The QCD running coupling

A. Deur

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Work done in collaboration with:

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- V. Burkert, J-P Chen and W. Korsch (experimental).

The strong coupling α_s

α_s is the most important quantity of QCD.

Well understood at high energy where it is small: $\alpha_s \sim 0.1 \Rightarrow$ pQCD.

Very active research to understand α_s at low energy where it is large: $\alpha_s \sim 1$.

Outline:

- What is a coupling constant?
- Why is it not constant? (Effective couplings in perturbative QFT.)
- α_s in pQCD.
- Extension of effective coupling to non-perturbative QCD.
- Experimental determination of the non-perturbative coupling.
- LFHQCD calculation. Comparison with data and other predictions.
- Application: determination of the hadron spectrum using α_s and LFHQCD.

Coupling constants

When charges are quantized: (coupling constant)$^{1/2}$ normalizes the fundamental charge to 1 (e.g. $\alpha_s=g^2/4\pi$; $\alpha=e^2/4\pi$).
⇒ set the magnitude of the force (classical description) or the probability amplitude to emit a quantum force vector (quantum field description).

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(2-body static case)

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α (QED), α_s (QCD), G_F (Weak Force), G_N (gravity).

Tells about the magnitude of the force.

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Amount of matter. e.g. color charges (QCD), electric charge (QED), …

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$$\text{Force} = \text{coupling constant} \times \text{charge}_1 \times \text{charge}_2 \times f(r)$$

(2-body static case)

$f(r) = \frac{e^{-mr}}{r^2}(1-mr)$ for linear theories.

m is the mass of the force carrier. m=0 for QCD, QED and gravity, m~85 GeV for Weak Force.

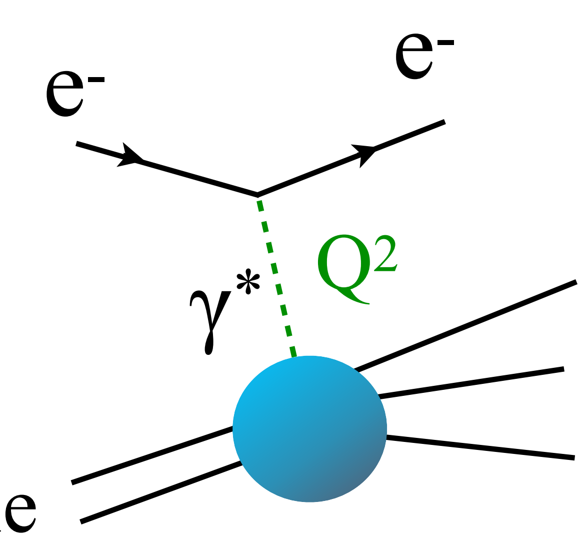
Effective couplings

Force=coupling constant×charge$_1$×charge$_2$×f(r) ← $\frac{1}{r^2}$

Classically (Faraday): $1/r^2$: dilution of the force flux as it spreads isotropically through space.

QFT: manifestation in the coordinate space of the propagator of the force carrier.

Ex: Electron scattering:

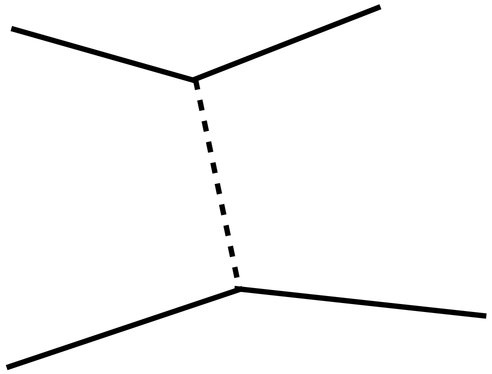


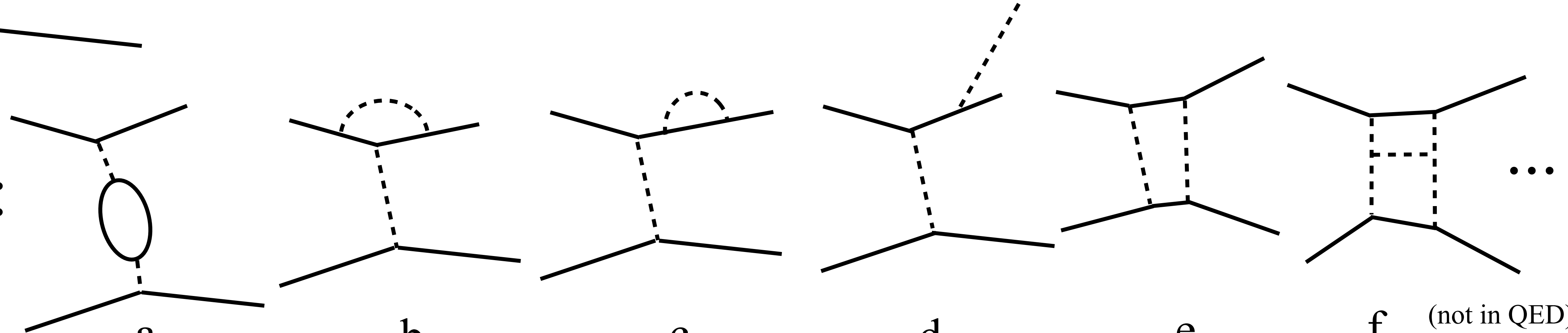
Target particle

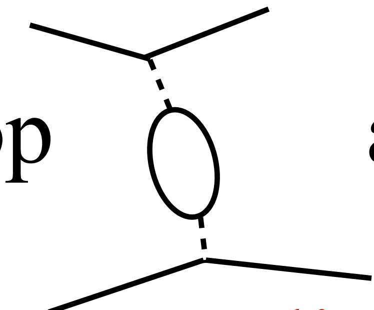
Virtual photon γ^* four-momentum squared: $-Q^2$.

In momentum space, scattering amplitude ∝ propagator ∝ $1/Q^2$.
⇒ Potential in coordinate space ∝ FT(amplitude) ∝ $1/r$.
⇒ Force ∝ $1/r^2$.

Effective couplings

But  is a first order approximation.

Higher orders: 

The loop  affects the propagator.

