Effective two-color dense QCD

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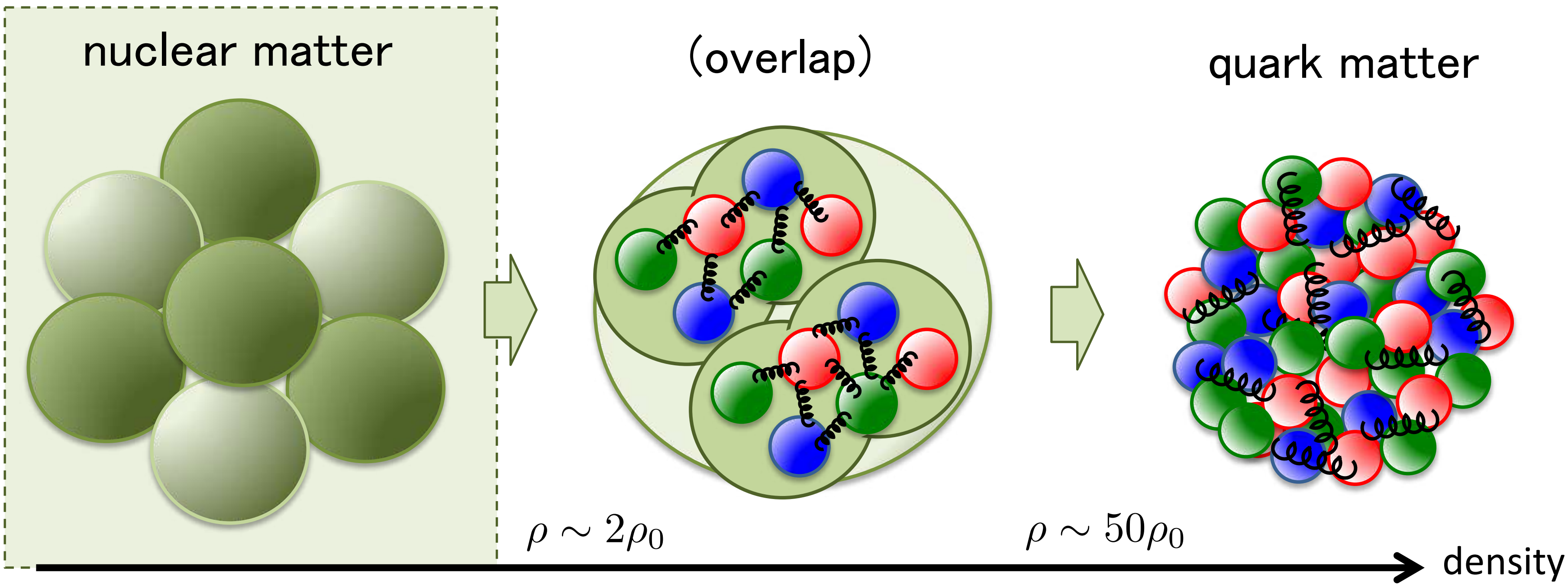
[1] D. S. and T. Kojo, Phys.Rev.D 100 (2019) 7, 076017
[2] T. Kojo and D. S., Phys.Rev.D 103 (2021) 9, 094008
[3] D. S. and T. Kojo, 2105.10538 [hep-ph]

1. Introduction

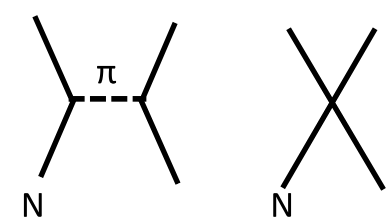
- **Hadron/QCD at density**

G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song and T. Takatsuka, Rept. Prog. Phys. 81 (2018) no.5, 056902



(ρ_0 : normal nuclear density)

– At lower density, hadronic degrees of freedom play an important role (baryon ChPT, etc.)



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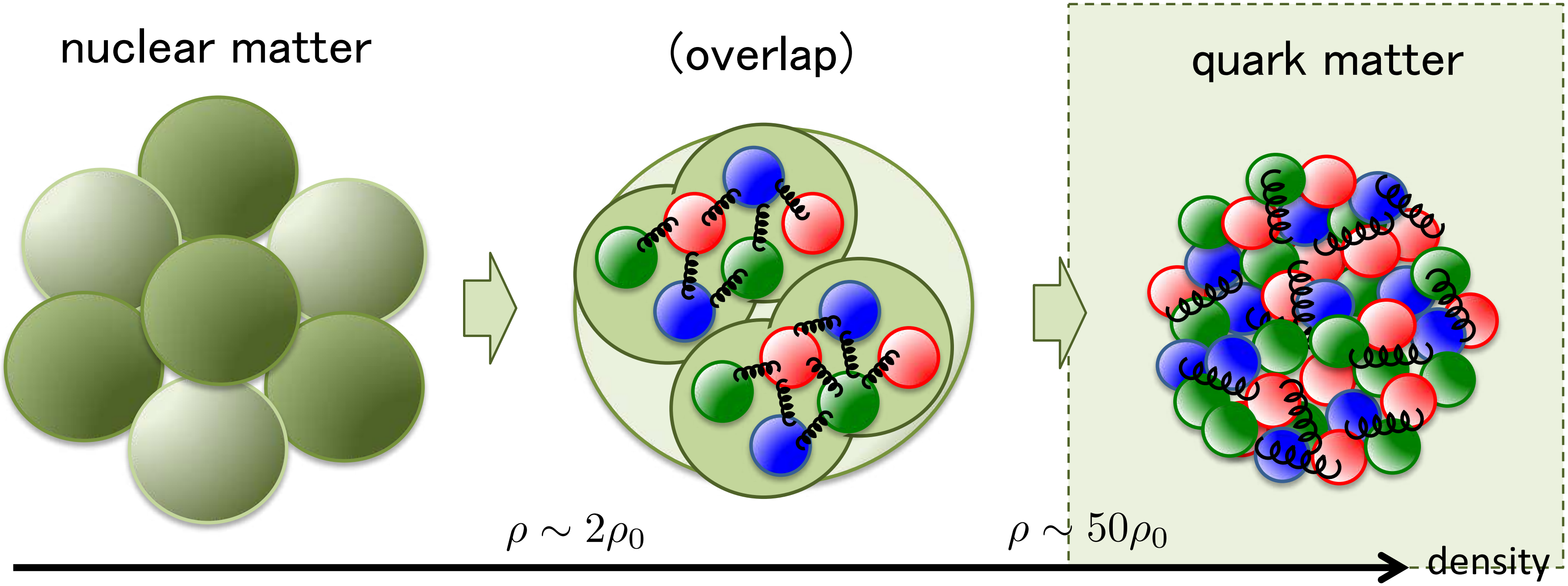
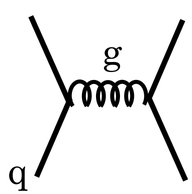


