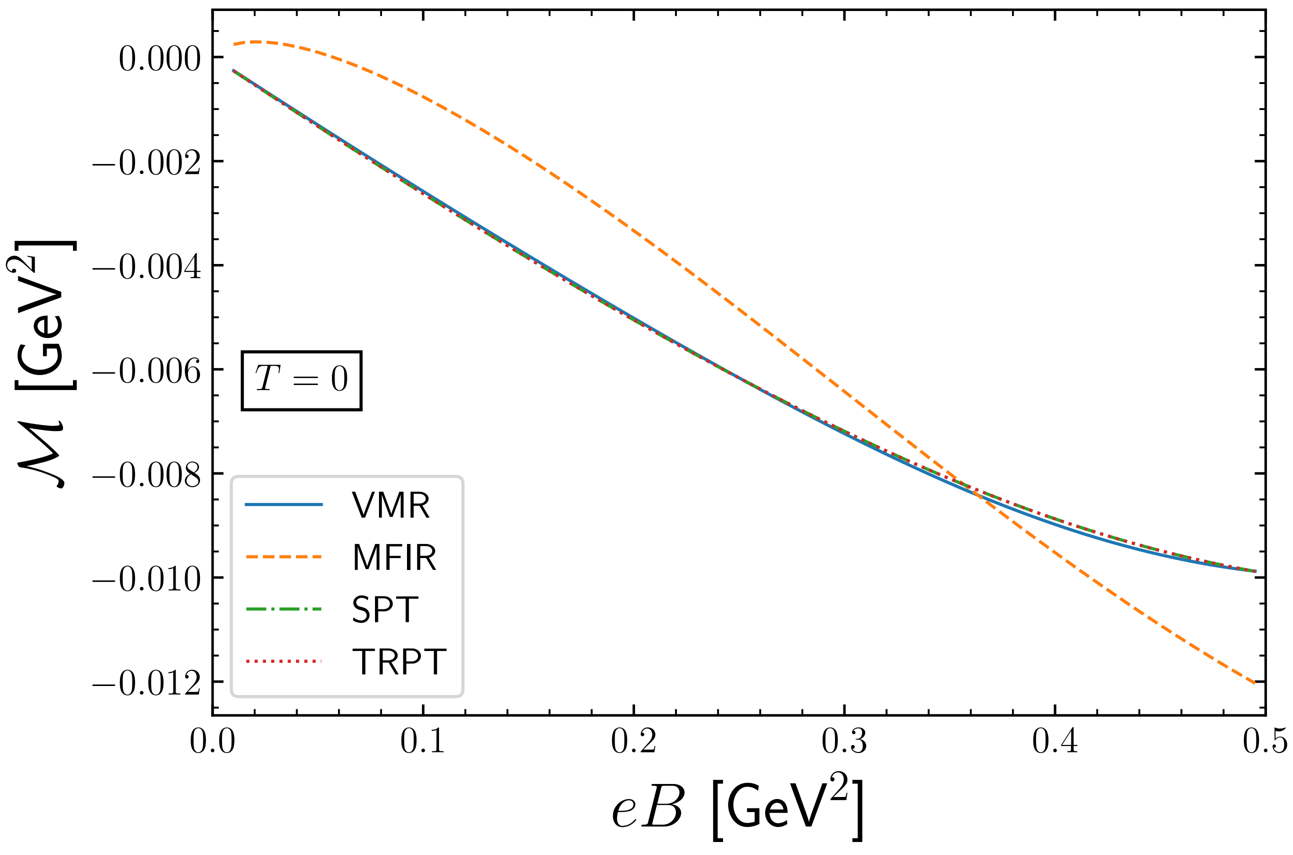
Pressure

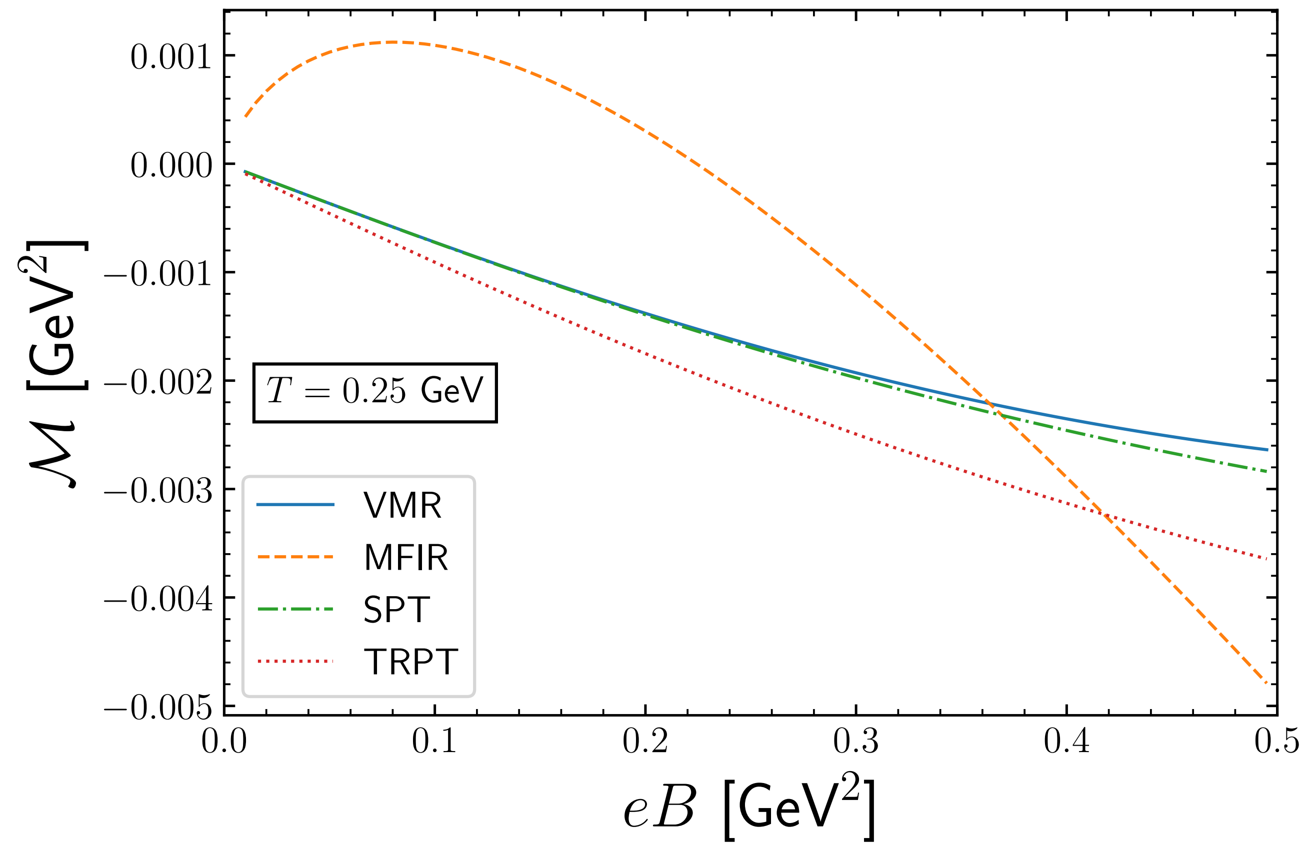




RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

Magnetization





$$\mathcal{M} = \frac{\partial P}{\partial(eB)}$$

Diamagnetic behavior?