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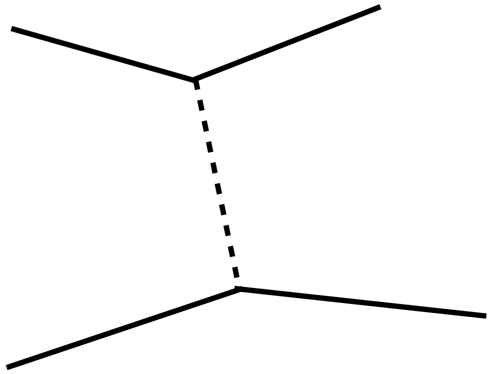
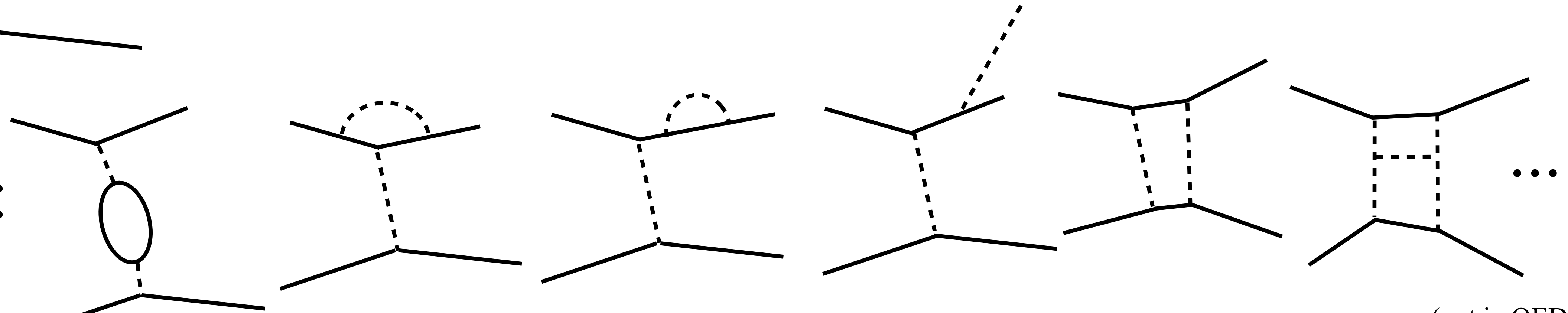
But  is a first order approximation.

Higher orders: 

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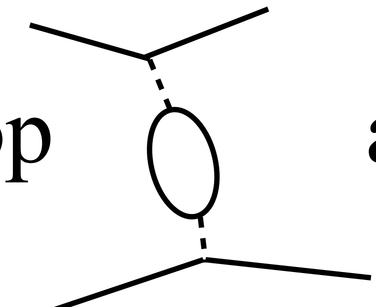
The loop  affects the propagator.

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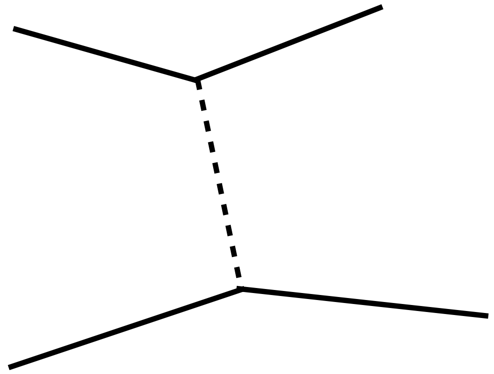
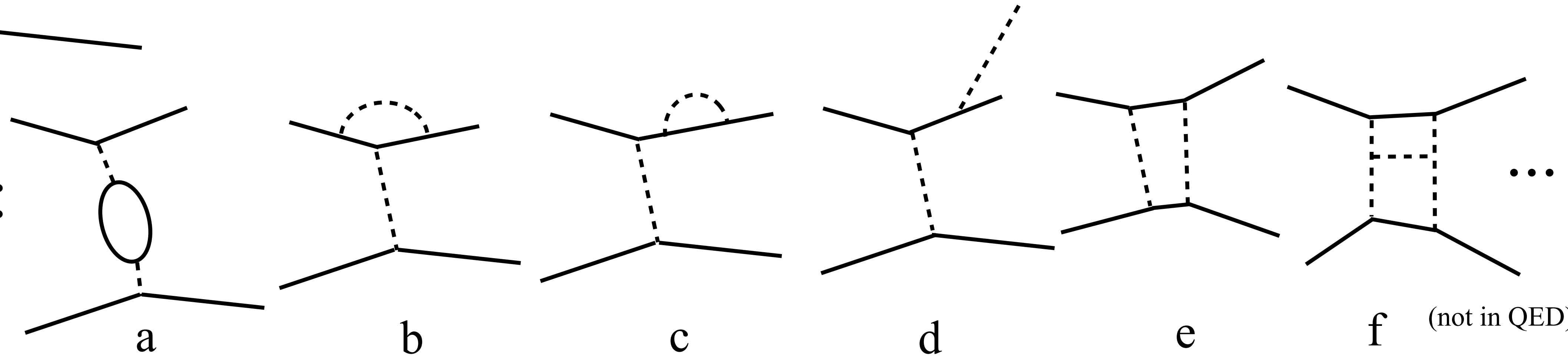
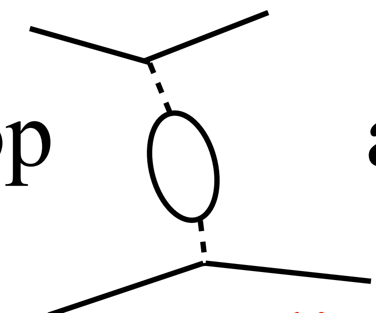
But  is a first order approximation.

Higher orders: 

The loop  affects the propagator.

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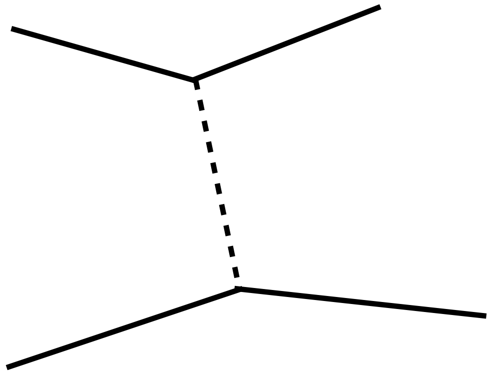
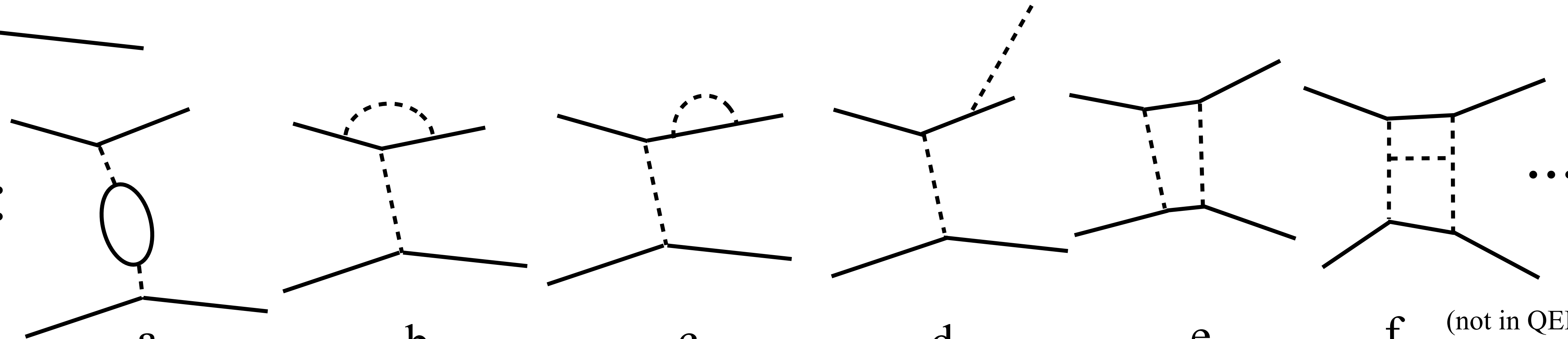
But  is a first order approximation.

