Bound states, form factors and distribution amplitudes in flavor physics

APCTP Focus Program in Nuclear Physics 2021 Part I:
Hadron properties in a nuclear medium from the quark and gluon degrees of freedom

July 14-16, 2021 (VIRTUAL)

apctp asia pacific center for theoretical physics

Bruno El-Bennich

Laboratório de Física Teórica e Computacional
Universidade Cidade de São Paulo

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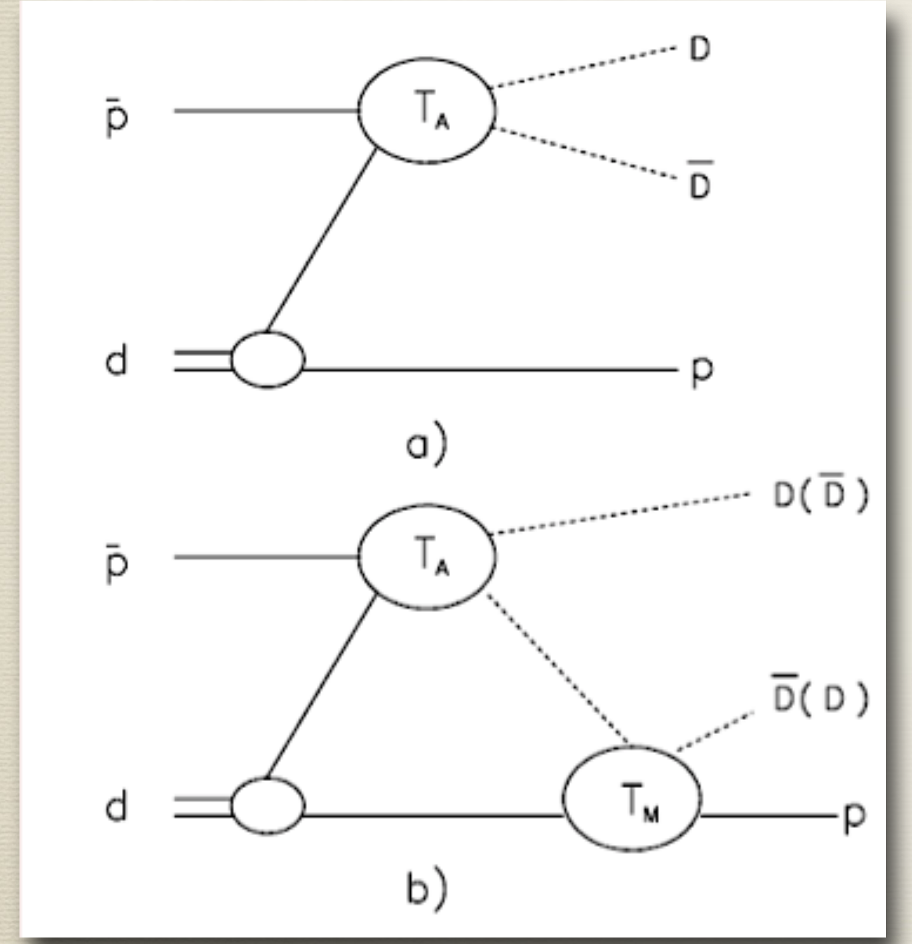
CNPq Conselho Nacional de Desenvolvimento Científico e Tecnológico

Antiproton annihilation on the deuteron

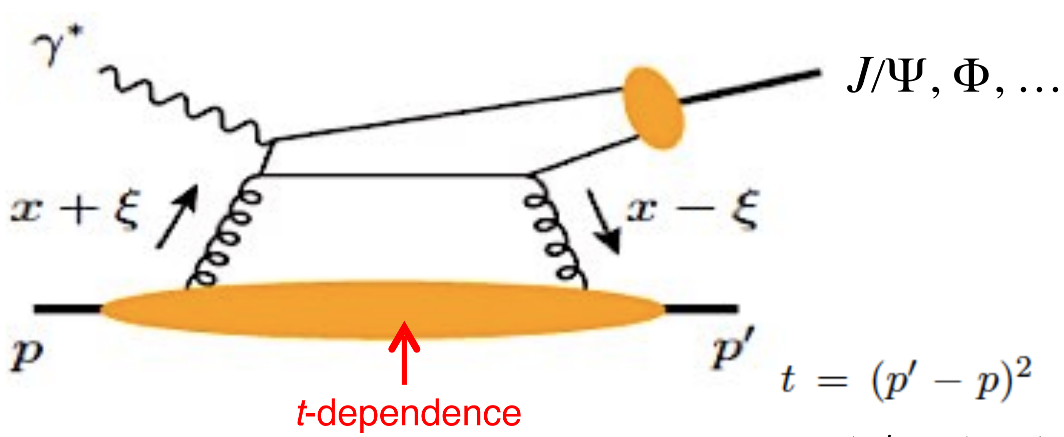
PANDA @ Facility for Antiproton and Ion Research (FAIR)



Spatial imaging of glue in a nucleon/nucleus
Jefferson Lab and Electron-Ion Collider

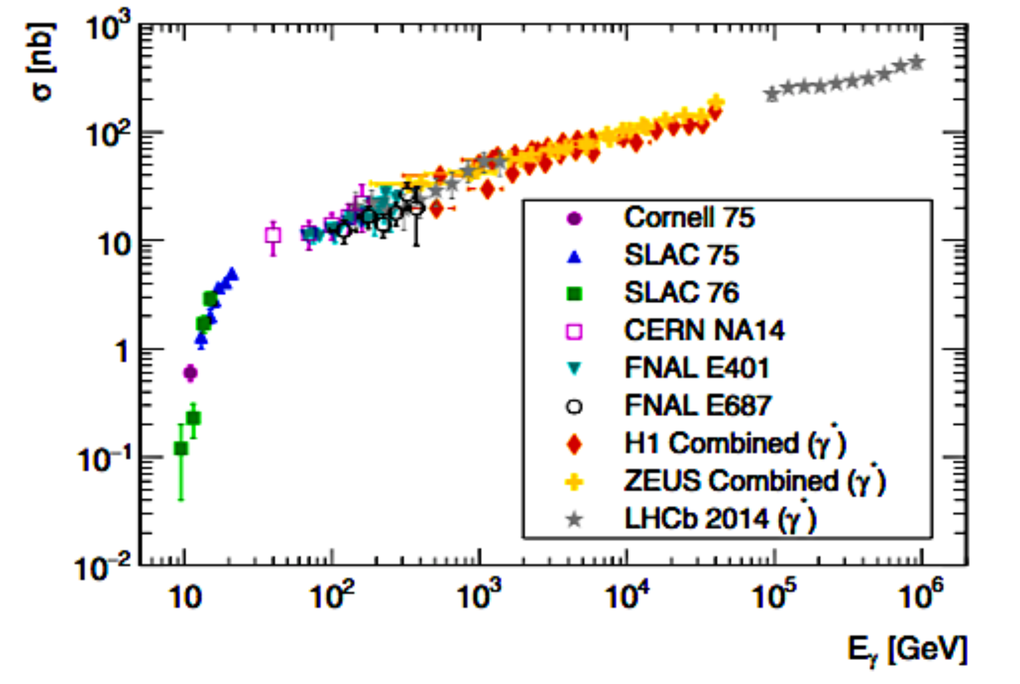
Exclusive photoproduction: hard-scattering mechanism $E_\gamma > 10$ GeV

S.J. Brodsky, E. Chudakov, P. Hoyer, J.M. Laget, Phys. Lett. B (2001)



$t = (p' - p)^2$

$\xi = (P' - P) \cdot n/2$



Fourier transform of *t*-dependence:

- Exclusive J/ψ production where the narrow quarkonium interacts by exchanging gluons with the nucleon's light quarks.
- Scattering amplitude allows for probing the energy-momentum tensor of the proton (nuclei) and can yield the spatial imaging of the glue density in proton.

