Vacuum and Zero Modes

Much ado about nothing: the LF vacuum

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Example for Zero-Mode Integral

- $\int \frac{d^2k}{(2\pi)^2} \frac{1}{(k^2-m^2+i\varepsilon)^2} = \frac{i}{4\pi m^2}$ (relevant for self-energy integrals)
- light-cone: $\int \frac{dk^+dk^-}{(2\pi)^2} \frac{1}{(2k^+k^- - m^2+i\varepsilon)^2}$

$\hookrightarrow$ pure zero-mode, i.e. (pole structure!) $k^+ = 0$ contribution, i.e.
$\int \frac{dk^-}{(2\pi)^2} \frac{1}{(2k^+k^- - m^2+i\varepsilon)^2} \propto \delta(k^+)$

- should we care?
- yes: higher-order diagrams contain terms with this pole structure
- how can we regularize this?
- point-splitting
- ε-coordinates

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point splitting (MB, Lenz, Thies; Brodsky, Lowdon, Mannheim,..

- $\int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2-m^2+i\varepsilon)^2} \longrightarrow \int \frac{d^2k}{(2\pi)^2}\frac{e^{i(x^+k^-+x^-k^+)}}{(k^2-m^2+i\varepsilon)^2}$
- evaluate integral for nonzero x^+

$\hookrightarrow$ take $x^+ \to 0$ limit after doing the integral

point splitting: results

- above procedure yields results that agree with equal time quantization - also in higher orders
- nonperturbative generalization (MB, S.Chabysheva, J.Hiller): evaluate $\langle 0|\phi(0)\phi(x)|0\rangle$ by inserting complete set of intermediate states
- application: constructing 'effective' Hamiltonians

introducing a momentum flow (Collins)

- $\int \frac{d^2k}{(2\pi)^2}\frac{1}{(k^2-m^2+i\varepsilon)^2} \longrightarrow \int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2-m^2+i\varepsilon}\frac{1}{(q-k)^2-m^2+i\varepsilon}$

$\hookrightarrow$ and take $q \to 0$ limit after doing the integral

- works to get correlation functions right but not clear how to implement this into a nonperturbative framework

ε coordinates

- key idea: go away from light front (just a little bit)

↪ quantize on a space-like hypersurface (F.Lenz; K.Hornbostel)

  - work in ∞ momentum frame, or
  - modify metric, e.g. $2k^+k^- \longrightarrow 2k^+k^- + \frac{2\varepsilon}{L}k^{-2}$

  ↪ on finite interval,

$$\int \frac{d^2k}{(2\pi)^2} \frac{1}{\left(k^2-m^2+i\varepsilon\right)^2} \to \frac{1}{L}\sum_{k^+}\int \frac{dk^-}{2\pi} \frac{1}{\left(2k^+k^- + \frac{2\varepsilon}{L}k^{-2} - m^2 + i\varepsilon\right)^2} =$$

$$\frac{i}{4\sqrt{2\varepsilon L}} \sum_{n=-\infty}^{\infty} \left[\frac{(2\pi n)^2}{2\varepsilon L} + m^2\right]^{-3/2}$$

- 'correct' answer only obtained in limit $L \to \infty$ first and then $\frac{\varepsilon}{L} \to 0$