Higher orders: 

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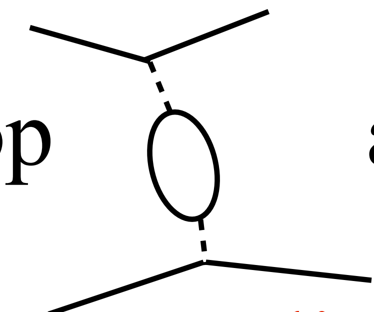
The loop  affects the propagator.

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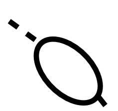
Loops such as  lead to infinite probability amplitudes. Theories need to be regularized and renormalized. (In fact, introduction of effective couplings is central to the renormalization procedure. This is how $\alpha(Q^2)$ and $\alpha_s(Q^2)$ are formally defined.)

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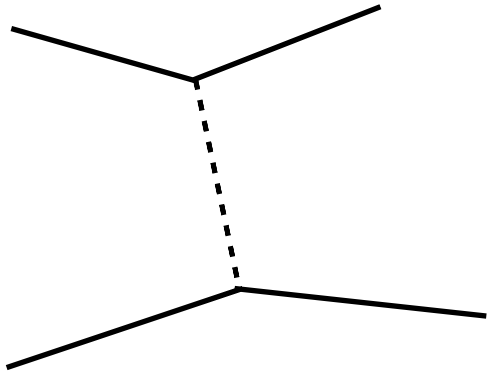
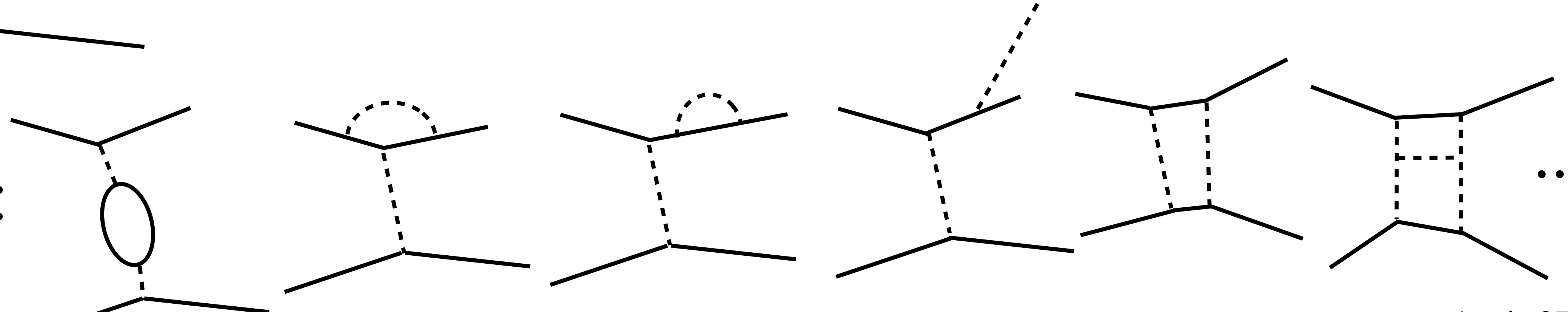
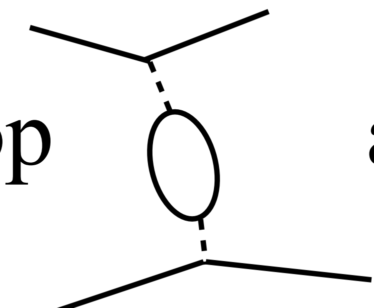
But  is a first order approximation.

Higher orders: 

The loop  affects the propagator.

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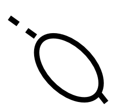
Loops such as  lead to infinite probability amplitudes. Theories need to be regularized and renormalized. (In fact, introduction of effective couplings is central to the renormalization procedure. This is how $\alpha(Q^2)$ and $\alpha_s(Q^2)$ are formally defined.)
⇒ Coupling depends on method: renormalization scheme dependence. They can also become gauge dependent: the coupling is not an observable anymore.

The strong coupling α_s at short distances (large Q^2)