Electroweak production of charmed mesons at EIC

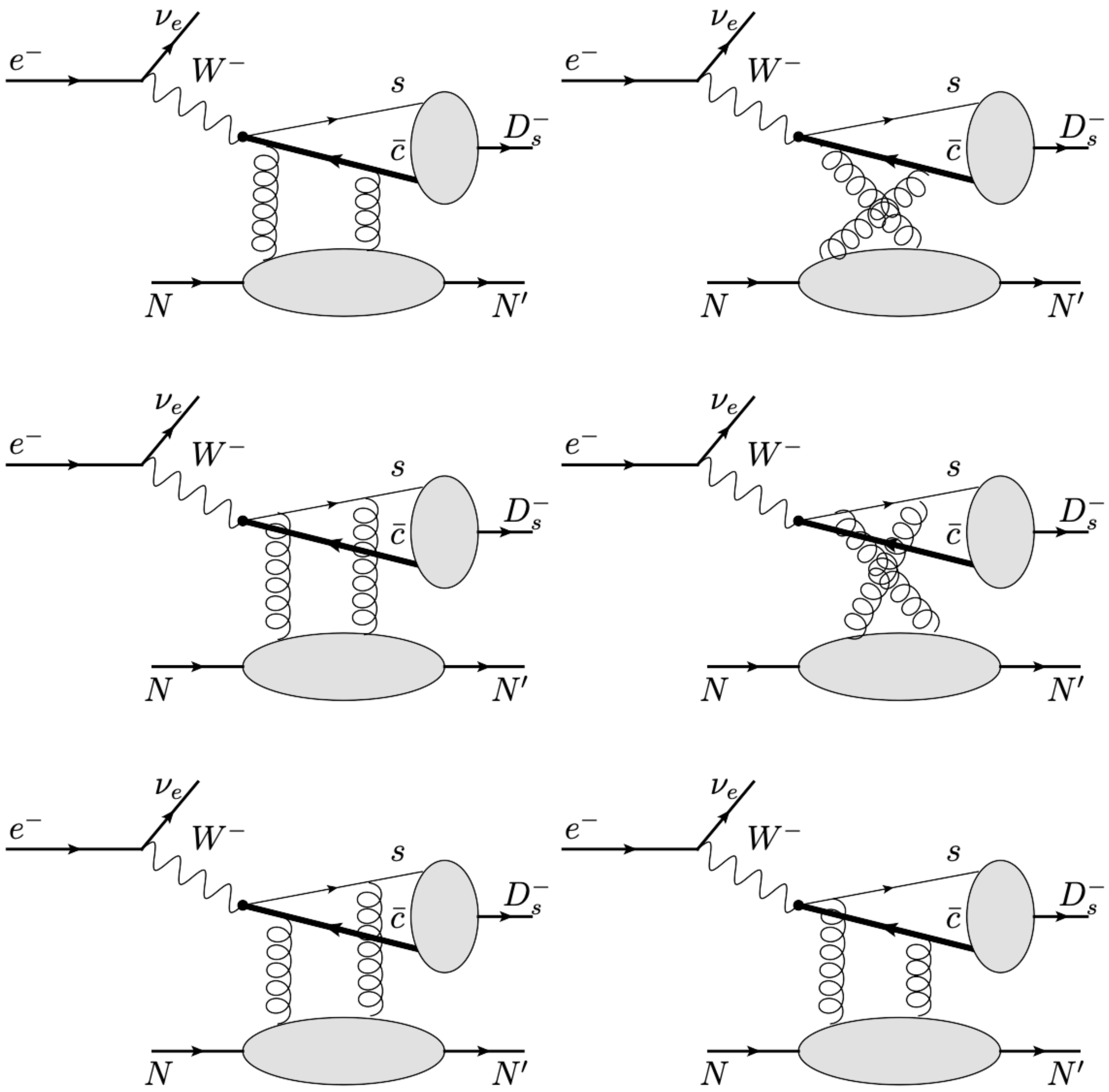


FIG. 1: Feynman diagrams for the factorized amplitude for the $e^- + N \to \nu_e + D_s^- + N'$ process involving the gluon GPDs; the thick line represents the heavy anti-quark $\bar{c}$.

B. Pire, L. Szymanowski, and J. Wagner, arXiv:2104.04944 (2021)

Heavy Quark Effective Theory & Effective Lagrangians

Heavy Quark Effective Theory

$$p_\mu = m_Q v_\mu + k_\mu$$

$$k \sim \Lambda_{\rm QCD}\,;\quad v^2 = 1$$

$$h_v(x) = e^{im_Q v\cdot x}\,\frac{1+\not v}{2}\,Q(x)\,, \qquad H_v(x) = e^{im_Q v\cdot x}\,\frac{1-\not v}{2}\,Q(x)$$

$$\mathcal{L}_Q = \bar{Q}(\not D - m_Q)Q = \underbrace{\bar{h}_v\, iv\cdot D h_v}_{\text{massless mode}} + \underbrace{\bar{H}_v(-iv\cdot D - 2m_Q)H_v}_{\text{massive mode}} + \underbrace{\bar{h}_v\, i\vec{\not D} H_v + \bar{H}_v\, i\vec{\not D} h_v}_{\text{interaction terms}}$$

$$H_v = \frac{1}{iv\cdot D + 2m_Q}\, i\vec{\not D} h_v = \frac{1}{2m_Q}\sum_{n=0}^{\infty}\left(-\frac{iv\cdot D}{2m_Q}\right)^n i\vec{\not D} h_v$$

$$\mathcal{L}_{\rm HQET} = \bar{h}_v\, iv\cdot D h_v + \frac{1}{2m_Q}\left[\bar{h}_v\,(i\vec{D})^2 h_v + c(\zeta)\frac{g}{2}\bar{h}_v\,\sigma_{\mu\nu}g^{\mu\nu}h_v\right] + \ \ldots$$

$$S(p) = i\,\frac{\not p + m_Q}{p^2 - m_Q^2} \ \xrightarrow{m_Q\to\infty}\ i\,\frac{1+\not v}{2v\cdot k} + \mathcal{O}\left(\frac{k}{m_Q}\right)$$

Heavy Quark Effective Theory

$$p_\mu = m_Q v_\mu + k_\mu$$
$$k \sim \Lambda_{\rm QCD}\,;\ \ v^2 = 1$$

$$h_v(x) = e^{im_Q v\cdot x}\,\frac{1+\not v}{2}\,Q(x)\,, \qquad H_v(x) = e^{im_Q v\cdot x}\,\frac{1-\not v}{2}\,Q(x)$$

HQET Lagrangian:

Expansion in α_s and m_Q^{-1} !

$$\mathcal{L}_{\rm HQET} = \mathcal{L}_0 + \frac{1}{m_Q}\mathcal{L}_1 + \frac{1}{m_Q^2}\mathcal{L}_2 + \ldots$$

$\mathcal{L}_0$ has spin-flavor symmetry,

$1/m_Q$ terms are symmetry breaking corrections.