(ρ_0 : normal nuclear density)

- In super dense medium, perturbative quarks and gluons govern the system (asymptotic freedom)



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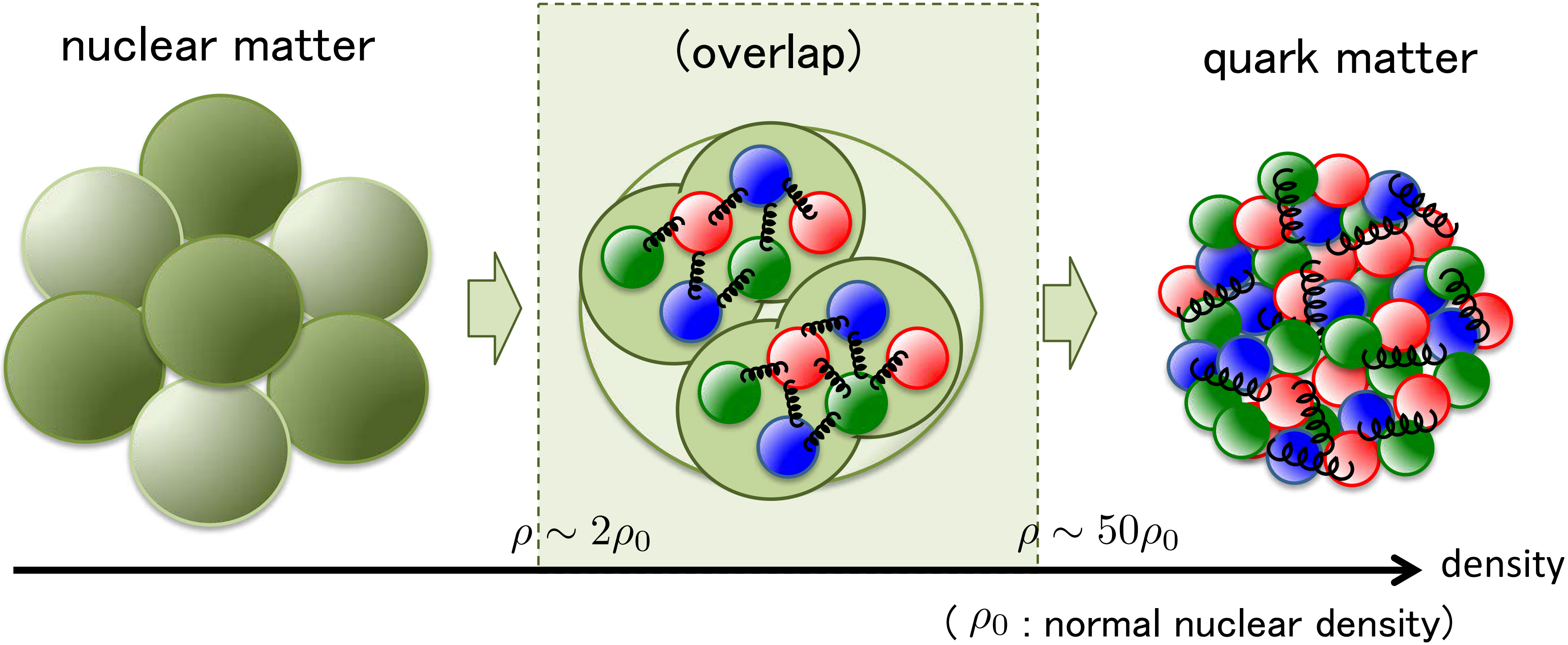


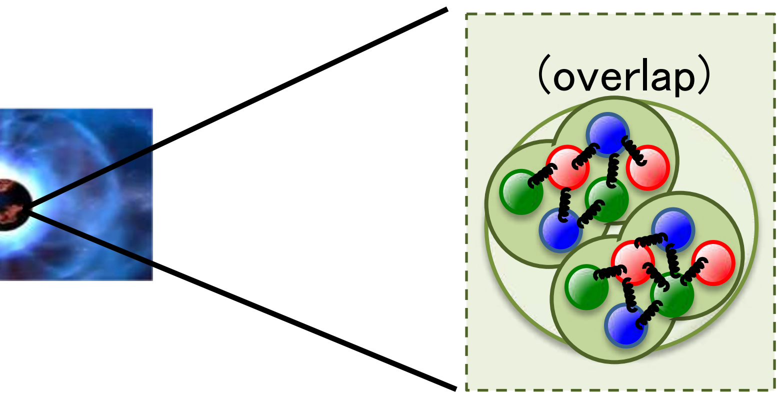
- What are the effective degrees of freedom at intermediate density ?
- Neither hadronic nor (perturbative) quarks and gluons description are suitable

1. Introduction

• Hadron/QCD at density

G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song and T. Takatsuka, Rept. Prog. Phys. 81 (2018) no.5, 056902

– QCD at intermediate density is necessary for study of "quark matter" inside neutron stars



$2\rho_0 \lesssim \rho \lesssim 10\rho_0$

– We need to understand the effective degrees of freedom, i.e. the effective theory of QCD at intermediate density

1. Introduction

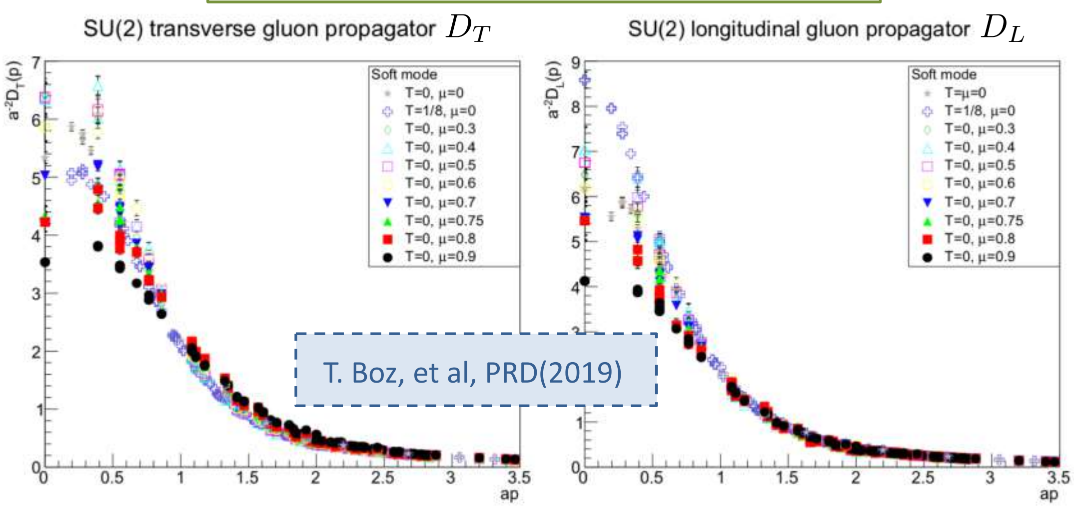
- **Lattice results at two-color dense QCD**
  - Lattice results are useful for constructing the effective theory