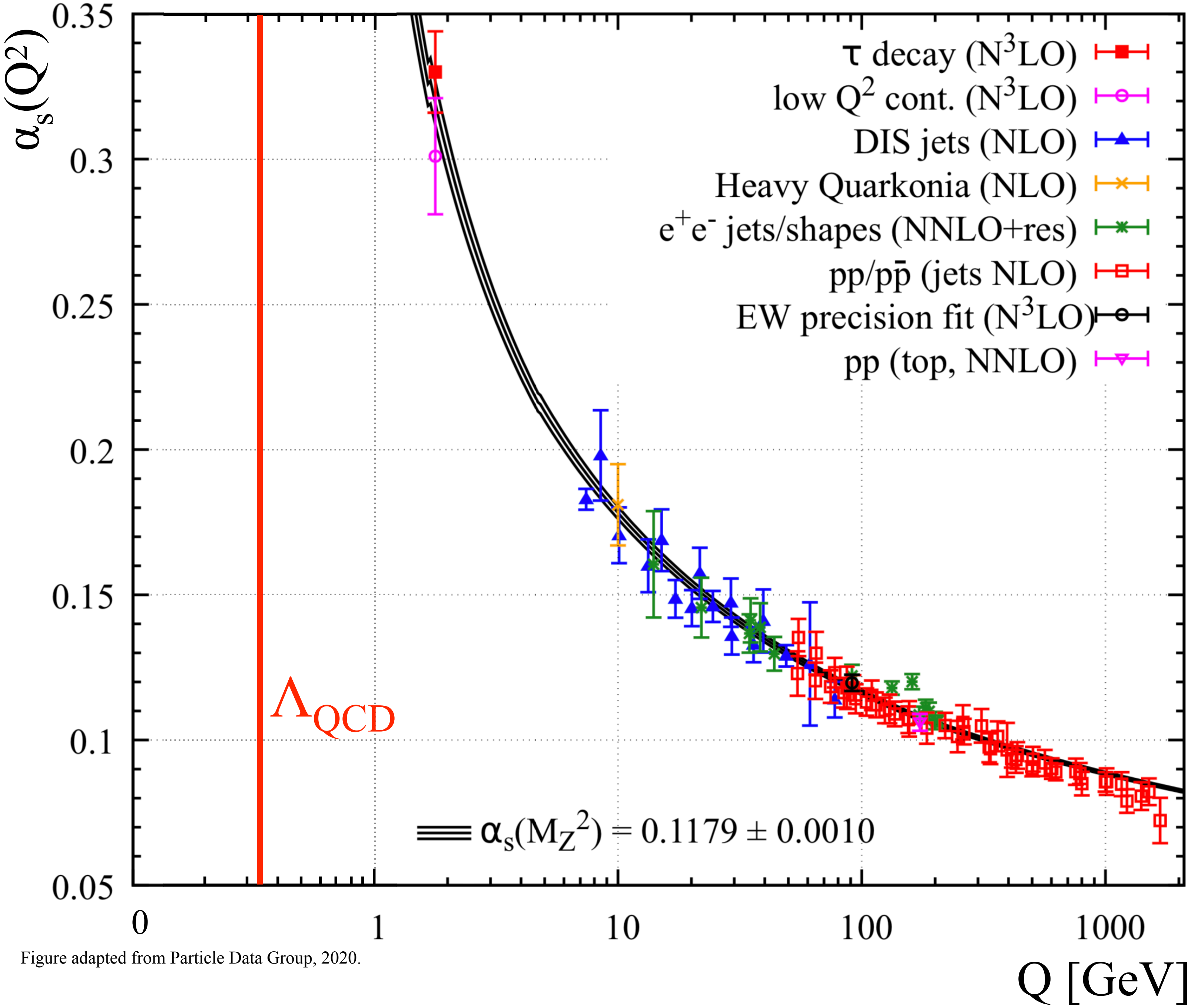
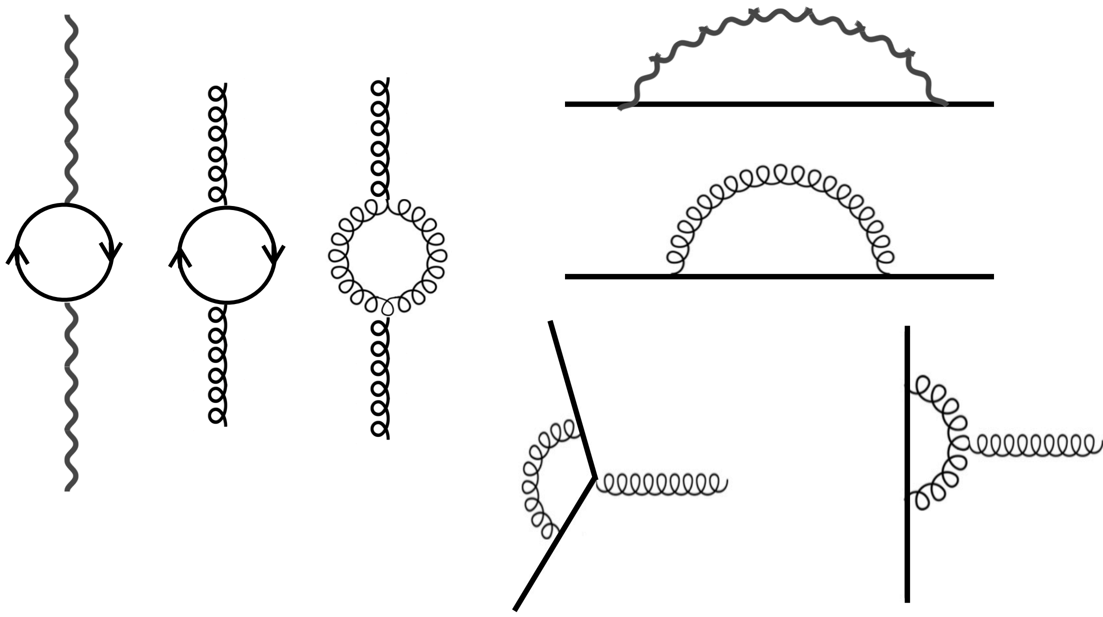


Figure adapted from Particle Data Group, 2020.

α_s is not constant due to loops in gluon propagator, fermion self-energy, and vertex corrections:



α_s becomes small at short distances (large Q^2)

⇒ Asymptotic freedom: perturbative treatment of QCD (pQCD). $\alpha_s(Q^2)$ is well defined within pQCD.

Large Q^2 determinations in excellent agreement with pQCD expectation.

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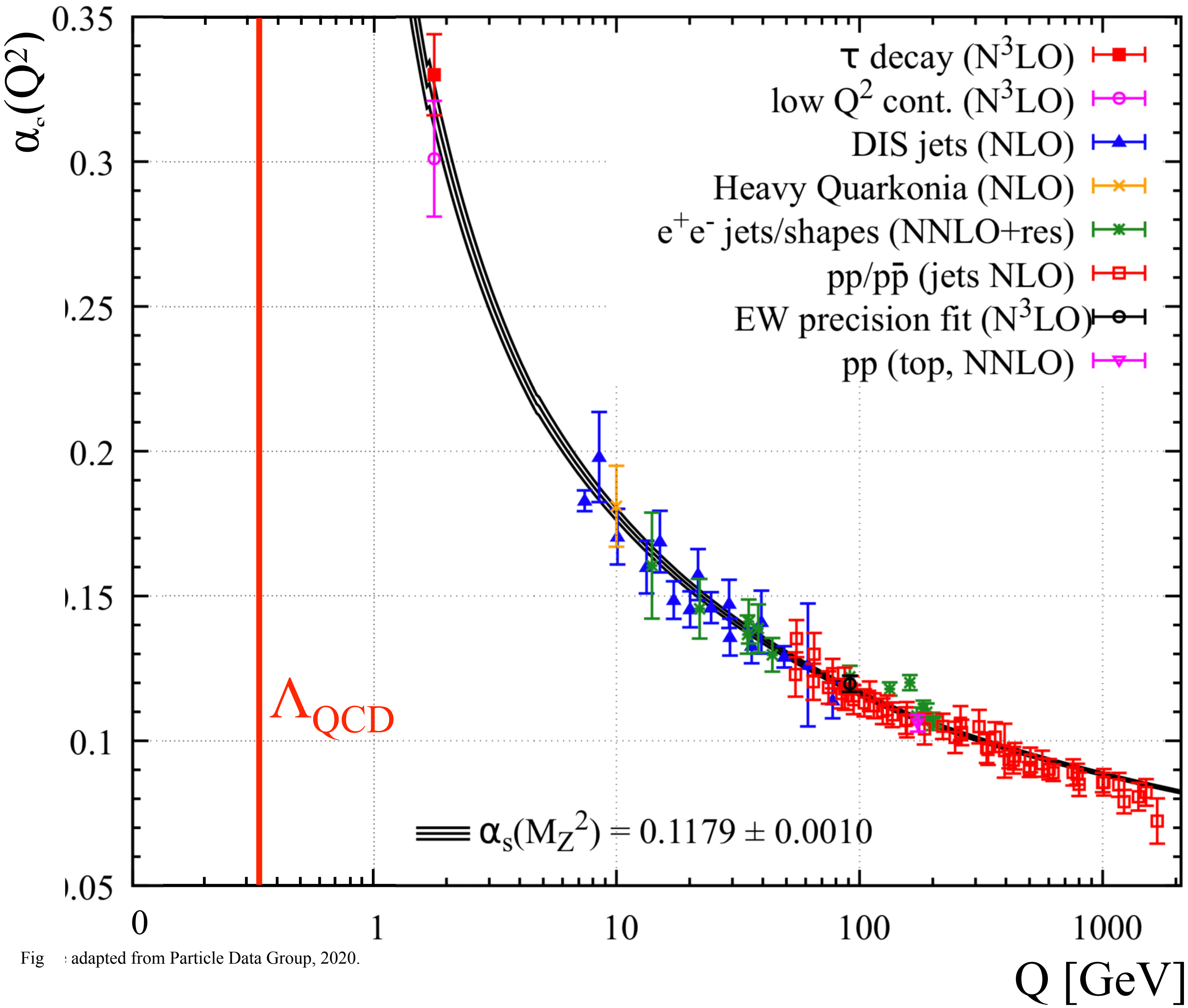