RLSF, S.S. Avancini, M.B.Pinto, T.E. restrepo and W.R. Tavares, PRD **103**, 056009 (2021)

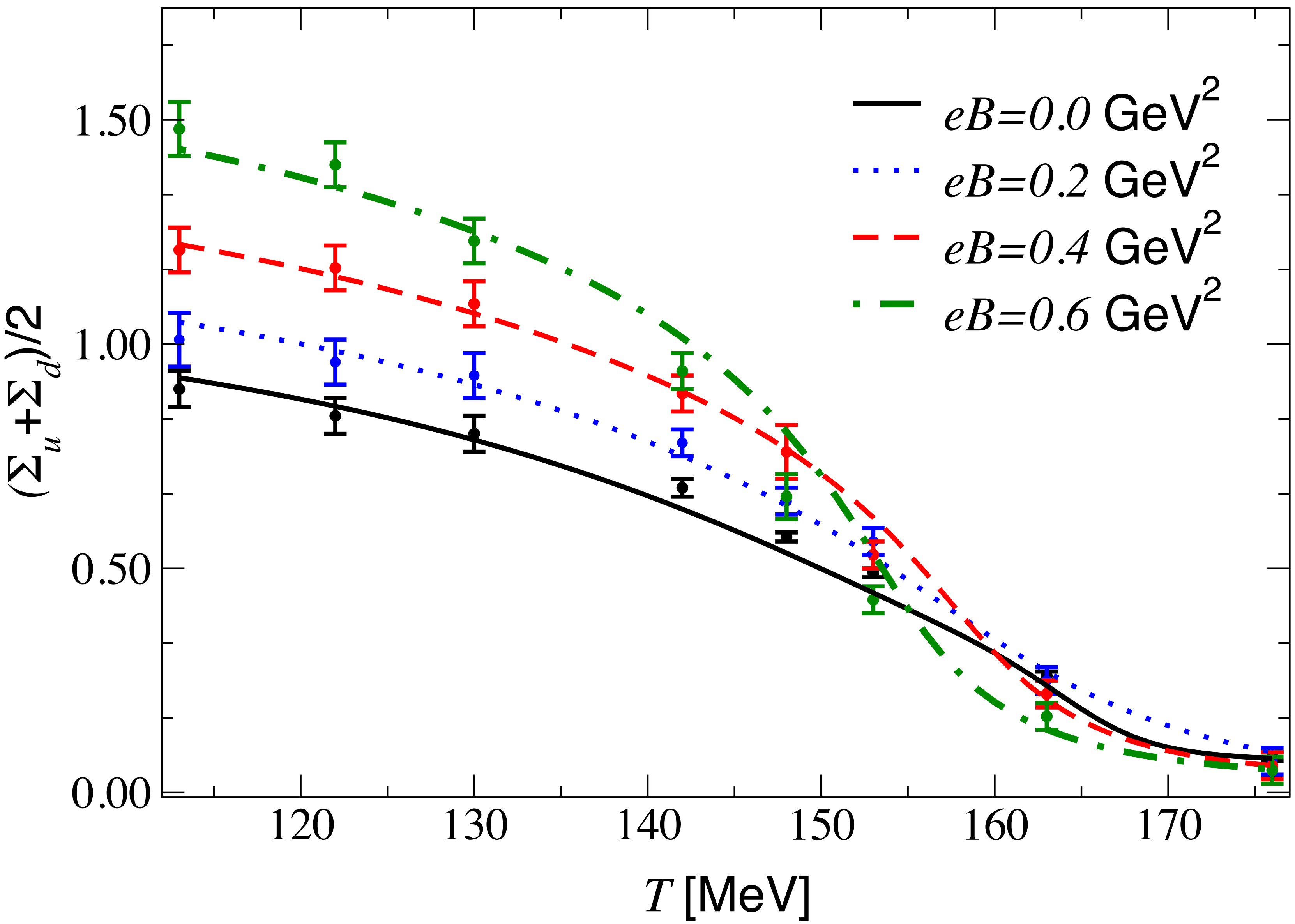
Renormalized magnetization

At this point, a digression concerning the magnetic character of the QCD vacuum is in order since **LQCD evaluations, at T = 0, have shown that the vacuum is paramagnetic in contradiction to our present findings.**