“lattice QCD = numerical experiments”

  - In $N_F = 2, N_C = 2$ QCD (QC$_2$D), lattice results at density have been reported by several groups ⇐ **no sign problem !**

e.g. G. Aarts 2016 J. Phys.: Conf. Ser. 706 022004

e.g. gluon propagator at density



T. Boz, et al, PRD(2019)

Strategy

Construct an effective theory at density reproducing this result

2. Effective theory

- **Massive Yang-Mills theory for gluons**
  - In vacuum the so-called **massive Yang-Mills (mYM) theory** is known to be reasonable as an effective theory of gluons at IR

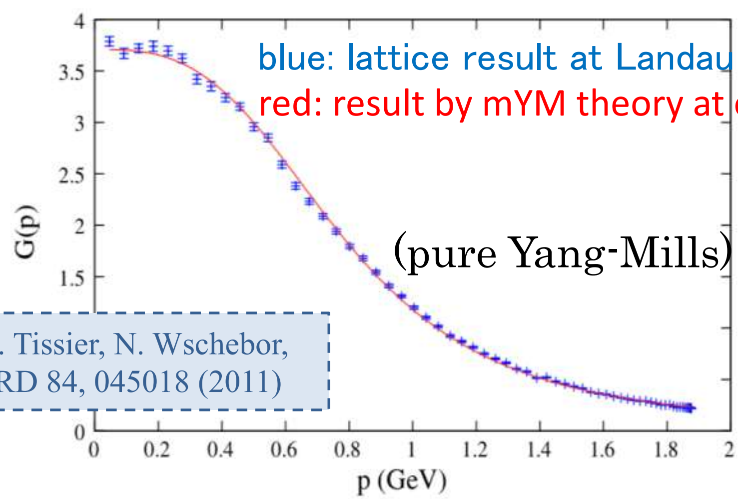
$$\mathcal{L}_{\rm gluon}^{\rm eff} = \mathcal{L}_{\rm YM} + \frac{m_g^2}{2} A_\mu^a A^{\mu a}$$

with $\mathcal{L}_{\rm YM} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{2\alpha}(\partial_\mu A^{\mu a})^2 + \bar{c}^a i \partial^\mu D_\mu c^a$

c.f. CF model
G. Curci and R. Ferrari, Nuovo Cim. A32, 151(1976)

original BRS invariance is broken

gluon propagator in vacuum



M. Tissier, N. Wschebor, PRD 84, 045018 (2011)

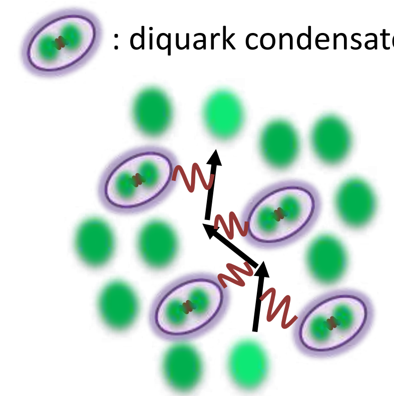
We employ mYM as the effective theory of gluons at finite density

2. Effective theory

- **Quasiparticle description for quarks in QC$_2$D**
  - The quark sector $\mathcal{L}^{\rm eff}_{\rm quark}$ at density in QC$_2$D is

$$\mathcal{L}^{\rm eff}_{\rm quark} = \bar{\psi}(i\not{D} - i\mu_q\gamma_4 - M_q)\psi - \psi^T \mathbf{\Delta}\psi$$

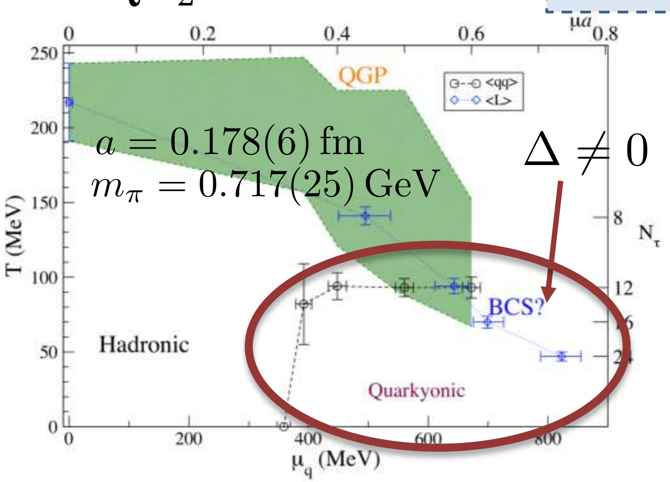
  - $\mathbf{\Delta}$ is a diquark condensate (gap) $\sim \langle \psi^T C\gamma_5\sigma^2\tau^2\psi\rangle$ as suggested by QC$_2$D lattice simulation



⇒ quasiparticle description of quarks in cold matter

QC$_2$D lattice result

Boz-Cotter-Fister-Mehta-Skullerud EPJ(2013)



for other works, e.g.,
Buividovich-Smith-Smekal PRD(2020)
Braguta-Ilgenfritz-Kotov-Molochkov-Nikolaev PRD(2016)
Iida-Itou-Lee JHEP(2020)