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Here, α_s is in $\overline{\text{MS}}$ scheme (most common choice. It is gauge-independent in this scheme)

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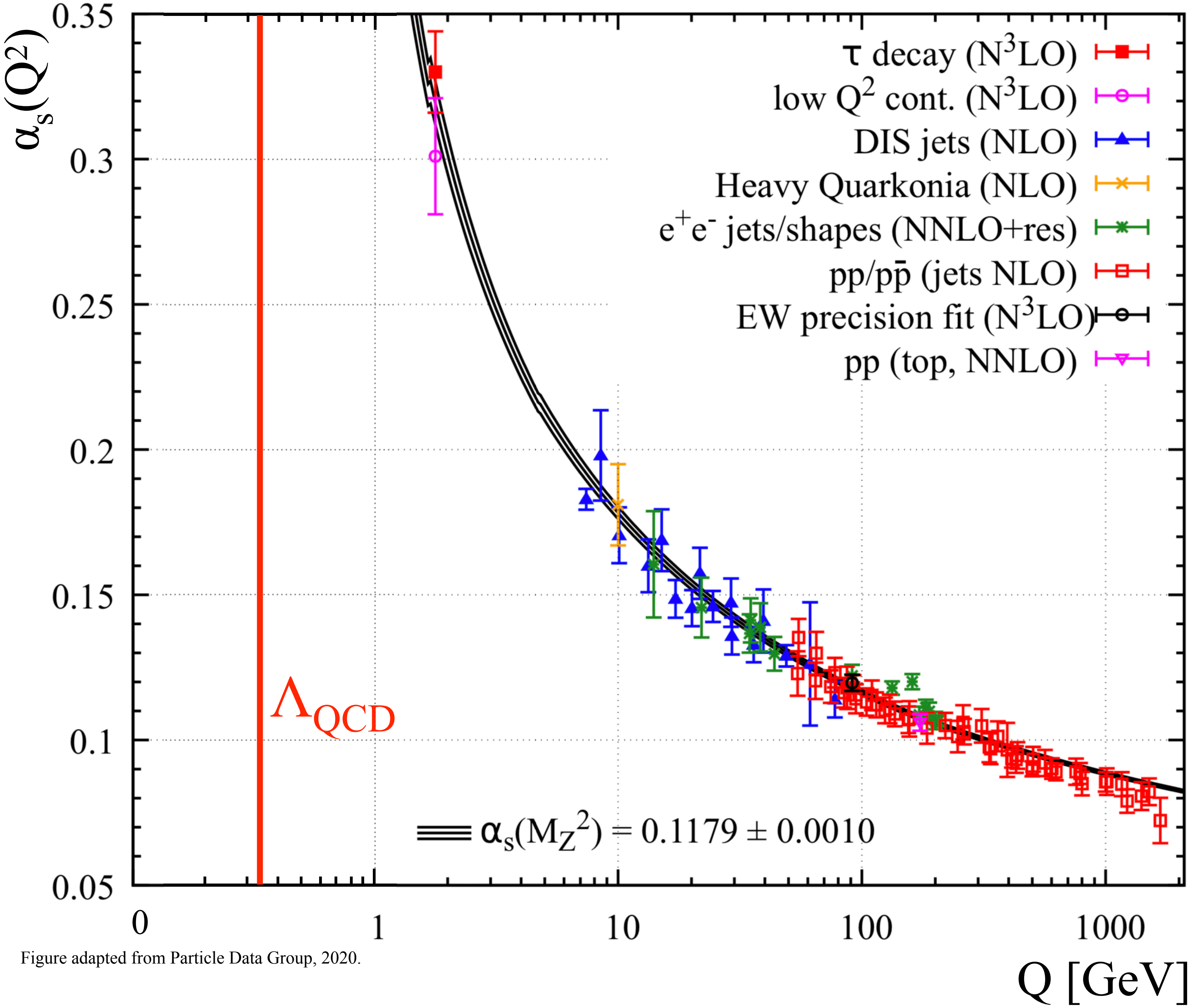

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$\alpha_s(Q^2)=f(Q^2/\Lambda^2_{QCD}) \Rightarrow$ needs data or non-perturbative methods to get $\alpha_s(Q^2)$, even in pQCD domain.

Lattice calculations: currently most accurate determination of $\alpha_s(M_Z^2)$.