The VMR can indeed be reconciled with the LQCD results provided that one uses the same definition for the renormalized magnetization

$$\mathcal{M}^r \cdot eB = \mathcal{M} \cdot eB - (eB)^2 \lim_{eB \to 0} \frac{\mathcal{M} \cdot eB}{(eB)^2}\bigg|_{T=0}$$

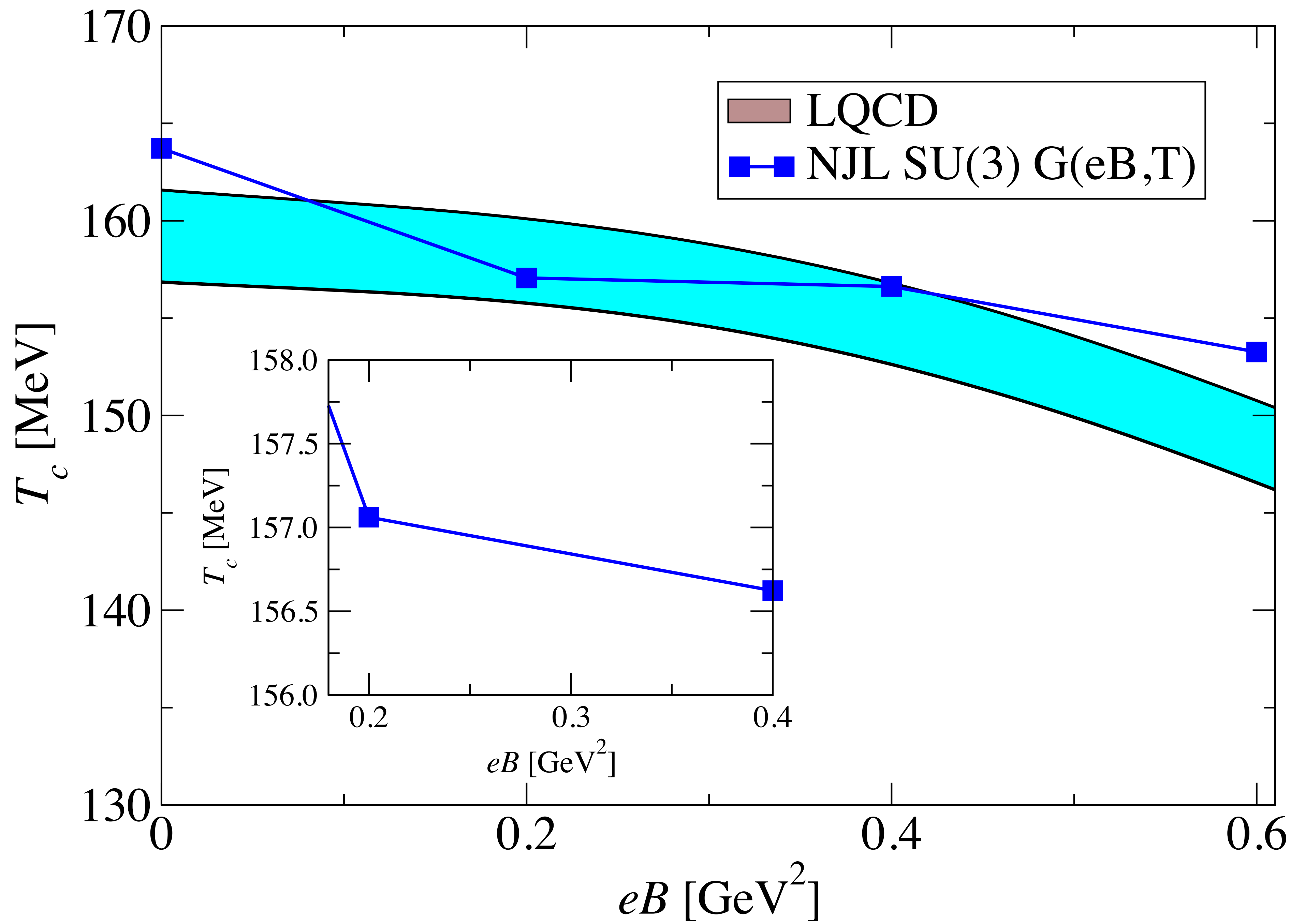
G(eB,T) in SU(3) NJL model



Lattice data: G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz and A. Schafer, Phys. Rev. D **86**, 071502(R) (2012)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, e-Print: 2104.11117 [hep-ph]