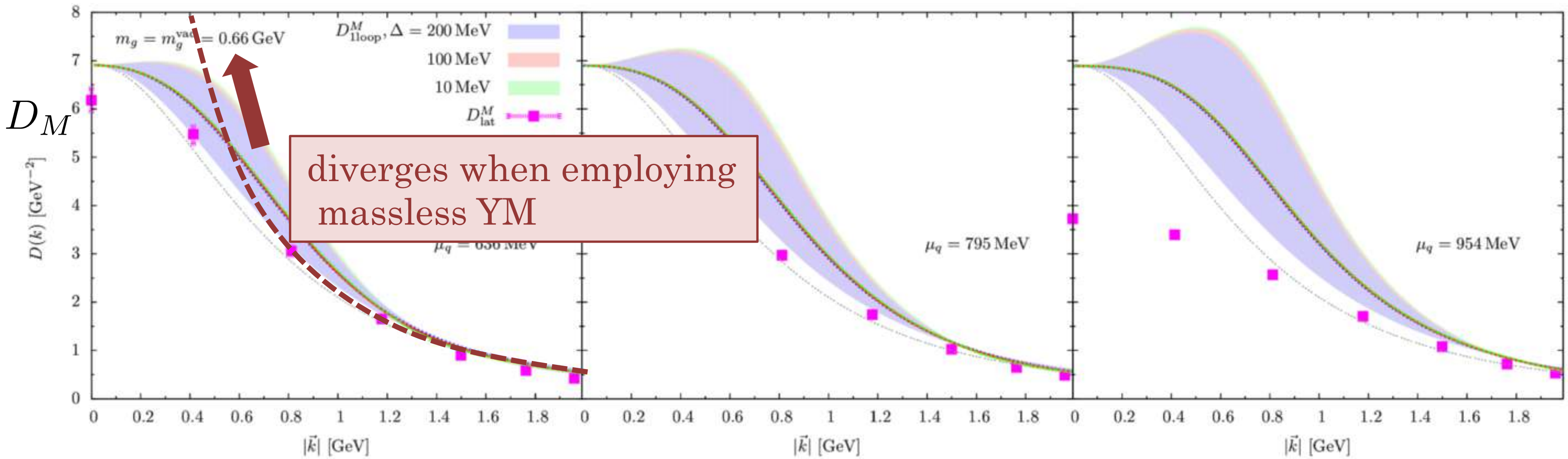
3. Results

- **Magnetic propagator** $D^{M}_{\mu\nu}$

DS-Kojo PRD(2019)
Kojo-DS PRD(2021)

– We employ the same parameters as used in vacuum

– We take $M_q = 0.1$ GeV and $\Delta = 10, 100, 200$ MeV



– Our result at $|\vec{k}| = 0$ is regulated due to the gluon mass m_g, consistent with the lattice data

– Still a discrepancy at small momentum is found

2. Effective theory

- **Summary of our effective theory**

Our effective theory of quarks and gluons at intermediate density

DS-Kojo, PRD(2019)
Kojo-DS, PRD(2021)
DS-Kojo, 2105.10538 [hep-ph]

$$\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm quark} + \mathcal{L}^{\rm eff}_{\rm gluon}$$

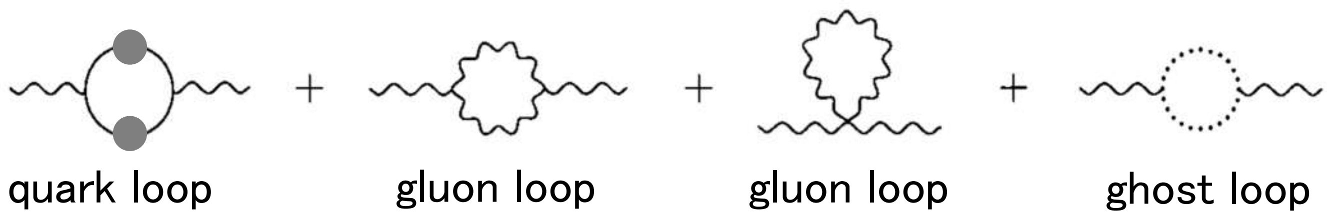
$$\begin{cases} \mathcal{L}^{\rm eff}_{\rm quark} = \bar{\psi}(i\not{D} - i\mu_q\gamma_4 - M_q)\psi - \psi^T \mathbf{\Delta}\psi \\ \mathcal{L}^{\rm eff}_{\rm gluon} = \mathcal{L}_{\rm YM} + \dfrac{m_g^2}{2} A_\mu^a A^{\mu a} \\ \text{with } \mathcal{L}_{\rm YM} = -\dfrac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} - \dfrac{1}{2\alpha}(\partial_\mu A^{\mu a})^2 + \bar{c}^a i\partial^\mu D_\mu c^a \quad (\alpha \to 0) \end{cases}$$

⇨ study gluon propagator at one-loop to compare with lattice QCD

3. Results

- **Gluon propagator in medium**
  - Gluon self energies at density are

$$\Pi_{\mu\nu} =$$



  - Gluon propagator is evaluate by the Dyson equation

$$D^{-1}_{\mu\nu} = D^{-1}_{0,\mu\nu} + \Pi_{\mu\nu}$$

  - Two types of the propagators are obtained (lack of Lorentz symmetry)

$$D_{\mu\nu} = D^{E}_{\mu\nu} + D^{M}_{\mu\nu}$$

electric → $D^{E}_{\mu\nu}$, magnetic → $D^{M}_{\mu\nu}$

3. Results

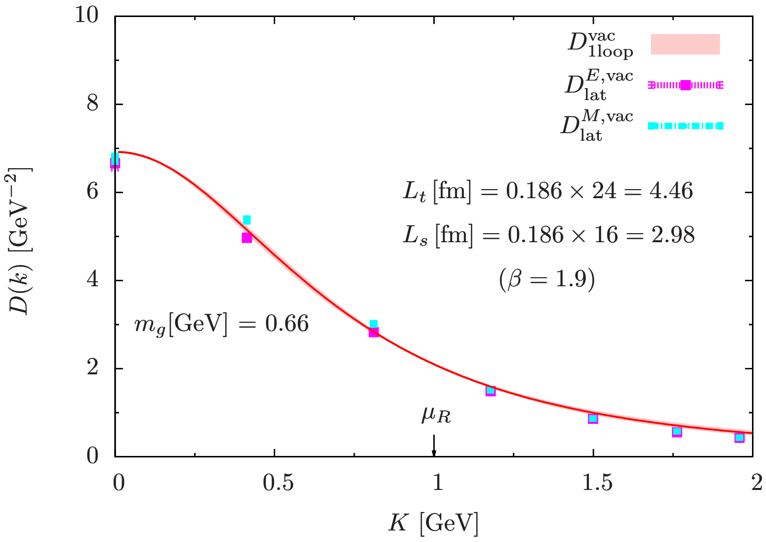
- **Fitting in vacuum** (before in medium)
  - Compare our gluon propagator at one loop with the lattice data
  - Input parameter are as follows

Lattice: T. Boz, O. Hajizadeh, A. Maas, J.-I. Skullerud: PRD99 (2019) no.7, 074514

$$\mu_R = 1\ \text{GeV}$$
$$M_q^{\text{vac}} = 0.3\ \text{GeV}$$
$$m_g = 0.66\ \text{GeV}$$
$$\alpha_s = 1-3$$