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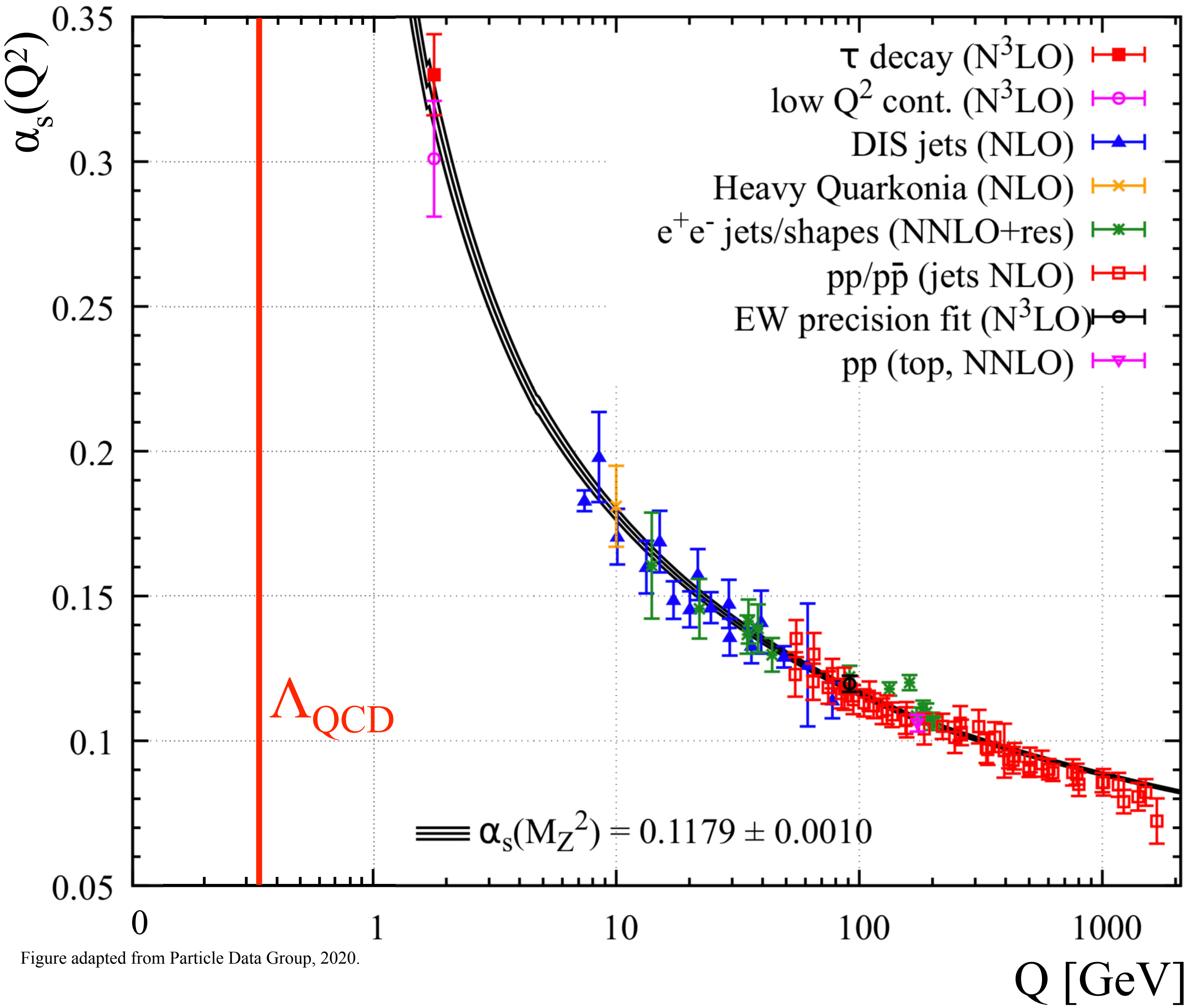

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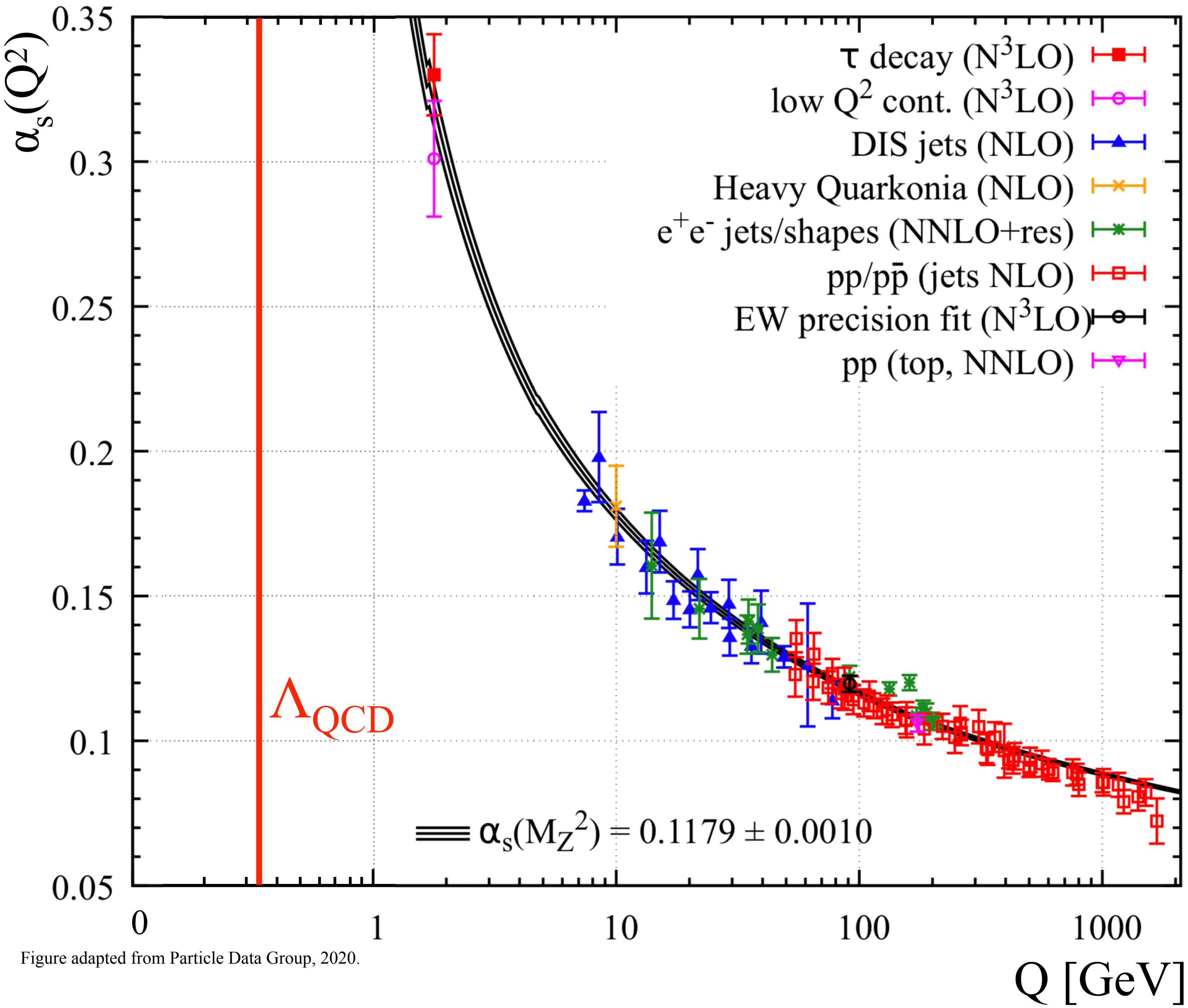

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Otherwise, $\alpha_s(Q^2)$ is extracted from data, e.g. Bjorken sum rule:

$$\int(g^p_1-g^n_1)dx = \frac{1}{6} g_A\left(1-\frac{\alpha_s}{\pi}-3.58\left(\frac{\alpha_s}{\pi}\right)^2-...\right)$$

$g^p_1(Q^2,x)$, $g^n_1(Q^2,x)$: proton & neutron longitudinal spin structure functions.
x: Bjorken-x
g_A: nucleon axial charge (well known).

After scheme-independent LO terms, pQCD series is expressed in a particular renormalization scheme. $\Rightarrow$ α_s also in that scheme.