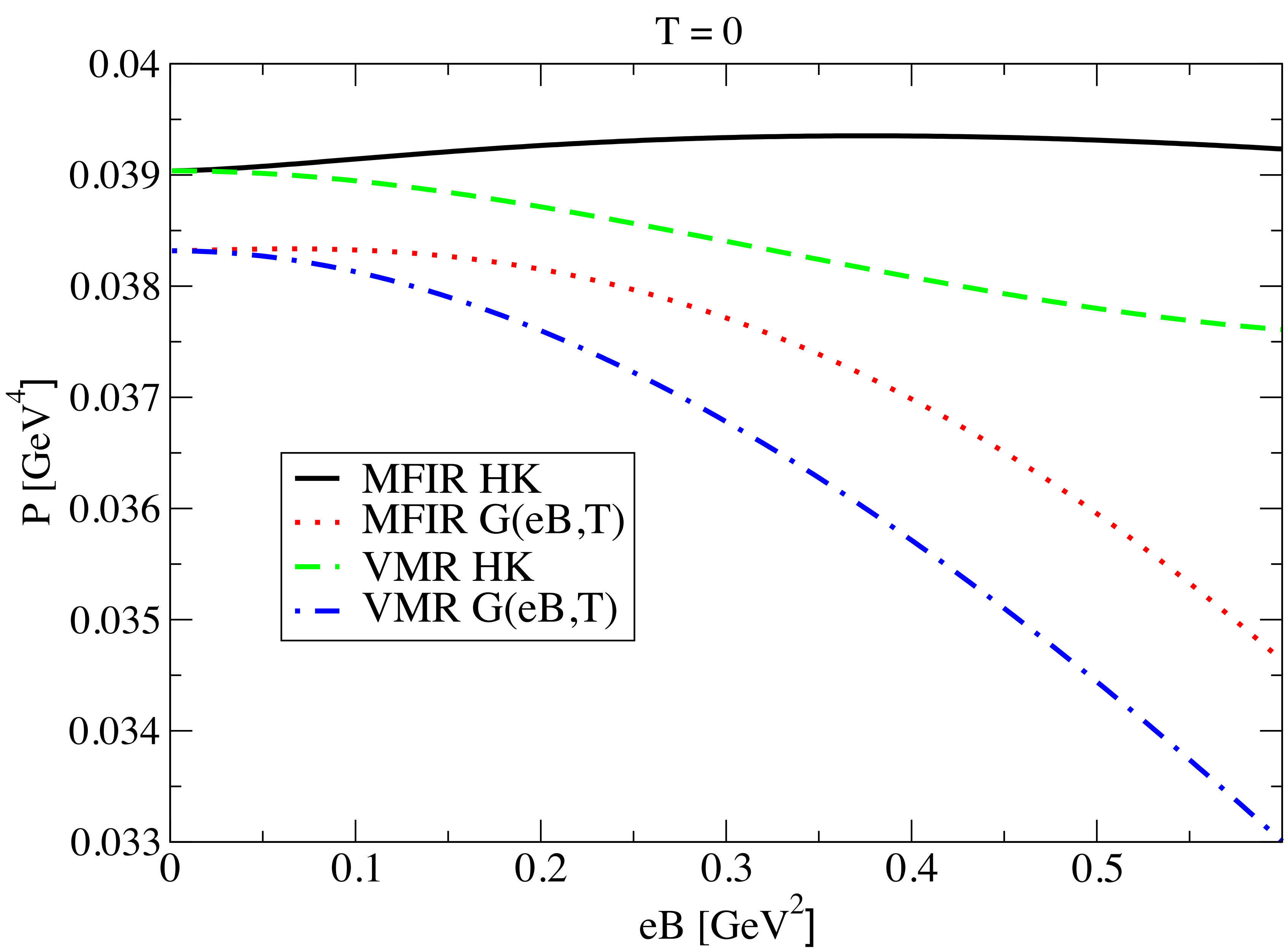
IMC - SU(3) NJL model + G(eB,T)



RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein,

e-Print: 2104.11117 [hep-ph]

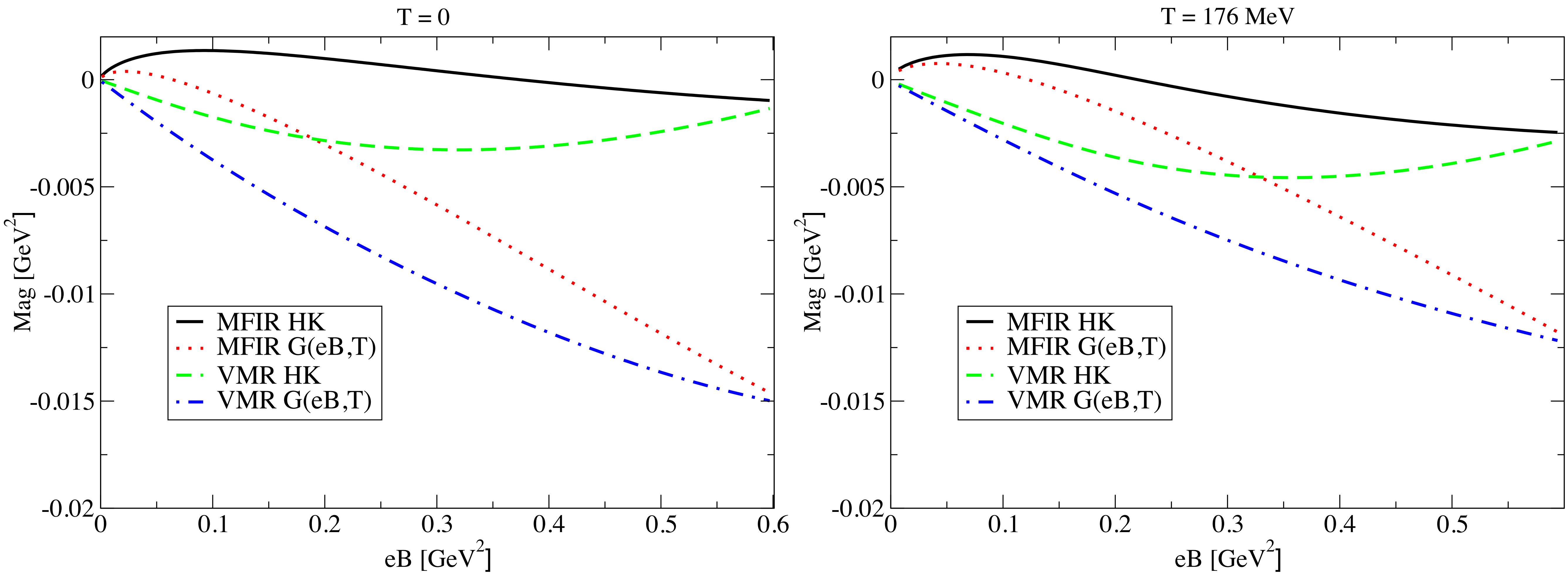
Pressure: MFIR X VMR in SU(3) NJL



HK parametrization, T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, e-Print: 2104.11117 [hep-ph]

