Longitudinal dynamics and chiral symmetry breaking: The quest for semiclassical wave equations in QCD

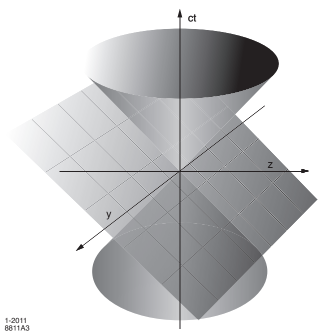
Guy F. de Téramond

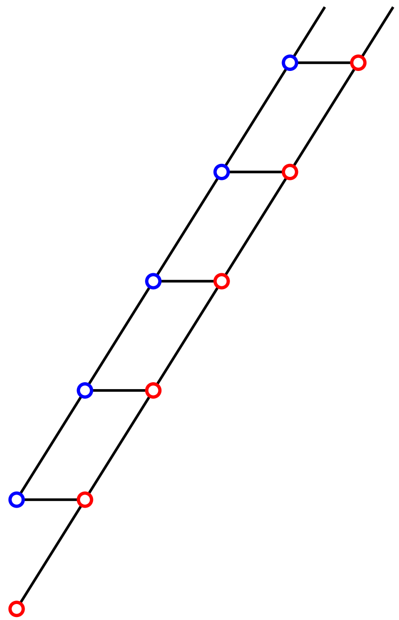
UCR

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{\lambda_N}{\pi} P \int_0^1 dx' \, \frac{\chi(x) - \chi(x')}{(x-x')^2} = M^2 \, \chi(x)$$

ILCAC Seminar Series

2 June 2021





Based on arXiv:2103.10950 [hep-ph] in collaboration with Stan Brodsky

See also: Y. Li and J. P. Vary [arXiv:2103.09993 [hep-ph]]

Contents

1 QCD (1+1): Quest for semiclassical wave equations in the large N limit

$SU(N)_C$ LF QCD in 1 + 1 dim: no transverse directions

- Gluons are not dynamical, no self-couplings of gluons and quarks have chirality but no spin
- Coupling g has dimensions of mass: Effective coupling strength is g/m
- Confining field theory for any coupling
- Include boundary terms from long range fields and constraints from LF EOM
- Analytic solutions exist in the weak and strong coupling limits
- A semiclassical light-front Schrödinger equation derived from first principles QFTh for large N

G. 't Hooft, NPB **75**, 461 (1974)

A. R. Zhitnitsky, PLB **165**, 405 (1985)

K. Hornbostel, S. J. Brodsky and H. C. Pauli, PRD **41**, 3814 (1990)

- Start with $SU(N)_C$ Lagrangian in QCD (1 + 1) [K. Hornbostel, PhD thesis, SLAC (1988)]

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

In the $A^+ = 0$ gauge

$$\mathcal{L}_{{\rm QCD}_{(1+1)}} = \psi_L^\dagger i \overleftrightarrow{\partial}_- \psi_L + \psi_R^\dagger i \overleftrightarrow{\partial}_+ \psi_R - m\big(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R\big) + \tfrac{1}{2}\big(\partial_- A^{-a}\big)^2 - g \psi_R^\dagger A^{-a} T^a \psi_R$$

- Express the hadron 2-momentum generator $P = (P^+, P^-)$, $P^\pm = P^0 \pm P^3$, in terms of the fields $\psi_{L,R}$ and A^- including x^+-contour integration

$$P^- = \int dx^- \left[\tfrac{1}{2}\big(\partial_- A^{-a}\big)^2 + \tfrac{m}{2}\big(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R\big)\right] + \int dx^+ \left[\psi_L^\dagger i \overleftrightarrow{\partial}_+ \psi_L - g \psi_L^\dagger A^{-a} T^a \psi_L\right]$$

$$P^+ = \int dx^- \psi_R^\dagger i \overleftrightarrow{\partial}_- \psi_R + \int dx^+ \left[\tfrac{1}{2}\big(\partial_- A^{-a}\big)^2 + \tfrac{m}{2}\big(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R\big)\right]$$