Renormalization conditions

$$\left(\Pi_{\text{vac}}(\mu_R) = \Pi_{\text{vac}}(0) = 0\right)$$



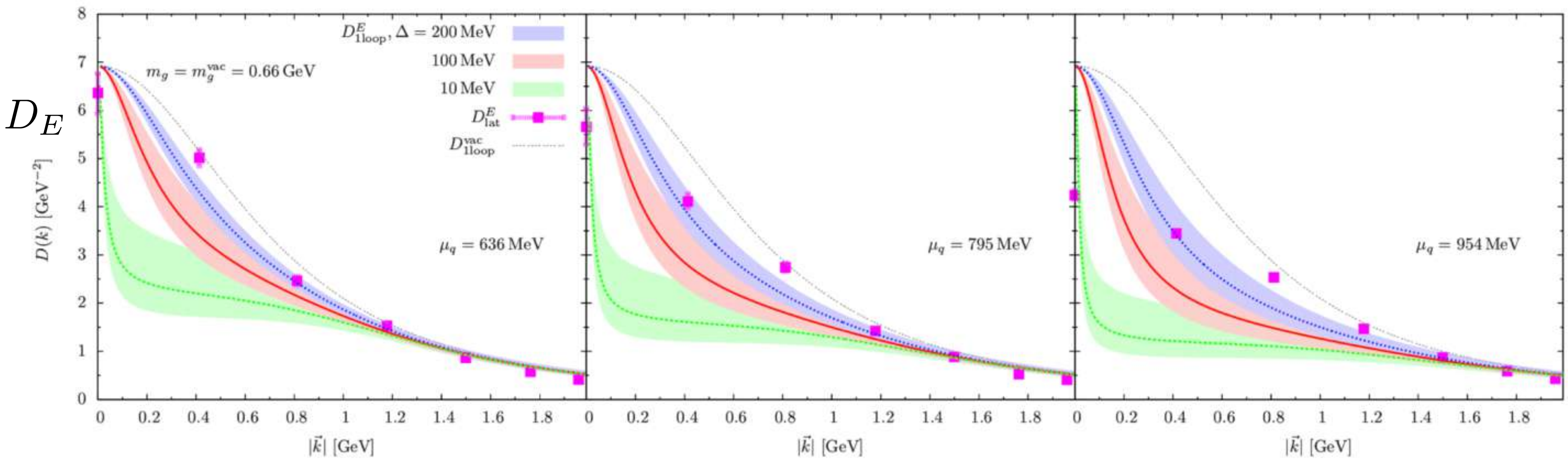
- Fitting seems to be good (α_s dependence is small)

3. Results

- **Electric propagator $D_{\mu\nu}^{E}$**

DS-Kojo PRD(2019)
Kojo-DS PRD(2021)

  - We employ the same parameters as used in vacuum
  - We take $M_q = 0.1$ GeV and $\Delta = 10, 100, 200$ MeV



  - Our result at $|\vec{k}| = 0$ is regulated due to the gluon mass m_g , consistent with the lattice data
  - Still a discrepancy at small momentum is found

3. Results

- **Electric propagator $D^E_{\mu\nu}$**

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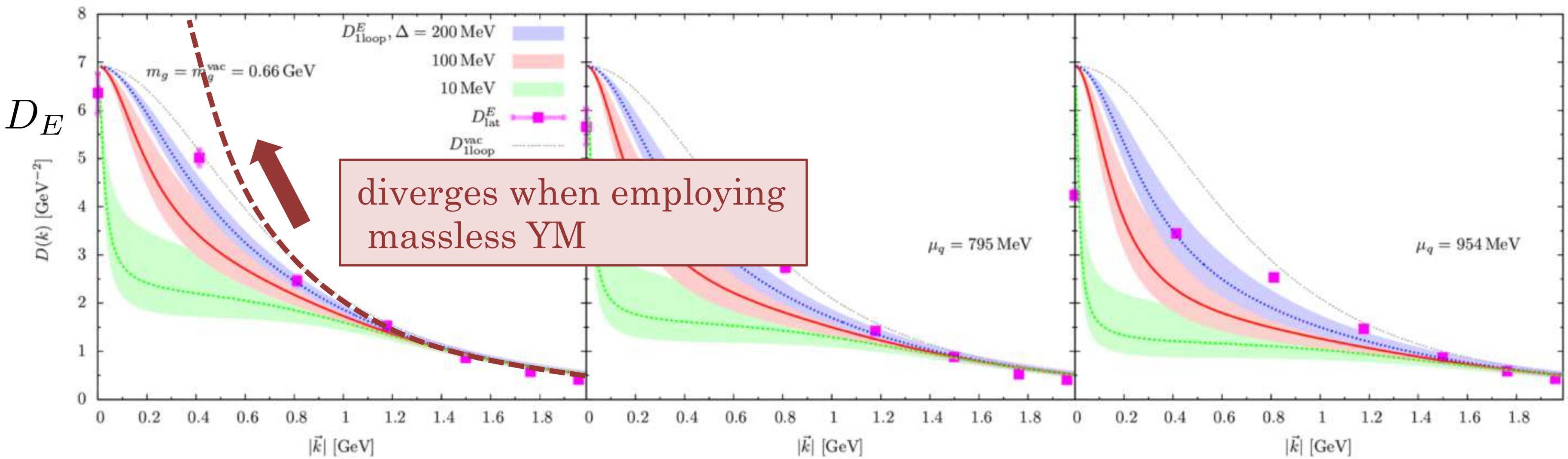

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3. Results

- **Magnetic propagator** $D^M_{\mu\nu}$

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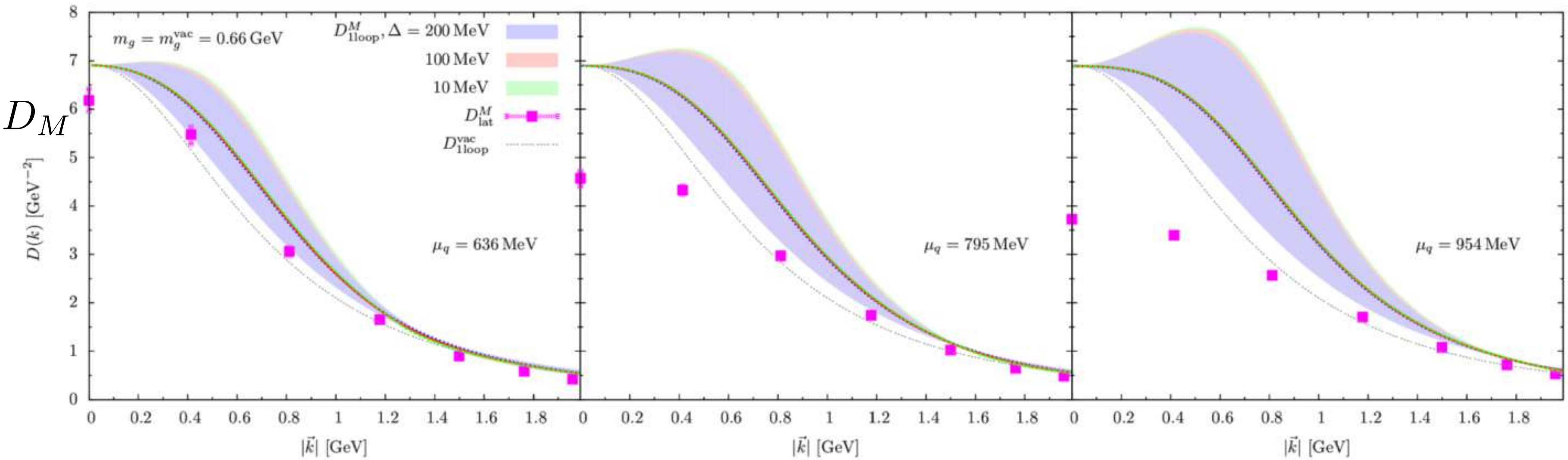

