Nuclear Equation of State and Neutron Stars

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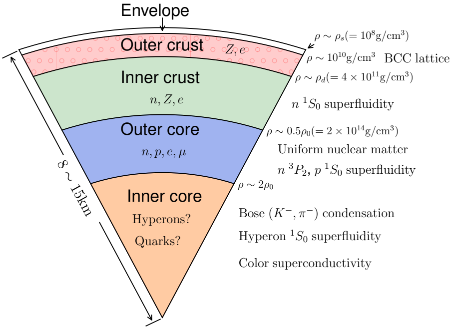
Neutron Stars

- Formed after core collapsing supernovae.
- Suggested by Walter Baade and Fritz Zwicky (1934) - Only a year after the discovery of the neutron by James Chadwick
- Jocelyn Bell Burnell and Antony Hewish observed pulsar in 1965.
- Neutron star is cold after 30s $\sim$ 60s of its birth
  - inner core, outer core, inner crust, outer crust, envelope
  - R : $\sim$ 10km
  - M : $1.2 \sim 2.x\ M_\odot$ $(2.14^{+0.1}_{-0.09}(2.08^{+0.07}_{-0.07})\,M_\odot$ PSR J0704+6620; $2.01 \pm 0.04\,M_\odot$ PSR J0348+0432 ; $1.97 \pm 0.04\,M_\odot$ PSR J614-2230)
  - 2×10^{11} earth g → General relativity
  - B field : $10^8 \sim 10^{12}$G.
  - Central density : $3 \sim 10\rho_0$ → Nuclear physics!!

- Inner structure of neutron stars



Envelope

Outer crust

Z, e

Inner crust

n, Z, e

Outer core

n, p, e, μ

Inner core

Hyperons?

Quarks?

$8 \sim 15\text{km}$

$\rho \sim \rho_s (= 10^8 \text{g/cm}^3)$

$\rho \sim 10^{10} \text{g/cm}^3$ BCC lattice

$\rho \sim \rho_d (= 4 \times 10^{11} \text{g/cm}^3)$

$n\ ^1S_0$ superfluidity

$\rho \sim 0.5\rho_0 (= 2 \times 10^{14} \text{g/cm}^3)$

Uniform nuclear matter

$n\ ^3P_2$, $p\ ^1S_0$ superfluidity

$\rho \sim 2\rho_0$

Bose (K^-, π^-) condensation

Hyperon $^1S_0$ superfluidity

Color superconductivity

- Neutron Stars:
  - Dense nuclear matter physics

- TOV equations for macroscopic structure

$$\begin{aligned} \frac{dp}{dr} &= -\frac{G(M(r)+4\pi r^3 p/c^2)(\epsilon+p)}{r(r-2GM(r)/c^2)c^2}, \\ \frac{dM}{dr} &= 4\pi\frac{\epsilon}{c^2}r^2, \end{aligned} \tag{1}$$

- Nuclear physics provide the information for ϵ and p.

