Vacuum and Zero Modes

The ultimate free lunch: the LF vacuum

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DLCQ

- periodic boundary conditions in $x^- = t - z$
- equal x^+ quantization conditions

early successes with DLCQ (1+1 dimensions)

- H. C. Pauli and S. J. Brodsky, "Discretized Light Cone Quantization: Solution to a Field Theory in One Space One Time Dimensions," Phys. Rev. D **32**, 2001 (1985).
- T. Eller, H. C. Pauli and S. J. Brodsky, "Discretized Light Cone Quantization: The Massless and the Massive Schwinger Model," Phys. Rev. D **35**, 1493 (1987).

$\hookrightarrow$ M. Burkardt, "The Virial Theorem and the Structure of the Deuteron in (1+1)-dimensional QCD on the Light Cone," Nucl. Phys. A **504**, 762 (1989).

early (high) optimism:

can we repeat this in 3+1 dimensions?

Discrete Light Cone Quantization (DLCQ)

DLCQ

- periodic boundary conditions in $x^- = t - z$
- equal x^+ quantization conditions

early (high) optimism:

can we repeat this in 3+1 dimensions?

some **supermassive** roadblocks

- renormalization
- gauge invariance
- vacuum (today's talk)
- these topics are likely interconnected...

Motivation: Zero Modes and the Vacuum

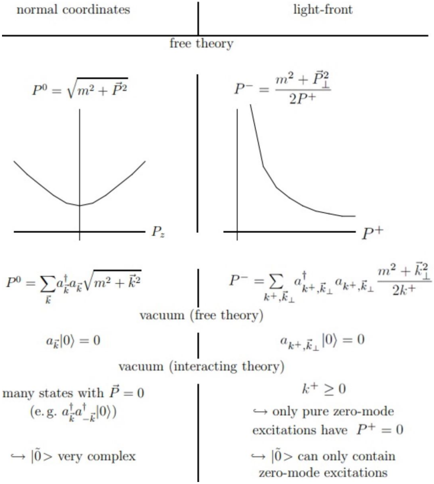
- P^+ conservation & P^+ purely kinematical

$\hookrightarrow$ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

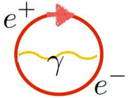
$\hookrightarrow$ also nondegenerate state of lowest P^-

$\hookrightarrow$ exact ground state of theory



Stan:

Instant-Form Vacuum in QED

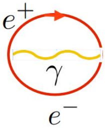


- Loop diagrams of all orders contribute
- Huge vacuum energy: $\rho_\Lambda^{QED} \simeq 10^{120} \rho_\Lambda^{Observed}$
- $\frac{E}{V} = \int \frac{d^3k}{2(2\pi)^3} \sqrt{\vec{k}^2 + m^2}$ Cut off the quadratic divergence at M_{Planck}
- Frame-dependent, acausal
- Divide S-matrix by disconnected vacuum diagrams
- In Contrast: Light-Front Vacuum trivial since plus momenta are positive and conserved: $k^+ = k^0 + k^3 > 0$

Stan:

Front-Form Vacuum in QED

$$P^+ = 0$$



$$\sum_i k_i^+ \neq P^+ = 0$$

$$k_i^+ > 0$$

- **All Light-Front Perturbative Vacuum Loop Amplitudes Vanish!**
- **Light-Front Vacuum is trivial since all plus momenta are positive and conserved.**
- **Zero modes (k^+=0) in vacuum allowed in some theories**
- **Zero contribution to Λ from QED LF Vacuum**
- **Instant Form gives same result if one normal-orders.**

Stan:

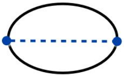
Front-Form Vacuum (P^μ=0)

All LF propagators have positive k^+

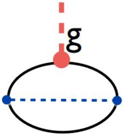
$$k^+ = k^0 + k^3 \geq 0 \text{ since } |\vec{k}| \leq k^0$$

P^+ Momentum Conserved

$P^+ = 0$



zero !!



zero !!

$$< 0|T^{\mu\nu}|0 >= 0$$

Graviton does not couple to LF vacuum!

- P^+ conservation & P^+ purely kinematical

↪ 'empty' or 'trivial' vacuum exact eigenstate of LF Hamiltonian

- nondegenerate state of lowest P^+

↪ also nondegenerate state of lowest P^-

↪ exact ground state of theory

issues with this result

- Higgs mechanism
- QCD vacuum:
  - lattice: $\langle 0|\bar{q}q|0\rangle \neq 0$
  - Gell-Mann, Oakes, Renner $f_\pi^2 m_\pi^2 = (m_u + m_d)\langle 0|\bar{q}q|0\rangle \neq 0$

possible resolutions

- $\langle 0|\bar{q}q|0\rangle \neq 0$ fake news!

