Mesons in nuclei and partial restoration of chiral symmetry

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Part I: Hadron properties in a nuclear medium from the quark and gluon degrees of freedom
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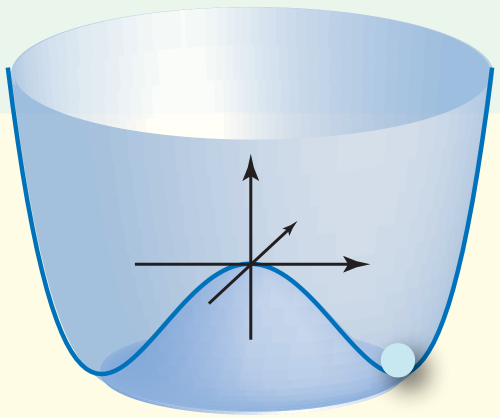
Introduction

chiral symmetry

- chiral symmetry, ChS, is a fundamental symmetry in QCD
- ChS is spontaneously broken by physical states
- ChSB is a **phase transition phenomenon**
- vacuum properties depend on environment



partial (incomplete) restoration of Chiral Symmetry

|quark condensate| does decrease in nuclear medium

- phenomenological analysis of pionic atom and low energy pi-A scattering
- 30-40 % reduction at saturation density, if believe linear extrapolation

K. Suzuki et al. PRL92, 072302, (04); Friedman et al., PRL93, 122302 (04); DJ, Hatsuda, Kunihiro, PLB 670, 109 (08).

we see that
such change of the vacuum properties can be observed by NG boson properties
and show possible in-medium changes of meson properties induced by the change of vacuum

Contents

- introduction
- Weinberg Glashow relation

  connects pion properties and quark condensate
- what we expect in partial restoration

  wavefunction renormalization

  η' meson in nuclear medium
- role of anomaly term in chiral symmetry breaking
- summary

Weinberg-Glashow Relation

Weinberg-Glashow relation in vacuum

Glashow, Weinberg, PRL 20 (1968) 224

$$F_\pi G_\pi^{1/2} = -\langle \bar{q} q \rangle$$

chiral limit relation
connects pion properties and quark condensate
G_π is not direct observable

pion decay constant

$$\langle 0 | A_\mu^a(x) | \pi^b(p) \rangle = \delta^{ab} i p_\mu F_\pi e^{-ip\cdot x}$$

coupling of pseudoscalar field to pion

$$\langle 0 | \phi_5^a(x) | \pi^b(p) \rangle = \delta^{ab} G_\pi^{1/2} e^{-ip\cdot x}$$

$$\phi_5^a = \tfrac{1}{2} \bar{q} i \gamma_5 \tau^a q$$

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equivalent to **Gell-Mann Oakes Renner relation**

PCAC relation

$$\partial^\mu A_\mu^a(x) = 2 m_q \phi_5^a(x)$$

$$F_\pi m_\pi^2 = 2 m_q G_\pi^{1/2}$$

taking pion matrix element

$$\langle 0 | \partial^\mu A_\mu(x) | \pi(p) \rangle = F_\pi p^2 = F_\pi m_\pi^2$$

$$2 m_q \langle 0 | \phi_5(x) | \pi(p) \rangle = 2 m_q G_\pi^{1/2}$$

eliminating unobservable G_π

Gell-Mann Oakes Renner relation

$$F_\pi^2 m_\pi^2 = -2 m_q \langle \bar{q} q \rangle$$

Weinberg-Glashow Relation

Derivation of Weinberg-Glashow relation

Glashow, Weinberg, PRL 20 (1968) 224

based on chiral Ward identity
consider the correlation function of axial vector current and pseudoscalar field

$$\Pi_5^{ab}(q) = \mathrm{F.T.}\ \partial^\mu \langle 0|\mathrm{T}[A_\mu^a(x)\phi_5^b(0)]|0\rangle,$$

take soft limit, q → 0   generator of chiral transformation

$$\Pi_5^{ab}(0) = \langle 0|[Q_5^a, \phi_5^b(0)]|0\rangle = -i\delta^{ab}\langle 0|\bar{q}q|0\rangle$$

pion decay constant

$$\langle 0|A_\mu^a(x)|\pi^b(p)\rangle = \delta^{ab} i p_\mu F_\pi e^{-ip\cdot x}$$

wavefunction normalization

$$\langle 0|\phi_5^a(x)|\pi^b(p)\rangle = \delta^{ab} G_\pi^{1/2} e^{-ip\cdot x}$$

hadronic description of correlation function
insert hadronic complete set
only pionic modes contribute thanks to pseudoscalar field

$$-iq^\mu \frac{\langle 0|A_\mu^a|\pi^c\rangle i\langle \pi^c|\phi_5^b|0\rangle}{q^2 - m^2 + i\epsilon} = \frac{q^2\, i\delta^{ab} F G^{1/2}}{q^2 - m^2 + i\epsilon} \to i\delta^{ab} F G^{1/2} \quad \text{for } m = 0$$

only Nambu-Goldstone mode contributes in the soft limit

$$F_\pi G_\pi^{1/2} = -\langle \bar{q}q\rangle$$

Weinberg-Glashow Relation

Weinberg-Glashow relation in medium

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

the derivation of WG relation is based on chiral Ward identity (operator relation)
the in-medium extension is straightforward

$$\Pi_5^{ab}(q) = \text{F.T. } \partial^\mu \langle \underline{\Omega} | \text{T}[A_\mu^a(x)\phi_5^b(0)] | \underline{\Omega} \rangle$$

in-medium ground state

take soft limit

$$\Pi_5^{ab}(0) = \langle \Omega | [Q_5^a, \phi_5^b] | \Omega \rangle = -i\delta^{ab} \langle \bar{q}q \rangle^*$$

calculate with in-medium hadronic quantities

$$\begin{aligned} \langle \Omega_\ell^b(k) | \phi_5^a(x) | \Omega \rangle &= \delta^{ab} G_\ell^{*1/2} e^{ik\cdot x}, \\ \langle \Omega | A_\mu^a(x) | \Omega_\ell^b(k) \rangle &= i\delta^{ab} [n_\mu (n\cdot k) N_\ell^* + k_\mu F_\ell^*] e^{-ik\cdot x}. \end{aligned}$$