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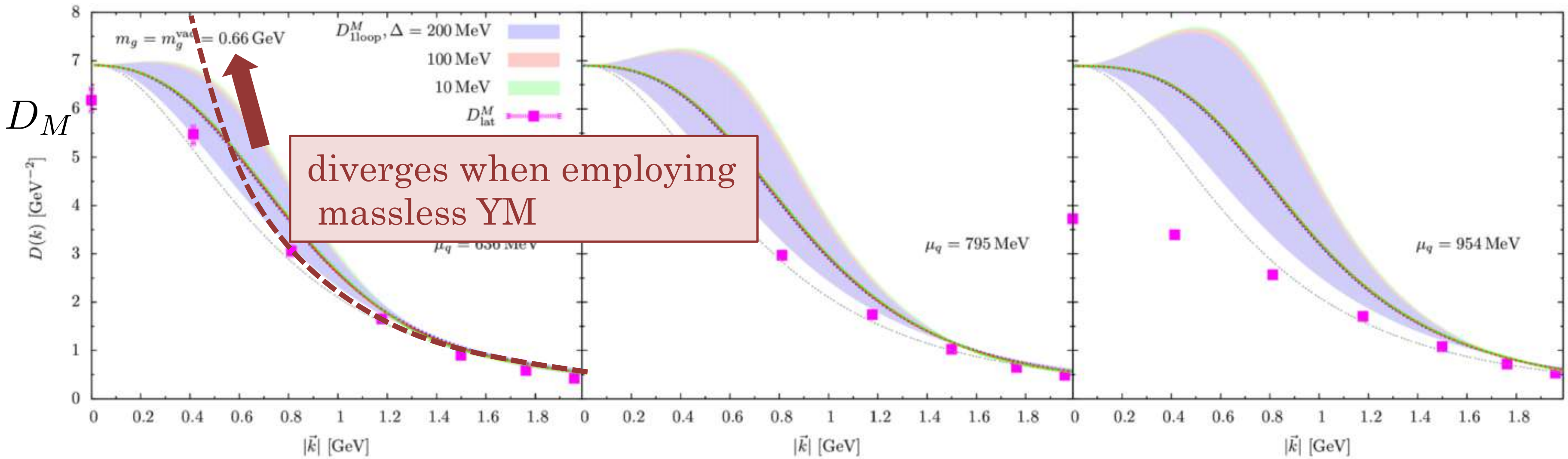

  - Our result at $|\vec{k}| = 0$ is regulated due to the gluon mass m_g, consistent with the lattice data
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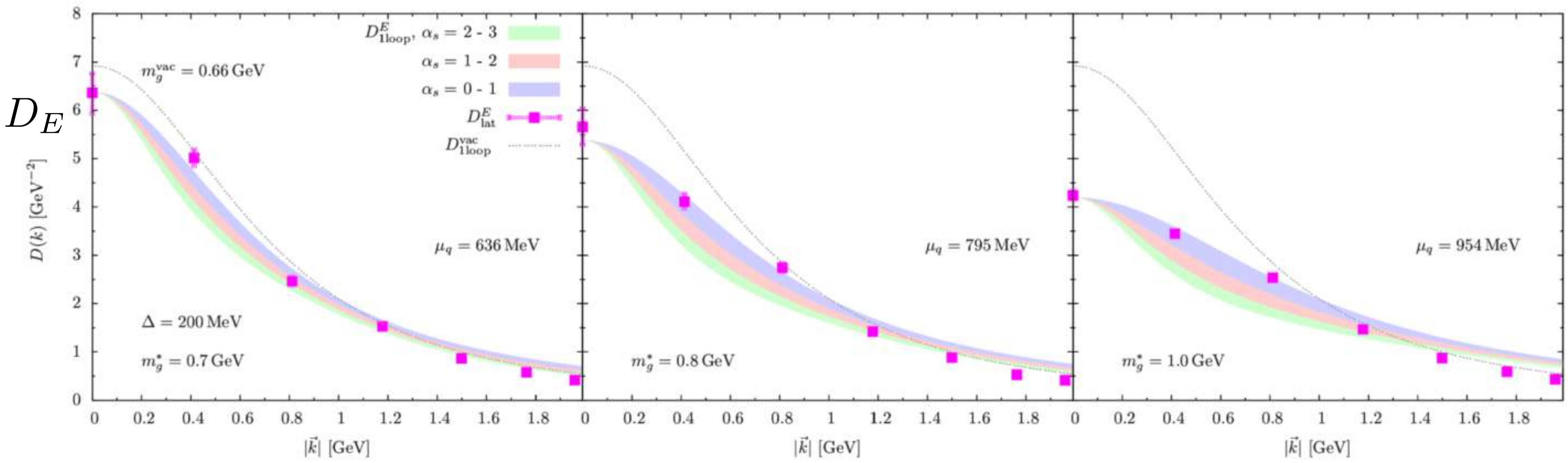
- **Electric propagator $D^E_{\mu\nu}$ (improved)**

DS-Kojo PRD(2019)
Kojo-DS PRD(2021)

  - We change the gluon mass m_g such that it reproduces the lattice data
  - We choose $\Delta = 200$ MeV



  - Comparison becomes better
  - We need to understand the enhancement of m_g at higher density

