

Photon initiated double parton scattering: a new light on the proton structure

Matteo Rinaldi

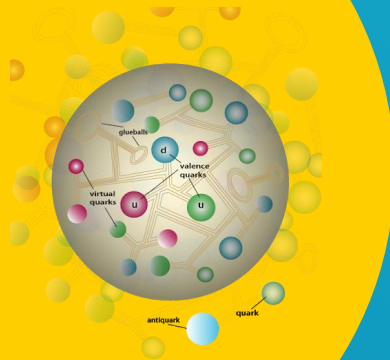
in collaboration with

Federico Alberto Ceccopieri

Marco Traini

Sergio Scopetta

Vicente Vento





Roadmap

INTRODUCTION

1

INTERPRETATION OF
LHC DATA

3

PREDICTIONS AND
OPPORTUNITIES

5

3D PROTON
STRUCTURE VIA
p-p DPS

2

DPS VIA
PHOTON-PROTON
INTERACTIONS

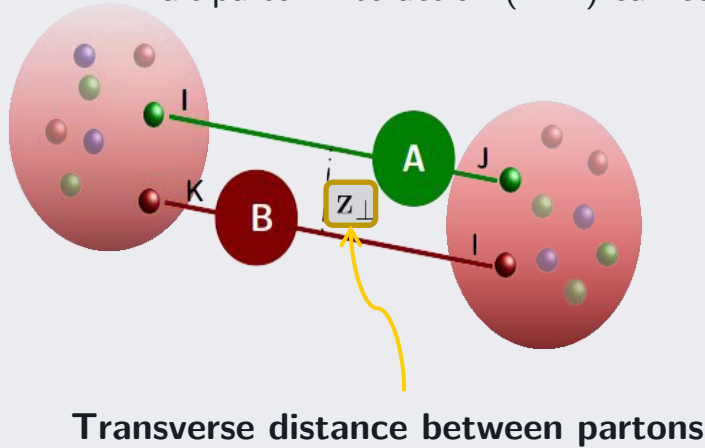
4

LATTICE DATA FOR
THE PION AND MODEL
COMPARISONS

6

1 Double Parton Scattering @LHC

Multiparton interaction (MPI) can contribute to the, pp and pA, cross section @ the LHC:



The cross section for a double parton scattering (DPS) event can be written in the following way:

N. Paver, D. Treleani, *Nuovo Cimento* 70A, 215 (1982)
 J.R. Gaunt et al, *JHEP* 07 (2014) 110, *JHEP* 01 (2016) 076

$$d\sigma \propto \int d^2z_{\perp} \overbrace{F_{ik}(x_1, x_2, \vec{z}_{\perp}; \mu_A, \mu_B) \cdot F_{jl}(x_3, x_4, \vec{z}_{\perp}; \mu_A, \mu_B)}^{\text{double PDF (dPDF)}}$$

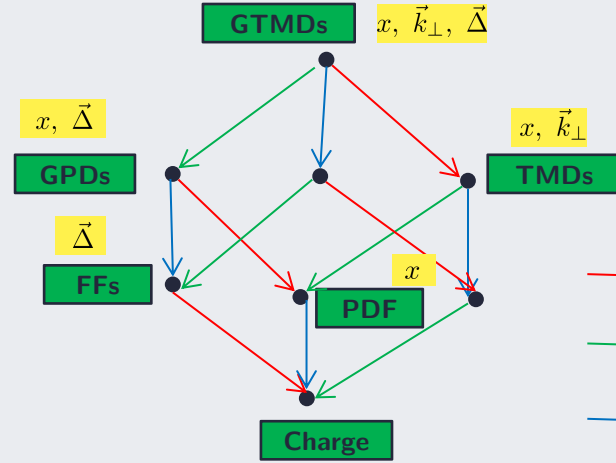
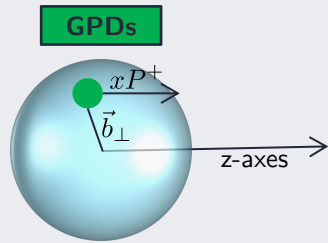
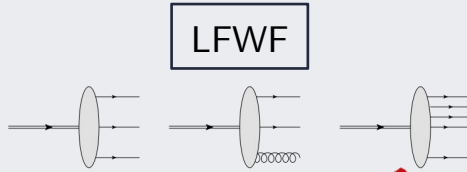
Momentum scales

Momentum fractions carried by the parton inside the proton

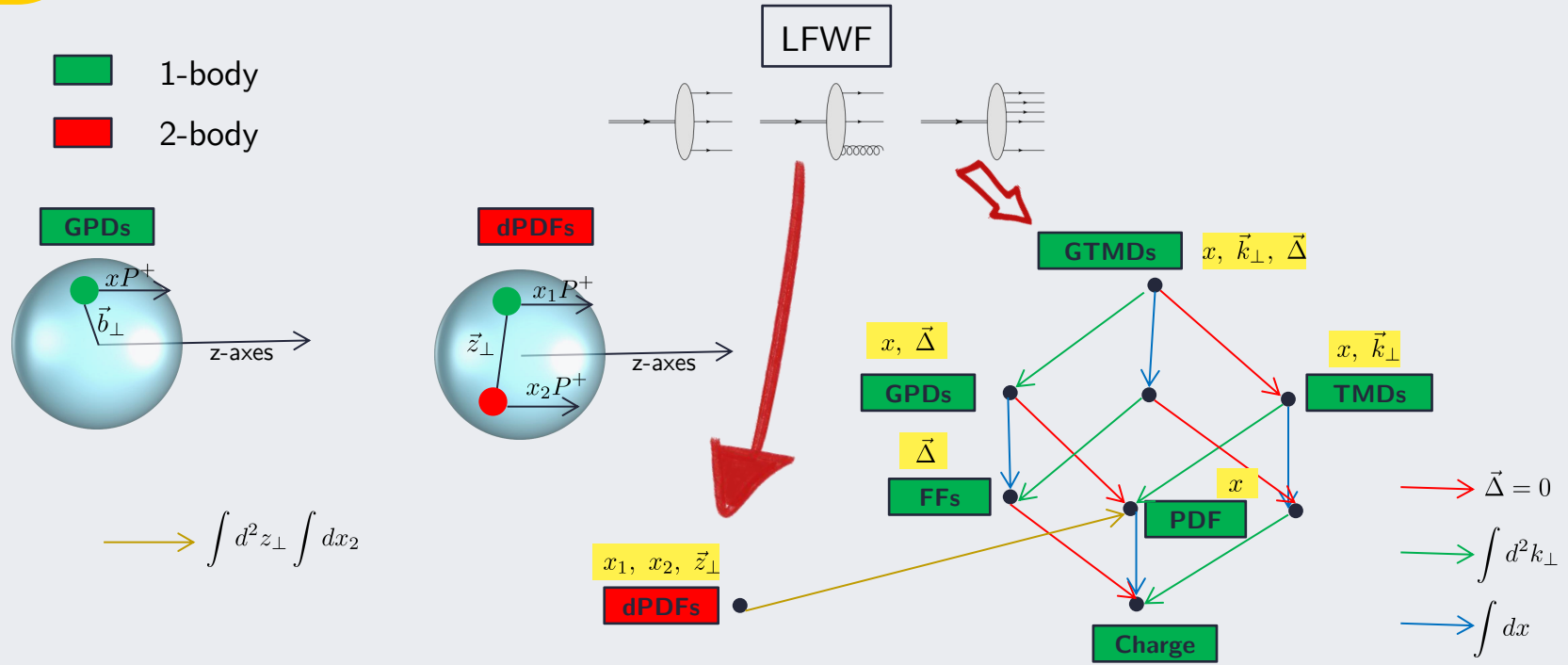
DPS processes are important for fundamental studies, e.g. the background for the research of new physics and to grasp information on the **3D PARTONIC STRUCTURE OF THE PROTON**

1 Multidimensional Pictures of Hadron

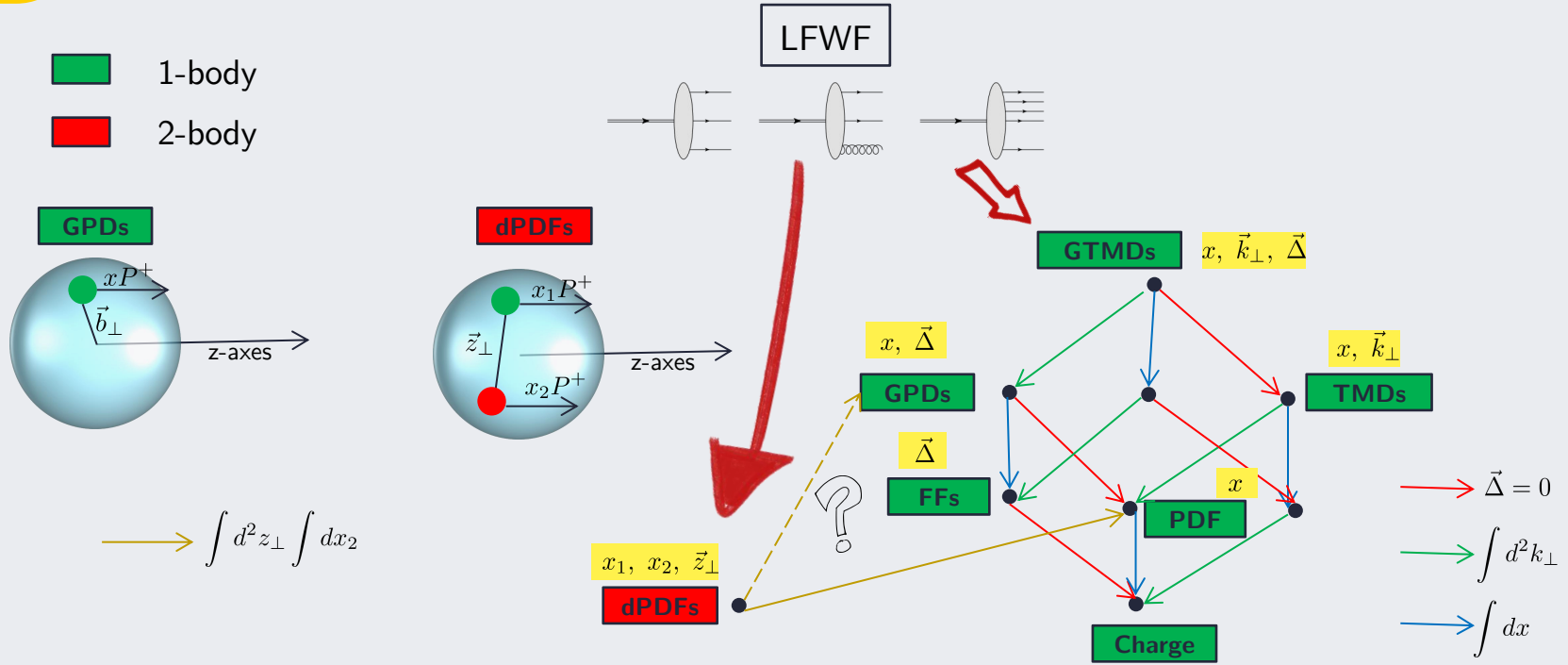
1-body



1 Multidimensional Pictures of Hadron




1 Multidimensional Pictures of Hadron



1 Double PDFs of the proton

$F_{ik}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)



1st uncorrelated scenario

$$F_{ik}(x_1, x_2, \vec{z}_\perp) \sim g(x_1, x_2) \tilde{T}(\vec{z}_\perp)$$

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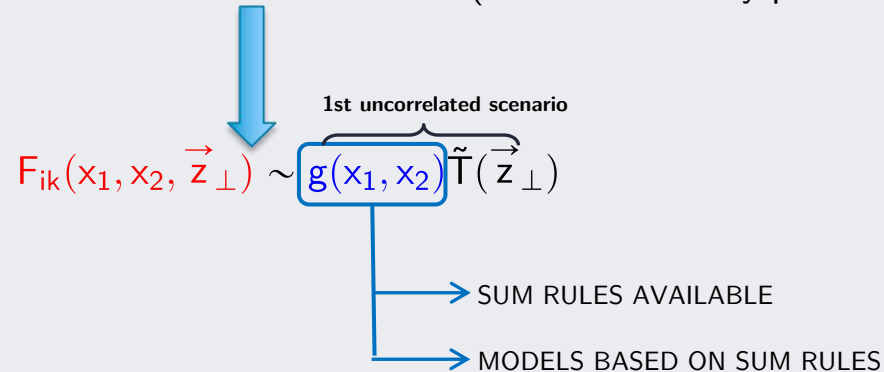
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SUM RULES AVAILABLE J.R. Gaunt et al, JHEP 03 (2010) 005,
EPJC 80 (2020) n°5, 468