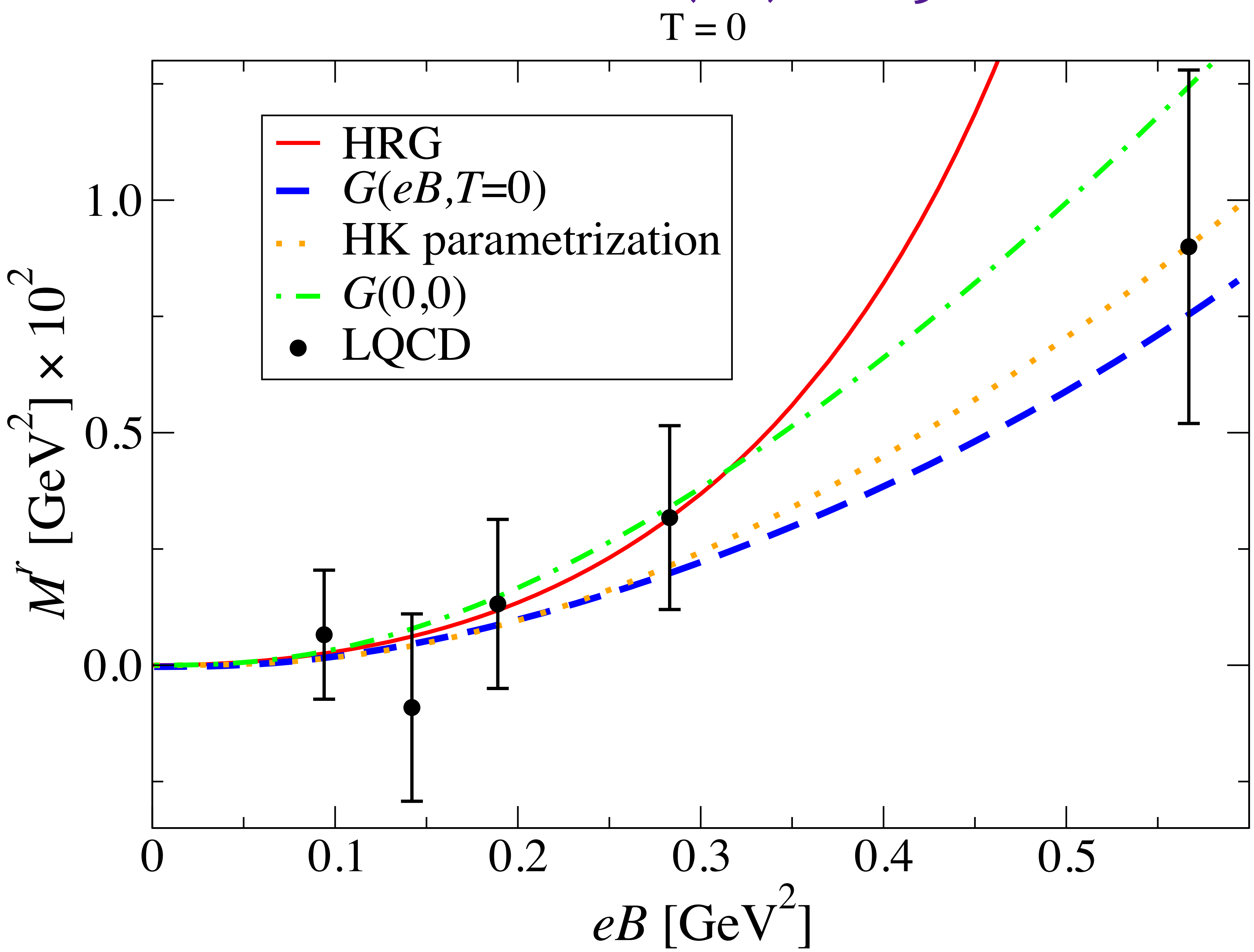
Magnetization: MFIR X VMR in SU(3)



HK parametrization, T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994)

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, e-Print: 2104.11117 [hep-ph]