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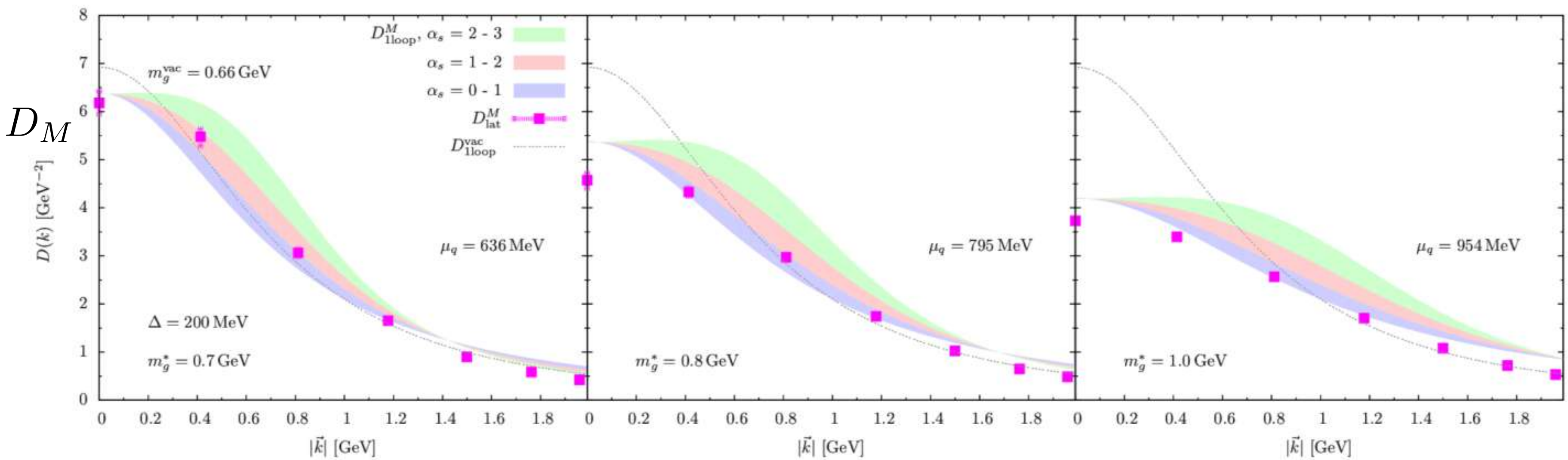


– Comparison becomes better

– We need to understand the enhancement of m_g at higher density

4. Conclusions

• Conclusions

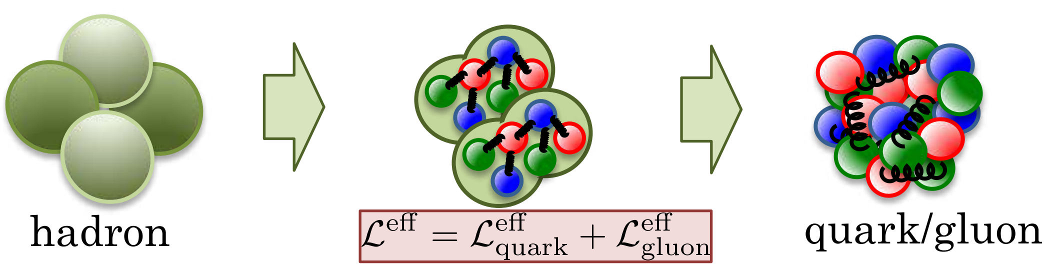
– We have constructed the effective theory $\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm quark} + \mathcal{L}^{\rm eff}_{\rm gluon}$ towards understanding of "quark matter" in neutron stars

$\mathcal{L}^{\rm eff}_{\rm quark}$: quasiparticle description of quarks with diquark condensates

$\mathcal{L}^{\rm eff}_{\rm gluon}$: massive Yang-Mills theory $\quad \mathcal{L}_{\rm mass} = \frac{m_g^2}{2} A^a_\mu A^{\mu a}$

– We have compared our model with the lattice results in QC_2D by focusing on the gluon propagator at density ⇨ qualitatively good!

– Our effective theory is expected to connect nuclear matter (hadronic) and perturbative quark matter

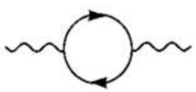


Thank you!

Back up

- **Note on quark loop**
  - We must regularize the divergence so as not to leave any UV artifacts

c.f., T. Kojo and G. Baym, PRD 89, no. 12, 125008 (2014)

$\Pi^{\mathrm{reg}} \equiv$  $-$ 

$$= \underbrace{\Pi_{\mathrm{med}}(\Delta; M_q)}_{\text{three-dimensional cutoff}} - \underbrace{\Pi^{\mathrm{count.}}_{\mathrm{vac}}(0; M_q^{\mathrm{vac}})}_{\text{dimensional regularization}}$$

three-dimensional cutoff

dimensional regularization

M_q : quark mass in medium

M_q^{vac} : constituent quark mass in the vacuum

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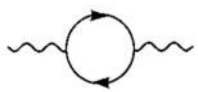
$\Pi^{\mathrm{reg}} \equiv$  $-$ 

$$= \Pi_{\mathrm{med}}(\Delta; M_q) - \Pi_{\mathrm{vac}}^{\mathrm{count.}}(0; M_q^{\mathrm{vac}})$$

$$= \underbrace{\Pi_{\mathrm{med}}(\Delta; M_q) - \Pi_{\mathrm{vac}}(0; \tilde{M}_q)}_{\text{three-dimensional cutoff}} + \underbrace{\Pi_{\mathrm{vac}}(0; \tilde{M}_q) - \Pi_{\mathrm{vac}}^{\mathrm{count.}}(0; M_q^{\mathrm{vac}})}_{\text{dimensional regularization}}$$

M_q : quark mass in medium

M_q^{vac} : constituent quark mass in the vacuum

$\tilde{M}_q = \sqrt{M_q^2 + \Delta^2}$ (in the present case)

The mWTI is preserved and no artifact appears!

Back up

- **Calculation of quark one-loop**
  - The self-energy is decomposed into electric and magnetic parts:

$$\Pi^{\rm reg}(k) = \bar{\Pi}^{\rm reg}_{q;M}(k) + \bar{\Pi}^{\rm reg}_{q;E}(k)$$

with

$$\bar{\Pi}^{\rm reg}_{q;M}(\vec{k}) = -\frac{g^2}{2}\int\frac{d^3q}{(2\pi)^3}\left\{\frac{C_N^{pp}+C_A^{pp}}{\epsilon_p(q_-)+\epsilon_p(q_+)}B_+ + \frac{C_N^{aa}+C_A^{aa}}{\epsilon_a(q_-)+\epsilon_a(q_+)}B_+ + 2\frac{C_N^{pa}+C_A^{pa}}{\epsilon_p(q_-)+\epsilon_a(q_+)}B_- - \frac{2}{\tilde{E}_{q_-}+\tilde{E}_{q_+}}\tilde{B}_-\right\} - |\vec{k}|^2\frac{g^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(\frac{\tilde{m}_q^2+x(1-x)|\vec{k}|^2}{M_q^2+x(1-x)\mu^2}\right),$$

$$\bar{\Pi}^{\rm reg}_{q;E}(\vec{k}) = g^2\int\frac{d^3q}{(2\pi)^3}\left\{\frac{C_N^{pp}-C_A^{pp}}{\epsilon_p(q_-)+\epsilon_p(q_+)}A_+ + \frac{C_N^{aa}-C_A^{aa}}{\epsilon_a(q_-)+\epsilon_a(q_+)}A_+ + 2\frac{C_N^{pa}-C_A^{pa}}{\epsilon_p(q_-)+\epsilon_a(q_+)}A_- - \frac{2}{\tilde{E}_{q_-}+\tilde{E}_{q_+}}\tilde{A}_-\right\} - |\vec{k}|^2\frac{g^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\left(\frac{\tilde{m}_q^2+x(1-x)|\vec{k}|^2}{M_q^2+x(1-x)\mu^2}\right)$$