Nuclear matter properties

- Nuclear equation of state at $T = 0\,\mathrm{MeV}$

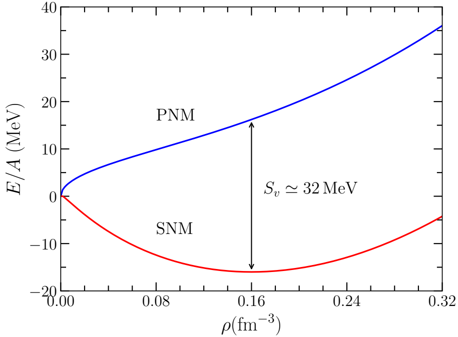


Figure: Energy per baryon for symmetric nuclear matter and pure neutron matter

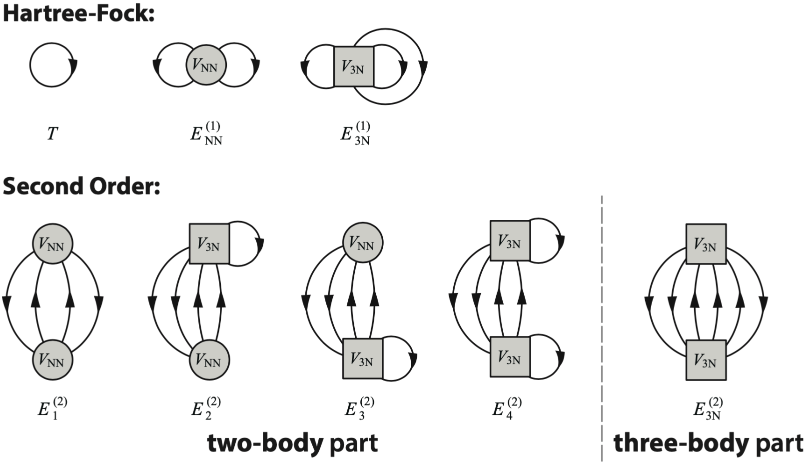
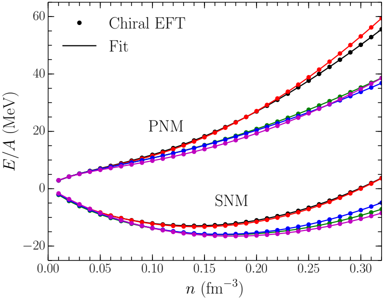


Figure: Many body diagrams for nuclear matter calculation (C. Drischler, Phd thesis)

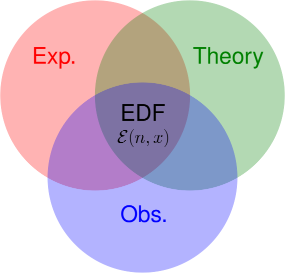
- Most neutron matter results can be fitted using the quadratic expansion.



$$\mathcal{E}(n,x) = \frac{1}{2m}\tau_n + \frac{1}{2m}\tau_p + (1-2x)^2 f_n(n) + \left[1-(1-2x)^2\right] f_s(n)\,, \qquad (2)$$

$$f_s(n) = \sum_{i=0}^{3} a_i n^{(2+i/3)}\,, \quad f_n(n) = \sum_{i=0}^{3} b_i n^{(2+i/3)} \qquad (3)$$

Neutron Star EOS constraints



- Experiments
  - SNM properties, Neutron skin thickness, binding energies
- Theory
  - Neutron matter calculations (QMC, MBPT, ..)
- Observation

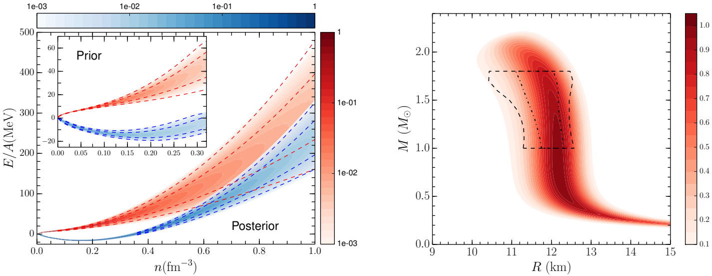
  Gravitation wave : tidal deformabilities, Moment of inertia, Nicer (mass-radius), maximum mass($M_{\max} > 2.0 M_{\odot}$)

EOS constraints

- Nuclear EOS constraints
  - Microscopic calculation(Pure neutron matter)
  - Nuclear structure: Neutron skin, binding energies of nuclei
  - Maximum mass of neutron stars($M_{\max} > 2.0 M_\odot$)
  - Gravitational wave: tidal deformabilities($\Lambda_{1.4}$)
  - (Moment of inertia)
  - NICER(Neutron Star Interior Composition Explorer): mass and radius

EOSs and *MR*

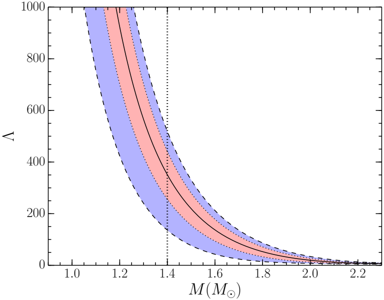
- Statistical uncertainties from EOSs (Theory + Experiment)



Y. Lim & J.W. Holt, PRL 2018.

Tidal deformability from EOSs

- $\Lambda_{1.4} = 350^{+169}_{-114}$(EOSs) vs $\Lambda_{1.4} = 190^{+380}_{-120}$ (LIGO).