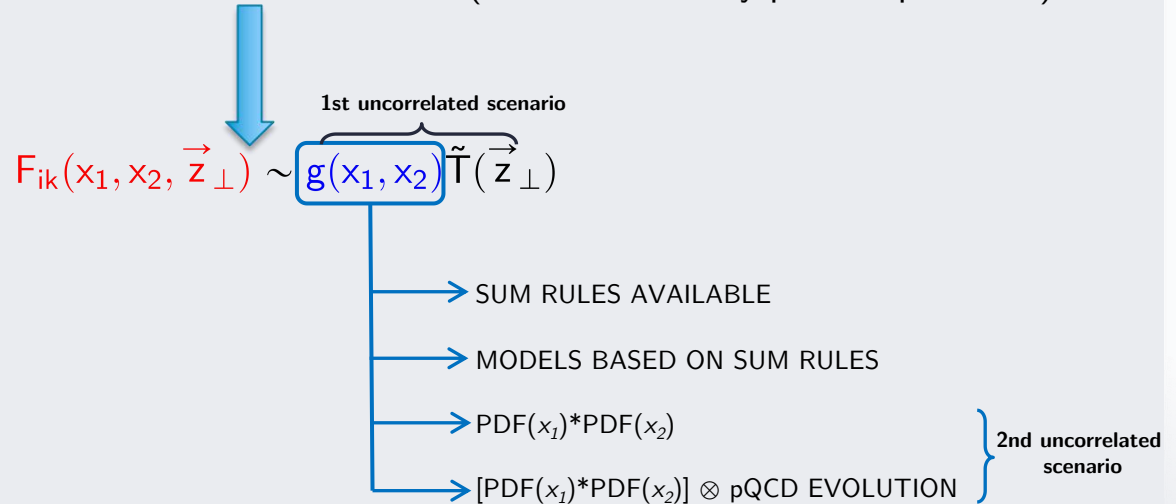
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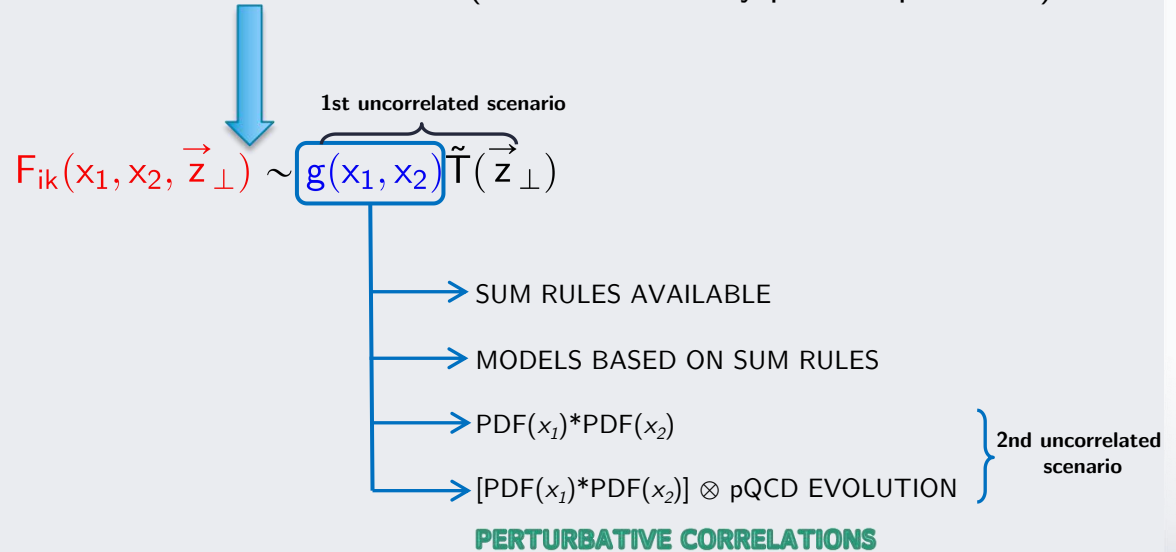
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J. R. Gaunt and W. J. Stirling, JHEP 03, 005 (2010)

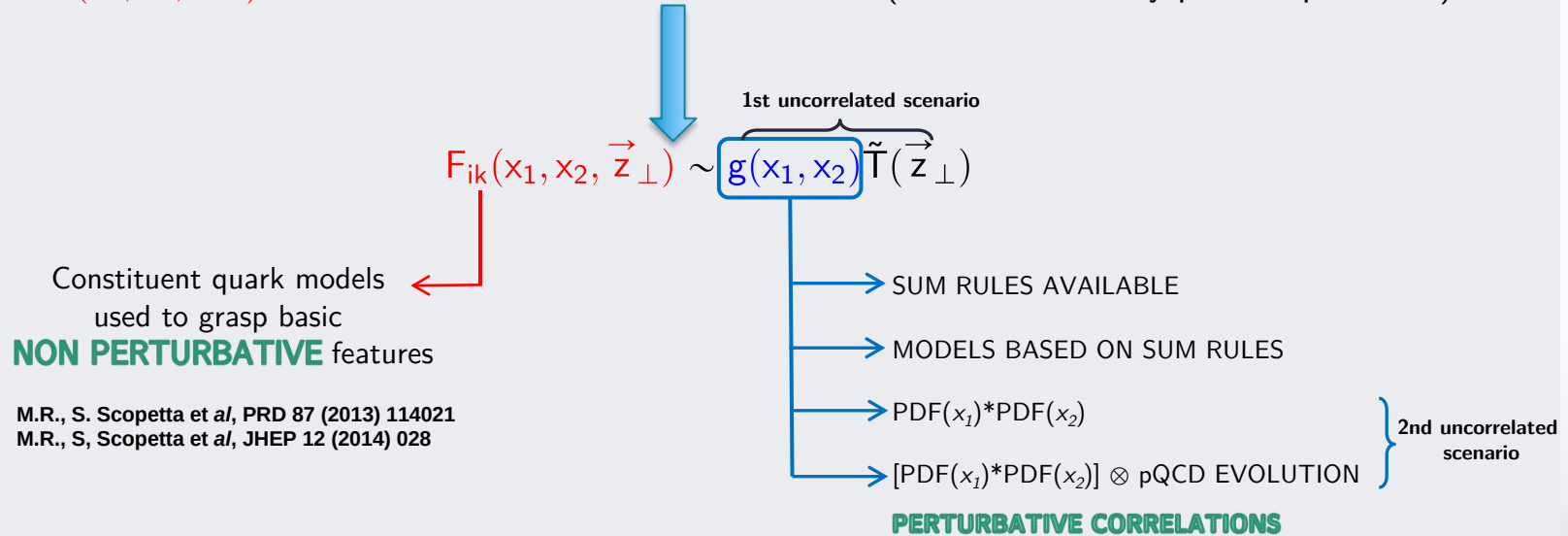
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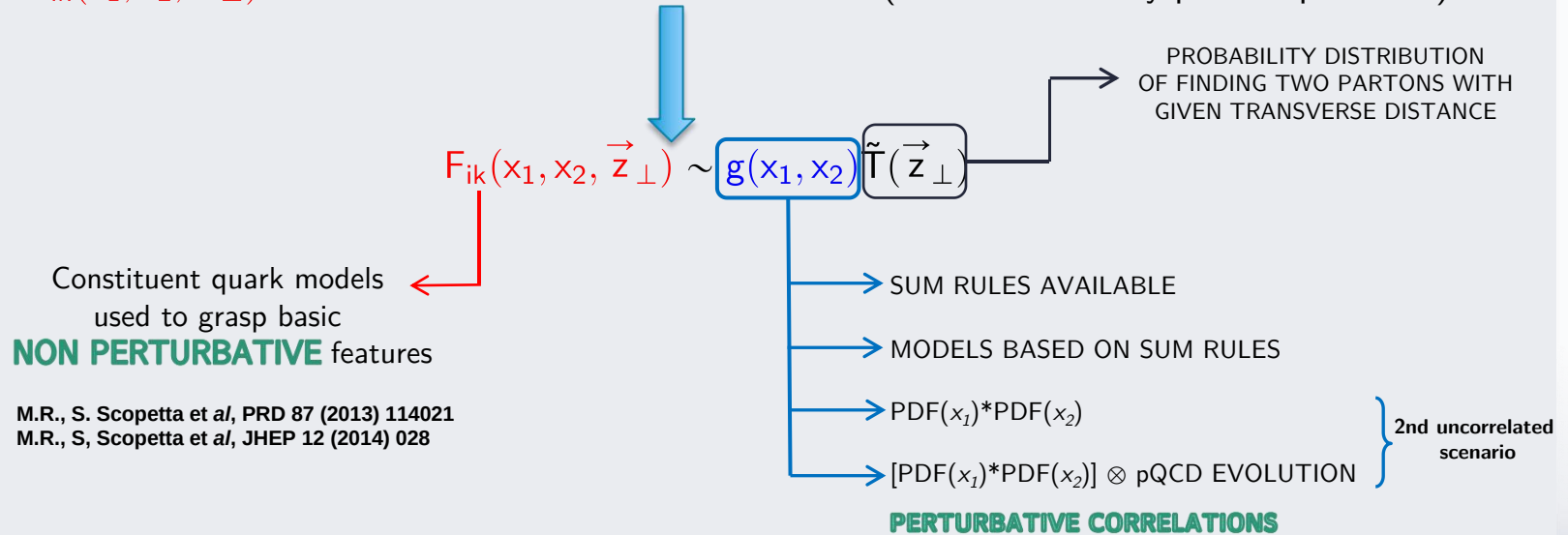
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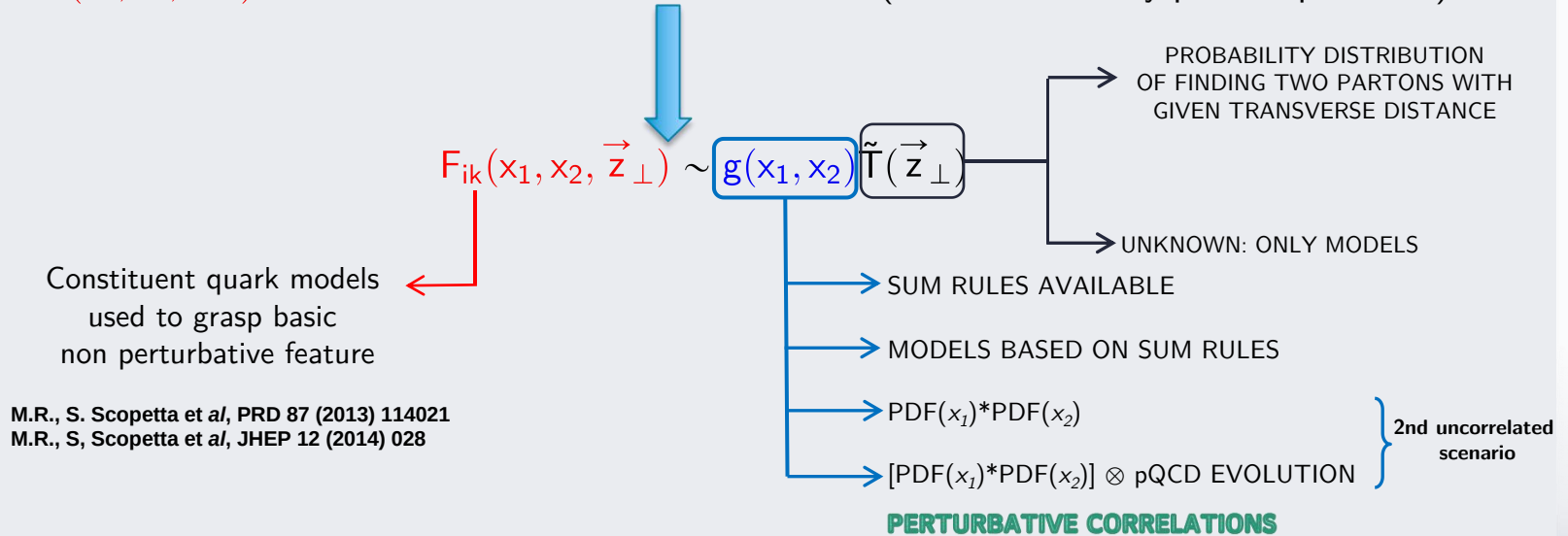
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1 Double PDFs of the proton

$F_{ij}(x_1, x_2, \vec{z}_\perp)$ is unknown. However @LHC kinematics (small x and many partons produced)

Some questions arise:

1) HOW CAN WE BE SURE OF THE ACCURACY OF SUCH APPROXIMATION?

dPDFs are non perturbative in QCD \longrightarrow DPCs cannot be accessed from QCD

2) WHICH INFORMATION ON THE PROTON STRUCTURE COULD BE ACCESSED FROM DPS?

M.R. S. Scopetta et al, PRD 87 (2013) 114021
M.R. S, Scopetta et al, JHEP 12 (2014) 028

PERTURBATIVE CORRELATIONS

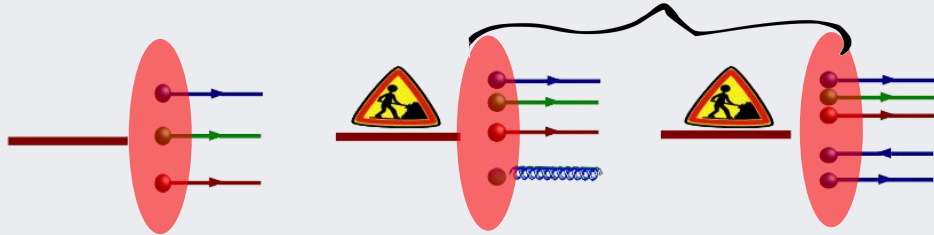
2 Double PDFs within the Light-Front

Extending the procedure developed in **S. Boffi, B. Pasquini and M. Traini, Nucl. Phys. B 649, 243 (2003)** for GPDs, we obtained the following expression of the **dPDF** in momentum space, often called **$_2$ GPDs**:

$$F_{ij}(x_1, x_2, k_{\perp}) = 3(\sqrt{3})^3 \int \prod_{i=1}^3 d\vec{k}_i \delta\left(\sum_{i=1}^3 \vec{k}_i\right) \underbrace{\Phi^*(\{\vec{k}_i\}, k_{\perp}) \Phi(\{\vec{k}_i\}, -k_{\perp})}_{\text{LF wave-function}}$$

Conjugate to z_{\perp} $\times \delta\left(x_1 - \frac{k_1^+}{P_+}\right) \delta\left(x_2 - \frac{k_2^+}{P_+}\right)$

M.R., S. Scopetta et al, JHEP 10 (2016) 063



$$\Phi(\{\vec{k}_i\}, \pm k_{\perp}) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_{\perp}}{2}, \vec{k}_2 \mp \frac{\vec{k}_{\perp}}{2}, \vec{k}_3\right)$$

2 Double PDFs within the Light-Front: GPDxGPD?

The **dPDF** is formally defined through the Light-cone correlator:

$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^2 dl_i^- e^{ix_i l_i^- p^+} \right] \langle p | O(z, l_1) | X \rangle \langle X | O(0, l_2) | p \rangle \Big|_{\substack{\vec{l}_{1\perp} = \vec{l}_{2\perp} = 0 \\ l_1^+ = l_2^+ = z^+ = 0}}$$

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle \langle p'|$$

$$F_{12}(x_1, x_2, \vec{k}_\perp) \sim f(x_1, 0, \vec{k}_\perp) f(x_2, 0, \vec{k}_\perp)$$