$$S(p) = i\,\frac{\not p + m_Q}{p^2 - m_Q^2} \overset{m_Q\to\infty}{\longrightarrow} i\,\frac{1+\not v}{2\,v\cdot k} + \mathcal{O}\left(\frac{k}{m_Q}\right)$$

Heavy-Meson Chiral Perturbation Theory

R. Casalbuoni et al. , Phys. Rept. **281**, 145 (1997)

$$\mathcal{L}_{\rm heavy} = -{\rm tr}_a {\rm Tr}[\bar{H}_a iv\cdot D_{ba} H_b] + \hat{g}\, {\rm tr}_a {\rm Tr}[\bar{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba} \gamma_5]$$

$$D^\mu_{ba} H_b = \partial_\mu H_a - H_b \frac{1}{2}[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba} \;;$$

$$\mathbf{A}^{ab}_\mu = \frac{i}{2}\left[\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right]_{ab} ;$$

$$H_a(v) = \frac{1+\not{v}}{2}\left[P^{*a}_\mu(v)\gamma_\mu - P^a(v)\gamma_5\right] ;$$

$$\xi = \exp(i\Phi/f^0_\pi) \;;$$

Φ is matrix of $N_f^2 - 1$ pseudo-Goldstone boson.

- Dynamics is constrained by heavy quark symmetry.
- Blind to the heavy quark flavor and spin.
- Heavy pseudoscalar and vector mesons are mass degenerate.

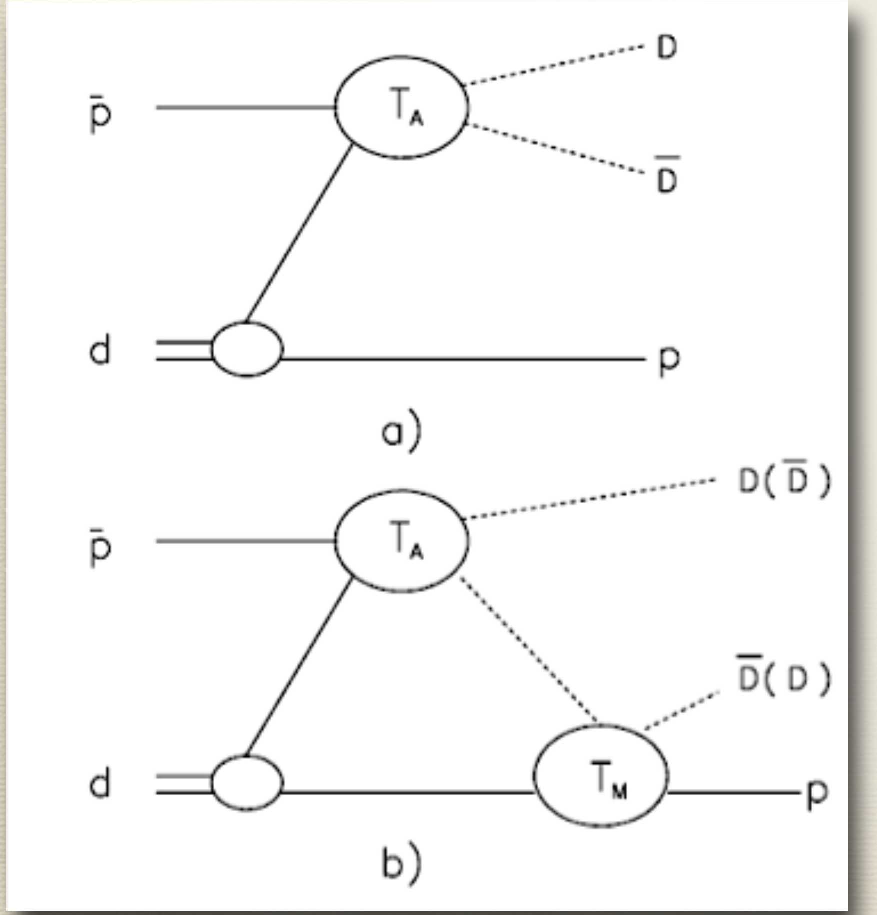
- Can be improved upon — take into account light degrees of freedom, chiral symmetry breaking $\Rightarrow$ HMChPT.

➤ Strong $H^* \to H\pi$ to extract effective heavy quark coupling

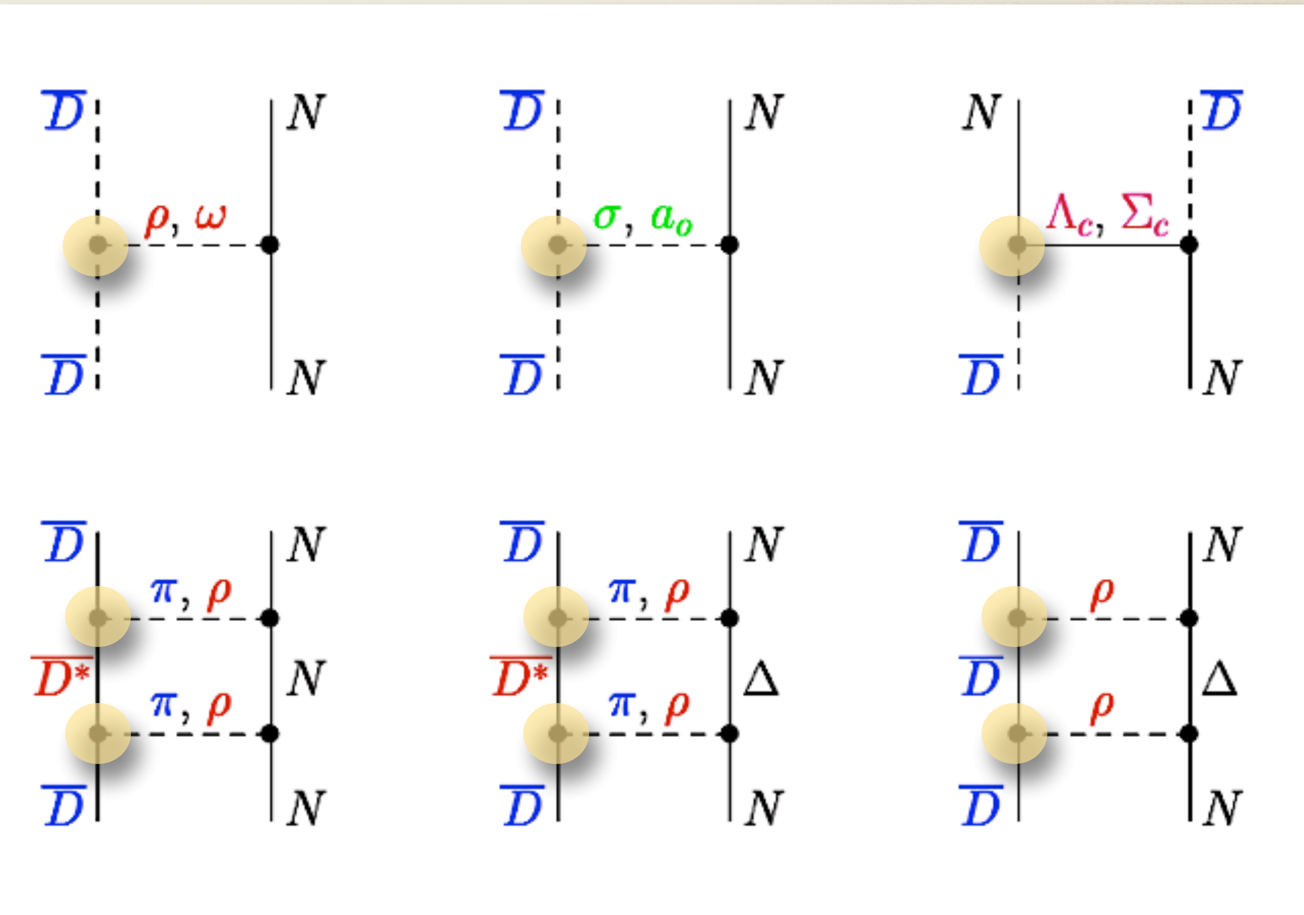
Phenomenological Heavy-Meson Lagrangians