Y. Lim & J.W. Holt, PRL 2018.

Probability distribution of central density I

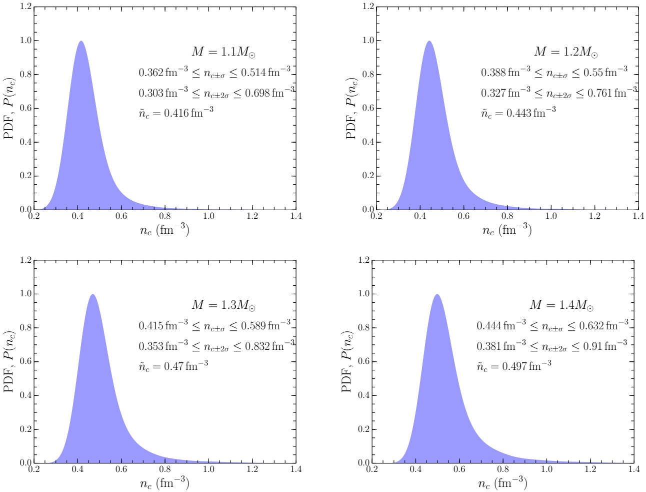


Figure: Lim & Holt, Eur. Phys. J. A 55, 209 (2019)

Probability distribution of central density II

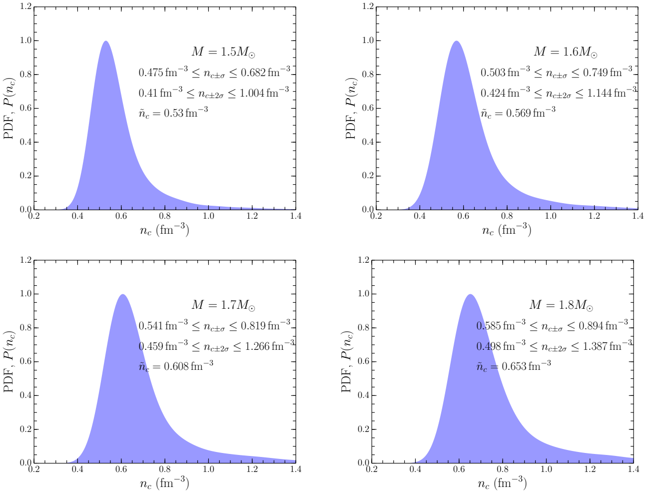


Figure: Lim & Holt, Eur. Phys. J. A 55, 209 (2019)