Lorentz invariance is lost due to the presence of nuclear matter

sum rule in chiral limit

$$\sum_\alpha \text{Re}\left[(N_\alpha^* + F_\alpha^*) G_\alpha^{*1/2} \right] = -\langle \bar{q}q \rangle^*$$

sum up all the zero modes

In-medium quark condensate

the sum rule can be simplified in the linear density approximation

DJ, Hatsuda, Kunihiro, PLB 670 (2008), 109.

linear density approximation

$$F_\pi^t G_\pi^{*1/2} = -\langle \bar{q} q \rangle^*$$

scaling law

$$\frac{F_\pi^t}{F_\pi} \frac{G_\pi^{*1/2}}{G_\pi^{1/2}} = \frac{\langle \bar{q} q \rangle^*}{\langle \bar{q} q \rangle}$$

phenomenological determination of in-medium condensate

pionic atom
πA scattering

$$\frac{b_1}{b_1^*} = \left(\frac{F_\pi^t}{F_\pi} \right)^2$$

πN scattering amplitude

$$\left(\frac{G_\pi^*}{G_\pi} \right)^{1/2} = \left(1 + \rho \left. \frac{\partial \mathcal{T}_{\pi N}^{(+)}}{\partial \omega^2} \right|_{\omega=0} \right)^{-1/2}$$

$$\frac{\langle \bar{q} q \rangle^*}{\langle \bar{q} q \rangle} = 1 - 0.37 \frac{\rho}{\rho_0}$$

30-40% reduction at saturation density takes place

this conclusion has been obtained in **linear density approximation**

→ **theoretical issue**:
density dependence of quark condensate beyond linear density approximation
phenomenological issue:
other phenomena expected by partial restoration

What we expect
in partial restoration of chiral symmetry
～phenomenological approach～

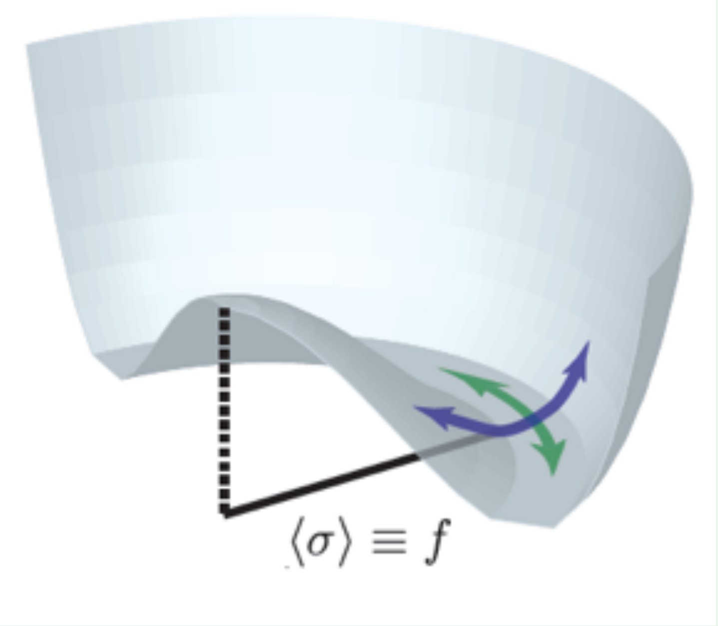
Wavefunction Renormalization

Wavefunction Renormalization

DJ, Hatsuda, Kunihiro, PRD63, 011901R (01).

In chiral effective theories,
Nambu-Goldstone boson fields are parametrized as angular variables in chiral space

ex. $$U = \exp\left[i \frac{\vec{\pi} \cdot \vec{\tau}}{f} \right]$$



Angular variables is **dimensionless**.
One needs an energy scale to **normalize** the NG boson field. The scale should be given by the strength of the spontaneous breaking, i.e. the **chiral condensate**, because this is only the energy scale available in the chiral limit.

When the partial restoration takes place with the change of the chiral condensate, we need to **renormalize the NG boson field**. Thus, wavefunction renormalization is one of the significant medium effects on the NG bosons.

Significance of wavefunction renormalization is also concluded by the energy dependence of the in-medium self-energy of the NG boson. The interaction of NG bosons should be given in chiral expansion at low-energy according to the low energy theorem.

$$\mathcal{T}_{\phi N}(q) \simeq m_q \mathcal{A} + q^2 \mathcal{B} + \cdots$$

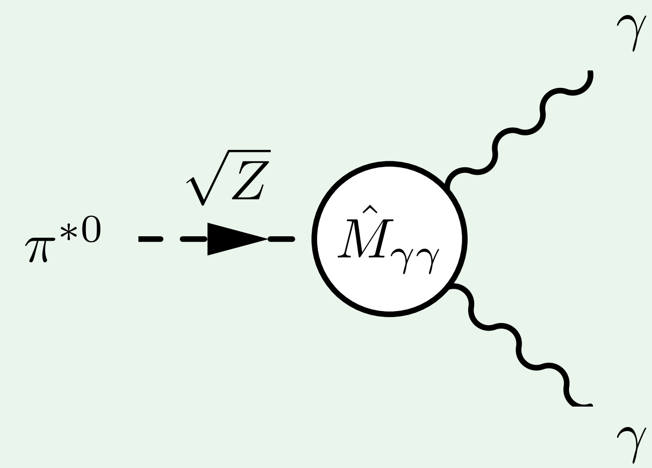
See also, Kolomeitsev, Kaiser, Weise, PRL90, 092501 (03).

$\pi^0 \to \gamma\ \gamma$ decay in nuclear medium

in-medium decay amplitude

Goda, DJ, PTEP 2014, 033D03 (2014).

$$M^*_{\gamma\gamma} = \sqrt{Z}\hat{M}_{\gamma\gamma}$$



Z wave function renormalization

$\hat{M}_{\gamma\gamma}$ 1-particle irreducible vertex correction

no correction in the linear density ← Meissner, Oller, Wirzbz, AnnPhys297, 27 (02)

in-medium change of the amplitude

$$\frac{M^*_{\gamma\gamma}}{M_{\gamma\gamma}} = \sqrt{Z}$$

$$\frac{\Gamma^*_{\gamma\gamma}}{\Gamma_{\gamma\gamma}} = Z \simeq 1 + 0.4\frac{\rho}{\rho_0}$$

in-medium chiral perturbation theory tells that

Z for pion is enhanced 40% in normal density

Goda, DJ, PTEP 2014, 033D03 (2014).
Huebsch, DJ. in preparation.