D-meson interactions with nucleons

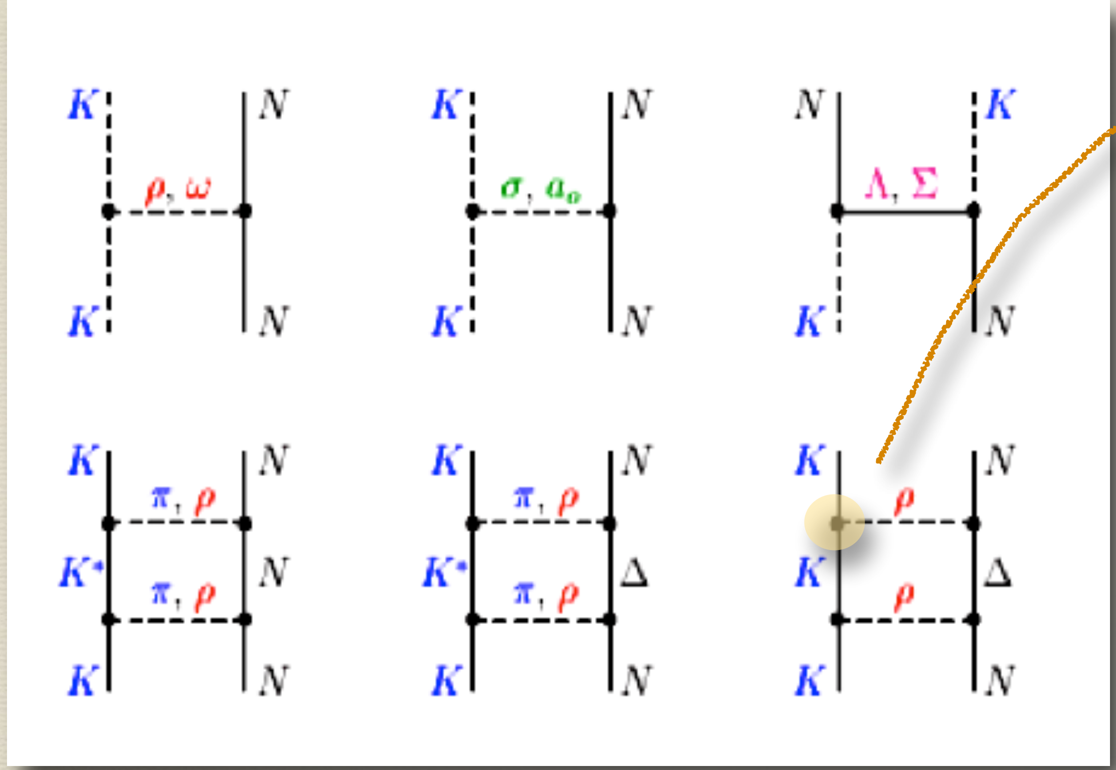
Antiproton annihilation on the deuteron (PANDA @ FAIR)

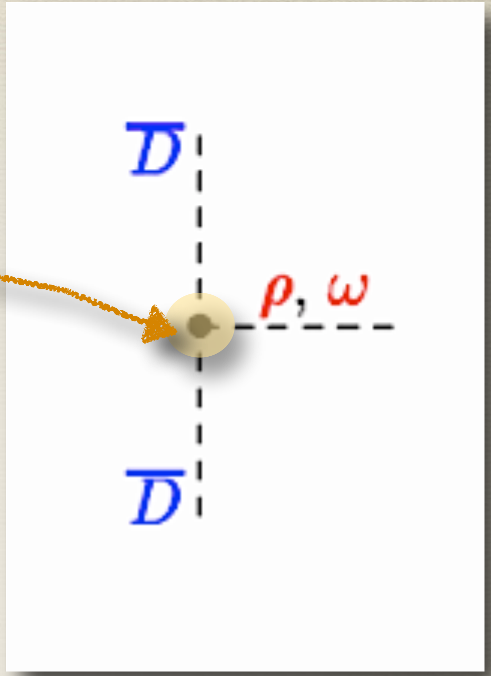


Meson exchange — effective Lagrangians



SU(4) symmetry used ....





Jülich model:

A. Müller-Groeling et al. NPA 513, 557 (1990)

M. Hoffmann et al. NPA 593, 341 (1995)

D. Hadjimichef, J. Haidenbauer and G. Krein, PRC 66 (2002)

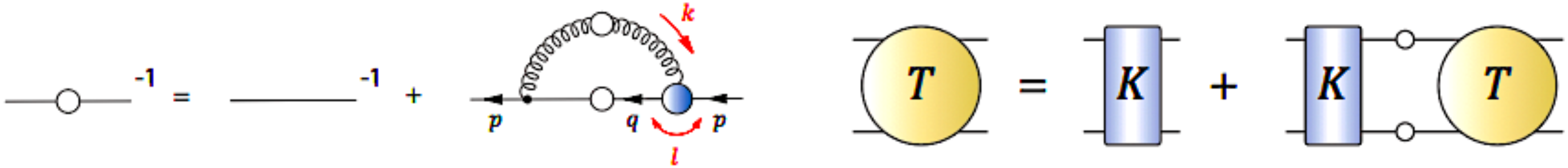
$$SU(4) \text{ symmetry: } g_{D\rho D} = g_{D\omega D} = g_{KK\rho} = \frac{1}{2} g_{\pi\pi\rho}$$