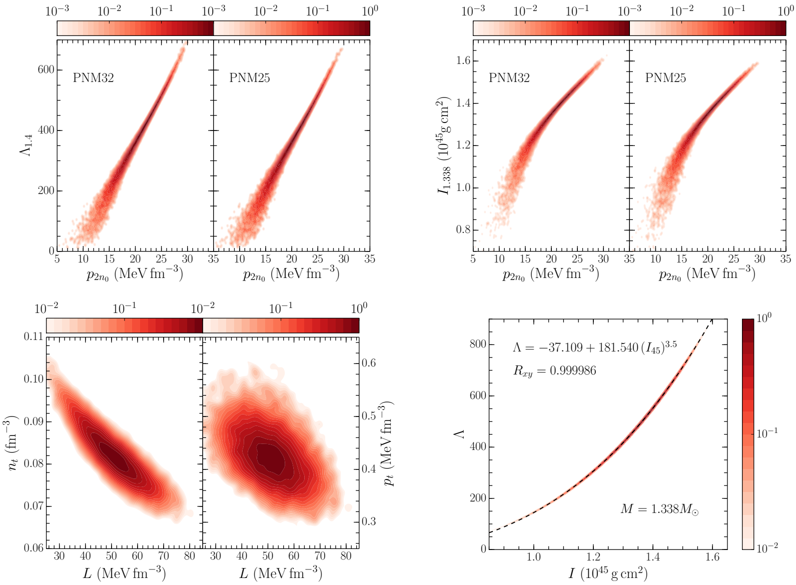


Figure: Lim & Holt, EPJA 2019

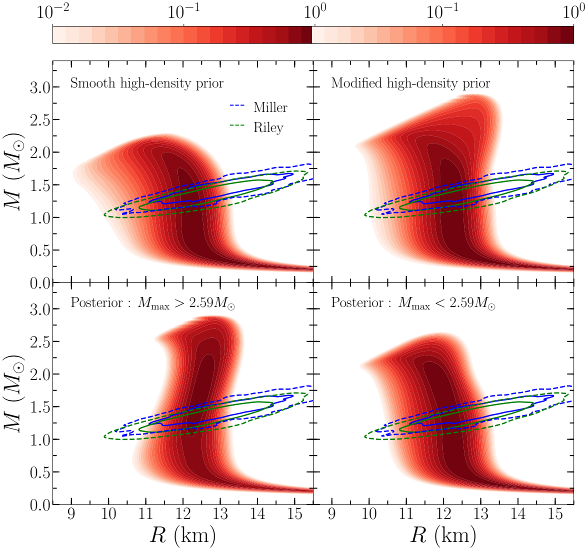


Figure: Mass radius confidence intervals, NICER, PNM, SNM, GW170817, GW190425, arXiv:2007.06526

- Mass radius of neutron stars using various constraints (Y.Lim and A. Schwenk *in preparation*)

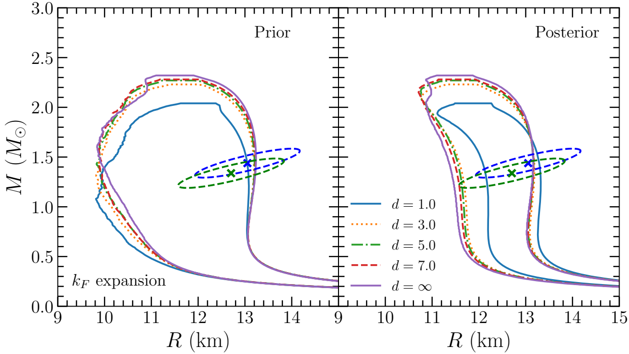


Figure: Mass radius confidence intervals, NICER(J003+0451), PNM, SNM, GW170817, $M_{\max} > 2.01$, NICER2(J0704+6620)

Nuclear Equation of State for Hot Dense Matter

What is it and why is it important?

- Nuclear EOS is thermodynamic relation for given ρ, Y_e, T with wide range of variables. $(1\,\mathrm{MeV} \simeq 10^{10}\,\mathrm{K})$
$(\rho = 10^4 \sim 10^{14}\mathrm{g/cm^3},\ Y_e = 0.01 \sim 0.65,\ T = 0.1 \sim 200\mathrm{MeV})$
- core collapsing supernova explosion, proto-neutron stars, and compact binary mergers involve neutron stars.

How can we construct EOS table ?

How can we construct EOS table ?
We need nuclear force model and numerical method.

Nuclear force model	Numerical technique
Skyrme Force model (non-relativistic potential model)	Liquid Drop(let) approach (LDM)
Relativistic Mean Field model (RMF)	Thomas Fermi Approximatoin (TF)
Finite-Range Force model	Hartree-Fock Approximation (HF)
	Nuclear Statistical Equilibrium (NSE)

- LS EOS ⇒ Skyrme force + LDM (without neutron skin)
- STOS ⇒ RMF + Semi TF (parameterized density profile)
- SHT ⇒ RMF + HARTREE
- HSB ⇒ RMF + NSE

Nuclear force model should be picked up to represent both finite nuclei and neutron star observation + Neutron matter calculation.