GPD

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Approximated by the product of two GPDs

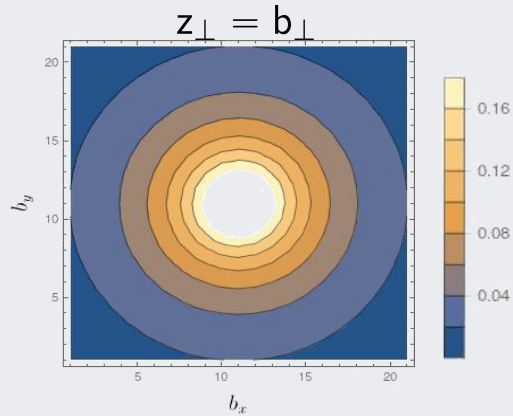
$$\int \frac{dp'^+ d\vec{k}'_\perp}{(2\pi)^4} \dots$$

$$F_{12}(x_1, x_2, \vec{z}_\perp) \sim f(x_1, 0, \vec{k}_\perp) f(x_2, 0, \vec{k}_\perp)$$

GPD

TO BE TESTED WITH MODELS

2 Information from Quark Models

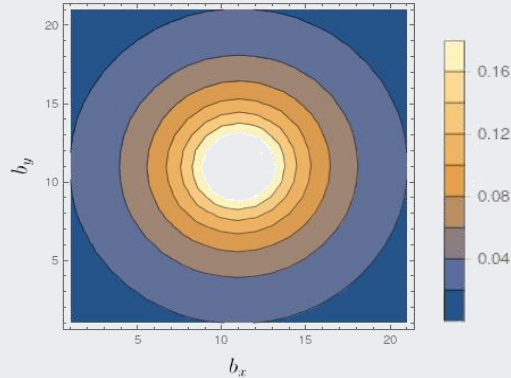


M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

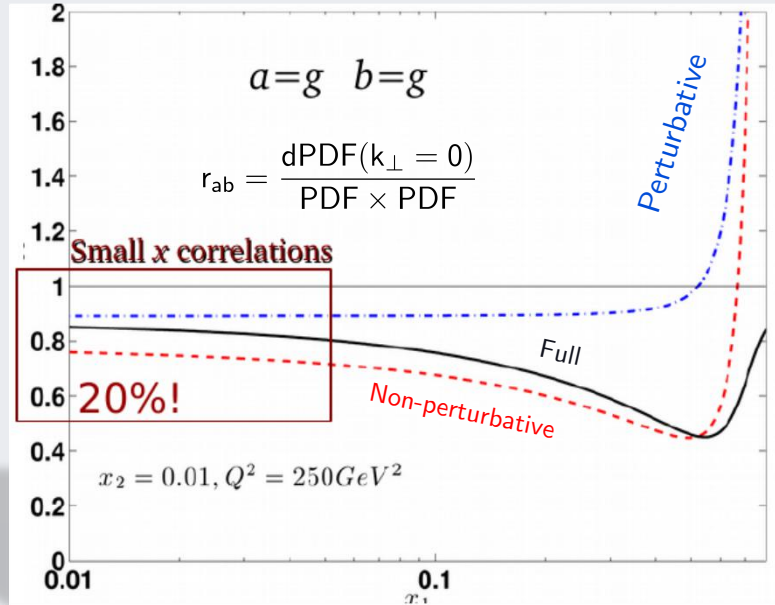
1) e.g. the distance distribution of **two gluons** in the proton

$$\langle z_{\perp}^2 \rangle_{x_1, x_2}^{ij} = \frac{\int d^2 z_{\perp} z_{\perp}^2 F_{ij}(x_1, x_2, z_{\perp})}{\int d^2 z_{\perp} F_{ij}(x_1, x_2, z_{\perp})}$$

2 Information from Quark Models



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

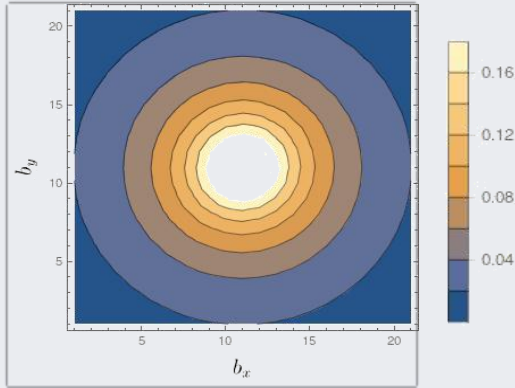


2) Correlations are important

M.R., S. Scopetta et al, JHEP 10 (2016) 063

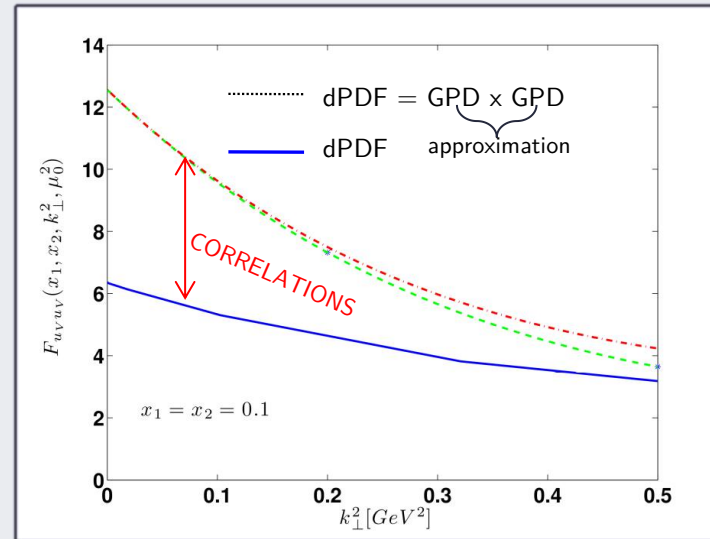
M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

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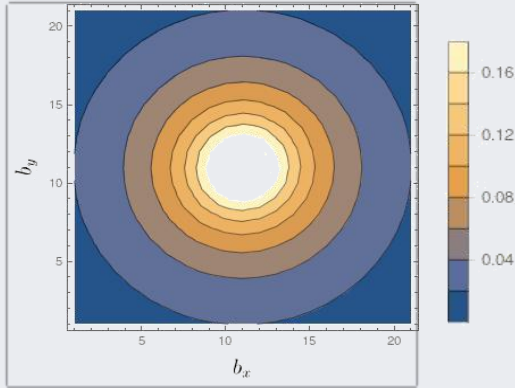


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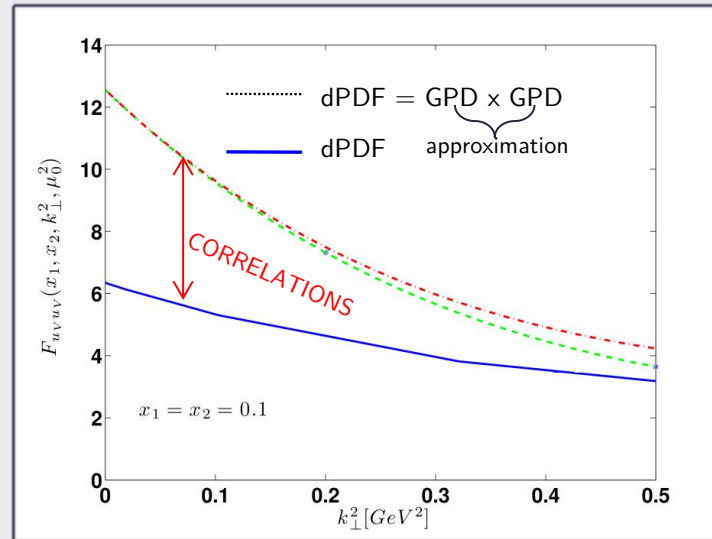
2 Information from Quark Models



M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

IS IT POSSIBLE TO ACCESS DOUBLE PARTON CORRELATIONS?

1) e.g. the distance distribution of two gluons in the proton



2) Correlations are important

M.R., S. Scopetta et al, JHEP 10 (2016) 063

M.R. and F. A. Ceccopieri PRD 95 (2017) 034040

3 Data and Effective Cross Section

A tool for the comprehension of the role of DPS in hadron-hadron collisions is the so called “effective X-section”.

$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_{\text{A}}^{\text{pp}} \sigma_{\text{B}}^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$

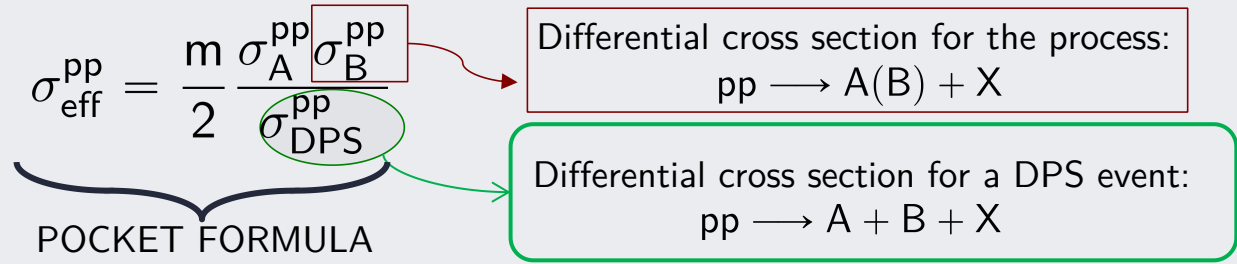
POCKET FORMULA

Differential cross section for the process:
 $pp \rightarrow A(B) + X$

Differential cross section for a DPS event:
 $pp \rightarrow A + B + X$

3 Data and Effective Cross Section

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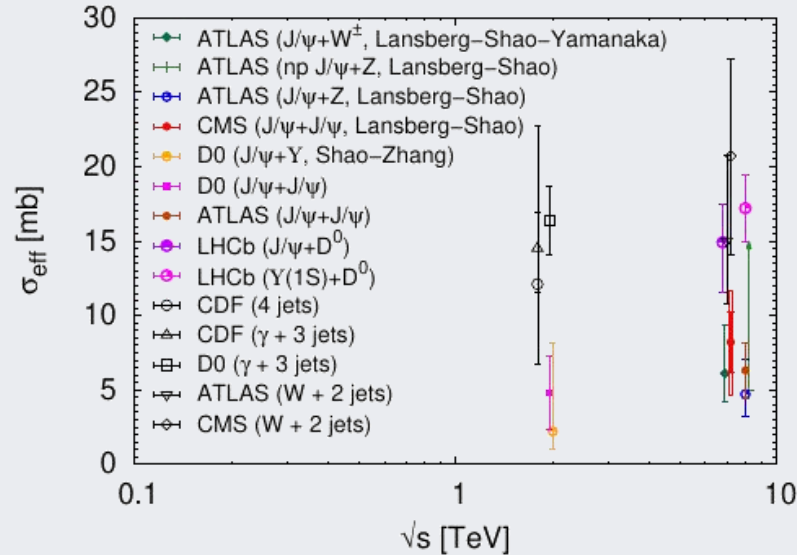
$$\sigma_{\text{eff}}(x_1, x_2, x_3, x_4) = \frac{\sum_{i,j,k,l} \text{color factors } C_{ik} C_{jl} F_i(x_1) F_j(x_2) F_k(x_3) \text{ PDF } F_l(x_4)}{\sum_{i,j,k,l} C_{ik} C_{jl} \int d^2 z_{\perp} F_{ij}(x_1, x_3, z_{\perp}) F_{kl}(x_3, x_4, z_{\perp})}$$

M.R., S. Scopetta et al, PLB 752

M. Traini, M.R., S. Scopetta and V. Vento, PLB 768 (2017)

3 Data and Effective Cross Section

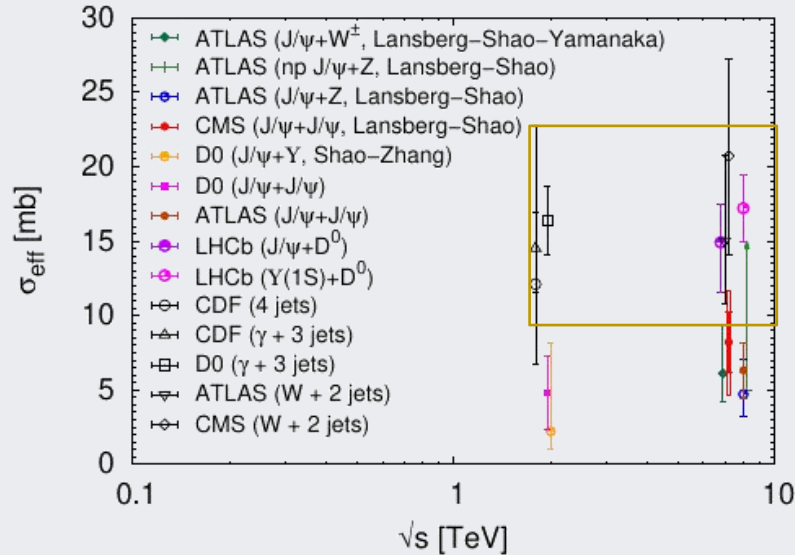
$$\sigma_{\text{eff}}^{\text{pp}} = \frac{m}{2} \frac{\sigma_A^{\text{pp}} \sigma_B^{\text{pp}}}{\sigma_{\text{DPS}}^{\text{pp}}}$$



J.P. Lansberg's slide
MPI-2019 workshop

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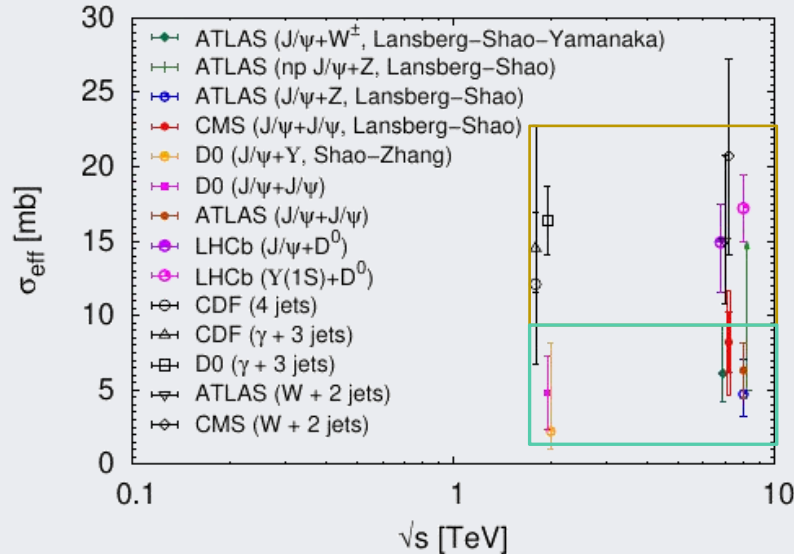
J.P. Lansberg's slide
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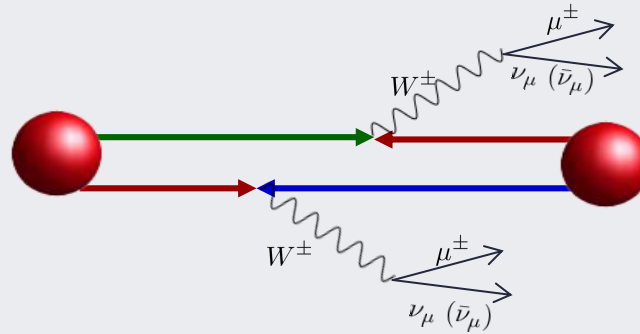
- SENSITIVE TO CORRELATIONS
- PROCESS DEPENDENT?
- CAN WE DIRECTLY ACCESS THESE CORRELATIONS?