Calculation of the Effective Theory Couplings

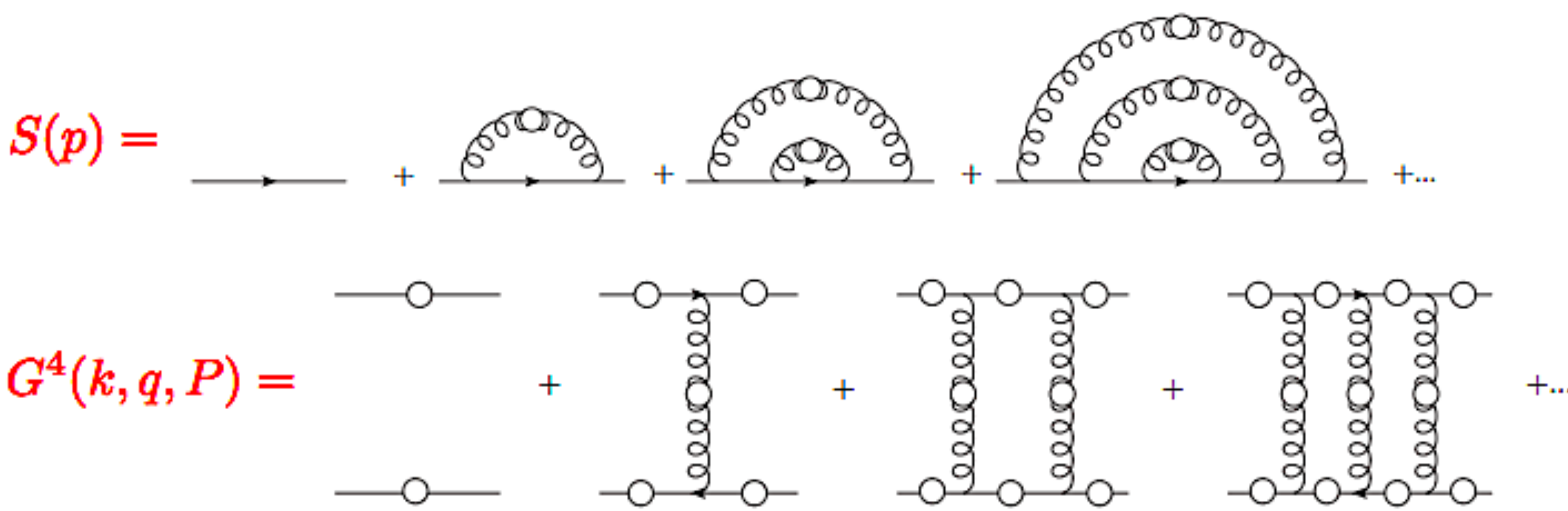
Caveat

- No assumption of heavy-quark symmetry is made.
- In particular, pseudoscalar and vector meson masses are not degenerate.
- We solve the gap equations (Dyson-Schwinger equations) for light and heavy quarks $\Longrightarrow$ dressed quark propagators with running mass $M(p)$.
- Solving the Bethe-Salpeter equation for flavored pseudoscalar and vector mesons and quarkonia, we obtain their wave functions in an improved ladder truncation.
- Although impulse approximation and truncations are employed, $\Lambda_{\rm QCD}/m_c$ contributions are systematically included.

Bethe-Salpeter Equations for QCD Bound States

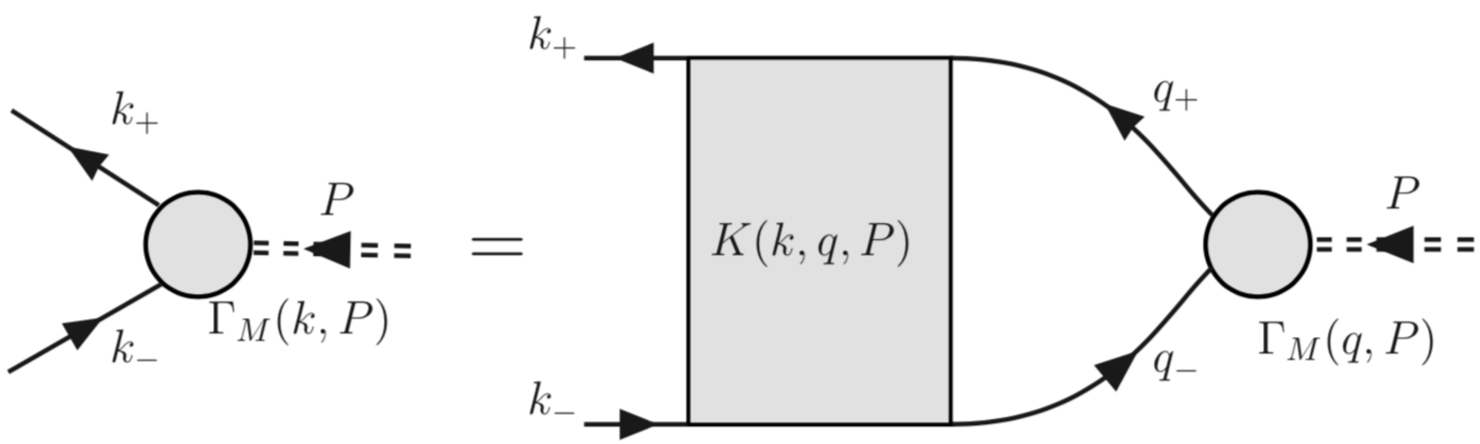


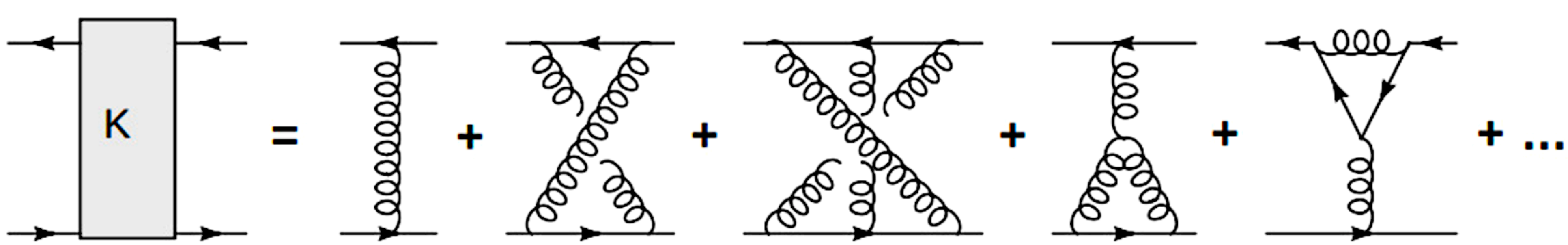
Rainbow-ladder truncation (leading symmetry-preserving approximation)



Bethe-Salpeter Equations for QCD Bound States

$$\left[\Gamma_M^{f\bar{g}}(k;P)\right]_{AB} = \int \frac{d^4q}{(2\pi)^4} \left[K^{f\bar{g}}(k,q;P)\right]_{AC,DB} \left[S_f(q_+)\Gamma_M^{f\bar{g}}(q;P)S_{\bar{g}}(q_-)\right]_{CD}$$