- Equations of motion from QCD (1 + 1) Lagrangian

$$i\partial_- \psi_L = \tfrac{1}{2} m \psi_R \tag{1}$$

$$-\partial_-^2 A^{-a} = g \psi_R^\dagger T^a \psi_R \equiv \tfrac{1}{2} g j^{+a} \tag{2}$$

$$i\partial_+ \psi_R = \tfrac{1}{2} g A^{-a} T^a \psi_R + \tfrac{1}{2} m \psi_L \tag{3}$$

$$\partial_+ \partial_- A^{-a} = g \psi_L^\dagger T^a \psi_L - \tfrac{1}{2} g f^{abc} \partial_- A^{-b} A^{-c} \equiv \tfrac{1}{2} g j^{-a} \tag{4}$$

- Only Eqs (3) and (4) are dynamical, (1) and (2) are constraint equations

- ψ_R only independent degree of freedom which is quantized in a box of length L (DLCQ)

- Interaction component of P^-, V, from inverting (2) for A^-

$$V = -\tfrac{1}{4} g^2 \int dx^- dy^- j^{+a}(x^-) \left| x^- - y^- \right| j^{+a}(y^-)$$

- *Note:* For a discussion of the interpolation of 't Hooft model between instant and front form of dynamics, see:

  B. Ma and C. R. Ji [arXiv:2105.09388 [hep-ph]]

- Mass spectrum computed from LF eigenvalue equation

$$P^+P^-|\chi(P^+)\rangle = M^2|\chi(P^+)\rangle$$

- For the $q\overline{q}$ state (large N) in the continuum limit $L \to \infty$ [Th. Eller, PhD thesis, Heidelberg U. (1987)]

$$\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)\chi(x) + \frac{\lambda_N}{\pi} P\int_0^1 dx' \, \frac{\chi(x) - \chi(x')}{(x-x')^2} = M^2\,\chi(x)$$

‘t Hooft equation with coupling $\lambda_N = g^2\left(N^2 - 1\right)/2N$

- For approximate analytic solution $\chi(x) \sim x^{\beta_1}(1-x)^{\beta_2}$ cancellation of singularities at $x = \epsilon$,

$$\left[\pi m_q^2 - \lambda_N + \pi\beta_1\lambda_N \cot(\pi\beta_1)\right]\epsilon^{\beta_1 - 1} = 0$$

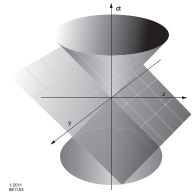
leads to $\beta_1 = \left(3m_q^2/\pi\lambda_N\right)^{1/2}$ by expanding $\cot(\beta\pi)$ for $m_q^2/\lambda_N << 1$ (same at $x = 1-\epsilon$)

- Meson mass computed from

$$\begin{aligned} M^2 &= \int_0^1 dx\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)\chi^2(x) + \frac{\lambda_N}{\pi}P\int_0^1 dx\int_0^1 dx' \,\frac{\chi(x)[\chi(x) - \chi(x')]}{(x-x')^2} \\ &= \sqrt{\frac{\pi\lambda_N}{3}}\,(m_q + m_{\overline{q}}) + \mathcal{O}\big((m_q + m_{\overline{q}})^2\big) \end{aligned}$$

- QCD(1+1) spontaneous chiral symmetry breaking in the limit $N \to \infty$ followed by the limit $m_q \to 0$

2 QCD (3+1): Quest for semiclassical wave equations $\mathrm{m_q} = 0$



- Start with $SU(N)_C$ Lagrangian in QCD (3 + 1) [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

$$\mathcal{L}_{\rm QCD} = \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Express the hadron 4-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, in terms of the dynamical field $\psi_+ = \Lambda_+ \psi$ $\left(\Lambda_\pm = \gamma^0\gamma^\pm\right)$ quantized in the null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned}
P^- &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \, \gamma^+ \frac{\left(i\nabla_\perp\right)^2 + m^2}{i\partial^+} \psi_+ + \text{interactions} \\
P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+ \\
\mathbf{P}_\perp &= \tfrac{1}{2}\int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\nabla_\perp \psi_+
\end{aligned}$$

- Semiclassical approximation: ψ_+ is the only independent degree of freedom which is quantized

$$\psi_+(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+>0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q,\lambda) e^{-iq\cdot x} + d_\lambda(q)^\dagger v_\alpha(q,\lambda) e^{iq\cdot x}\right]$$