K+ - nucleus scattering revisited

Aoki, DJ, PTEP2017,103D01(17)

thanks to large mean free path (5 fm),

K^+A elastic scattering is expected to be described by a single step process

(multiple steps effects are small)

expectation $\sigma_{K^+A} \simeq A\,\sigma_{K^+N}$ $\quad p_{\rm lab.} < 800\ {\rm MeV/c}$

(small inelasticity)

breakdown of linear density approx.

- ratio of cross sections is larger than 1

$$\frac{\sigma_{K^{+\,12}{\rm C}}}{6\sigma_{K^+d}} > 1.0$$

Bugg, et al. Phys.Rev. **168**, 1466 (1968)
Weise, Nuovo Cim. **A102,** 265 (1989)

should be smaller than 1, if one considers nuclear shadowing effect

- Tρ approximation is broken down

Friedman, Gal, Phys.Rept. 425, 89 (2007)

$$2m_K^+ V_{\rm opt} \simeq -\rho\, T_{K^+N}$$ linear density approx.

K+optical potential

$p_{\rm lab}$	$V_{\rm opt}$		Re b_0(fm)	Im b_0(fm)
488	$t\rho$	**"experiment"**	$-0.203(26)$	0.172(7)
	$t_{\rm free}\rho$	**"expectation"**	-0.178	0.153

$$T_{K^+N} = -\frac{4\pi E_{\rm c.m.}}{M_N} b_0$$

fit by experiments

free KN

in-medium K+N amplitude is enhanced in 15%

K^+ - nucleus scattering revisited

in-medium K^+N amplitude is enhanced in 15%

possible explanations
- nucleon-nucleon correlation
- nucleon "swelling"
- mass reduction of vector meson

etc.

WFR can be counted as one of the corrections beyond linear density

$$2mV_{\mathrm{opt}} = -Z\rho T_{K^+N}$$

Kolomeitsev, Kaiser, Weise, PRL90, 092501 (03)

ChPT leading order calculation of KN scattering amplitude

$$Z = 1 + 0.1\frac{\rho}{\rho_0}$$

Aoki, DJ, PTEP2017,103D01(17)

more systematic calculation is necessary based on in-medium chiral perturbation theory

I=0 KN amplitude has ambiguity at low energy

Expectations in partial restoration

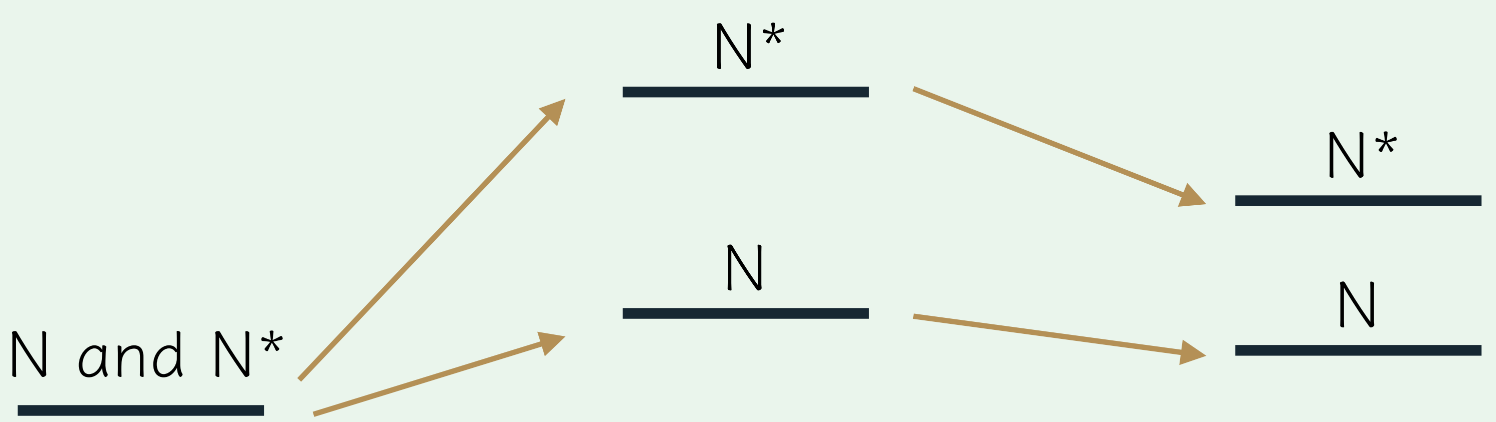
- reduction of mass difference between chiral partners

- chiral partner : transformed by chiral symmetry

  π - σ、 ρ - a_1、 N-N*(1535) etc.

- chiral partners should degenerate in chiral restoration limit
- reduction of the mass difference in partial restoration
  (more precisely, their spectral functions tend to degenerate)

parity partner   nucleon N and excited nucleon N*



$\langle \bar{q}q \rangle = 0$ $\langle \bar{q}q \rangle \neq 0$ partial restoration

η

η meson couples to N-N*(1535)
$\rightarrow$ probes chiral symmetry of N and N*

DJ, Nagahiro, Hirenzaki, PRC66, 045202 (02).
Nagahiro, DJ, Hirenzaki, PRC68, 035805 (03); NPA761, 92 (05)
DJ, Kolomeitsev, Nagahiro, Hirenzaki, NPA811,158 (08)

one of the difficulties is that bound states have a large width.

η' meson

as an example of in-medium reduction of mass which
is generated by spontaneous breaking of chiral symmetry

η' meson and chiral symmetry

η' is a PS meson having 1 GeV mass.