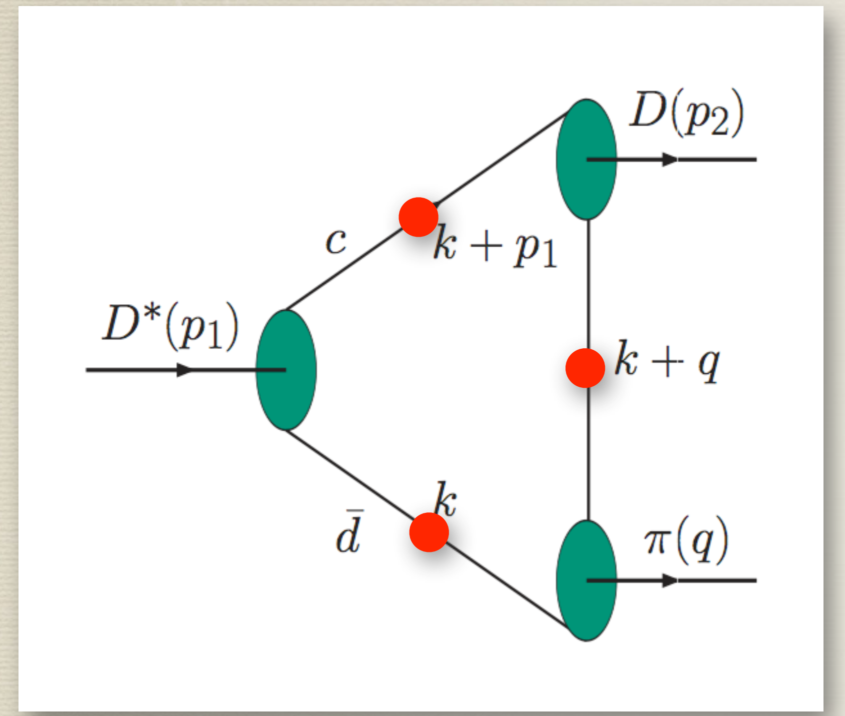


Meson Spectrum

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	$f_M^{\text{exp./lQCD}}$	ϵ_r^f [%]
$\pi(u\bar{d})$	0.136	0.140	2.90	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(s\bar{u})$	0.494	0.494	0.0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0.0
$D_u(c\bar{u})$	$1.867^{+0.008}_{-0.004}$	1.870	0.11	$0.144^{+0.001}_{-0.001}$	0.150(0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	f_M^{lQCD}	ϵ_r^f [%]
$B_u(b\bar{u})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(b\bar{s})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.50
$B_c(b\bar{c})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	7.28
$\eta_b(b\bar{b})$	$9.383^{+0.005}_{-0.004}$	9.398	0.16	$0.520^{+0.009}_{-0.009}$	0.472(4)	10.17

Strong decays: $D^* \to D\pi$



	CLEO	DSE	QCDSR	Lattice
$g_{D^* D\pi}$	$17.9 \pm 0.3 \pm 1.9$	16.5 ± 2	14.0 ± 1.5	20 ± 2
				16.23 (1.71)

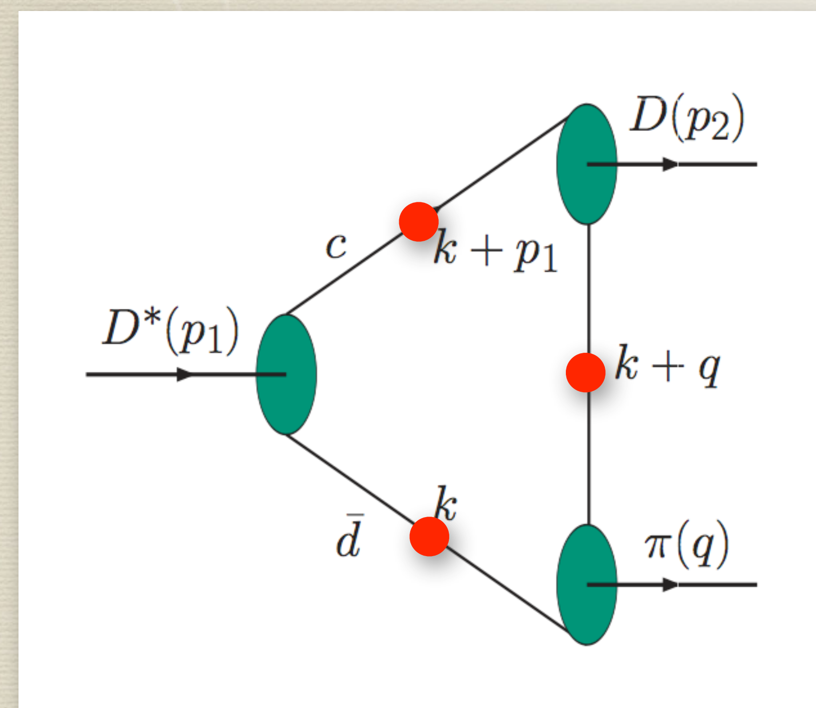
B. E., M.A. Ivanov and C.D. Roberts (2012)

Coupling yields D^* width

$$A(D^* \to D\pi) = \epsilon_\mu^{\lambda_{D^*}}(p_{D^*}) M^\mu(p_D^2, p_{D^*}^2) := \epsilon_\mu^{\lambda_{D^*}}(p_{D^*}) p_D^\mu \, g_{D^* D\pi}$$

$$M^\mu(p_D^2, p_{D^*}^2) = N_c \, \mathrm{tr} \int^\Lambda \frac{d^4k}{(2\pi)^4} \bar{\Gamma}_D(k; -P_D) S_c(k + P_{D^*}) i\Gamma_{D^*}^\mu(k; P_{D^*}) S_u(k) \bar{\Gamma}_\pi(k; -Q_\pi) S_u(k + Q_\pi)$$

Strong decays: $D^* \to D\pi$



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B. E., M.A. Ivanov and C.D. Roberts (2012)

$$A(D^* \to D\pi) \quad = \quad \epsilon_\mu^{\lambda_{D^*}}(p_{D^*}) M^\mu(p_D^2, p_{D^*}^2 \ldots$$

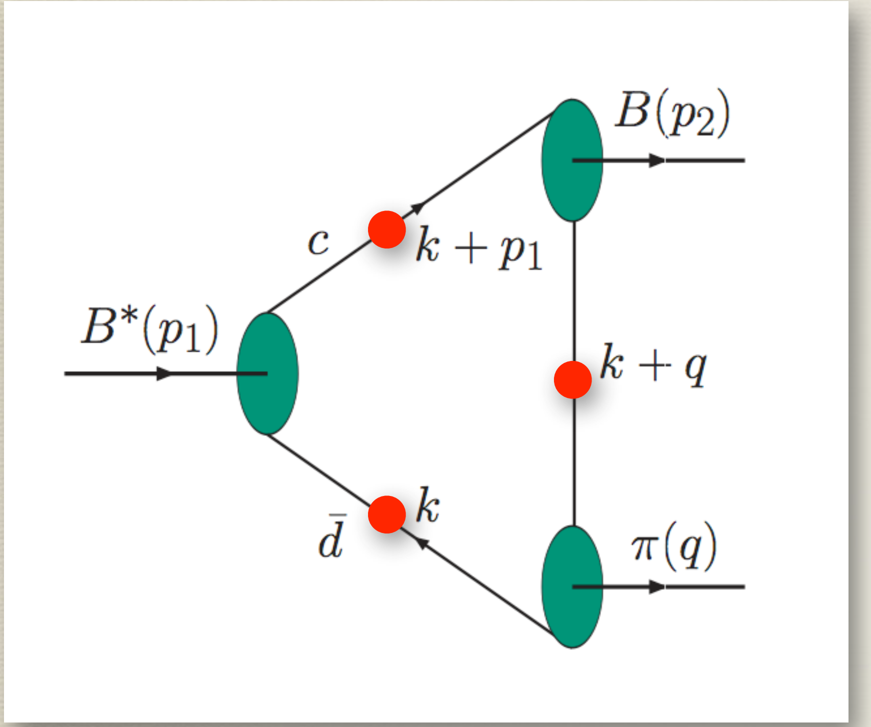
$$M^\mu(p_D^2, p_{D^*}^2) \quad = \quad N_c \, \mathrm{tr} \int^\Lambda \frac{d^4k}{(2\pi)^4} \bar{\Gamma}_D(k; \ldots \; Q_\pi)$$

Similarly: $D_s^* \to DK$

$$g_{D_s^* DK} = 20^{+2.5}_{-1.7}$$

B. E., M.A. Ivanov and C.D. Roberts (2012)

Strong decays: $B^* \to B\pi$ (*analogy*)



This amplitude can be used for $m_\pi^2 \to 0$ to extract $\hat{g}$ at leading order in HMChPT:

$$\hat{g} = \frac{g_{B^*B\pi}}{2\sqrt{m_B m_{B^*}}} f_\pi$$

	DSE model	Lattice in static limit $(n_f = 2)$
$\hat{g}$	0.37 ± 0.04	$0.44 \pm 0.03^{+0.07}_{-0.0}$

DSE-BSE: B. E., M.A. Ivanov and C.D. Roberts (2011)

LQCD: D. Bećirević, B. Blossier, E. Chang and B. Haas (2009)

$$\mathcal{L}_{\text{heavy}} = -\text{tr}_a \text{Tr}[\bar{H}_a iv \cdot D_{ba} H_b] + \hat{g}\, \text{tr}_a \text{Tr}[\bar{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba} \gamma_5]$$

The value obtained from D^* decay is: $\hat{g}_c = 0.56^{+0.07}_{-0.03} \longrightarrow \Lambda_{\text{QCD}}/m_c$ corrections are important!