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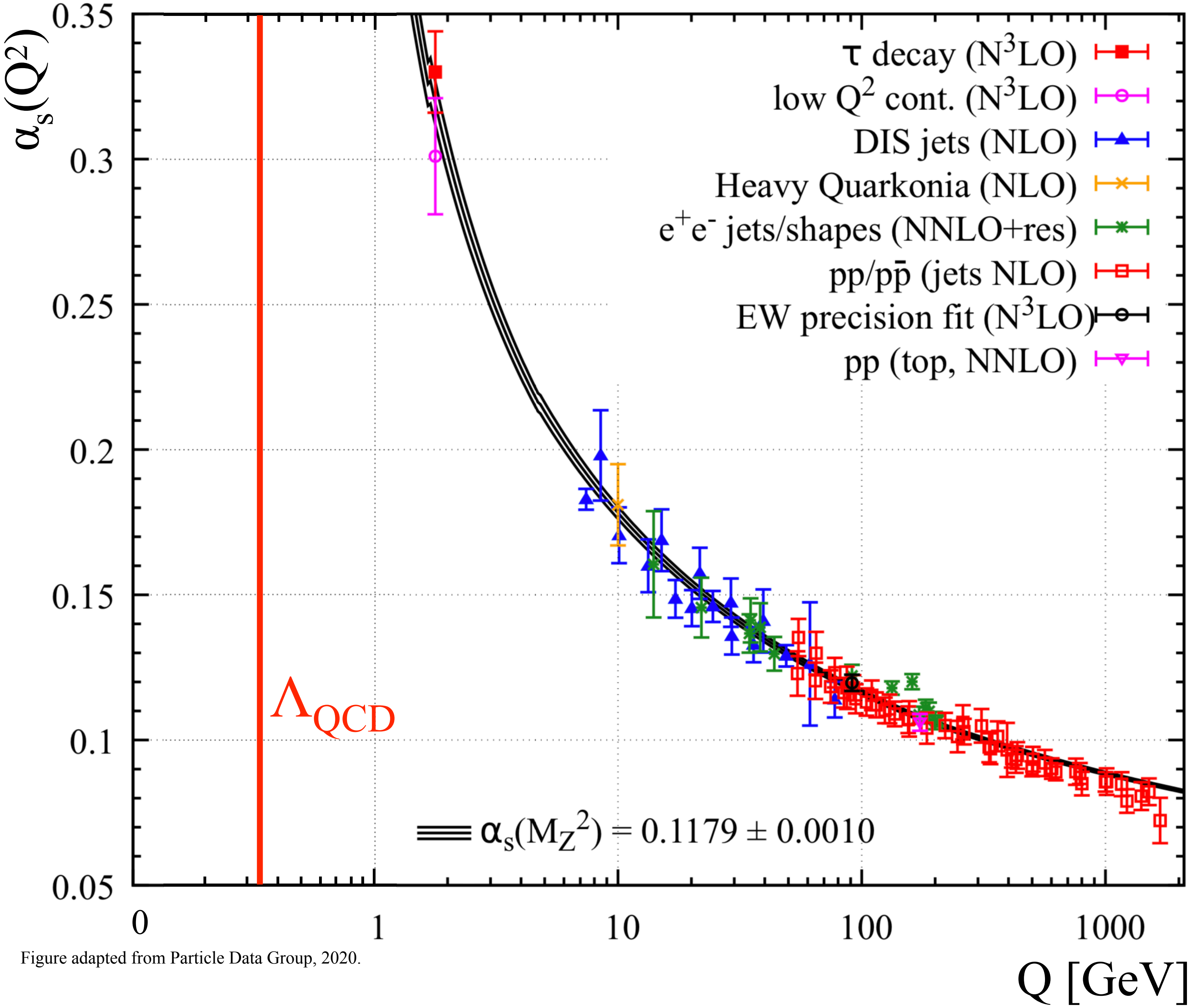

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At $Q^2 \lesssim 1 GeV^2$, pQCD cannot be used to define α_s: if pQCD is trusted, $\alpha_s \rightarrow \infty$ when $Q \rightarrow \Lambda_{QCD}$.

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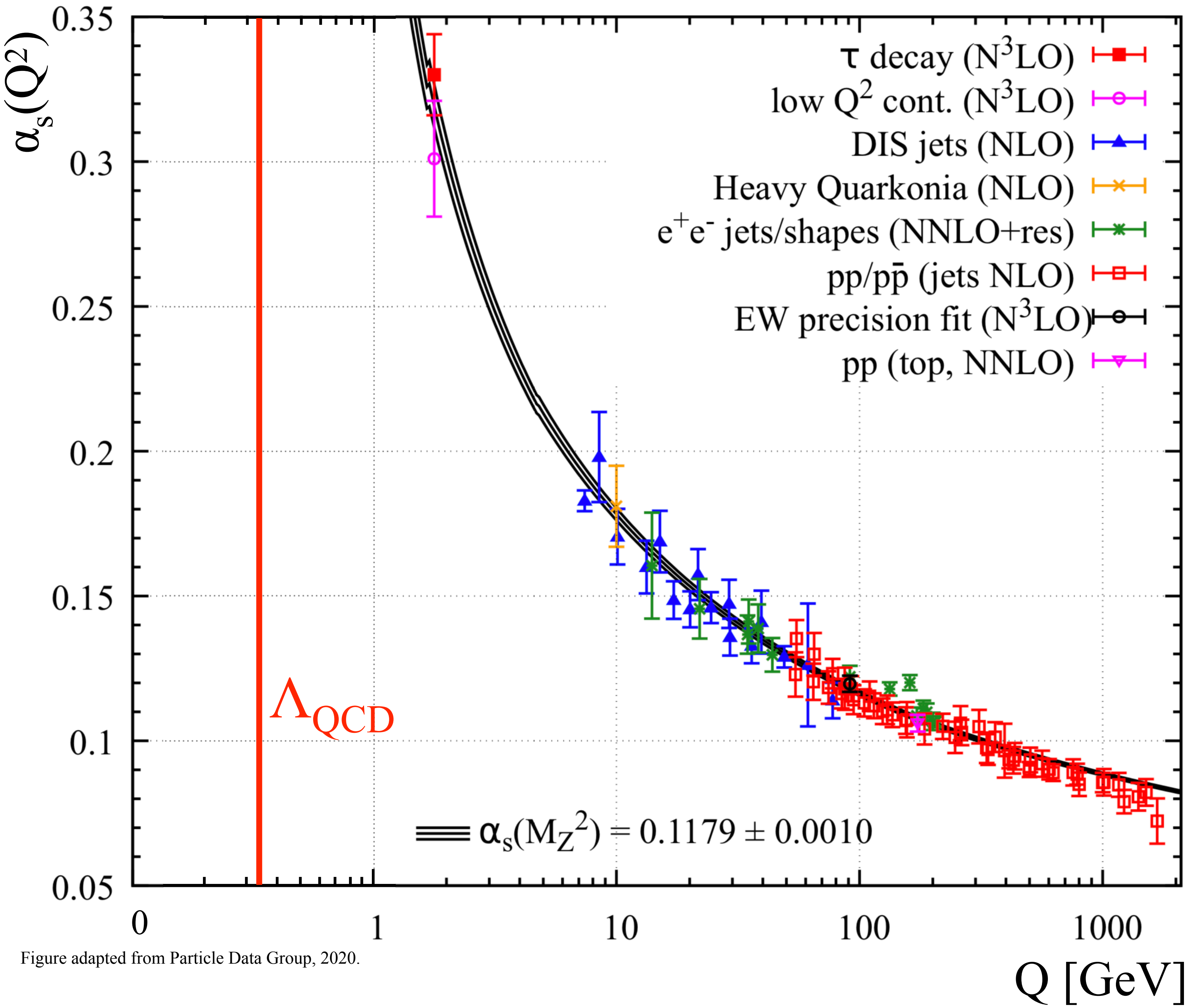

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Definition and computation of α_s at long distance?