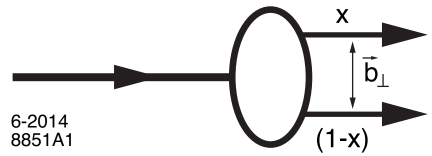
- Gluons with small virtualities assumed non-dynamical and incorporated in the confinement potential

- Mass spectrum computed from LF eigenvalue equation

$$\left(P^+P^- - \mathbf{P}_\perp^{\mathbf{2}}\right)|\psi\left(P^+, \mathbf{P}_\perp\right)\rangle = M^2|\psi\left(P^+, \mathbf{P}_\perp\right)\rangle$$

- For a two-parton bound state we factor out the longitudinal $X(x)$ and orbital $e^{iL\varphi}$ dependence from ψ

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



with invariant impact LF variable $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$, $x = k^+/P^+$ and $L = \max|L^z|$

- In the limit of zero-quark masses $m_q \to 0$ the longitudinal modes $X(x)$ decouple and we find

$$M^2 = \int d\zeta\, \phi^*(\zeta)\sqrt{\zeta}\left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta}\frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right)\frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta\, \phi^*(\zeta)\, U(\zeta)\, \phi(\zeta)$$

where the effective potential U incorporates all interactions, including those from higher Fock states

- The Lorentz invariant equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ becomes a LF wave equation for ϕ

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)}$$

- Critical value $L = 0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE: It has identical structure of AdS WE with $z = \zeta$

3 QCD (3+1): Quest for semiclassical wave equations $\mathbf{m_q \neq 0}$

- How to account for quark masses in the semiclassical approximation? Simple ansatz based on the off-shell dependence of the Gaussian LFWF on the invariant mass (IM) which controls the bound state
- For a $q\overline{q}$ bound state this amounts to $\frac{\mathbf{k}_\perp^2}{x(1-x)} \to \frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}$ in the WF to include the LF kinetic energy with quark masses: It is also the IM squared $s = (p_q + p_{\overline{q}})^2$ of the $q\overline{q}$ pair
- Substitution leads to the longitudinal wave function [S. J. Brodsky and GdT, Erice Lectures (2007)]

$$\chi_{IM}(x) = \mathcal{N} \exp\Big(-\frac{1}{2\lambda}\Big[\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\Big]\Big)$$

  and the hadronic mass shift

$$\Delta M^2 = \int_0^1 dx \left[\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right]\chi^2(x), \quad \chi(x) = \frac{X(x)}{\sqrt{x(1-x)}}$$

- CSB ansatz: Instead of IM ansatz choose $\chi_{CSB}(x) \sim x^a(1-x)^b$ to incorporate CSB breaking [T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, PRD **87**, 056001 (2013)]
- Problem: IM or CSB ansatz do not include longitudinal dynamics

Longitudinal LF dynamics

- We start from the combined semiclassical LF transverse and longitudinal wave equations for mesons

  [S. S. Chabysheva and J. R. Hiller, AnnPhys. **337**, 143 (2013)]

  Y. Li, P. Maris, X. Zhao and J. P. Vary, PLB **758**, 118-124 (2016)]

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2 \phi(\zeta)$$

$$\left(\frac{m_q^2}{x} + \frac{m_q^2}{1-x} + U_\parallel(x)\right)\chi(x) = M_\parallel^2\, \chi(x)$$

  in the approximation where transverse and longitudinal dynamics are separated: $M^2 = M_\perp^2 + M_\parallel^2$

- Successful results from HLFQCD not modified, even for heavy quark masses, as long as transverse and longitudinal dynamics can be separated [H. G. Dosch, GdT and S. J. Brodsky, PRD **95**, 034016 (2017)]

- Chabysheva and Hiller identify the longitudinal potential with 't Hooft's large-N_C QCD (1+1) potential