Representative EOSs

- LS EOS (Lattimer Swesty 1991)
  Use Skyrme type potential with Liquid droplet approach
  - Consider phase transition, several K
- STOS EOS (H. Shen, Toki, Oyamastu, Sumiyoshi 1998), new version (2011)
  Use RMF with TF approximation and parameterized density profile (PDP)
  - Old : awkward grid spacing
  - New : finer grid spacing, adds Hyperon($\Lambda, \Sigma^{+,-,0}$)
- SHT EOS (G. Shen, Horowitz, Teige 2010)
  Use RMF with Hartree approximation
- HSB (M. Hempel and J. Schaffner-Bielich). 2010, 2012
  - Use Relativistic mean field model (TM1, TMA, FSUgold)
  - Nuclear statistical equilibrium (alpha, deutron, triton, helion)

Domains for a supernova simulation

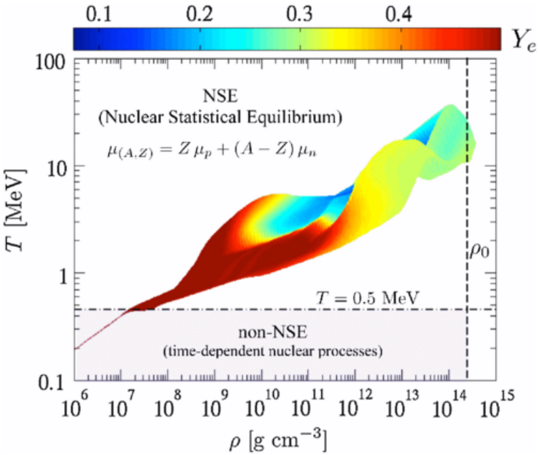


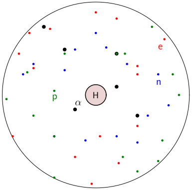
Figure from Oertel et al., Rev. Mod. Phys. 89, 015007.

Items in EOS tables

1 - Total pressure P
2 - Total free energy per baryon f
3 - Total entropy per baryon s
4 - Total internal energy per baryon e
5 - Neutron chemical potential
6 - Proton chemical potential
7 - Neutron mass fraction (external to nuclei)
8 - Proton mass fraction
9 - Alpha particle mass fraction
10 - Baryon pressure
11 - Baryon free energy per baryon
12 - Baryon entropy per baryon
13 - Baryon internal energy per baryon
14 - Nuclei filling factor u
15 - Baryon density inside heavy nucleus
16 - dP/dn
17 - dP/dT
18 - dP/dYe
19 - ds/dT
20 - ds/dYe
21 - Mass number of heavy nucleus
22 - Proton fraction of heavy nucleus
23 - Number of neutrons in neutron skin of heavy nucleus
24 - Baryon density of nucleons external to heavy nucleus and alpha particles
25 - Proton fraction of nucleons external to heavy nucleus and alpha particles
26 - Out of whackness: $\mu_n - \mu_p - \mu_e + 1.293$ MeV

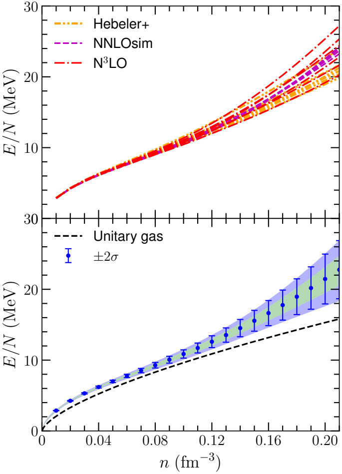
Calculation of EOS

- Schematic picture of inhomogeneous nuclear matter(neutron star crust)



- Liquid Drop Model(Fast and accurate)
- Most difficult part: inhomogeneous matter, low temperature
- Adopt state-of-the-art neutron matter results
  -ex) MBPT(Drischler *et al.*, PRL 2019), QMC(Tews *et al.*, PRC 2016)

- New energy density functional for the nuclear EOS. (Y. Lim, S. Huth, and A. Schwenk *in preparation*)



Free energy

Total free energy density consists of

$$F = F_N + F_o + F_\alpha + F_d + F_t + F_h + F_e + F_\gamma \tag{4}$$

where F_N, F_o, F_α, F_e, and F_γ are the free energy density of heavy nuclei, nucleons out the nuclei, alpha particles, electrons, and photons.