1 GeV is a **typical mass scale** of hadrons

looks **nothing special**

$\eta'(958)$ $\quad I^G(J^{PC}) = 0^+(0^{-+})$

Mass $m = 957.78 \pm 0.06$ MeV
Full width $\Gamma = 0.198 \pm 0.009$ MeV

special mesons are π, K, η

light mass because they are Nambu-Goldstone bosons associated with ChSB

chiral symmetry in QCD

octet axial currents are (almost) conserved

$$\partial^\mu A_\mu^{(8)} = \frac{i}{\sqrt{3}}(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d - 2m_s \bar{s}\gamma_5 s) \qquad \textbf{(small) PCAC}$$

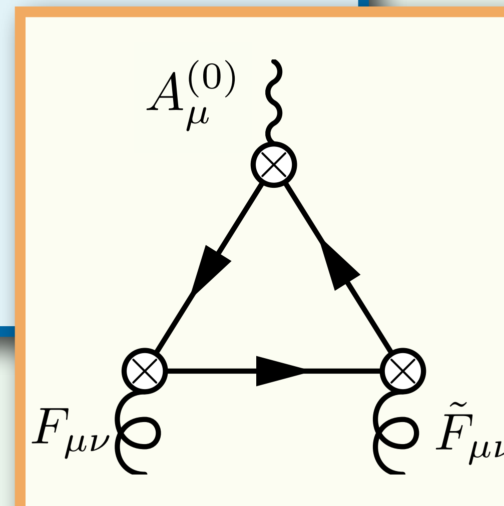
octet chiral symmetry is spontaneously broken $\quad \mathrm{SU}(3)_L \otimes \mathrm{SU}(3)_R \to \mathrm{SU}(3)_V$

singlet axial current is NOT conserved due to quantum anomaly

$$\partial^\mu A_\mu^{(0)} = 2i(m_u \bar{u}\gamma_5 u + m_d \bar{d}\gamma_5 d + m_s \bar{s}\gamma_5 s) + \frac{3\alpha_s}{8\pi} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

(small) PCAC **anomaly**

singlet chiral symmetry is **always broken by anomaly explicitly**



η' fails to become a NG boson, and is not necessarily massless,
when ChSB takes place

η' meson in chiral restoration

DJ, Nagahiro, Hirenzaki, PRC85 (12) 032201(R)

Group theoretical argument of $\mathrm{SU}_L(3) \otimes \mathrm{SU}_R(3)$

no matter how $U_A(1)$ is

Scalar and Pseudoscalar mesons

$(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})$ $\quad \bar{q}_i^L q_j^R \quad \bar{q}_i^R q_j^L$

both octet and singlet contain

$\pi, K, \eta_8, \eta_0 \qquad \sigma, a_0, \kappa, f_0$

parity eigenstate

9 scalar and 9 pseudoscalar

$\bar{q}_i \gamma_5 q_j \, , \, \bar{q}_i q_j$

Vector and Axial vector mesons

$(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ $\quad \bar{q}_i^L q_j^L \quad \bar{q}_i^R q_j^R$

$3 \times 3^{\mathrm{bar}} = 8 + 1$ flavor single is independent

parity eigenstate

8 vector and 8 axial vector

$\bar{q}_i \gamma_\mu q_j \, , \, \bar{q}_i \gamma_5 \gamma_\mu q_j$

if chiral symmetry is manifest or restored,
π, η_8 and η_0 should degenerate even though anomaly is there

dynamical argument is given by Lee and Hatsuda

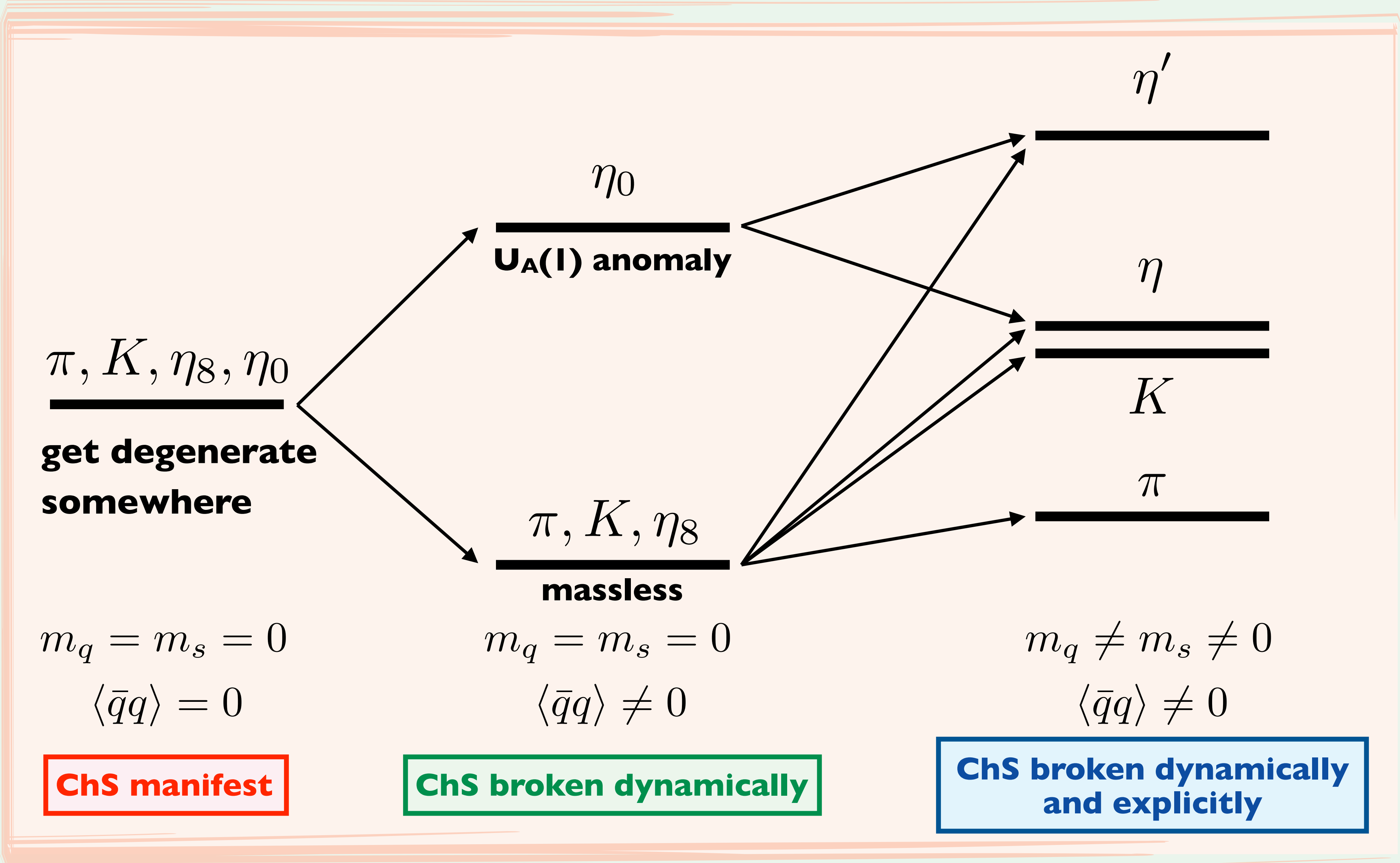
Lee, Hatsuda, PRD54, 1871 (1996)