- Recent application of ‘t Hooft QCD (1 + 1) model to holographic QCD

  M. Ahmady, H. Dahiya, S. Kaur, C. Mondal, R. Sandapen and N. Sharma [arXiv:2105.01018 [hep-ph]]

4 Quest for a transverse potential

Superconformal algebraic structure in HLFQCD

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

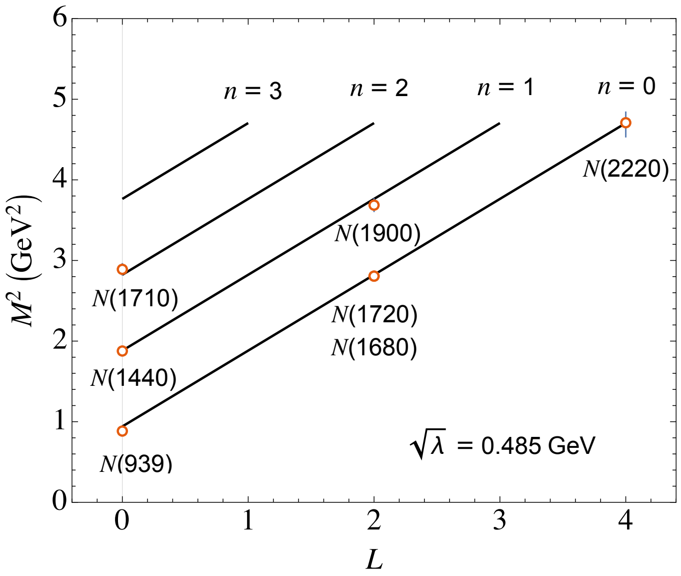
- Superconformal algebra underlies in HLFQCD the scale invariance of the QCD Lagrangian. It leads to the introduction of a scale in the Hamiltonian maintaining the action conformal invariant

- It incorporates a connection between mesons, baryons and tetraquarks underlying the $SU(N)_C$ representation properties: $\overline{N} \to N \times N$

- It leads to the baryon bound-state equations [GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L+1)\right)\psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4(L+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L\right)\psi_- = M^2\psi_-$$



- Eigenvalues

$$M^2 = 4\lambda(n + L + 1)$$

- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2), \quad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$

- Superconformal algebra also leads to LF bound-state wave equations for mesons and nucleons

  [H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L_M^2}{4\zeta^2} + \lambda_M^2\,\zeta^2 + 2\lambda_M(L_M-1)\right)\phi_M = M^2\phi_M$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L_B^2}{4\zeta^2} + \lambda_B^2\,\zeta^2 + 2\lambda_B(L_B+1)\right)\phi_B = M^2\phi_B$$

with eigenvalues

$$M_M^2 = 4\lambda\,(n + L_M)$$

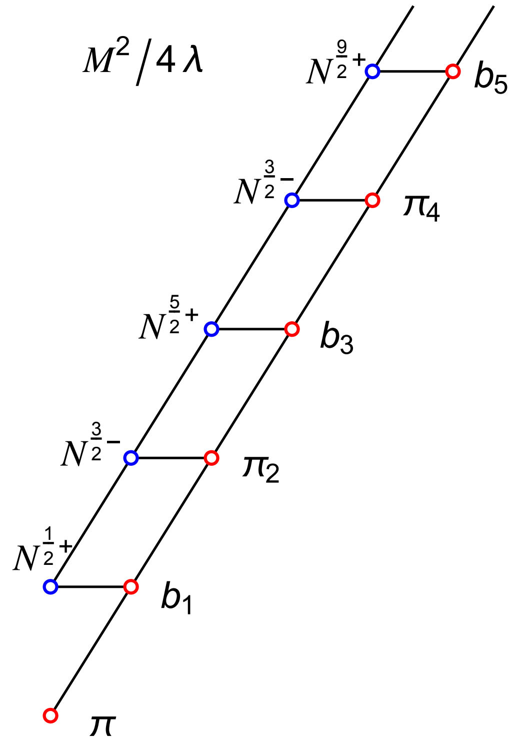
$$M_B^2 = 4\lambda\,(n + L_B + 1)$$

and the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes)

and the remarkable relation $\boxed{L_M = L_B + 1}$

- Special role of the pion as a unique state of zero energy
- Expected accuracy $1/N_C^2 \sim 10\%$
- Identify transverse LF potential for mesons