- $F_N = F_{bulk,i} + F_{coul} + F_{surf} + F_{trans}$
- $F_o = F_{bulk,o}$
- α, d, t, h particles : Non-interacting Boltzman gas
- e, γ : treat separately

For $F_{bulk,i}$, $F_{bulk,o}$, and F_{surf}, we use the same force model.
F_{surf} from the semi infinite nuclear matter calculation

The is the modification of LPRL (1985), LS (1991, No skin)

- Consistent calculation of surface tension
- Deuteron, triton, helion
- The most recent parameter set

Free energy minimization

For fixed independent variables (ρ, Y_p, T), we have the 11 dependent variables $(\rho_i, x_i, r_N, z_i, u, \rho_o, x_o, \rho_\alpha, \rho_d, \rho_t, \rho_h)$.

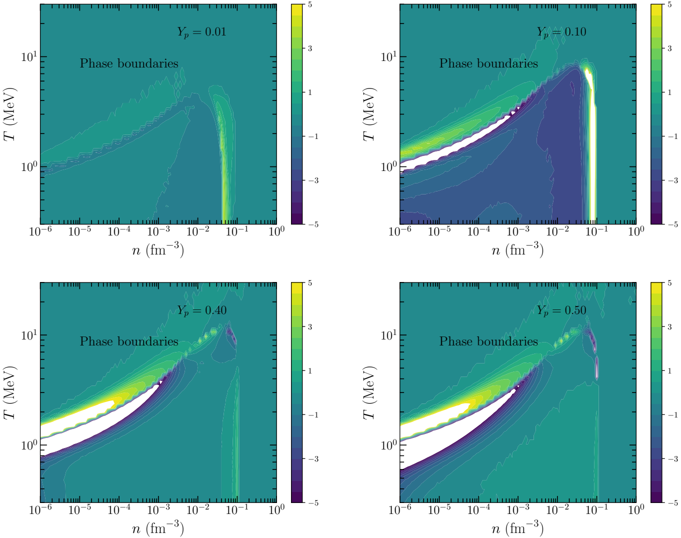
where i heavy nuclei, o nucleons outside, x proton fraction, u filling factor, and ν_n neutron skin density.

From baryon and charge conservation, we can eliminate x_o and ρ_o.

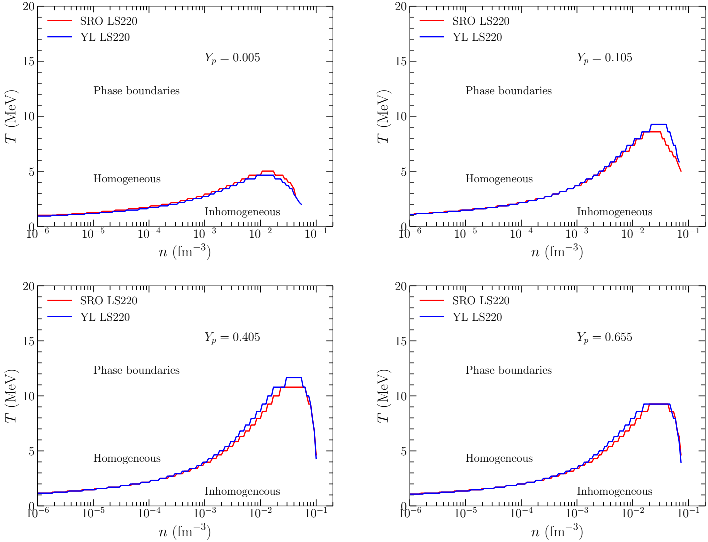
Free energy minimization,
$\frac{\partial F}{\partial \rho_i} = \frac{\partial F}{\partial x_i} = \frac{\partial F}{\partial r_N} = \frac{\partial F}{\partial z_i} = \frac{\partial F}{\partial u} = \frac{\partial F}{\partial \rho_\alpha} = \frac{\partial F}{\partial \rho_d} = \frac{\partial F}{\partial \rho_t} = \frac{\partial F}{\partial \rho_h} = 0.$

- Finally, we have 6 equations to solve and 6 unknowns.
  $z = (\rho_i, \ln(\rho_{no}), \ln(\rho_{po}), x_i, \ln(u), z_i)$.