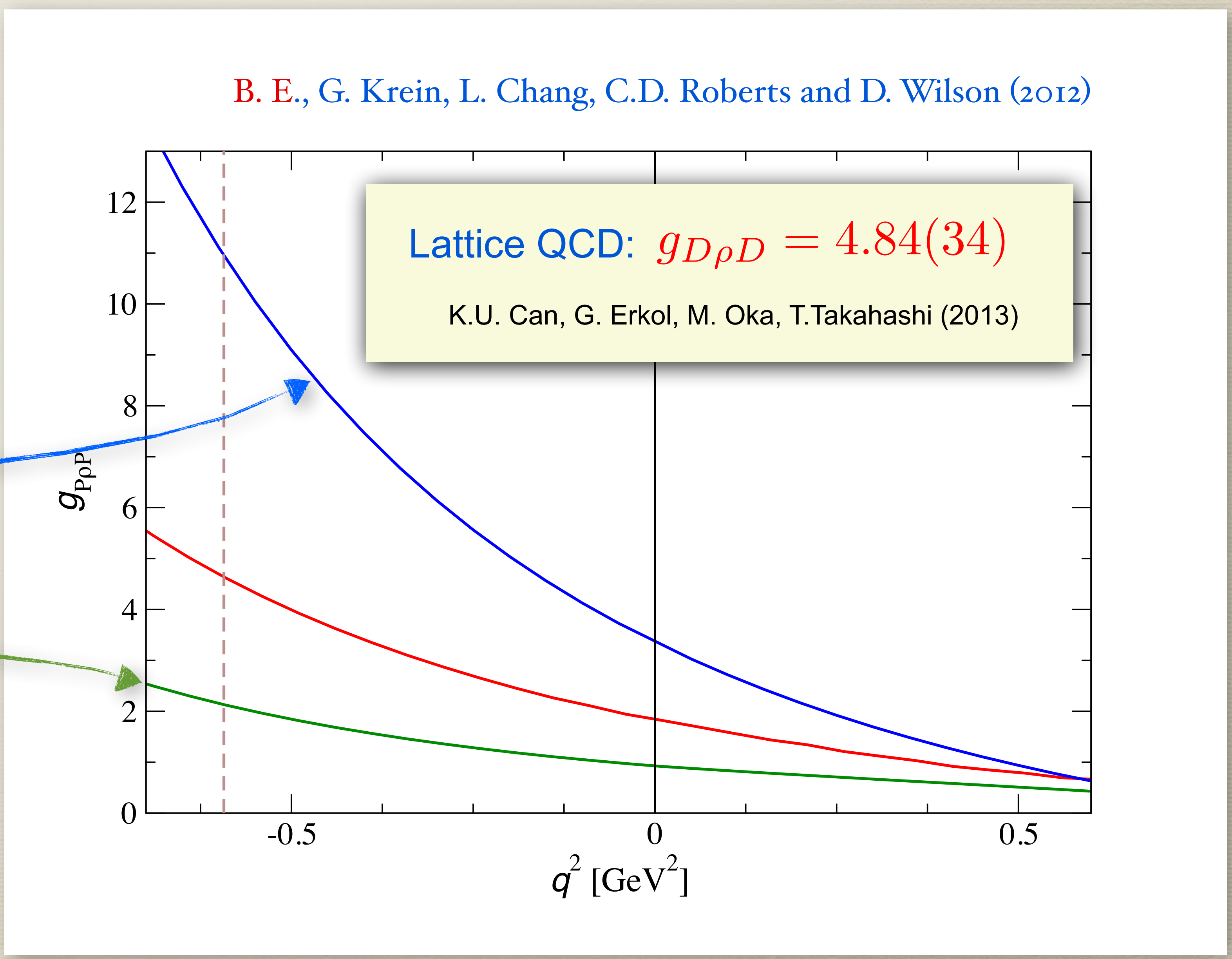
Flavor SU(3), SU(4), sensible symmetries?

$$g_{D\rho D} \neq g_{K\rho K} \neq \frac{1}{2} g_{\pi\rho\pi}$$

Define $\zeta_\rho := \dfrac{g_{D\rho D}(q^2)}{g_{K\rho K}(q^2)}$

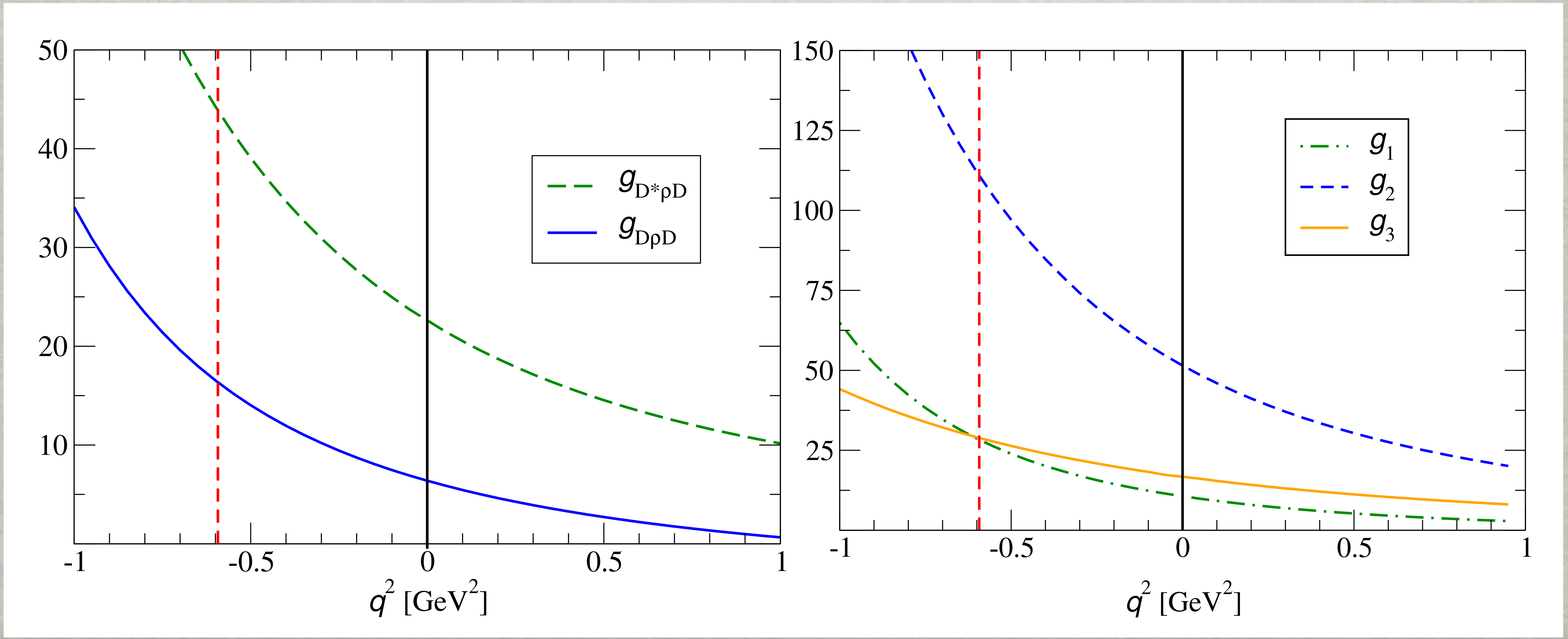
Ratio measures the effect of SU(4) breaking ≈ 300%

SU(3) breaking ≈ 20–30%



A single universal coupling of the *D* to the ρ mesons ?