$$U_\perp(\zeta) = \lambda^2\,\zeta^2 + 2\lambda(L-1)$$



5 Quest for a longitudinal potential

- Holographic principles and the symmetries from superconformal algebra determine the confinement potential in the LF transverse coordinates for mesons and baryons of arbitrary spin for $m_q = 0$
- An extension of HLFQCD is required to incorporate LF longitudinal dynamics for $m_q \neq 0$
- We adopt here the effective longitudinal potential of Li, Maris, Zhao and Vary (LMZV)

$$U_{||}(x) = -\sigma^2 \partial_x \left(x(1-x)\,\partial_x\right)$$

- It leads to an oscillator potential in the longitudinal and transverse directions establishing a connection of the longitudinal scale σ with the transverse scale $\lambda \to \lambda_Q$ in the limit of heavy quark masses

$$\sigma = \lambda_Q/(m_Q + m_{\overline{Q}})$$

- It leads for the lowest eigenfunction to a solution identical to the 't Hooft model for small quark masses, therefore incorporating the breaking of chiral symmetry

$$M_\pi^2 = \sigma(m_q + m_{\overline{q}}) + \mathcal{O}\big((m_q + m_{\overline{q}})^2\big)$$

- One could examine other possible effective potentials, for example using the constituent rest frame [A. P. Trawiński, S. D. Głazek, S. J. Brodsky, GdT and H. G. Dosch, PRD **90**, 074017 (2014)]

- We compute the longitudinal mass for an arbitrary WF $\chi(x)$ expanding in terms of the complete functional basis generated by the LMZV potential, $\chi(x) = \sum_\kappa C_\kappa \chi_\kappa^{\alpha,\beta}(x)$, where

$$\chi_\kappa^{\alpha,\beta}(x) = N x^{\alpha/2}(1-x)^{\beta/2} P_\kappa^{(\alpha,\beta)}(1-2x),$$

with $\alpha = 2m_q/\sigma$ and $\beta = 2m_{\overline{q}}/\sigma$

- We have

$$M_{\|}^2/\sigma^2 = \int_0^1 dx\, \chi(x)\Big(-\partial_x \left(x(1-x)\partial_x\right) + \frac{1}{4}\Big[\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\Big]\Big)\chi(x)$$
$$= \sum_\kappa C_\kappa^2\, \nu^2(\kappa,\alpha,\beta)$$

with eigenvalues

$$\nu^2(\kappa,\alpha,\beta) = \tfrac{1}{4}(\alpha+\beta+2\kappa)(2+\alpha+\beta+2\kappa)$$

- For the invariant mass ansatz

$$\mathcal{N} \exp\Big(-\frac{\sigma^2}{8\lambda}\Big[\frac{\alpha^2}{x} + \frac{\beta^2}{1-x}\Big]\Big) = \sum_\kappa C_\kappa\, \chi_\kappa(x)$$

a rapid convergence is found, with the lowest-index Jacobi polynomial giving the dominant contribution

	$\kappa=0$	$\kappa=1$	$\kappa=2$	$\kappa=3$	$\kappa=4$	$\kappa=5$	$\kappa=6$
$C(u\overline{d})$	0.998	0	0.055	0	0.010	0	-0.003
$C(u\overline{s})$	0.967	-0.231	0.100	-0.006	-0.009	0.013	-0.016
$C(s\overline{s})$	0.998	0	0.038	0	-0.045	0	-0.024
$C(u\overline{c})$	0.958	-0.267	0.097	-0.012	-0.003	0	-0.007
$C(c\overline{c})$	0.999	0	0.016	0	-0.020	0	-0.003