η' meson in chiral restoration

DJ, Nagahiro, Hirenzaki, PRC85 (12) 032201(R)

When chiral symmetry is restored... as a consequence of $\mathrm{SU}_L(3) \otimes \mathrm{SU}_R(3)$

9 PS π, K, η_8, η_0 9 S σ, a_0, κ, f_0 get degenerate



mass difference between η and η' stems from chiral symmetry breaking with help of $U_A(1)$ anomaly

in order for $U_A(1)$ anomaly to affect the η' mass,
chiral symmetry is **necessarily** broken spontaneously and/or explicitly.

η' meson in nuclear matter

DJ, Nagahiro, Hirenzaki, PRC85 (12) 032201(R)

the mass gap of η' and η is generated by chiral symmetry breaking assisted by anomaly

η' mass should get reduced when chiral symmetry is restored in nuclear medium

simple order estimation

η'-η mass difference (400 MeV) be dependent on quark condensate linearly

partial restoration of ChS take place with 30% at ρ_0

we expect strong η' mass reduction $\Delta m_{\eta'} \sim 120$ MeV @ $\rho = \rho_0$

chiral effective theories tell similar results.

linear sigma model Sakai, DJ, PRC88 (13) 064906

$\Delta m_{\eta'} \sim 80$ MeV @ $\rho = \rho_0$

$\Delta(m_{\eta'} - m_{\eta}) \sim 130$ MeV @ $\rho = \rho_0$

NJL model P. Costa, M. C. Ruivo, and Y. L. Kalinovsky, PLB560, 171 (03). Nagahiro, Takizawa, Hirenzaki, PRC74,045203 (2006)

$\Delta m_{\eta'} \sim 150$ MeV @ $\rho = \rho_0$

Interpretation of η' mass reduction

interpretation of in-medium η' mass reduction in nuclear physics

Sakai, DJ, PRC88 (13) 064906; PTEP2017, 013D01 (17).

(a part of) nucleon mass is generated also by spontaneous breaking of ChS

$$m_N = g\langle\sigma_0\rangle$$

→ presence of strong coupling σNN

this is the origin of the strong scalar attraction in NN interaction

chiral symmetry breaking generates a part of eta' meson with help of anomaly

$$m_{\eta_0}^2 - m_{\eta_8}^2 = 6B\langle\sigma_0\rangle$$

→ presence of strong coupling ση'η'

B term : anomaly effect

strong attraction in η'-N with sigma exchange

this provides a strong scalar potential for η' in nucleus

DJ, Masutani, Hirenzaki, PTEP 2019, 053D02 (2019)