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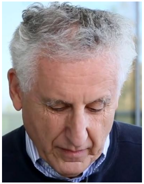

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- or: LF formalism is fake!

↪ Dirac (& 'do-nothing coneheads') made it up!

↪ Sorry Stan...

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- maybe there is a 3^{rd} option ...

SSB in $1+1$ dimensions?

- no spontaneous symmetry breaking (SSB) in 1+1 (S.Coleman)
- however not valid for $N_C \longrightarrow \infty$ as Hartree-Fock approx. becomes exact

$\hookrightarrow$ SSB possible

't Hooft model

- $QCD_{1+1}(N_C \longrightarrow \infty)$
- LF quantization & gauge

$$M_n^2 \phi_n(x) = \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \phi_n(x) + \frac{g^2 C_F}{\pi} \int_0^1 dy \frac{\phi_n(x) - \phi_n(y)}{(x-y)^2}$$

- M^2 meson mass; x $(1-x)$ momentum fraction carried by q $(\bar{q})$
- trivial vacuum, lowest Fock sector for meson exact as $N_C \rightarrow \infty$
- infinite 'tower' of solutions
- lowest meson state $M_\pi^2 \propto m_q$

$\hookrightarrow$ hint that $\langle 0|\bar{q}q|0\rangle \neq 0$

- meson spectrum confirmed by Li, Wilets, Birse in ET/BS (1986)

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Zhitnitzky PLB 165B (1985) 405, Sov.JNP 43, 999; 44, 139 (1984)

- GMOR: $\lim_{m_q \to 0}\langle 0|\bar{q}q|0\rangle = -\frac{N_C}{\sqrt{12}}\sqrt{\frac{g^2 C_F}{\pi}}$
- confirmed by ET calculation: M. Li, PRD34 (1986) 3888
- nonperturbative analytic expression for $\langle 0|\bar{q}q|0\rangle$ valid for all m_q: MB&N.Uraltsev, PRD 63 (2001) 014004

free lunch?

- Solving LF wave functions from diagonalizing LF Hamiltonian based on trivial vacuum yields same rusults (incl. condensate numbers - using GMOR) as complicated ET calculation!!!!!

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free lunch?

- Solving LF wave functions from diagonalizing LF Hamiltonian based on trivial vacuum yields same rusults (incl. condensate numbers - using GMOR) as complicated ET calculation!!!!!
- Does that mean the vacuum is trivial or that it is not trivial?!?

explicit LF calculations MB, F.Lenz, M.Thies, PRD 65 (2002) 125002

- vacuum condensate $\langle 0|\bar{q}(0)q(0)|0\rangle$ ill-defined (UV-divergent)
- employ point-splitting in LF time x^+, i.e.
  $\langle 0|\bar{q}(0)q(0)|0\rangle \longrightarrow \langle 0|\bar{q}(0)Wq(\varepsilon)|0\rangle$ with $\varepsilon^2 \neq 0 \quad \Rightarrow \quad \varepsilon^+ \neq 0$
- W Wilson line gauge link
- same as heavy-light correlator: for straight Wilson line, W represents a 'static' heavy quark

↪ relate $\langle 0|\bar{q}(\varepsilon)Wq(0)|0\rangle$ to properties of heavy-light mesons (calculated using LF quantization: masses, decay constants)

- reproduced $\langle 0|\bar{q}(0)q(0)|0\rangle$ from GMOR (Zhitnitsky)
- take $\varepsilon^\pm \to 0$ (subtract free-field divergence)
- condensate only from zero-modes $k^+ \longrightarrow 0$

implications for LF vacuum (QCD_{1+1} only)

- condensates (properly regularized) nonzero
- don't affect hadron structure/dynamics in QCD_{1+1}

↪ fine to pretend that vacuum is trivial (QCD_{1+1})

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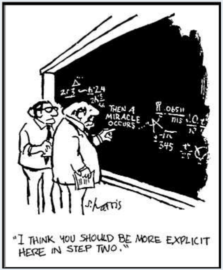
physics behind this fairy tale:

- zero-modes high-energy degrees of freedom

$\hookrightarrow$ parton degrees not enough energy to excite zero-mode sector

$\hookrightarrow$ LF Hamiltonian works like effective Hamiltonian



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- Is it fine to pretend that vacuum is trivial in more complicated theories, such as ϕ^n, QCD_{3+1}, ...?

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- unfortunately not!
- can this be 'fixed'?
- maybe!
- can it be fixed by introducing a single zero mode?
- no! Need ∞ many modes in infinitesimal vicinity of $k^+ = 0$
- single zero mode (as in DLCQ) may be able to explain (static) Higgs effect but not dynamical chiral symmetry breaking!