J.P. Lansberg's slide
MPI-2019 workshop

4 Same sign W 's production at the LHC

M. R. et al, Phys.Rev.
D95 (2017) no.11,
114030



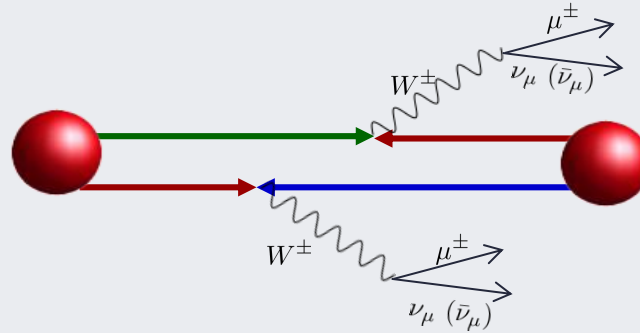
In this channel, the single parton scattering (usually dominant w.r.t to the double one) starts to contribute to higher order in strong coupling constant.



“Same-sign W boson pairs production is a promising channel to look for signature of double Parton interactions at the LHC.”

4 Same sign W 's production at the LHC

M. R. et al, Phys.Rev.
D95 (2017) no.11,
114030



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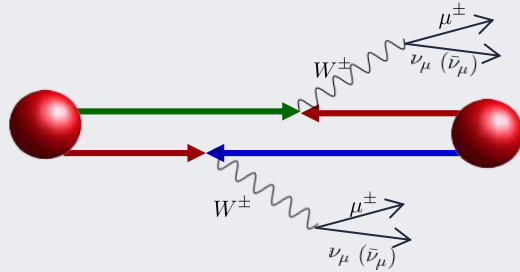


Can double parton correlations be observed for the first time in the next LHC run ?

4 Same sign W's production at the LHC

M. R. et al, Phys.Rev.
D95 (2017) no.11,
114030

Kinematical cuts



$$\begin{aligned}
 &pp, \sqrt{s} = 13 \text{ TeV} \\
 &p_{T,\mu}^{\text{leading}} > 20 \text{ GeV}, \quad p_{T,\mu}^{\text{subleading}} > 10 \text{ GeV} \\
 &|p_{T,\mu}^{\text{leading}}| + |p_{T,\mu}^{\text{subleading}}| > 45 \text{ GeV} \\
 &|\eta_\mu| < 2.4 \\
 &20 \text{ GeV} < M_{\text{inv}} < 75 \text{ GeV} \text{ or } M_{\text{inv}} > 105 \text{ GeV}
 \end{aligned}$$

DPS cross section:

$$\frac{d^4 \sigma_{pp \rightarrow \mu^\pm \mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2 \vec{b}_\perp F_{ij}(x_1, x_2, \vec{b}_\perp, M_W) F_{kl}(x_3, x_4, \vec{b}_\perp, M_W) \frac{d^2 \sigma_{ik}^{pp \rightarrow \mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2 \sigma_{jl}^{pp \rightarrow \mu^\pm X}}{d\eta_2 dp_{T,2}} \mathcal{I}(\eta_i, p_{T,i})$$

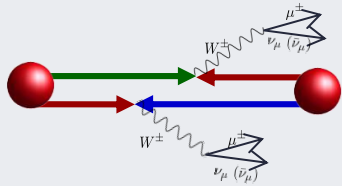
In order to estimate the role of double parton correlations we have used as input of dPDFs:

1) Longitudinal and transverse correlations arise from the relativistic CQM model describing three valence quarks

2) These correlations propagate to sea quarks and gluons through pQCD evolution

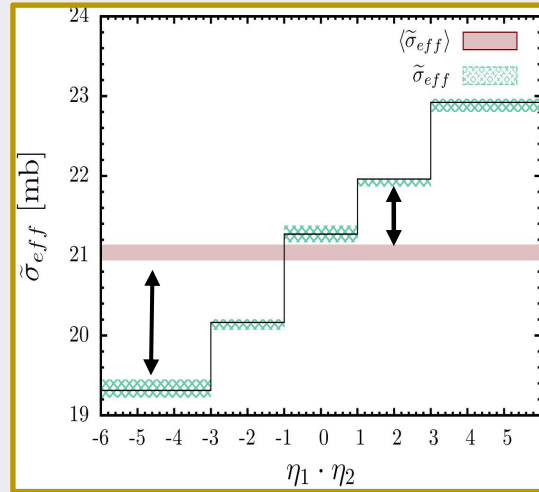
4 Same sign W's production at the LHC

M. R. et al, Phys.Rev.
D95 (2017) no.11,
114030



$$\eta_1 \cdot \eta_2 \simeq \frac{1}{4} \ln \frac{x_1}{x_3} \ln \frac{x_2}{x_4}$$

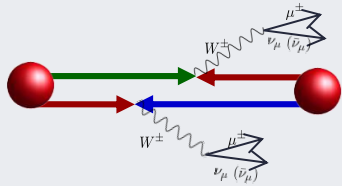
$$\langle \tilde{\sigma}_{eff} \rangle = 21.04^{+0.07}_{-0.07} (\delta Q_0)^{+0.06}_{-0.07} (\delta \mu_F) \text{ mb} .$$



Difference $\left[\updownarrow \right]$ between green and red line is due to correlations effects

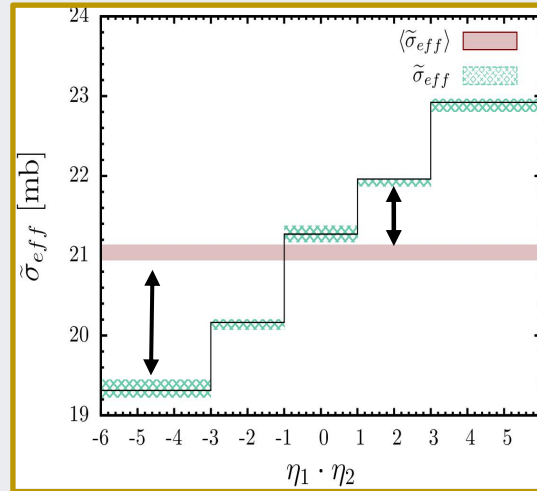
4 Same sign W's production at the LHC

M. R. et al, Phys.Rev. D95 (2017) no.11, 114030



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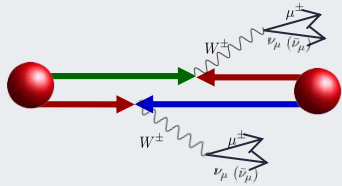
x- dependence of effective x-section
 M.Rinaldi et al PLB 752,40 (2016)
 M. Traini, M. R. et al, PLB 768, 270 (2017)

Assuming that the results of the first and the last bins can be distinguished if they differ by 1 sigma, we estimated that:

$$\mathcal{L} = 1000 \text{ fb}^{-1}$$

is necessary to observe correlations
 * to be updated to new CMS cuts

4 Same sign W 's production at the LHC



In Ref. **S. Cotogno et al, JHEP 10 (2020) 214**, it has been shown that several experimental observables are sensitive to **double spin correlations**.

The LHC has the potential to access this new information!

IN THIS CHANNEL, WE ESTABLISHED THE POSSIBILITY TO OBSERVE, FOR THE FIRST TIME, TWO-PARTON CORRELATIONS IN THE NEXT LHC RUN!

5 Clues from data?

If dPDFs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2 \rightarrow \text{Effective form factor (EFF)}$$

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If dPDFs factorize in terms of PDFs then

$$\sigma_{\text{eff}}^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2$$

→ Effective form factor (EFF)

EFF can be formally defined as
FIRST MOMENT of dPDF
in momentum space

$$T(k_{\perp}) \propto \int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$

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If dPDFs factorize in terms of PDFs then $\sigma_{\text{eff}}^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2$ → Effective form factor (EFF)

EFF can be formally defined as
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k_{\perp} is the conjugate variable to z_{\perp} . In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \frac{d}{dk_{\perp}} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

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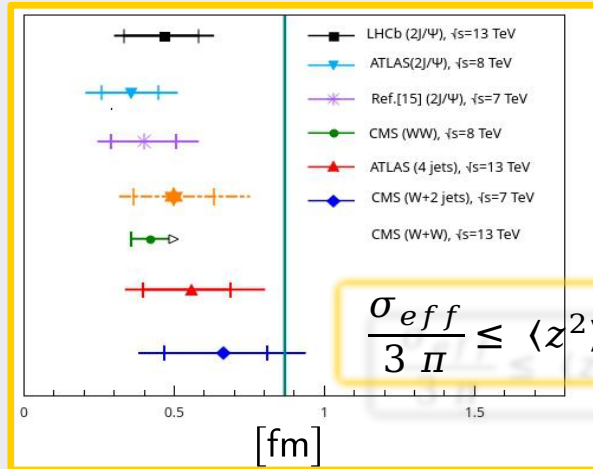
k_{\perp} is the conjugate variable to z_{\perp} . In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \left. \frac{d}{dk_{\perp}} T(k_{\perp}) \right|_{k_{\perp}=0}$$

$$\int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$



DPS processes:
The vertical line stands for the transverse proton radius



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If dPDFs factorize in terms of PDFs then $\sigma_{\text{eff}}^{-1} = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2 \rightarrow$ Effective form factor (EFF)

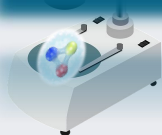
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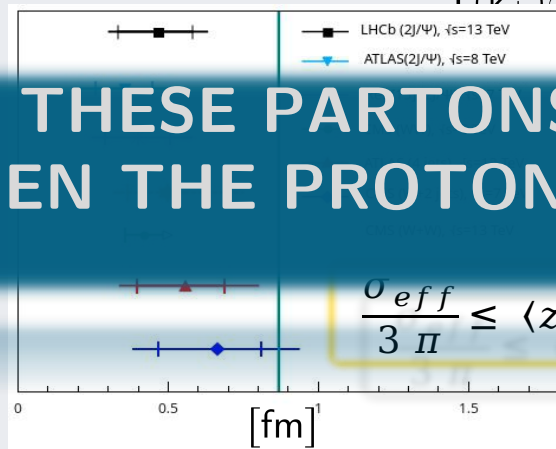
k_{\perp} is the conjugate variable to z_{\perp} . In analogy with the charge form factor:

$$\langle z_{\perp}^2 \rangle \propto \frac{d}{dk_{\perp}^2} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

THE DISTANCE OF THESE PARTONS SEEMS TO BE SMALLER THEN THE PROTON RADIUS



the transverse proton radius



5 Clues from data?

If dPDFs factorize in terms of PDFs then $\sigma_{\text{eff}}^{-1} = \int \frac{d^2k_{\perp}}{(2\pi)^2} \boxed{T(k_{\perp})}^2 \rightarrow \text{Effective form factor (EFF)}$

EFF can be formally defined as **FIRST MOMENT** of dPDF in momentum space

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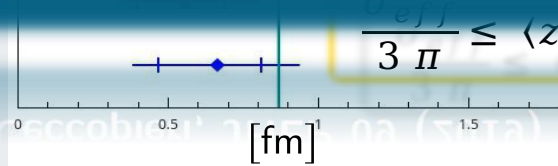
$$\langle z_{\perp}^2 \rangle \propto \frac{d}{dk_{\perp}} T(k_{\perp}) \Big|_{k_{\perp}=0}$$

$$T(k_{\perp}) \propto \int dx_1 dx_2 \tilde{F}(x_1, x_2, k_{\perp})$$



HOWEVER FROM PROTON-PROTON COLLISIONS ONLY RANGES CAN BE ACCESSED

M.R. and F. A. Ceccopieri, JHEP 09 (2019) 097

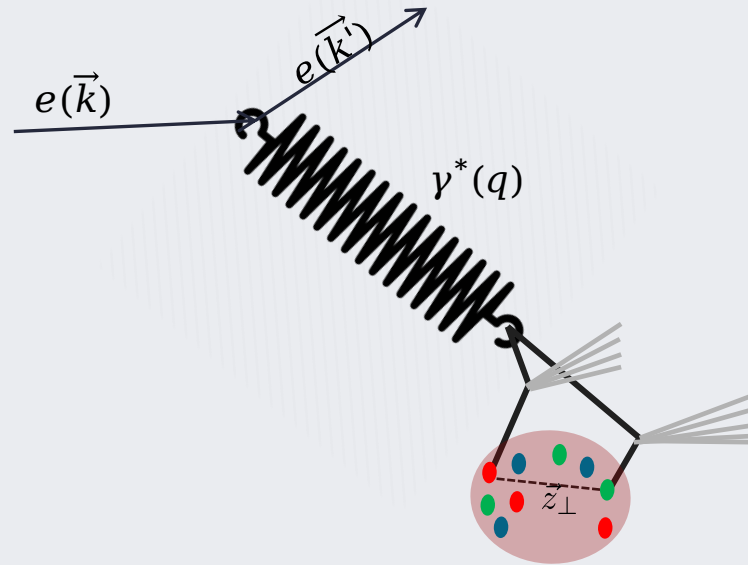


$$\frac{3}{\pi} \leq \langle z_{\perp}^2 \rangle \leq \pi$$



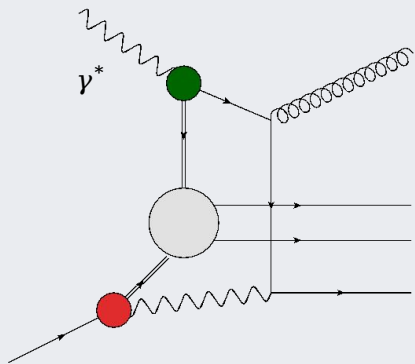
6 New Idea: DPS via γ -p interaction

We consider the possibility offered by a DPS process involving a photon FLUCTUATING in a quark-antiquark pair interacting with a proton:



6 New Idea: DPS via γ -p interaction

In order to study the impact of the DPS contribution to a process initiated via photon-proton interactions we evaluated the 4-JET photoproduction at HERA (S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008))



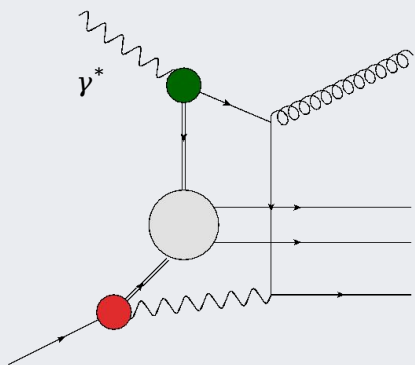
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- 1) **G. Abbiend et al, Phys. Commun 67, 465 (1992)**
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It has been shown that the agreement with data improves if MPI are included in the Monte Carlo

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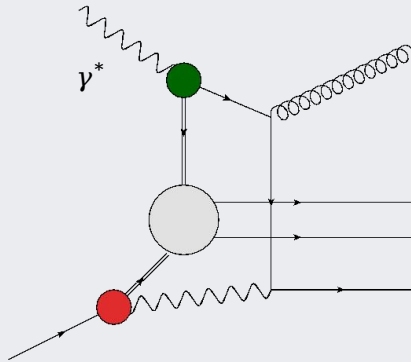
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WE EVALUATE THE DPS CONTRIBUTION TO THIS PROCESS

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*Single Parton Scattering (SPS)

For this first investigation, we make use of the
POCKET FORMULA:

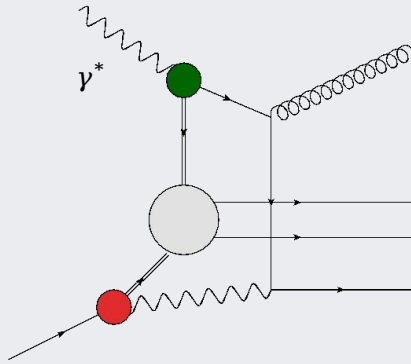
$$d\sigma_{\text{DPS}}^{4j} = \frac{1}{2} \sum_{ab,cd} \int dy dQ^2 \frac{f_{\gamma/e}(y, Q^2)}{\sigma_{\text{eff}}^{\gamma p}(Q^2)} \times \left. \begin{array}{l} \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b}) \\ \times \int dx_{p_c} dx_{\gamma_d} \underbrace{f_{c/p}(x_{p_c})}_{\text{p-PDF}} \underbrace{f_{d/\gamma}(x_{\gamma_d})}_{\gamma\text{-PDF}} d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d}) \end{array} \right\} \begin{array}{l} \text{SPS}^* \\ \times \\ \text{SPS} \end{array}$$

Flux Factor
P. Nason et al, PLB319
339 (1993)

(M. Gluck et al. PRD46, 1973 (1992))
(J. Pumplin et al. JHEP 07, 012 (2002))

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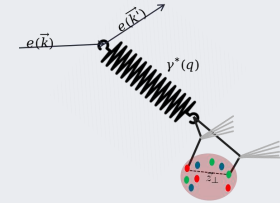
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$$\times \left. \begin{aligned} & d\hat{\sigma}_{ab}^{2j}(x_{pa}, x_{\gamma b}) \quad \left. \begin{array}{l} \text{SPS*} \\ \times \end{array} \right\} \\ & d\hat{\sigma}_{cd}^{2j}(x_{pc}, x_{\gamma d}) \quad \left. \begin{array}{l} \text{SPS} \end{array} \right\} \end{aligned} \right\} \times$$

PDF (M. Gluck et al. PRD46, 1973 (1992)
2002)

The main quantity we have to evaluate is:
 $\sigma_{eff}^{\gamma p}(Q^2)$

6 The γ -p effective cross section

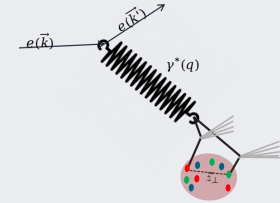


The expression of this quantity is very similar to the proton-proton collision case and can be formally derived by comparing the product of SPS cross sections and the DPS one obtained in **Gaunt, JHEP 01, 042 (2013)** and describing a DPS from a vector bosons splitting with given Q^2 virtuality

$$[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} \overset{\text{Proton EFF}}{\boxed{T_p(k_{\perp})}} T_{\gamma}(k_{\perp}; Q^2)$$

M. R. and F. A. Ceccopieri, arXiv:2103.13480

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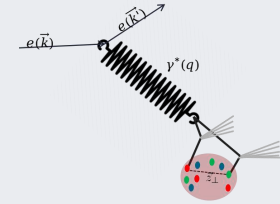


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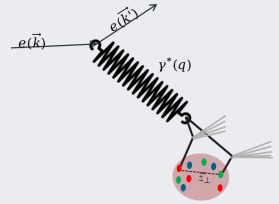


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The full DPS cross section depends on the amplitude of the splitting photon in a $q \bar{q}$ pair. The latter can be formally described within a Light-Front (LF) approach in terms of LF wave functions:

6 The γ -p effective cross section

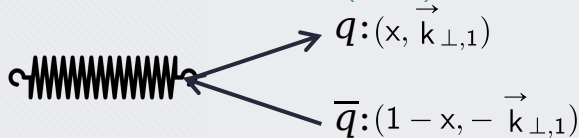


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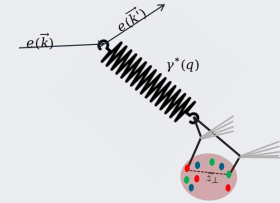
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$$f_{q,\bar{q}}^{\gamma}(x, \vec{k}_{\perp}; Q^2) = \int d^2 k_{\perp,1} \psi_{q\bar{q}}^{\dagger\gamma}(x, \vec{k}_{\perp,1}; Q^2) \times \psi_{q\bar{q}}^{\gamma}(x, \vec{k}_{\perp,1} + \vec{k}_{\perp}; Q^2)$$



M. R. and F. A. Ceccopieri, arXiv:2103.13480

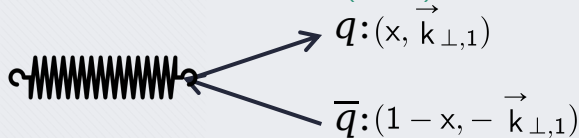
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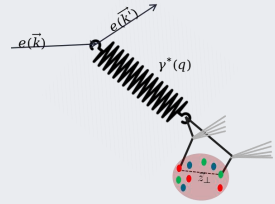
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Similar definition of a meson dPDF

M. R. and F. A. Ceccopieri, arXiv:2103.13480

M. R. et al., EPJC78, 781 (2018)

6 The γ -p effective cross section

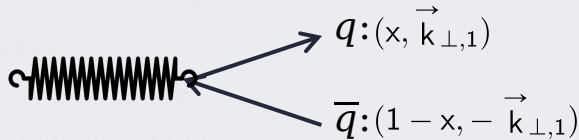


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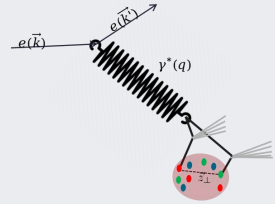
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$$T_{\gamma}(k_{\perp}; Q^2) = \frac{\sum_q \int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp}; Q^2)}{\sum_q \int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp} = 0; Q^2)}$$



M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The γ -p effective cross section



The expression of this quantity is very similar to the case and can be formalized in terms of cross sections and the DPS on collision cross sections DPS from a

1 INGREDIENTS OF THE CALCULATIONS

$$[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2k_{\perp}}{(2\pi)^2}$$

$$T_p(k_{\perp}) \text{ proton EFF}$$

$$\int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp}; Q^2)$$

2

$$f_{q,\bar{q}}^{\gamma}(x, \tilde{k}_{\perp}; Q^2) = \int d^2k_{\perp}$$

$$\psi/\gamma \text{ Photon WF}$$

$$\int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp} = 0; Q^2)$$

3

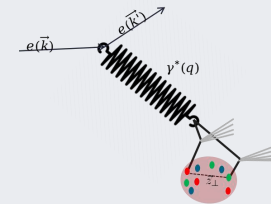


$$q: (1-x, k_{\perp,1})$$

$$\int dx f_{q,\bar{q}}^{\gamma}(x, k_{\perp} = 0; Q^2)$$

M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The γ -p effective cross section



For the proton EFF use has been made of three choices:

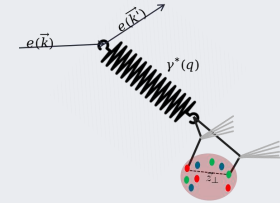
1 $[\sigma_{\text{eff}}^{\gamma p}(Q^2)]^{-1} = \int \frac{d^2 k_{\perp}}{(2\pi^2)} T_p(k_{\perp}) T_{\gamma}(k_{\perp}; Q^2)$

2 $T_p(k_{\perp})$ proton EFF

3 ψ/γ Photon WF

M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The γ -p effective cross section



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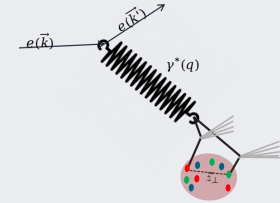
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1) G1: $e^{-\alpha_1 k_{\perp}^2}$ $\alpha_1 = 1.53 \text{ GeV}^{-2} \implies \sigma_{\text{eff}}^{\text{pp}} = 15 \text{ mb}$

6 The γ -p effective cross section

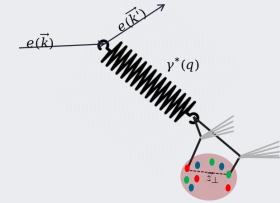


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- 3 ψ/γ Photon WF

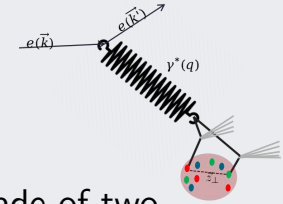
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- 3) S: $\left(1 + \frac{k_{\perp}^2}{m_g^2}\right)^{-4}$, $m_g^2 = 1.1 \text{ GeV}^2 \implies \sigma_{\text{eff}}^{\text{pp}} = 30 \text{ mb}$

B. Blok et al, EPJC74, 2926 (2014)

M. R. and F. A. Ceccopieri, arXiv:2103.13480

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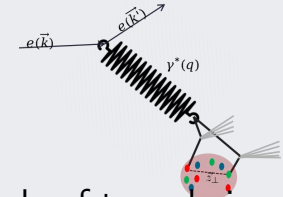
3 ψ_{γ} Photon WF

For the photon W.F. use has been made of two choices representing two extreme cases:

1) QED at LO (S.J. Brodsky et al. PRD50, 3134 (1994)):

$$\psi_{q, \bar{q}}^{\lambda=\pm}(x, k_{1\perp}; Q^2) = -e_f \frac{\bar{u}_q(k) \gamma \cdot \varepsilon^{\lambda} v_{\bar{q}}(q - k)}{\sqrt{x(1-x)} \left[Q^2 + \frac{k_{1\perp}^2 + m^2}{x(1-x)} \right]}$$

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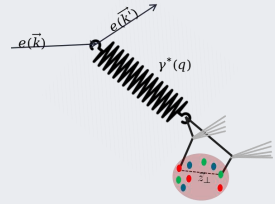
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2) Non-Pertubative (NP) effects (E.R.Arriola et al, PRD74,054023 (2006))

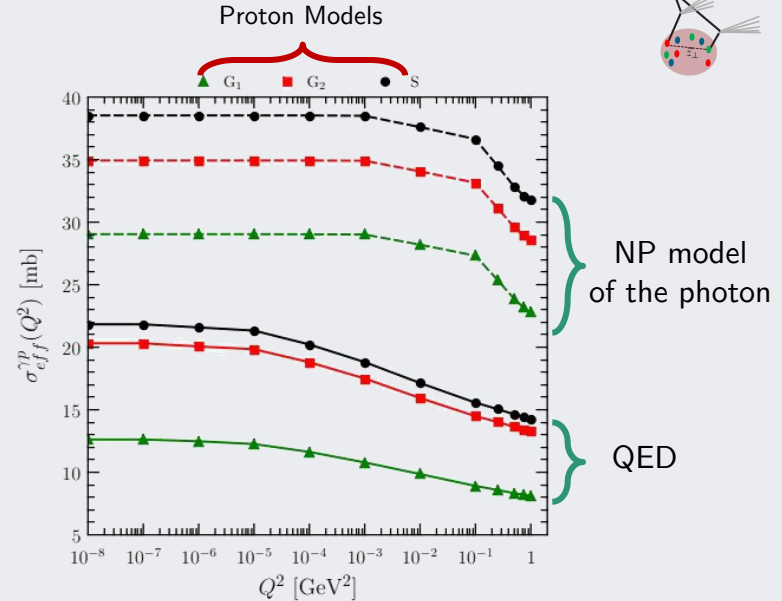
$$\psi_A^{\gamma}(x, k_{1\perp}; Q^2) = \frac{6(1 + Q^2/m_{\rho}^2)}{m_{\rho}^2 \left(1 + 4 \frac{k_{1\perp}^2 + Q^2 x(1-x)}{m_{\rho}^2} \right)^{5/2}}$$