remark

we could have repulsive vector potential like ω exchange in nuclear force

we have no Weinberg-Tomozawa type interaction for η'N channel

induced by vector meson exchange

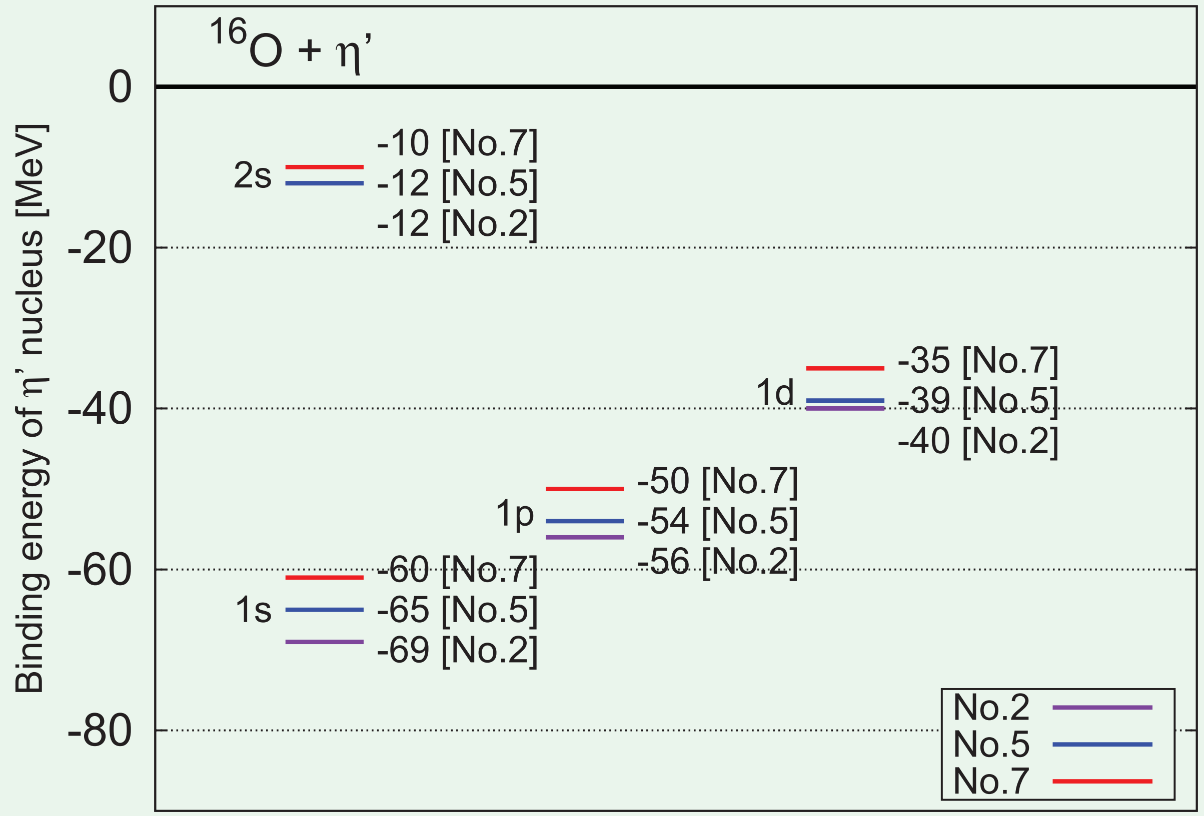
Possible bound state spectra

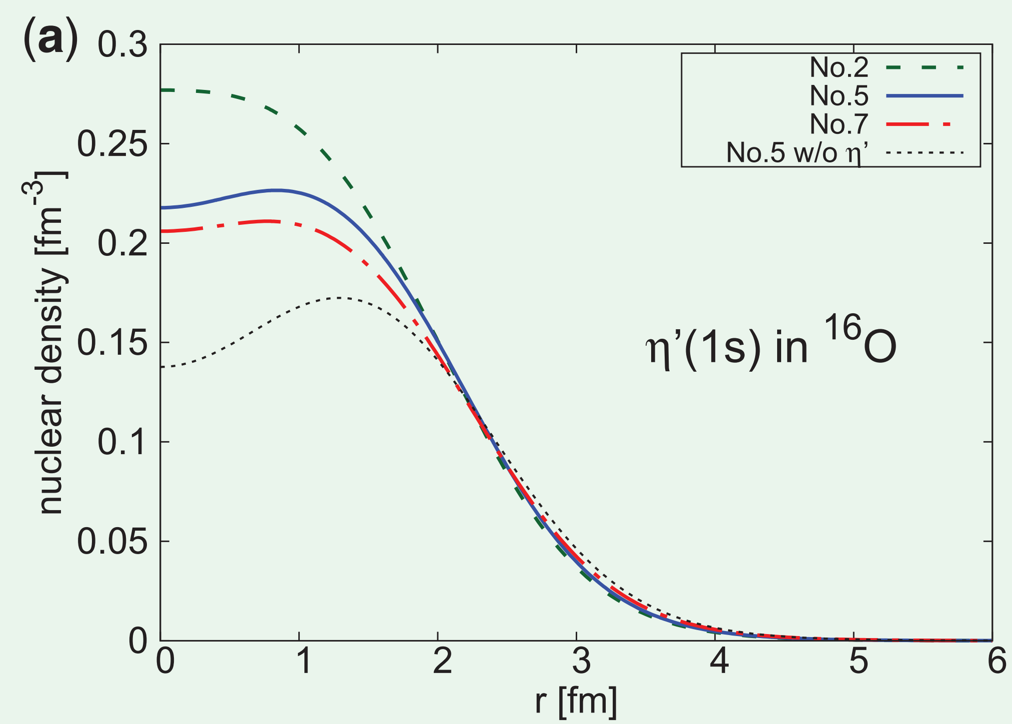
DJ, Nagahiro, Hirenzaki, PRC85 (12) 032201(R)

mass reduction in nuclear matter provides a scalar potential in finite nucleus

relativistic mean field theory (RMF)

DJ, Masutani, Hirenzaki, PTEP 2019, 053D02 (2019)





four bound states of η' meson in Oxygen nucleus

currently,
no distinct structure was observed in formation experiments at GSI and SPring8

Tanaka et al. (η-PRiME/Super-FRS Collaboration), PRL117 (16) 202501; PRC97 (18) 015202

N. Tomida et al. [LEPS2/BGOegg], Phys. Rev. Lett. 124, 202501 (2020)

Role of anomaly term for ChS breaking

Kono (Tokyo Metropolitan University)
Jido (Tokyo Institute of Technology)
Kuroda, Harada (Nagoya University)

Role of anomaly term for ChS breaking

question whether anomaly term would play minor role for ChS breaking
chiral symmetry is broken without anomaly term

Kono, DJ, Kuroda, Harada,
PTEP accepted
DOI: 10.1093/ptep/ptab084

SU(3) linear σ model

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}[\partial_\mu M \partial^\mu M^\dagger] - \frac{\mu^2}{2}\mathrm{Tr}[MM^\dagger] - \frac{\lambda}{4}\mathrm{Tr}[(MM^\dagger)^2] - \frac{\lambda'}{4}\left(\mathrm{Tr}[MM^\dagger]\right)^2 - A\mathrm{Tr}[\chi M^\dagger + \chi^\dagger M] + \sqrt{3}B(\det M + \det M^\dagger)$$

explicit ChS breaking
flavor symmetry breaking

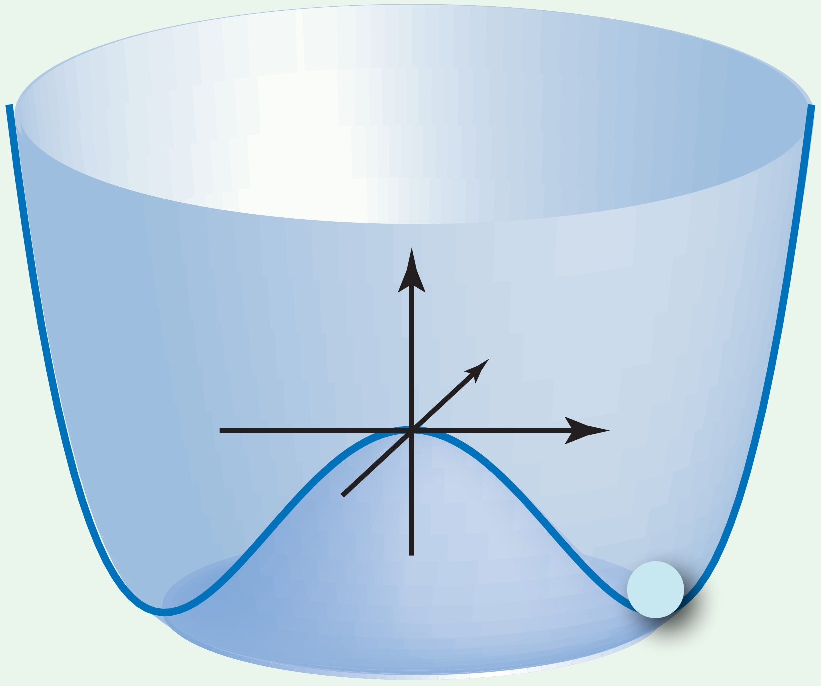
anomaly term
breaks $U_A(1)$ symmetry

- **$\mu^2 < 0$ → spontaneous breaking**

SU(3) NJL model

$$\mathcal{L} = \bar{q}(\partial\!\!\!/ - m)q + g_s\left[\left(\bar{q}\frac{\lambda_a}{2}q\right)^2 + \left(\bar{q}i\gamma_5\frac{\lambda_a}{2}q\right)^2\right] + \frac{g_d}{2}\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\}$$