Results : Relative pressure difference bewteen the current code & LS SkM*



Phase Boundary



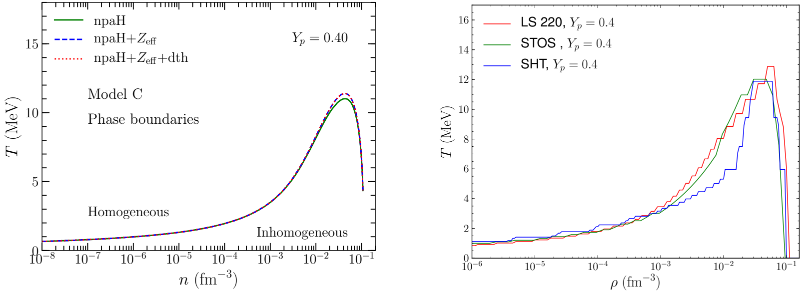


Figure: Phase boundaries using Model C from EOS table (Left) and representative EOSs (Right)

Simulation

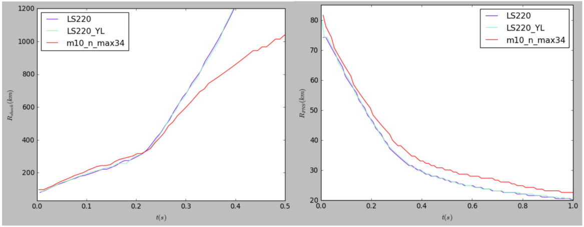


Fig.1 Case with heating factor

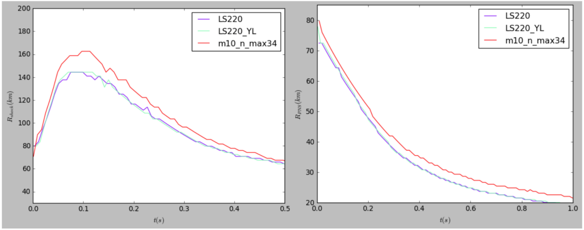
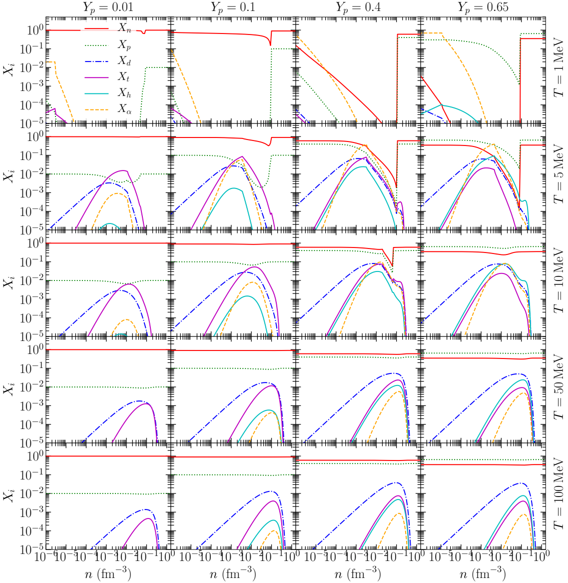


Fig.2 Case with no heating factor

Particle fraction



Summary

Neutron star and Nuclear equation of state

- Nuclear Model : Energy density functional (Exp. + Theory + Obs.)
  - should be consistent with finite nuclei,
  - neutron matter calculation
  - maximum mass of neutron stars, gravitational wave, MR(NICER), (moment of inertia in the future)
- Numerical Method: Liquid Drop Model
  - fast and accurate
  - surface tension, critical temperature, effective charge (Screening from outside particles)
  - deuteron, triton, helion

Thank you for your attention !