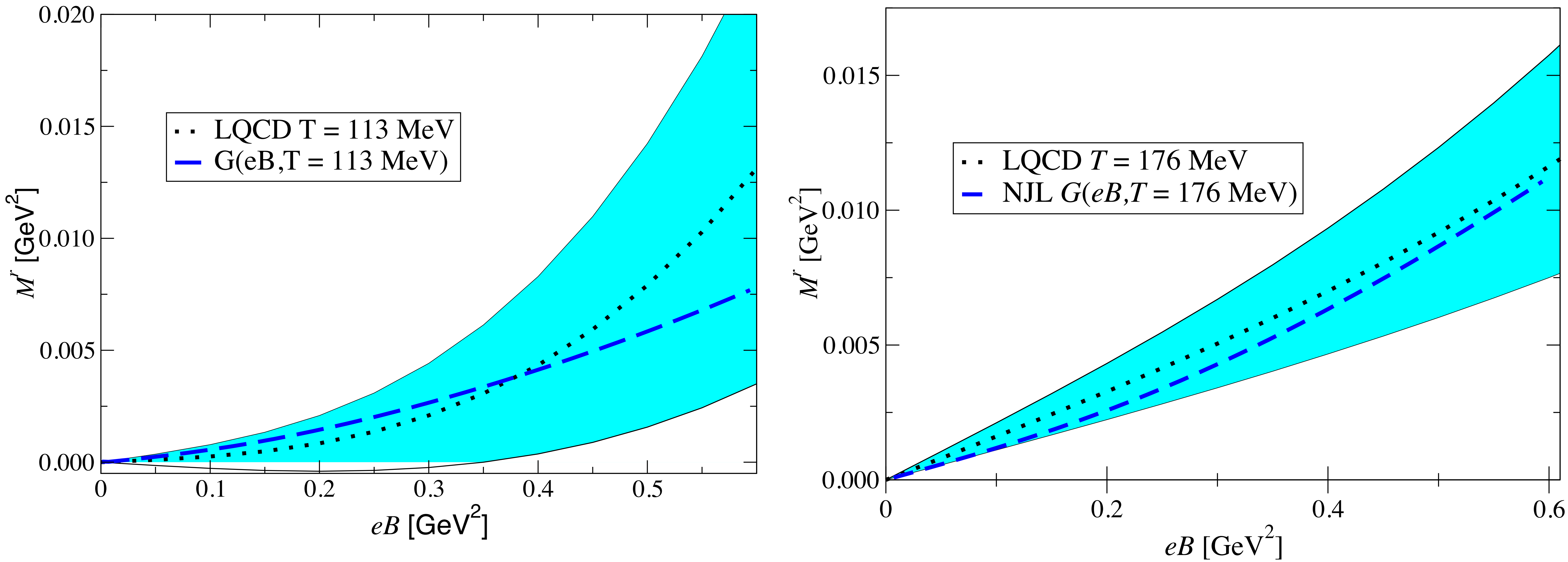
Renormalized magnetization: VMR in SU(3) NJL



Lattice data, G. Bali, F. Bruckmann, G. Endrodi, F. Gruber, and A. Schaefer, JHEP 04, 130 (2013).

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, e-Print: 2104.11117 [hep-ph]

Renormalized magnetization: VMR in SU(3) NJL



Lattice data, G. S. Bali, F. Bruckmann, G. Endrodi, and A. Schafer, Phys. Rev. Lett. **112**, 042301 (2014).

RLSF, W.R.Tavares, S.S. Avancini, M.B.Pinto, V.S.Timoteo and G. Krein, e-Print: 2104.11117 [hep-ph]

Conclusions

✓ The thermo-magnetic dependence of G(B,T) is obtained by fitting lattice QCD predictions for the chiral transition order parameter

✓ **<u>MFIR/VMR scheme avoid some unphysical results</u>**, and this choice of regularization provide to us some different results from most of the regularizations prescriptions of the current literature.

✓ SU(3) NJL model + G(eB,T) is in agreement with lattice simulations: indicating a paramagnetic behavior for the QCD vacuum

Thank you for your attention!