- **$g_s > g_{s\,\mathrm{crit}}$ → spontaneous breaking**



with anomaly term, ChS can be broken even if above conditions is not satisfied

Anomaly driven symmetry breaking

chiral limit just for simplicity

Kono, DJ, Kuroda, Harada, PTEP accepted
DOI: 10.1093/ptep/ptab084

NJL model

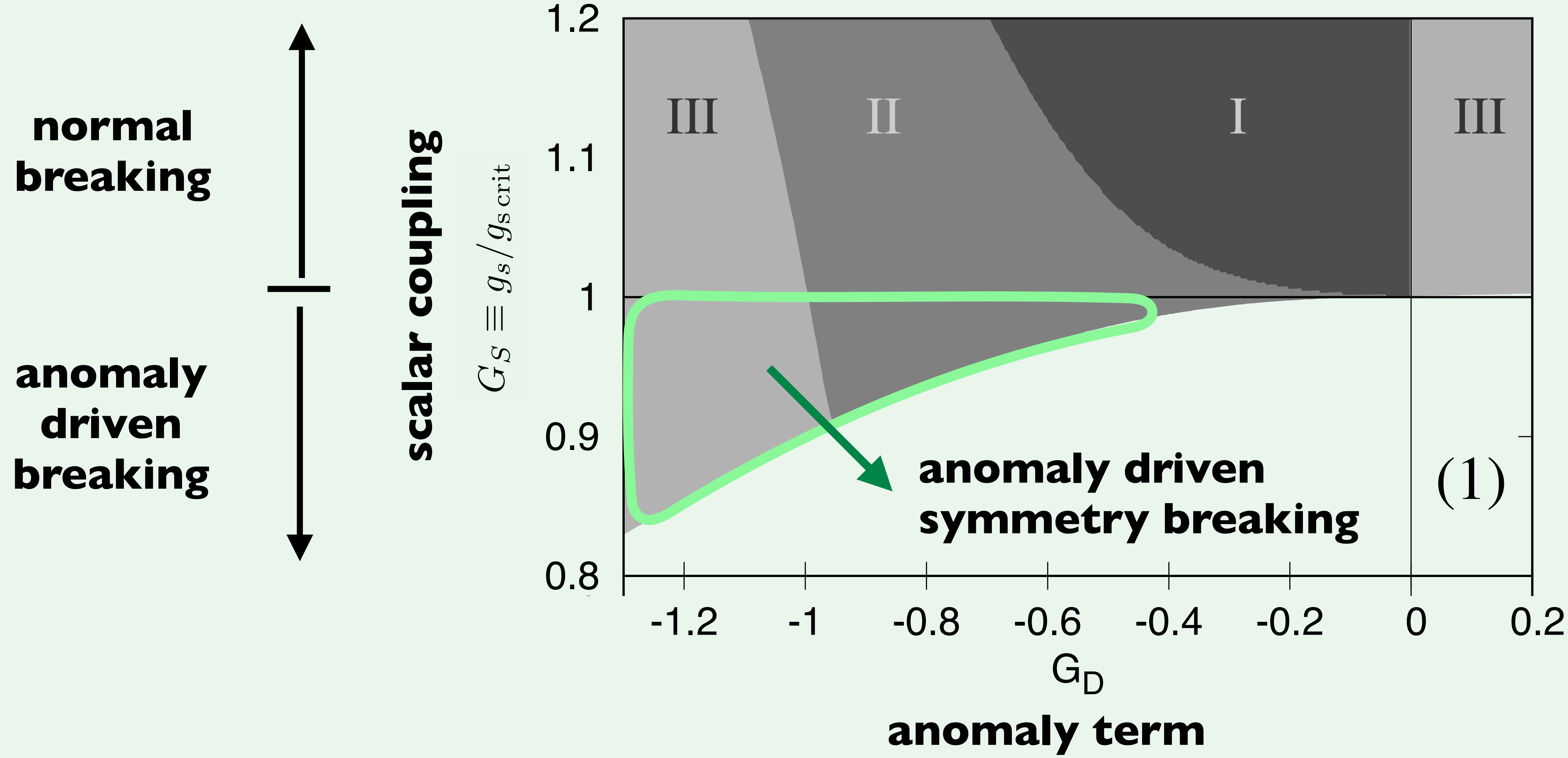
$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + g_s\left[\left(\bar{q}\frac{\lambda_a}{2}q\right)^2 + \left(\bar{q}i\gamma_5\frac{\lambda_a}{2}q\right)^2\right] + \frac{g_d}{2}\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\}$$

chiral limit

phase diagram (SSB takes place in gray areas)

$$G_S = g_s\left(\frac{3\Lambda^2}{2\pi^2}\right)$$

$$G_D = g_d\Lambda\left(\frac{3\Lambda^2}{2\pi^2}\right)^2$$



anomaly driven symmetry breaking can take place with sufficiently large anomaly term

Anomaly driven symmetry breaking

chiral limit just for simplicity

Kono, DJ, Kuroda, Harada,
PTEP accepted
DOI: 10.1093/ptep/ptab084

NJL model

$$\mathcal{L} = \bar{q}(\not\partial - \not{m})q + g_s\left[\left(\bar{q}\frac{\lambda_a}{2}q\right)^2 + \left(\bar{q}i\gamma_5\frac{\lambda_a}{2}q\right)^2\right] + \frac{g_d}{2}\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\}$$