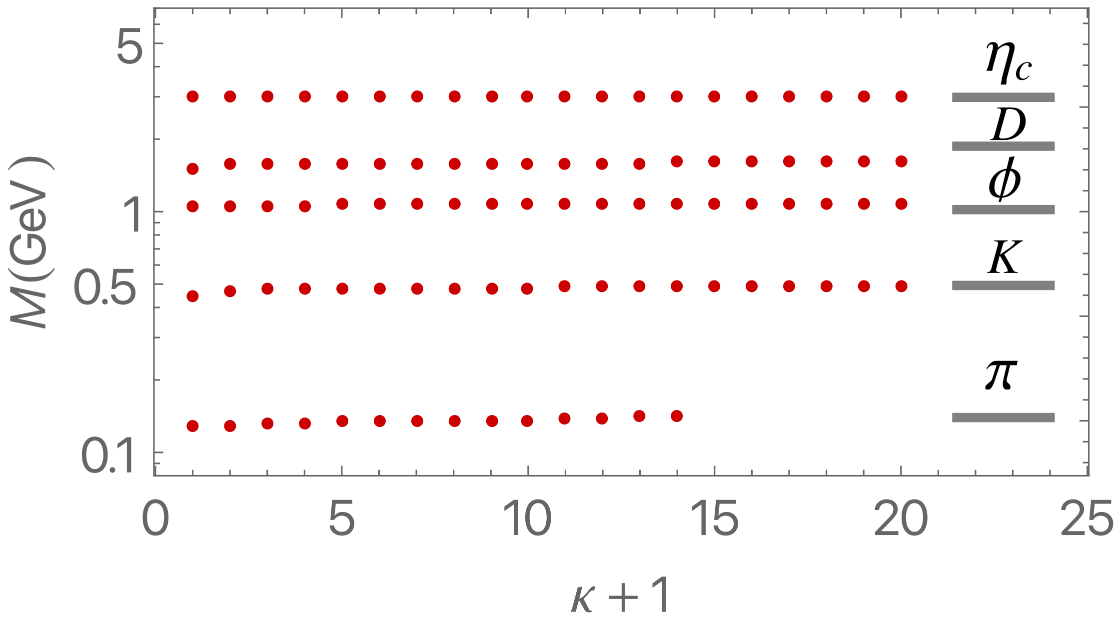


Figure 1: Numerical evaluation of ground state meson masses for $m_u = m_d = 28$ MeV and $m_s = 326$ MeV

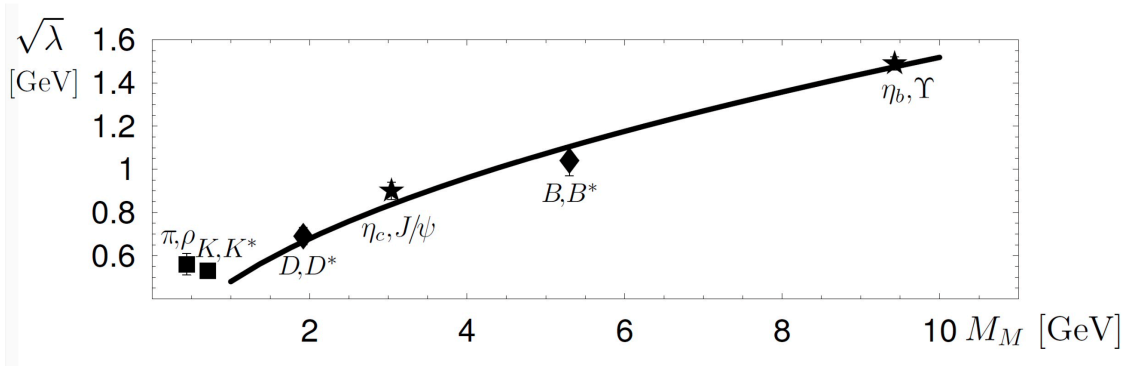


Figure 2: The value $\sigma = 0.24$ GeV is computed from HQET [M. Nielsen et al. Phys. Rev. D **98**, 034002 (2018)]