- Divergences are subtracted
- No finite terms violating the mWTI appear

Back up

- **Calculation of quark one-loop**
  - The detail of the definition

$$C_N^{pp} - C_A^{pp} = \frac{1}{2}\left(1 - \frac{(E_{q_-} - \mu_q)(E_{q_+} - \mu_q) + |\Delta|^2}{\epsilon_p(q_-)\epsilon_p(q_+)}\right) \qquad C_N^{pp} + C_A^{pp} = \frac{1}{2}\left(1 - \frac{(E_{q_-} - \mu_q)(E_{q_+} - \mu_q) - |\Delta|^2}{\epsilon_p(q_-)\epsilon_p(q_+)}\right)$$

$$C_N^{aa} - C_A^{aa} = \frac{1}{2}\left(1 - \frac{(E_{q_-} + \mu_q)(E_{q_+} + \mu_q) + |\Delta|^2}{\epsilon_a(q_-)\epsilon_a(q_+)}\right) \qquad C_N^{aa} + C_A^{aa} = \frac{1}{2}\left(1 - \frac{(E_{q_-} + \mu_q)(E_{q_+} + \mu_q) - |\Delta|^2}{\epsilon_a(q_-)\epsilon_a(q_+)}\right)$$

$$C_N^{pa} - C_A^{pa} = \frac{1}{2}\left(1 + \frac{(E_{q_-} - \mu_q)(E_{q_+} + \mu_q) - |\Delta|^2}{\epsilon_p(q_-)\epsilon_a(q_+)}\right) \qquad C_N^{pa} + C_A^{pa} = \frac{1}{2}\left(1 + \frac{(E_{q_-} - \mu_q)(E_{q_+} + \mu_q) + |\Delta|^2}{\epsilon_p(q_-)\epsilon_a(q_+)}\right)$$

$$C_N^{ap} - C_A^{ap} = \frac{1}{2}\left(1 + \frac{(E_{q_-} + \mu_q)(E_{q_+} - \mu_q) - |\Delta|^2}{\epsilon_a(q_-)\epsilon_p(q_+)}\right) \qquad C_N^{ap} + C_A^{ap} = \frac{1}{2}\left(1 + \frac{(E_{q_-} + \mu_q)(E_{q_+} - \mu_q) + |\Delta|^2}{\epsilon_a(q_-)\epsilon_p(q_+)}\right)$$

$$A_+ = 1 + \frac{\vec{q}_- \cdot \vec{q}_+ + m_f^2}{E_{q_-} E_{q_+}}\ , \quad A_- = 1 - \frac{\vec{q}_- \cdot \vec{q}_+ + m_f^2}{E_{q_-} E_{q_+}}$$

$$B_+ = 2\left(1 - \frac{m_f^2 |\vec{k}|^2 + (\vec{k}\cdot\vec{q}_-)(\vec{k}\cdot\vec{q}_+)}{|\vec{k}|^2 E_{q_-} E_{q_+}}\right)\ , \quad B_- = 2\left(1 + \frac{m_f^2 |\vec{k}|^2 + (\vec{k}\cdot\vec{q}_-)(\vec{k}\cdot\vec{q}_+)}{|\vec{k}|^2 E_{q_-} E_{q_+}}\right)$$

$$E_q = \sqrt{|\vec{q}|^2 + m_q^2} \qquad \tilde{m}_q = \sqrt{m_q^2 + |\Delta|^2}$$

Back up

- **One loop in vacuum**
  - Gluon propagator with the dimensional regularization

$$D^{\mathrm{vac}}(k) = \frac{Z_g^{\mathrm{overall}}}{K^2 + m_g^2 + \Pi_{\mathrm{vac}}(k)}$$

renormalization condition

$$\left(\Pi_{\mathrm{vac}}(\mu_R) = \Pi_{\mathrm{vac}}(0) = 0\right)$$

with $\Pi_{\mathrm{vac}}(k) = \Pi_{\mathrm{YM}}(k) + \Pi_q^{\mathrm{vac}}(k)$

$$\Pi_{\mathrm{YM}}(k) = \frac{g^2 K^2}{192\pi^2}\Bigg\{111 s^{-1} - 2s^{-2} + (2 - s^2)\ln(s) + 2(s^{-1}+1)^3(s^2 - 10s + 1)\ln(1+s)$$

$$+(4s^{-1}+1)^{3/2}(s^2 - 20s + 12)\ln\left(\frac{\sqrt{4+s} - \sqrt{s}}{\sqrt{4+s} + \sqrt{s}}\right) - (s \leftrightarrow \mu_R^2/m_g^2)\Bigg\}, \quad \left(s = K^2/m_g^2\right)$$

$$\Pi_q^{\mathrm{vac}}(k) = -K^2 \frac{g^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\frac{(M_q^{\mathrm{vac}})^2 + x(1-x)K^2}{(M_q^{\mathrm{vac}})^2 + x(1-x)\mu_R^2}$$

- Z_g^{overall} is an unphysical constant matching the overall normalization
- m_g is the “renormalized” gluon mass

Back up

- **One loop in vacuum**
  - Gluon propagator with the dimensional regularization

$$D^{\rm vac}(k) = \frac{Z_g^{\rm overall}}{K^2 + m_g^2 + \Pi_{\rm vac}(k)}$$

renormalization condition

$$\left(\Pi_{\rm vac}(\mu_R) = \Pi_{\rm vac}(0) = 0\right)$$

with $\Pi_{\rm vac}(k) = \Pi_{\rm YM}(k) + \Pi_q^{\rm vac}(k)$

$$\Pi_{\rm YM}(k) = \frac{g^2K^2}{192\pi^2}\left\{111s^{-1} - 2s^{-2} + (2 - s^2)\ln(s) + 2(s^{-1}+1)^3(s^2 - 10s + 1)\ln(1+s)\right.$$
$$\left. + (4s^{-1}+1)^{3/2}(s^2 - 20s + 12)\ln\left(\frac{\sqrt{4+s}-\sqrt{s}}{\sqrt{4+s}+\sqrt{s}}\right) - (s \leftrightarrow \mu_R^2/m_g^2)\right\}, \quad \left(s = K^2/m_g^2 \right)$$

$$\Pi_q^{\rm vac}(k) = -K^2\frac{g^2}{2\pi^2}\int_0^1 dx\, x(1-x)\ln\frac{(M_q^{\rm vac})^2 + x(1-x)K^2}{(M_q^{\rm vac})^2 + x(1-x)\mu_R^2}$$ (→change in medium!)

- $Z_g^{\rm overall}$ is an unphysical constant matching the overall normalization
- m_g is the “renormalized” gluon mass