chiral limit

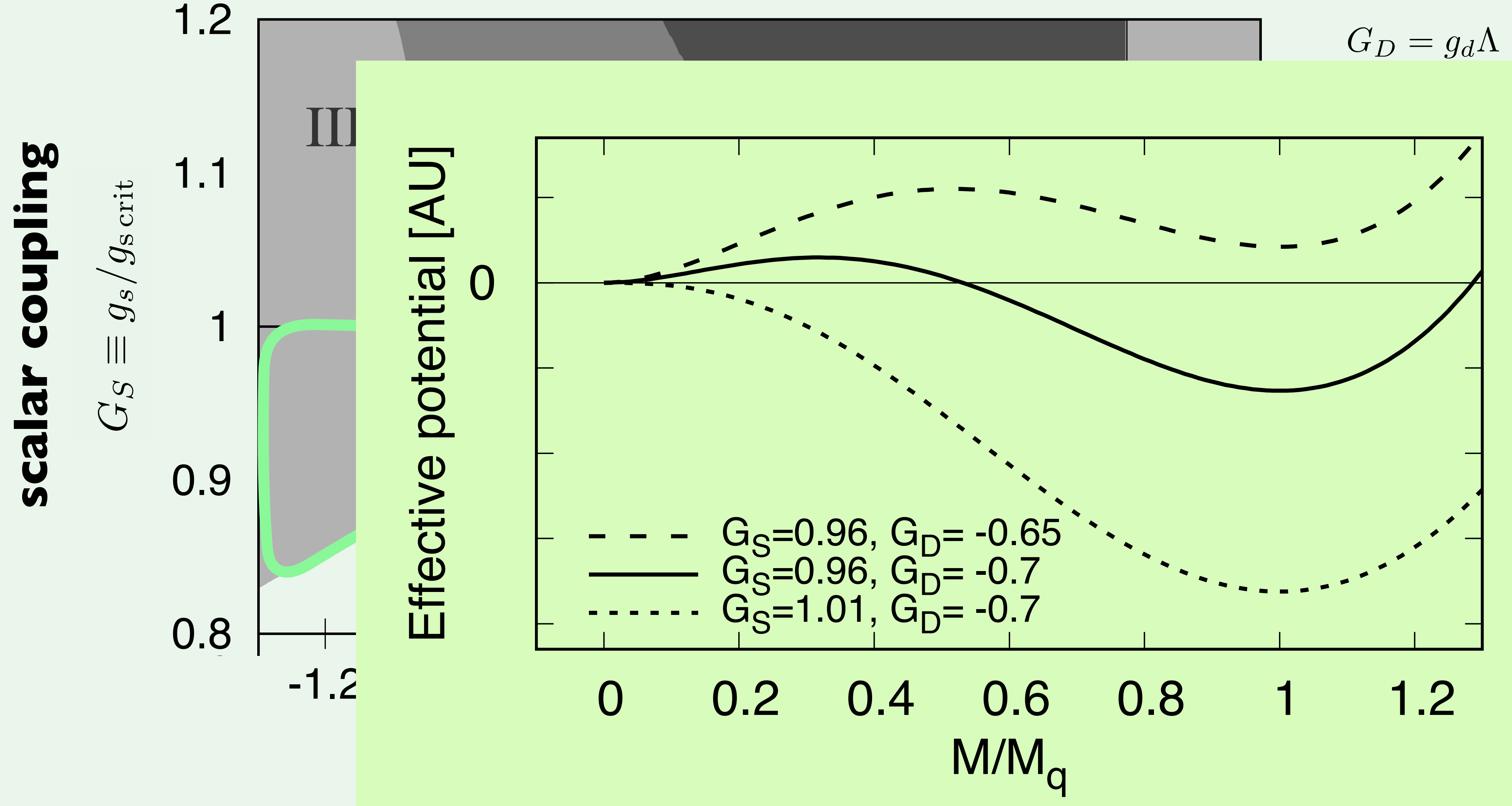
phase diagram (SSB takes place in gray areas)

$$G_S = g_s\left(\frac{3\Lambda^2}{2\pi^2}\right)$$

$$G_D = g_d\Lambda\left(\frac{3\Lambda^2}{2\pi^2}\right)^2$$

normal breaking

anomaly driven breaking



anomaly driven symmetry can take place with sufficiently large anomaly term

Anomaly driven symmetry breaking

linear sigma model

Kono, DJ, Kuroda, Harada, PTEP accepted
DOI: 10.1093/ptep/ptab084

chiral limit

vacuum condition at tree level + mass formula

relation between σ and η' masses and μ^2

$$6\mu^2 = m_{\eta_0}^2 - 3m_{\sigma_0}^2$$

where σ is the chiral partner of pion

usual breaking	$\mu^2 < 0$	$m_{\sigma_0}^2 > m_{\eta'}^2/3$
anomaly driven breaking	$\mu^2 > 0$	$m_{\sigma_0}^2 < m_{\eta'}^2/3$

Anomaly driven symmetry breaking

linear sigma model

Kono, DJ, Kuroda, Harada,
PTEP accepted
DOI: 10.1093/ptep/ptab084

chiral limit

vacuum condition at tree level + mass formula

relation between σ and η' masses and μ²

$$6\mu^2 = m_{\eta_0}^2 - 3m_{\sigma_0}^2$$

where σ is the chiral partner of pion

usual breaking	$\mu^2 < 0$	$m_{\sigma_0}^2 > m_{\eta'}^2/3$
anomaly driven breaking	$\mu^2 > 0$	$m_{\sigma_0}^2 < m_{\eta'}^2/3$

off chiral limit

usual breaking	$\mu^2 < 0$	$m_{\sigma_0} > 840\ \mathrm{MeV}$
anomaly driven breaking	$\mu^2 > 0$	$m_{\sigma_0} < 840\ \mathrm{MeV}$

$$m_{\eta_8} = 550\ \mathrm{MeV}$$
$$m_{\eta_0} = 958\ \mathrm{MeV}$$
$$m_{\pi} = 140\ \mathrm{MeV}$$

anomaly driven symmetry breaking is possible,
if singlet sigma mass is smaller than 840 MeV.

Summary

- partial restoration of chiral symmetry = reduction of magnitude of quark condensate
  in-medium modification of pion properties → change of quark condensate
  change of quark condensate → possible property changes of other hadrons

- expectations when partial restoration of chiral symmetry takes place in nuclear matter
  - wave function renormalization of NG bosons
  - reduction of mass difference of chiral partners
  - reduction of (a part of) hadron mass

- role of the anomaly term in spontaneous breaking of chiral symmetry
  - anomaly driven breaking is possible
  - such a case, sigma as a chiral partner of pion should have a lighter mass (than 840 MeV)
  - this could shed light on nature of a chiral partner of pion