B.E., M.A. Paracha, E. Rojas, C.D. Roberts, PRD 95 (2017)



Lattice QCD: $g_{D^*\rho D^*} = 5.94(56)$

K.U. Can, G. Erkol, M. Oka, T. Takahashi (2013)

DSE-BSE: $g^1_{D^*\rho D^*}(0) = 10.5$

Consequences for *DN* cross sections?

The integrated $D\rho D$ interaction is enhanced by about 40% compared with an $SU(4)$ prediction for the coupling/form factor.

Large value value for the interaction strength entails an enhanced cross section in *DN* scattering ($I = 1$ cross section inflated by a factor 4–5).

Possible novel charmed resonances or bound states in nuclei?

Light-Front Distribution Amplitudes

QCD factorization involves matrix elements which are convolution integrals:

$$\langle\pi^+\pi^-|(\bar{u}b)_{\mathrm{V-A}}(\bar{d}u)_{\mathrm{V-A}}|\bar{B}_d\rangle \to \int_0^1 d\xi du dv\, \Phi_B(\xi)\, \Phi_\pi(u)\, \Phi_\pi(v)\, T(\xi,u,v;m_b).$$

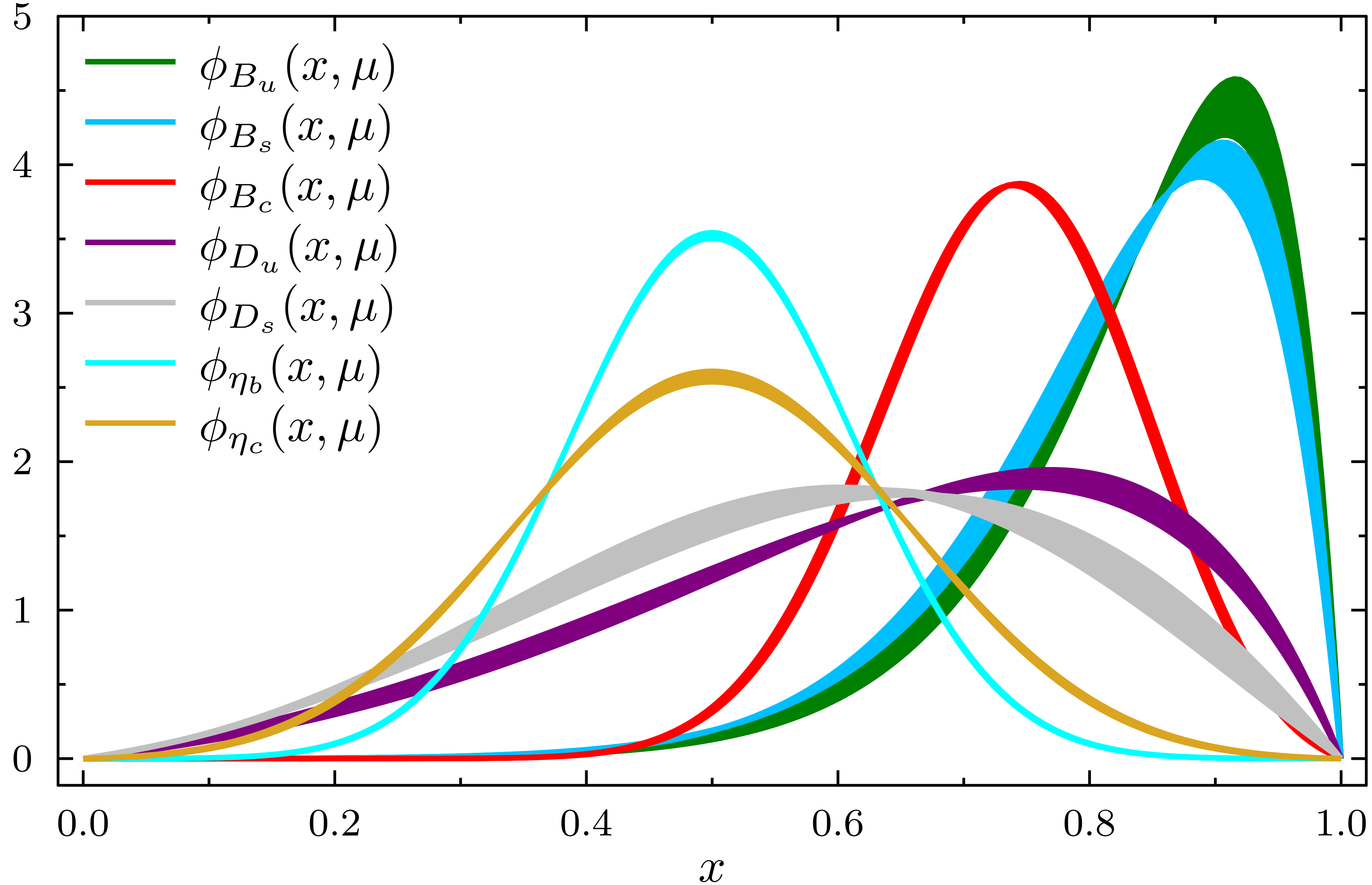
The integrals are over a (hard) scattering kernel $\boldsymbol{T}\,(\xi,u,v,m)$ and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

$$\varphi_\pi(x;\tau) = \varphi_\pi^{\mathrm{asy}}(x)\left[1 + \sum_{j=2,4,\dots}^{\infty} a_j^{3/2}(\tau)\, C_j^{(3/2)}(2x-1)\right]$$

$$\varphi_\pi^{\mathrm{asy}}(x) = 6x(1-x)$$

- LCDA until recently poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, RQCD Collaboration, Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.
- Recent results using DSE-BSE calculations projected on light front: Serna et al. (2020).

Light-Front Distribution Amplitudes



F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Light-Front Distribution Amplitudes

Only valid for full QCD !

Application to HQET requires the use of heavy-quark expansion of charm and bottom propagator in the Bethe-Salpeter equation.

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Conclusions & Progress

- Over the years, much progress was made from QCD based modeling toward nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying *chiral symmetry* and *Poincaré covariance*.

- Good reproduction of charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.

- Improvements in Bethe-Salpeter kernels beyond ladder truncation underway … $\Longrightarrow$ needed for scalar and axialvector channels and their higher radially excited states, as well as better control of quark correlation functions on complex plane.

- The calculation of light-front distribution amplitudes for vector quarkonia and heavy-light mesons is currently being concluded.

- Couplings between heavy and light mesons employed in effective field theories were obtained in impulse approximation with simple BSA models. Improvements with full BSA for D and D^* and beyond leading approximation underway.