M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The γ -p effective cross section



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M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The 4 jet DPS cross section

The HERA KINEMATICS:

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

$$E_T^{\text{jet}} > 6 \text{ GeV}$$

Transverse energy of the jets

$$|\eta_{\text{jet}}| < 2.4$$

Pseudorapidity

$$Q^2 < 1 \text{ GeV}^2$$

Photon virtuality

$$0.2 \leq y \leq 0.85$$

Inelasticity

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The ZEUS collaboration quoted an integrated total 4-jet cross section of 200 pb

S. Checkanov et al. (ZEUS), Nucl. Phys B792, 1 (2008)

6 The 4 jet DPS cross section

KINEMATICS:

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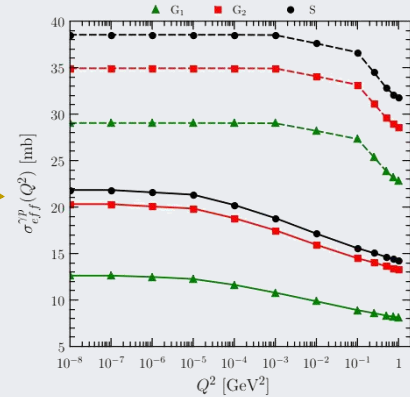
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$$\times \int dx_{p_a} dx_{\gamma_b} f_{a/p}(x_{p_a}) f_{b/\gamma}(x_{\gamma_b}) d\hat{\sigma}_{ab}^{2j}(x_{p_a}, x_{\gamma_b})$$

$$\times \int dx_{p_c} dx_{\gamma_d} f_{c/p}(x_{p_c}) f_{d/\gamma}(x_{\gamma_d}) d\hat{\sigma}_{cd}^{2j}(x_{p_c}, x_{\gamma_d})$$



M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The 4 jet DPS cross section

KINEMATICS

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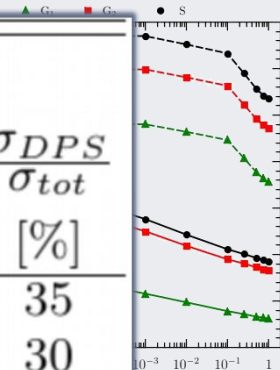
$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.8$$

		σ_{DPS} [pb]			
		$Q^2 \leq 10^{-2}$	$10^{-2} \leq Q^2 \leq 1$	$Q^2 \leq 1$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
photon		[GeV ²]	[GeV ²]	[GeV ²]	[%]
NP model	G ₁	47	25	71	35
	G ₂	39	20	59	30
	S	35	18	54	27
QED	G ₁	117	72	189	95
	G ₂	73	44	117	58
	S	64	41	109	54

proton



6 The 4 jet DPS cross section

KINEMATICS

$$E_T^{\text{jet}} > 6 \text{ GeV}$$

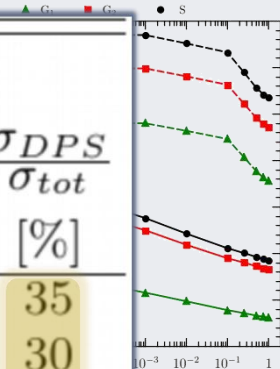
$$|\eta_{\text{jet}}| < 2.4$$

$$Q^2 < 1 \text{ GeV}^2$$

$$0.2 \leq y \leq 0.8$$

		σ_{DPS} [pb]			
		$Q^2 \leq 10^{-2}$	$10^{-2} \leq Q^2 \leq 1$	$Q^2 \leq 1$	$\frac{\sigma_{DPS}}{\sigma_{tot}}$
photon		[GeV ²]	[GeV ²]	[GeV ²]	[%]
NP model	G ₁	47	25	71	35
	G ₂	39	20	59	30
	S	35	18	54	27
QED	G ₁	117	72	189	95
	G ₂	73	44	117	58
	S	64	41	109	54

proton



6 The 4 jet DPS cross section

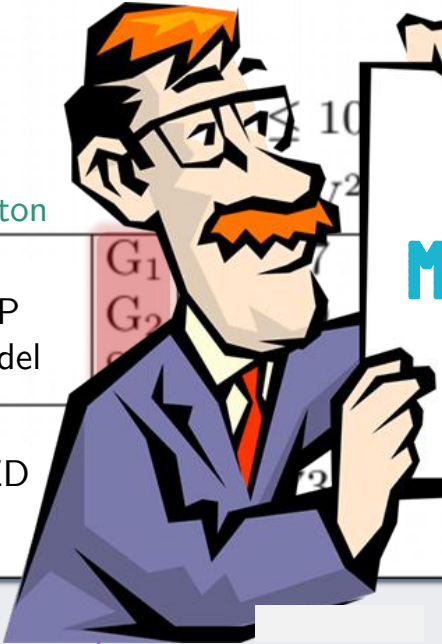
KINEMATICS

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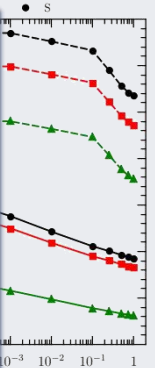
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	σ_{DPS} [pb]	$Q^2 \leq 1$ [GeV ²]	$\frac{\sigma_{DPS}}{\sigma_{tot}}$ [%]
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6 The 4 jet DPS cross section

KINEMATICS

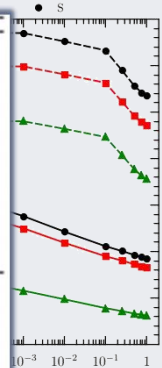
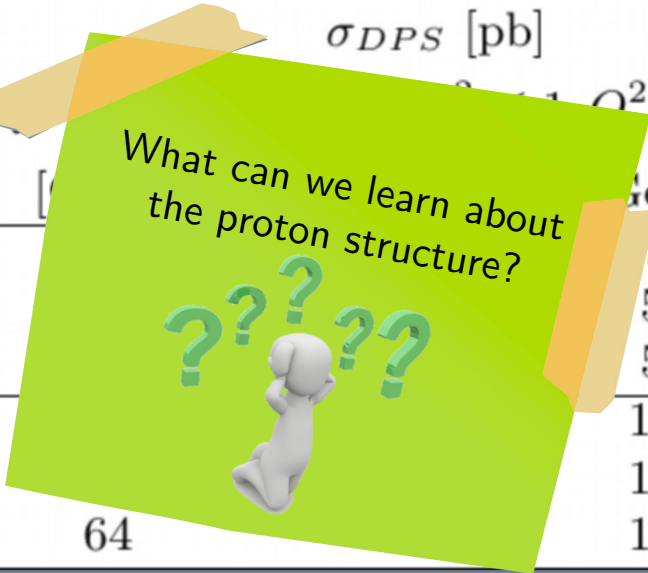
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		$\sigma_{DPS} [\text{pb}]$	$\frac{\sigma_{DPS}}{\sigma_{tot}} [\%]$
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	G ₂	59	30
	S	54	27
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	G ₂	117	58
	S	109	54



6 The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of
Fourier Transform of the EFF:

$$\tilde{F}(z_{\perp})$$

The probability of finding a parton pair at distance

$$z_{\perp}$$

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$$\left[\sigma_{\text{eff}}^{\gamma p}(Q^2) \right]^{-1} = \int d^2 z_{\perp} \tilde{F}_2^p(z_{\perp}) \tilde{F}_2^{\gamma}(z_{\perp}; Q^2)$$

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If we can measure the dependence of the effective-cross section on the photon VIRTUALITY

M. R. and F. A. Ceccopieri, arXiv:2103.13480

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We could access for the first time the mean transverse distance between partons in the proton



M. R. and F. A. Ceccopieri, arXiv:2103.13480

6 The effective cross section: a key for the proton structure

The effective cross section can be also written in terms of Fourier Transform of the EFF:

$$\left[\sigma_{\text{eff}}^{\gamma p}(\dots) \right]$$

Is it possible to get this measure?



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M. R. and F. A. Ceccopieri, arXiv:2103.13480

6

The effective cross section: a key for the proton structure

To test if in future a dependence of the effective cross section on the photon virtuality, we considered again the 4 JET photoproduction:

M. R. and F. A. Ceccopieri, arXiv:2103.13480

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To test if in future a dependence of the effective cross section on the photon virtuality could be observed, we considered again the 4 JET photoproduction:

1) We divided the integral of the cross section on Q^2 in two intervals:

$$Q^2 \leq 10^{-2} \quad \text{and} \quad 10^{-2} \leq Q^2 \leq 1 \quad \text{GeV}^2$$

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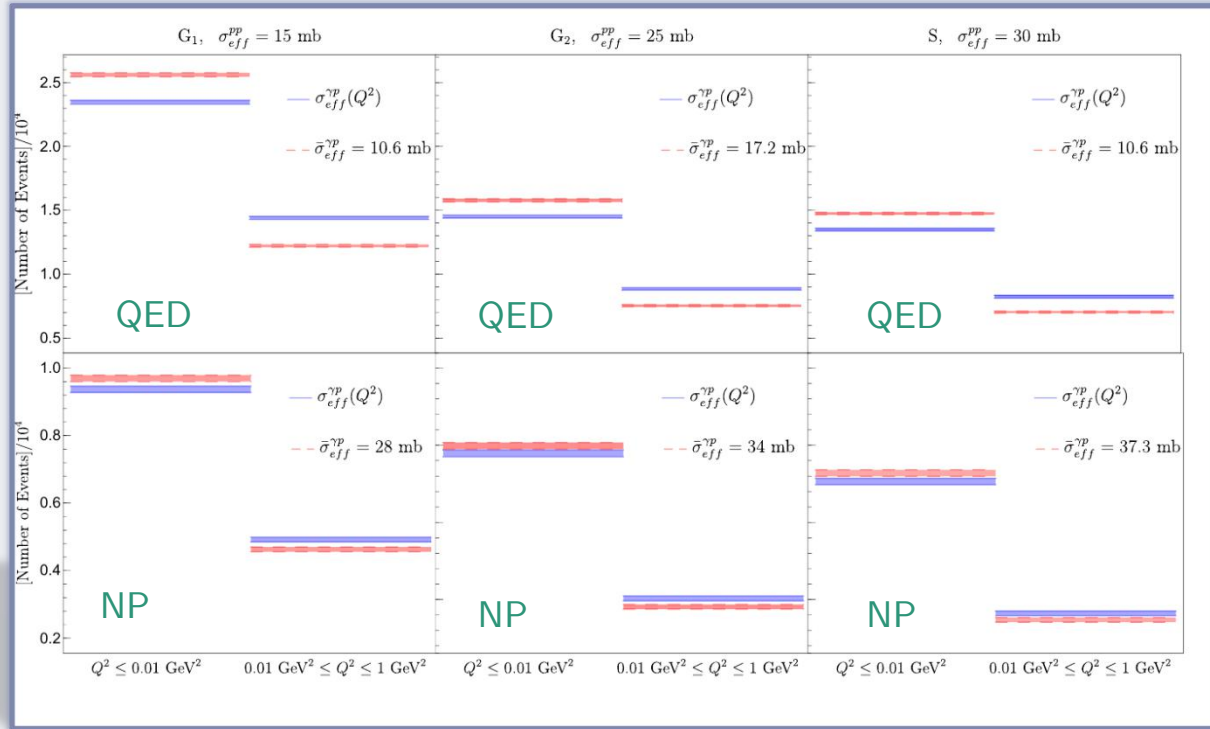
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3) We estimate the minimum luminosity to distinguish the two cases

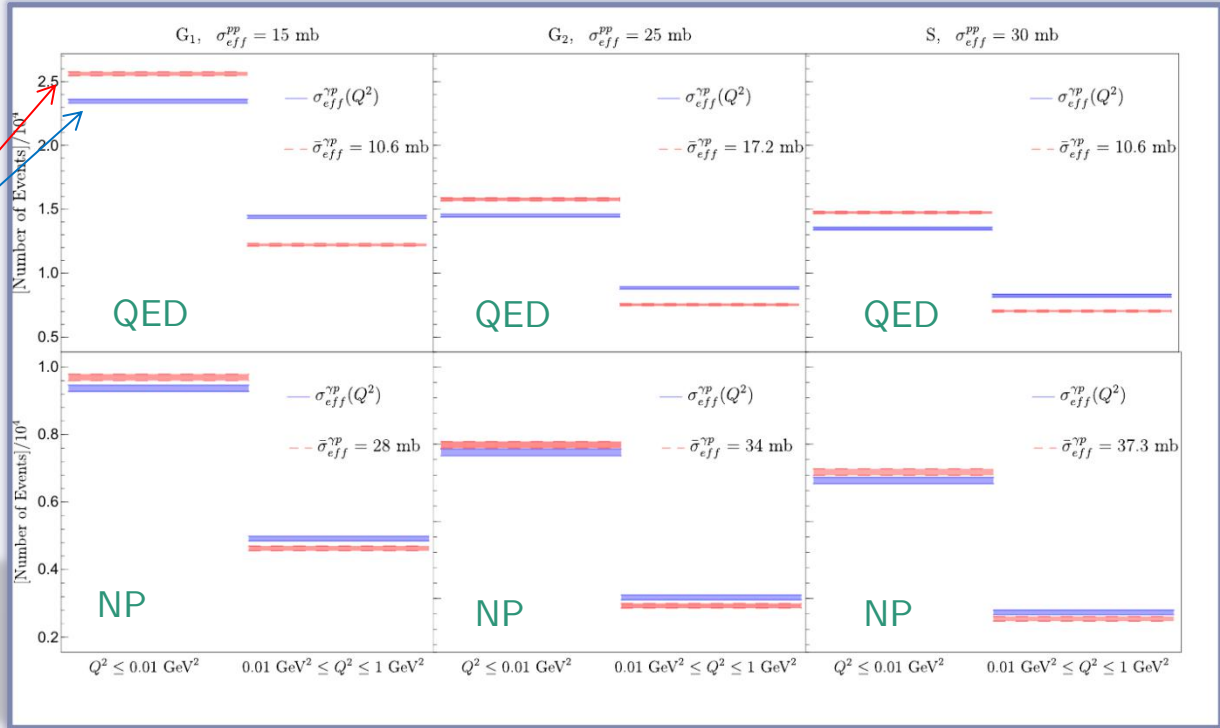
6 The effective cross section: a key for the proton structure