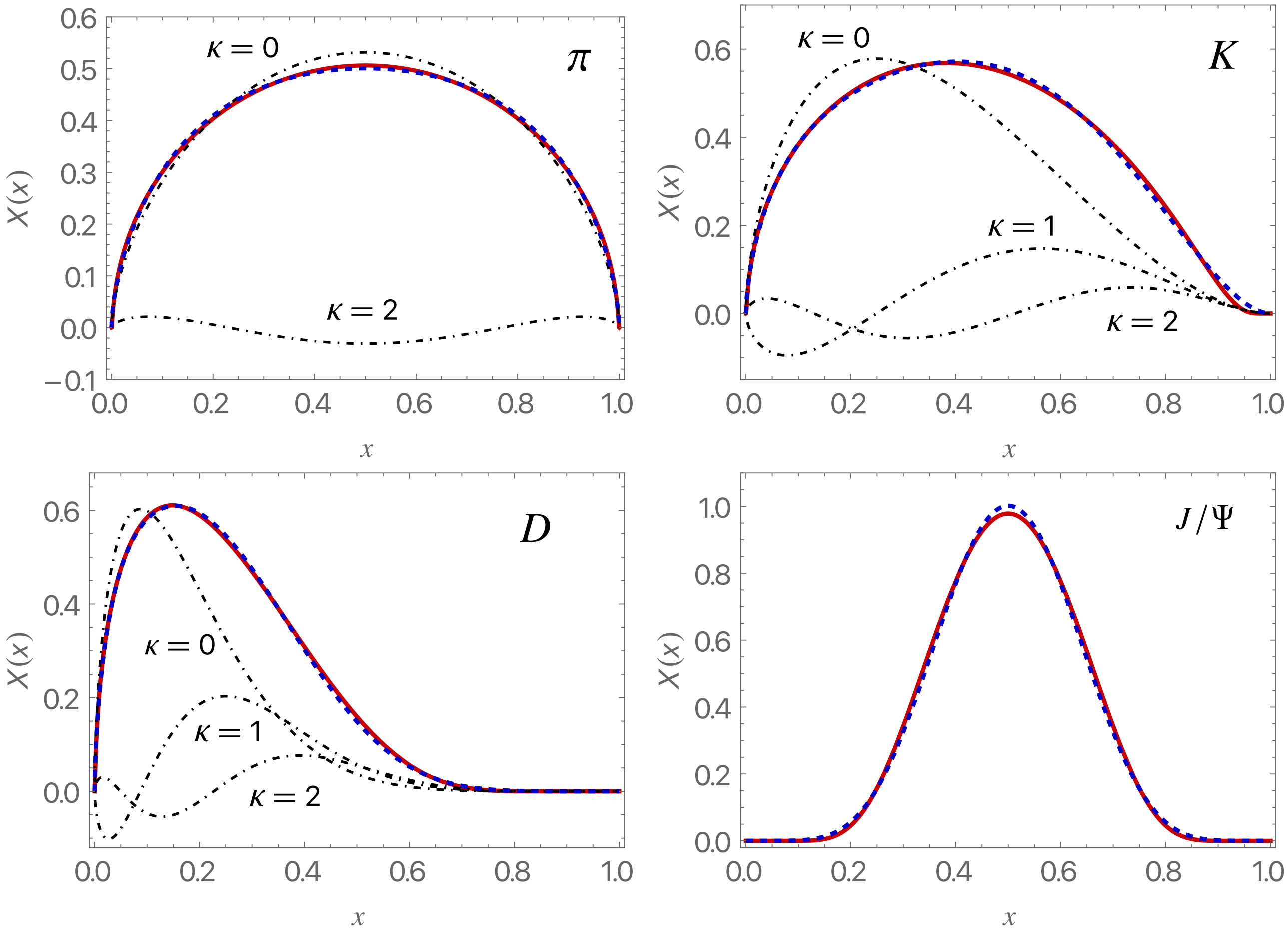


Figure 3: Light-front distribution amplitudes $X(x)$ for the π, K, D and J/Ψ mesons: the red curve is the invariant mass (IM) result, dot dashed black curves are individual modes in the IM expansion, dashed blue curve represent the sum of modes in the figure

6 Outlook

- I have presented in this talk a quest of essential elements that should incorporate semiclassical LF wave equations to describe fundamental aspects which are not obvious from the QCD Lagrangian, such as confinement and chiral symmetry breaking. The present approach also incorporates other basic nonperturbative properties such as the emergence of a mass scale and the connection between mesons and baryons. The implementation of superconformal algebra determines uniquely the form of the transverse confining interaction for mesons, nucleons and tetraquarks.

- The pion plays a special role as the hadronic state of zero mass in the chiral limit. Since it does not have a baryonic partner, the pion breaks the meson-baryon supersymmetry. Its transverse kinetic and potential energy cancels exactly, as required by the superconformal structure of the transverse LF Hamiltonian, therefore its mass is generated by the longitudinal dynamics. In contrast, the proton mass (as well as the mass of radial and orbital hadron excited states) is generated by the addition of its transverse kinetic and potential energies with a small contribution from the longitudinal dynamics, in agreement with the Regge phenomenology of the hadron mass spectrum.