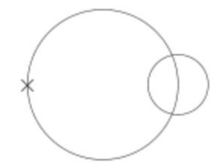
$\langle 0|\phi^2|0\rangle$

- LF: no particles popping out of vacuum (→SJB)

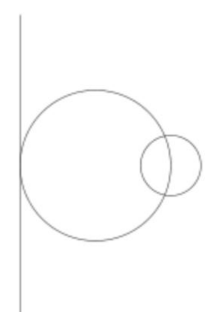
↪ LF: no contribution to $\langle 0|\phi^2|0\rangle$ beyond 1 loop

- cov. calc.: contribution to $\langle 0|\phi^2|0\rangle$ to all orders!
- discrepancy!
- relevant since corresponding tadpoles contribute to self-energy!

example for diagram that contributes to $\langle 0|\phi^2|0\rangle$, but cannot be generated by LF Hamiltonian



example for contri to self-energy, that cannot be generated by H_{LF}



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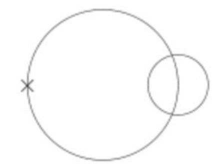


$\int dk^- \frac{\Pi(k^2)}{(k^2-m^2+i\varepsilon)^n}$

- issue arises for all integrals of above type!
- $\Pi(k^2)$ same pole structure as $\frac{1}{k^2-m^2+i\varepsilon}$

↪ $\delta(k^+)$ S.-J. Chang & S.-K. Ma, PR 180 (1969) 1506; T.-M. Yan,PRD 7 (1973) 1780

key integral (see e.g. Peskin & Schröder)

$$I_n \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - \mathcal{M}^2 + i\varepsilon\right)^n} = \frac{c_n}{\left(\mathcal{M}^2\right)^{n-2}} \neq 0$$

- $\int dk^- \frac{1}{\left(2k^+k^- - k_\perp^2 - \mathcal{M}^2 + i\varepsilon\right)^n} = 0$ for $k^+ \neq 0$

$\hookrightarrow$ $\int dk^- \frac{1}{\left(2k^+k^- - k_\perp^2 - \mathcal{M}^2 + i\varepsilon\right)^n} \sim \delta(k^+) \frac{1}{\left(\mathcal{M}^2\right)^{n-1}}$

- pure zero-mode contribution!
- when 'going slightly away' from LF, zero-mode contribution arises from ∞ number of modes in vicinity of $k^+ = 0$

bad news

- LF calc. misses whole class of diagrams: generalized tadpoles
- improper treatment of zero modes

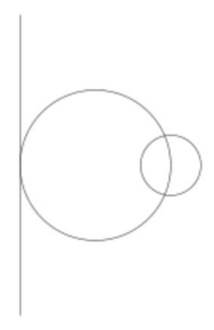
good news MB, PRD (1993)

- all of the missed diagrams only contribute constants

$\hookrightarrow$ can be taken care of by renormalization

$\hookrightarrow$ $m_{eff}^2 = m^2 + \lambda\langle 0|\phi^2|0\rangle$

example for contri to self-energy, that cannot be generated by H_{LF}



determining m_{eff}^2 nonperturbatively

- only match physical quantities during renorm.
- determine $\lambda\langle 0|\phi^2|0\rangle$ by point-splitting in LF time & inserting complete set of states (MB, S.Chabysheva, J.Hiller, PRD (2016))

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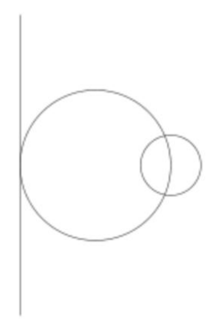


effective LF Hamiltonian P^-_{eff}

- zero-modes high-energy (k^-) degrees of freedom

$\hookrightarrow$ plausible that 'integrating out' zero modes leads to $P^- \longrightarrow P^-_{eff}$

- by construction, P^-_{eff} containe no zero-mode degrees of freedom!

Structure of Light Front Vacuum Sector Diagrams

- 'The concerns raised in this paper thus carry over to dressed light front vacuum graphs as well and cannot be ignored.'
- 'Since in analog to $\langle\Omega|\phi^2|\Omega\rangle$ the light front circle at infinity contribution to $\langle\Omega|\bar{\psi}\psi|\Omega\rangle$ is nonzero, in the light front the circle at infinity contributes to the cosmological constant.'
- 'It is this circle at infinity contribution that is then paramount in the light front vacuum sector, to thus make the off-shell Feynman diagram approach with its non-zero value for light front vacuum graphs the correct one.'

no tadpoles!?