Proton →
h
o
t
o
n ↓



6 The effective cross section: a key for the proton structure

With an integrated luminosity of 200 pb^{-1} we can separate:



7 Double PDFs of pions and lattice data

The dPDF expression, at the hadronic scale, in term of the meson $q\bar{q}$ wave function:

$$f_2(x, k_\perp) = \frac{1}{2} \sum_{h,h'} \int \frac{d^2 k_{1\perp}}{2(2\pi)^3} \psi_{h,h'}(x, \vec{k}_{1\perp}) \underbrace{\psi_{h,h'}^*(x, \vec{k}_{1\perp} + \vec{k}_\perp)}_{\text{Meson-Wave function}}$$

Parton helicities

Intrinsic parton momentum

Meson-Wave function

M. R., S. Scopetta, M. Traini and V.Vento, EPJC 78, no. 9,782 (2018)

W. Broniovski et al, PRD 101 (2020) n°1. 014019

S. Scopetta et al, EPJC80 (2020) n°10, 909

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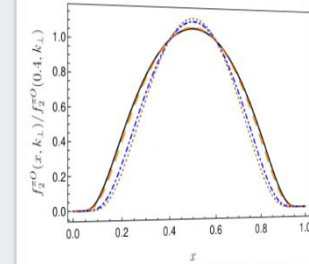
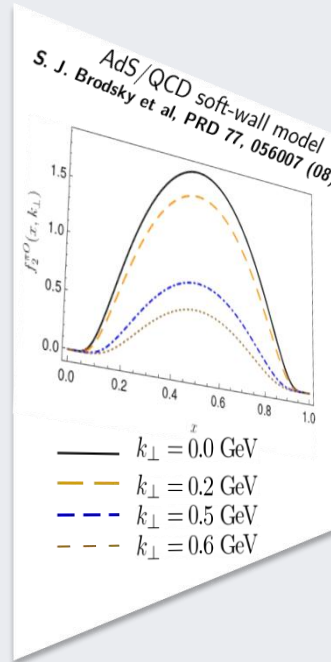
M. R., S. Scopetta, M. Traini and V.Vento, EPJC 78, no. 9,782 (2018)

1) Also for pion, model calculations indicate that factorization does not work!

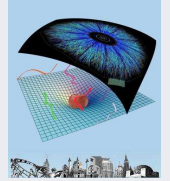
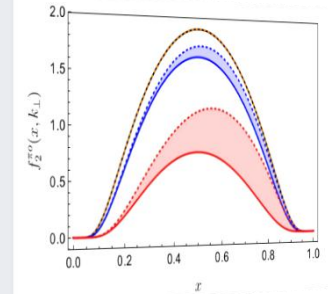
2) The approximation:

$$f_2(x, \Delta_\perp) \sim H(x, 0, \Delta_\perp) F_\pi(\Delta_\perp^2)$$

does not work



Sensitive to correlations

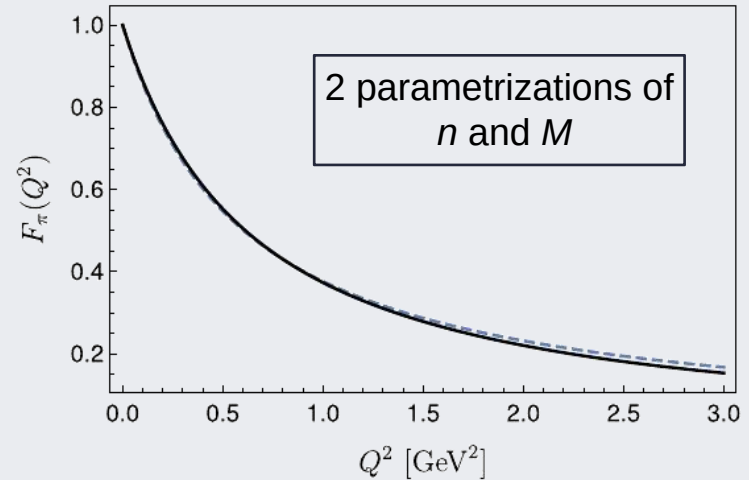


7 Double PDFs of pions and lattice data

In **G.S. Bali et al, JHEP 12, 061 (2018)** a first analysis of the moments of dPDFs within the lattice QCD have been discussed. In **M.R., EPJC 80 (2020) n°7, 678** lattice data have been compared with pion quark models.

$$F_{\pi}(Q^2) \sim \left[1 + \frac{Q^2}{M^2} \right]^{-n}$$

FIT OF LATTICE DATA FOR THE FORM FACTOR



7 Double PDFs of pions and lattice data

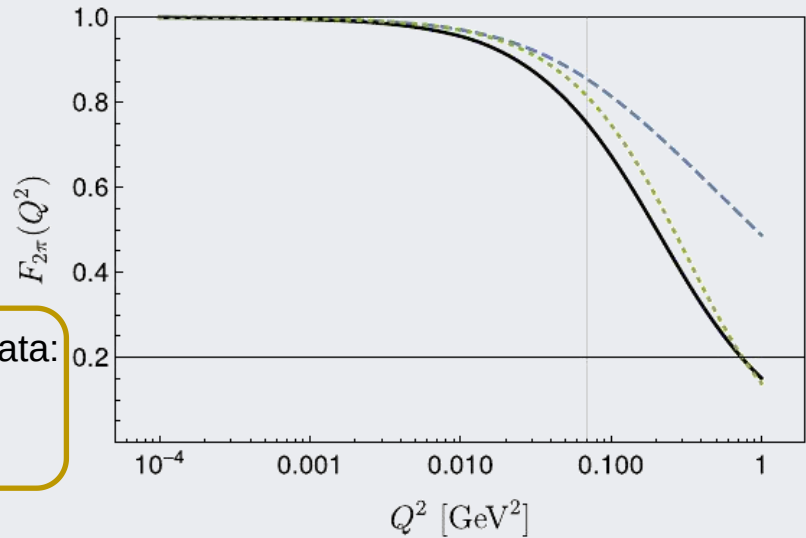
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THE EFFECTIVE FORM FACTOR:

1) Full line: $F_{2\pi}(Q^2) \sim \left[1 + \langle d^2 \rangle \frac{Q^2}{6n} \right]^{-n}$

mean transverse distance fitted from lattice data:

$$\sqrt{\langle d^2 \rangle} \sim 1.046 \text{ fm}$$



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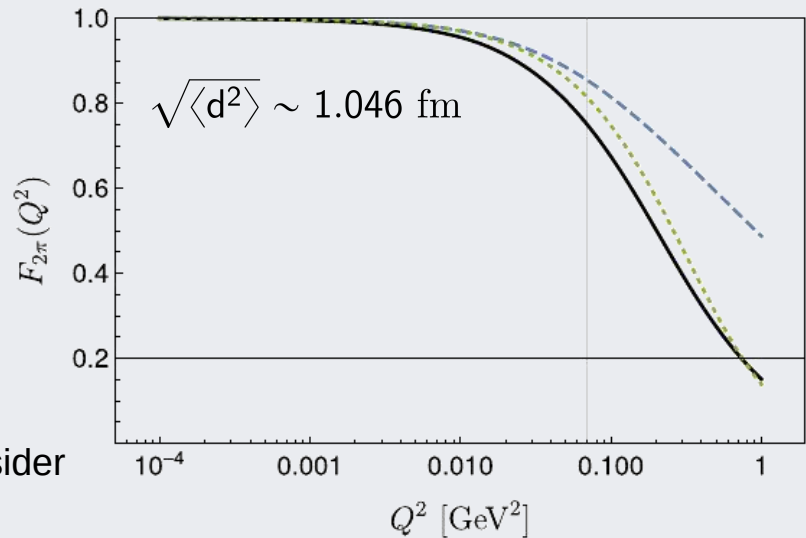
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However, results have been obtained in the pion rest frame! $\vec{p} = 0$

Therefore in order to compare these results with LF models (IMF) calculations we consider

$$Q^2 \ll m_\pi^2, \quad m_\pi \sim 0.3 \text{ GeV}$$



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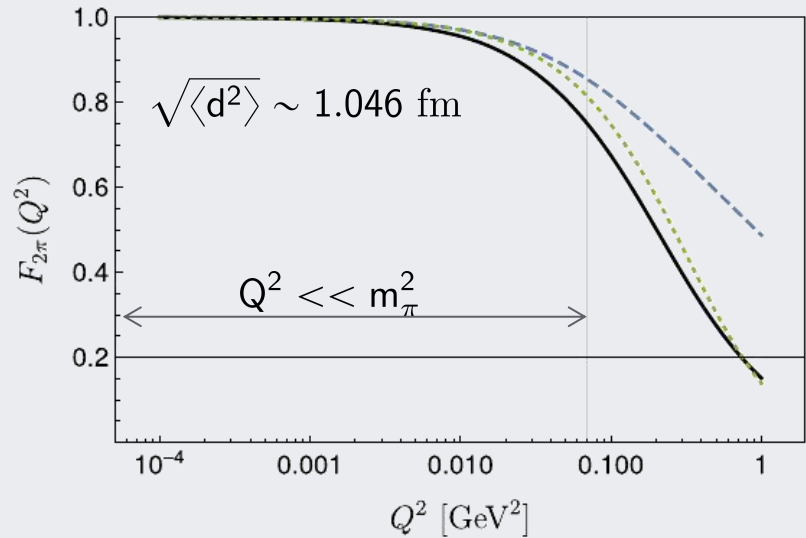
THE EFFECTIVE FORM FACTOR:

1) Full line: $F_{2\pi}(Q^2) \sim \left[1 + \langle d^2 \rangle \frac{Q^2}{6n} \right]^{-n}$

2) Dashed line:

$$F_{2\pi}(Q^2) \sim \frac{(m_\pi + E_q)^2}{4m_\pi E_q} F_\pi (2m_\pi E_q - 2m_\pi^2)^2$$

1-body approximation in the pion rest frame and $E_q = \sqrt{m_\pi^2 + Q^2}$



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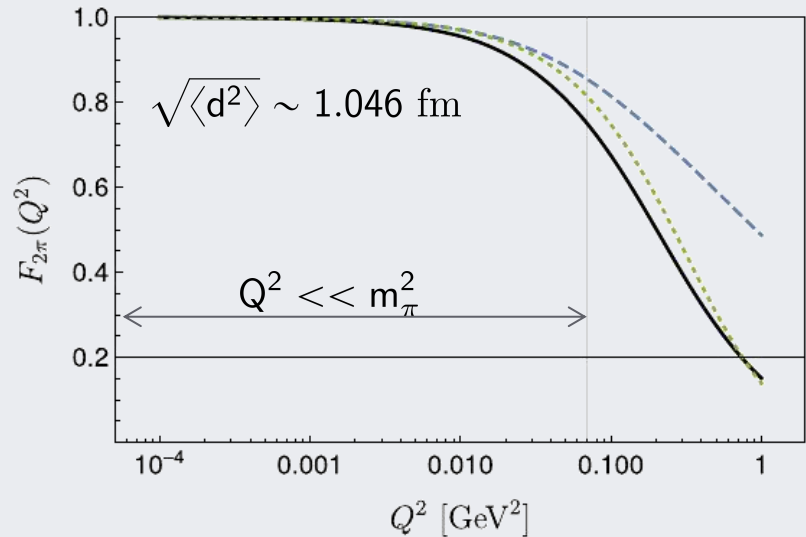
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3) Dotted line: $F_{2\pi}(Q^2) \sim F_\pi(Q^2)^2$
 1-body approximation in the IMF

R
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T



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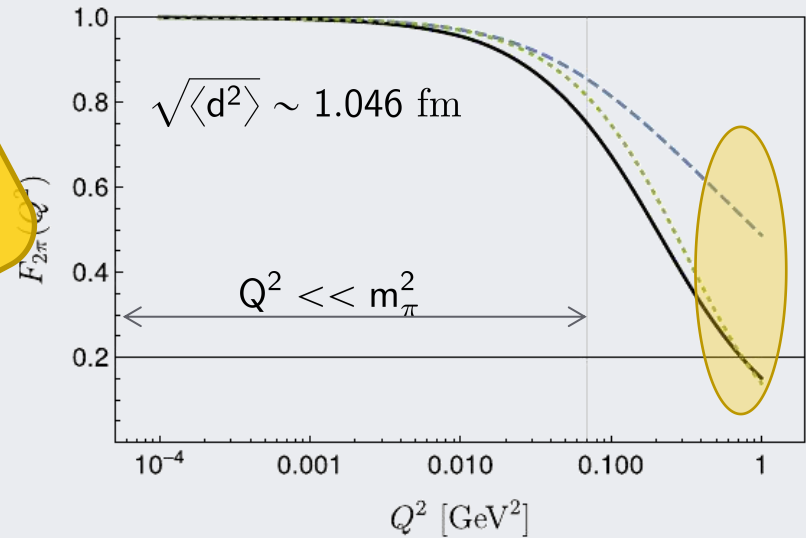
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3) Dotted line: $F_{2\pi}(Q^2) \sim F_\pi(Q^2)^2$

CORRELATIONS ARE IMPORTANT



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I considered HOLOGRAPHIC models:

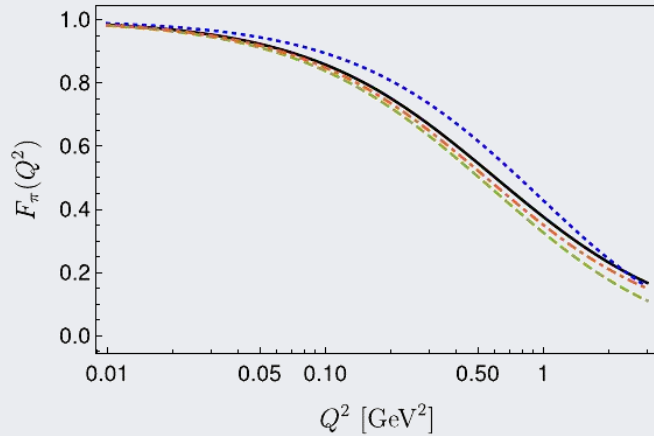
- 1) $\psi_0(x, \mathbf{k}_{\perp,1}) \propto \frac{4\pi}{\kappa_0 \sqrt{x(1-x)}} e^{-\frac{m_0^2 + \mathbf{k}_{\perp,1}^2}{x(1-x)2\kappa_0^2}}$
 - ORIGINAL MODEL
 - S.J. Brodsky, G.F. de Teramond, PRD 77, 056007 (2008)
 - $\kappa_0 = 0.548 \text{ GeV}; \quad m_0 = 0.33 \text{ GeV}$

- 2) $\psi_s(x, \mathbf{k}_{\perp,1}) = S(x, \mathbf{k}_{\perp,1})\psi_0(x, \mathbf{k}_{\perp,1})$
 - DYNAMICAL SPIN MODEL
 - M. Ahmady, F. Chishtie, R. Sandapen, PRD 95(7), 074008 (2017)

- 3) $\psi_U^\tau(x, \mathbf{k}_{\perp,1}) = 8\pi \frac{\sqrt{q_\tau(x)f(x)}}{1-x} e^{\frac{2f(x)}{(1-x)^2} \mathbf{k}_{\perp,1}^2}$
 - UNIVERSAL MODEL
 - G.F. de Teramond et al., PRL. 120(18), 182001 (2018)
 - $\tau = 2 \rightarrow |q\bar{q}\rangle \quad \tau = 4 \rightarrow |q\bar{q}q\bar{q}\rangle$

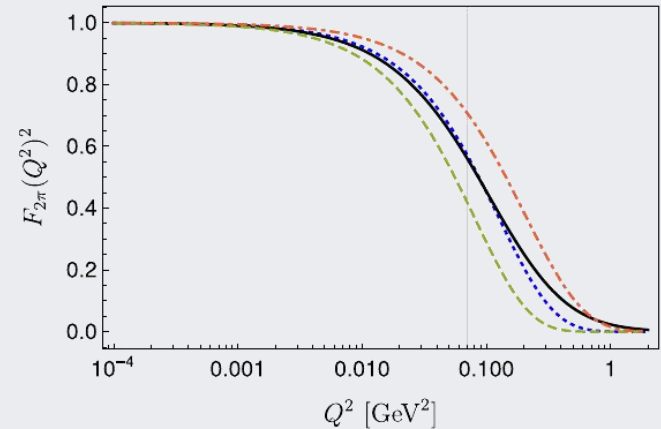
7 Double PDFs of pions and lattice data

THE FORM FACTOR:



- Lattice
- ⋯ Original
- - Spin
- · - Universal

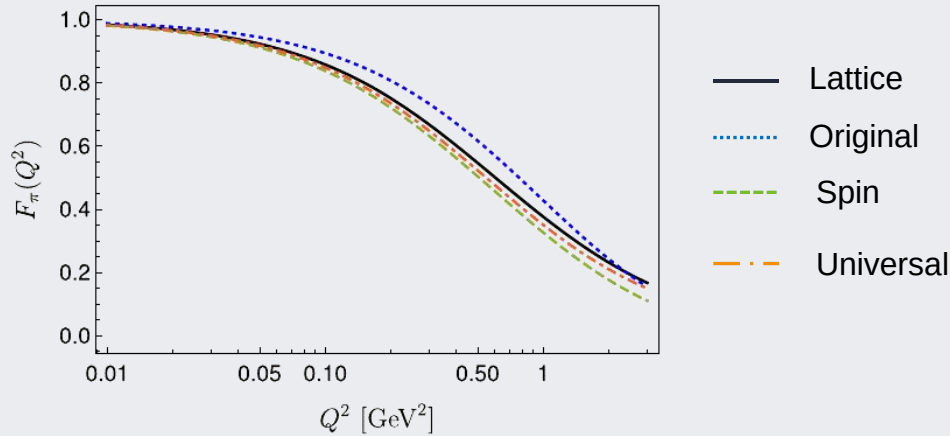
THE EFFECTIVE FORM FACTOR:



M.R., EPJC 80 (2020) n°7, 678

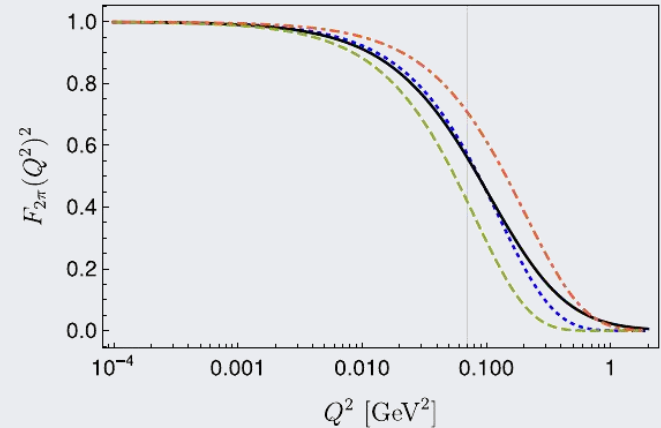
7 Double PDFs of pions and lattice data

THE FORM FACTOR:



Improved models are close to data

THE EFFECTIVE FORM FACTOR:

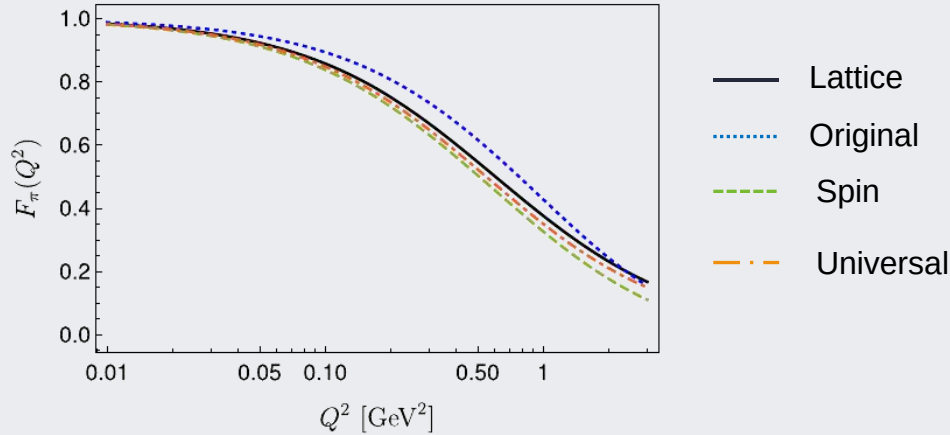


Original model is the close to data!

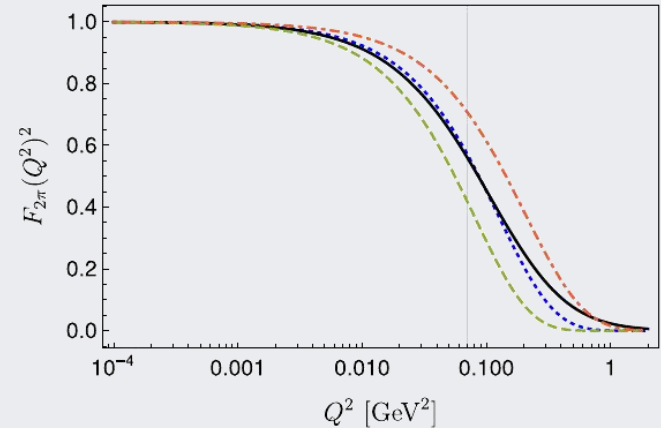
M.R., EPJC 80 (2020) n°7, 678

7 Double PDFs of pions and lattice data

THE FORM FACTOR:



THE EFFECTIVE FORM FACTOR:

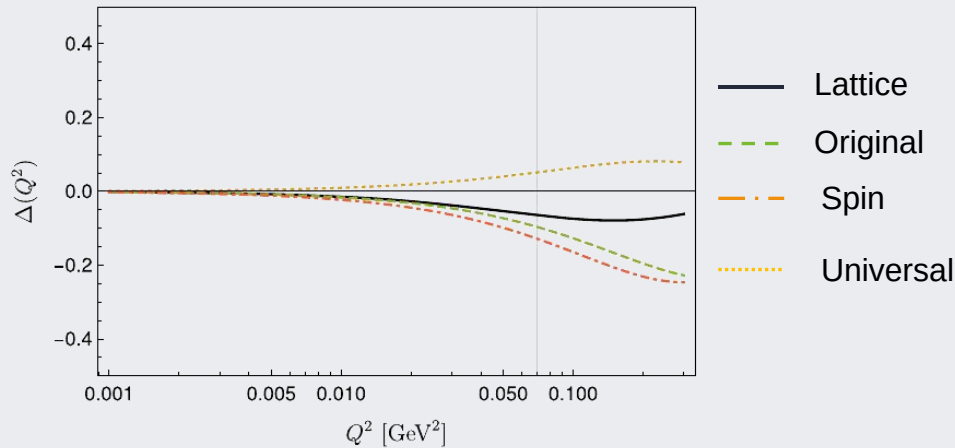


	Original model	Dynamical spin $A = B = 1$	Universal model	Lattice (A)
$\sqrt{\langle d^2 \rangle}$ [fm]	0.968	1.207	0.767	1.046 ± 0.049

M.R., EPJC 80 (2020) n°7, 678

7 Double PDFs of pions and lattice data

CORRELATIONS? $\Delta(Q^2) = F_{2\pi}(Q^2) - F_\pi(Q^2)^2$



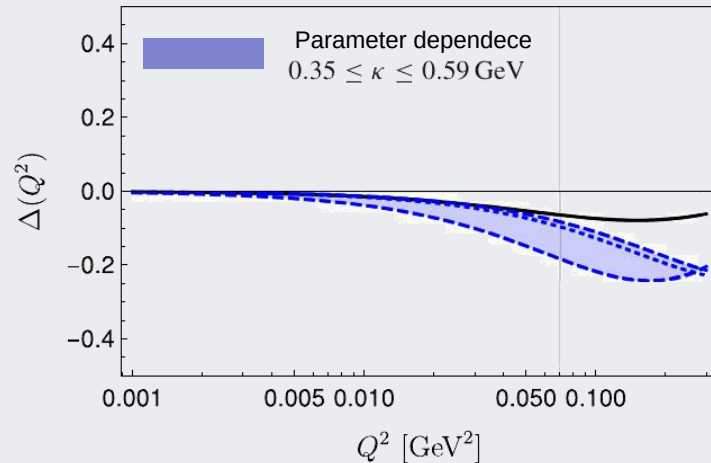
DOES THE DPS UNDERLYING PHYSICS
REQUEST SOME IMPROVEMENTS?

M.R., EPJC 80 (2020) n°7, 678

7 Double PDFs of pions and lattice data

$$\Delta(Q^2) = F_{2\pi}(Q^2) - F_{\pi}(Q^2)^2$$

ORIGINAL MODEL



DOES THE DPS UNDERLYING PHYSICS
REQUEST SOME IMPROVEMENTS?

YES!

If we change the parameter of the model we can get a better description of the FF but we lose the agreement with the EFF and viceversa. In fact the correlation distribution is not well reproduced.

7 Double PDFs of pions and lattice data

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SOLUTION:

add an explicit $|\bar{q}q \bar{q}q\rangle$ Fock state!

$$\psi(x_1, x_2, x_3, \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}, \mathbf{k}_{\perp 3})$$

M.R., EPJC 80 (2020) n°7, 678

7 Double PDFs of pions and lattice data

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The model of

G.F. de Teramond et al., PRL. 120(18), 182001 (2018)

already contains: $\tau = 4 \rightarrow |q\bar{q}q\bar{q}\rangle$
but only the contribution at the PDF and GPD
level has been investigated. The full dependence
should be studied \rightarrow This analysis could help in
improving the model

M.R., EPJC 80 (2020) n°7, 678

7 Double PDFs of pions and lattice data

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New information from:

G.S. Bali et al, JHEP 02 (2021) 067
"Double parton distributions in the pion from lattice QCD"

C. Zimmermann, "Double parton distributions in the nucleon on the lattice QCD" arXiv: 2106.03451

$$\psi(x_1, x_2, x_3, \mathbf{k}_{\perp 1}, \mathbf{k}_{\perp 2}, \mathbf{k}_{\perp 3})$$

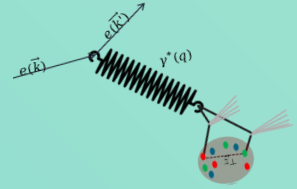
The model of

G.F. de Teramond et al., PRL. 120(18), 182001 (2018)

already contains: $\tau = 4 \rightarrow |q\bar{q}q\bar{q}\rangle$
but only the contribution at the PDF and GPD
level has been investigated. The full dependence
should be studied \rightarrow **This analysis could help in
improving the model**

M.R., EPJC 80 (2020) n°7, 678

CONCLUSIONS



- 1) We investigated the impact of correlations in DPS proton-proton collisions to learn something new on the parton structure of the proton
- 2) We demonstrated that in p-p collisions only some limited information on the proton can be obtained
- 3) We proposed to consider DPS initiated via photon-proton interactions by showing that:
 - * DPS can contribute also in this case. Cross section of the 4 jet photo production strongly affected
 - * The dependence of $\sigma_{\text{eff}}^{\gamma p}(Q^2)$ on the Q^2 can unveil the mean distance of partons in the proton
 - * We show that by increasing the luminosity such a dependence can be exposed in future facilities such as the **Electron Ion Collider**
 - * In the future could be interesting to study other processes with different final states such as those associated to the **QUARKONIUM PRODUCTION**