- naively tadpole issue absent
- k^- from Dirac numerators can cancel one propagator:
  $k^- = p^- - \frac{(p_\perp - k_\perp)^2 + \lambda^2}{2(p^+ - k^+)} - \frac{(p-k)^2 - \lambda^2}{2(p^+ - k^+)}$
  - cancels one denominator
  - 'canonical term' (incl. instantaneous)

↪ self-energies contain pieces with same pole structure as generalized tadpoles

↪ condensates matter!

- renormalization can fix it...! (e.g. vertex mass $\neq$ kin. mass)

self-energies

$$\Sigma \sim \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} + m}{k^2 - m^2 + i\varepsilon} \frac{1}{(p-k)^2 - \lambda^2 + i\varepsilon}$$

vertices

$$\Sigma \sim \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} - \frac{\not{\Delta}}{2} + m}{\left(k - \frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \Gamma \frac{\not{k} + \frac{\not{\Delta}}{2} + m}{\left(k + \frac{\Delta}{2}\right)^2 - m^2 + i\varepsilon} \frac{1}{(p-k)^2 - \lambda^2 + i\varepsilon}$$

renormalization

zero-modes essential for renormalization (rotational invariance)

MB & A. Langnau, PRD 44 (1991) 3857; A.Langnau & MB, PRD 47 (1993) 3452

higher twist sum-rules

- $\delta(x)$ contributions to twist-3 PDFs

↪ not probed in DIS

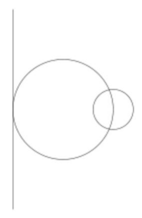
↪ apparent 'violations' of twist-3 sum rules & Lorentz invariance relations (σ-term sum rule, Burkhardt-Cottingham sum rule, ...)

F. Aslan & MB, "Singularities in Twist-3 Quark Distributions," PRD **101** (2020) 016010

$J = 0$ fixed poles

- diagrams that result in $\delta(x)$ contributions to PDFs also result in ν-independent contributions to Compton amplitude

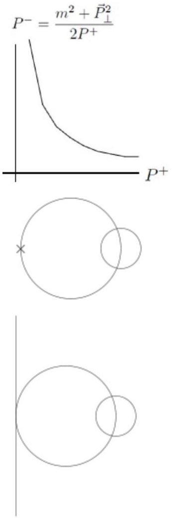
↪ $J = 0$ fixed poles S. J. Brodsky, F. E. Close and J. F. Gunion, "Compton Scattering And Fixed Poles In Parton Field Theoretic Models," Phys. Rev. D **5**, 1384 (1972).



Summary

- naively LF vacuum trivial
- apparent contradiction with pheno & lattice
- regularization (point splitting in ε^+) yields nonzero condensates
- consistent with covariant in QCD_{1+1} & ϕ^n
- $P^- \longrightarrow P^-_{eff}$ embodies effect of zero modes on non-zero modes

$\hookrightarrow$ 'vacuum condensate' contributions essential for equivalence of LF with ET field theory



- can a P^-_{eff} formalism work in gauge theories, i.e. can one decouple $x = 0$ from small x?
- can one relate vacuum condensates to in-hadron condensates?
- how do zero modes affect hadron masses?
- how can one calcualate such effects non-perturbatively?
- how are zero-modes related to the color glass condensate?
- how do zero-modes affect dispersion relations?
- what really is the connection to $J = 0$ fixed poles? Zero modes?
- what is the light-cone interpretation of subtractions in dispersion relations? Is that also a zero-mode thing?
- this question includes the interpretation of the so-called 'D-term' which arises as a subtraction in dispersion relations for twist-2 GPDs (related to pressure distribution)
- how does the structure of the vacuum affect nucleon structure, such as twist-3 PDFs and GPDs?

J.Collins, LC workshop 2018

- considered $\int d^2x \langle 0|\phi^2(0)\phi^2(x)|0\rangle e^{iqx}$
- for $q^+ = 0$ same pole structure as generalized tadpoles
- naively vanishes for $q^+ = 0$

$\hookrightarrow$ regulated by taking $q^+ \neq 0$

- support only for $0 < k^+ < q^+$ with k^+, $q^+ - k^+$ momentum of one of the particles created by $\phi^2|0\rangle$
- $\lim_{q^+\to 0}$ yields finite result
- in terms of k^+, rep. of $\delta(k^+)$

connection of singularities in twist-3 GPDs/PDFs

- pole structure similar to above vacuum corellator
- in GPDs $q^+ \neq 0$, 'regulates' $\delta(x)$ present in PDFs
- rep. of $\delta(x)$ as $q^+ \to 0$