α_s at long distance (low Q^2)

Prescription: Define effective couplings from an observable's perturbative series truncated to first order in α_s. G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

This definition is in analogy to α, QED's coupling definition (Gell-Mann Low coupling).

Proposed for pQCD. Can be extended to non-perturbative QCD.

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Ex: Bjorken sum rule:

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Nucleon axial charge.

pQCD corrections (gluon bremsstrahlung)

Higher Twists: $1/Q^{2n}$ corrections.

Non-perturbative quantities. Express correlations between parton distributions and confinement forces.

$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A\left(1 - \frac{\alpha_{g1}}{\pi}\right)}$$

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This means that additional short distance effects, and long distance confinement force and parton distribution correlations are now folded into the definition of α_s.

Analogy with the original coupling constant becoming an effective coupling when short distance quantum loops are folded into its definition.

α_s at long distance (low Q^2)

The effective coupling is then:

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However these α_s can be related (Commensurate Scale Relations).

S. J. Brodsky & H. J. Lu, PRD 51 3652 (1995)
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Such definition of α_s using a particular process is equivalent to a particular choice of renormalization scheme.

(process dependence) $\Leftrightarrow$ (scheme dependence)

$\alpha_{g1} = \alpha_s$ in the "g_1 scheme".
Relations between g_1 scheme and other schemes are known in pQCD domain, e.g.
$\Lambda_{g_1} = 2.70\Lambda_{\overline{MS}} = 1.48\Lambda_{MOM} = 1.92\Lambda_V = 0.84\Lambda_\tau$.

α_s at long distance (low Q^2)

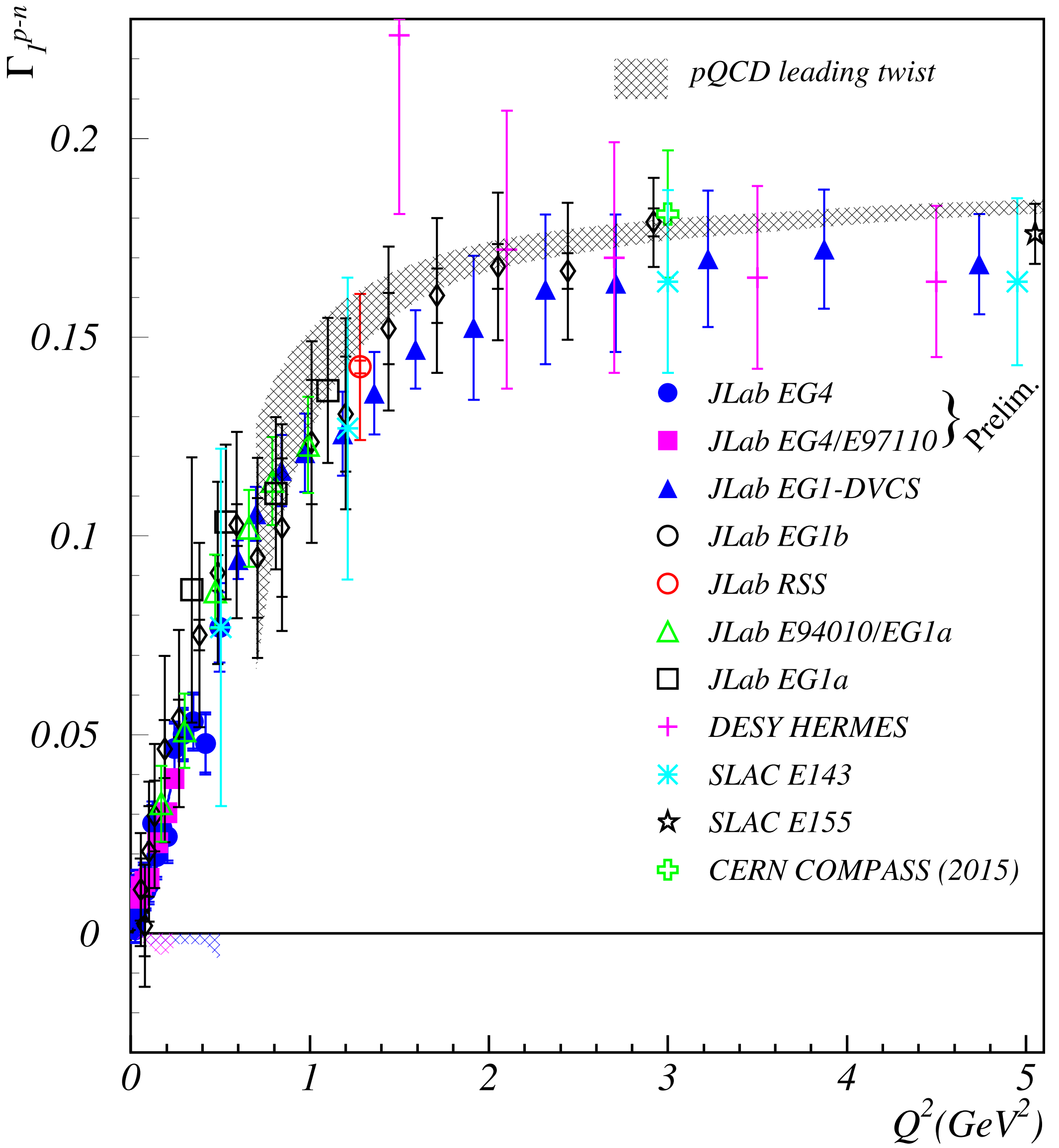
Advantages of extracting α_s from the Bjorken Sum Rule:

- Bjorken sum rule: simple perturbative series.
- Data exist at low, intermediate, and high Q^2.
- Rigorous Sum Rules dictate the behavior of α_{g1} in the unmeasured $Q^2 \to 0$ and $Q^2 \to \infty$ regions.

$\Rightarrow$ We can obtain α_{g1} at **any** Q^2.

α_{g1} from the Bjorken Sum data

Bjorken sum Γ_1^{p-n} measurements



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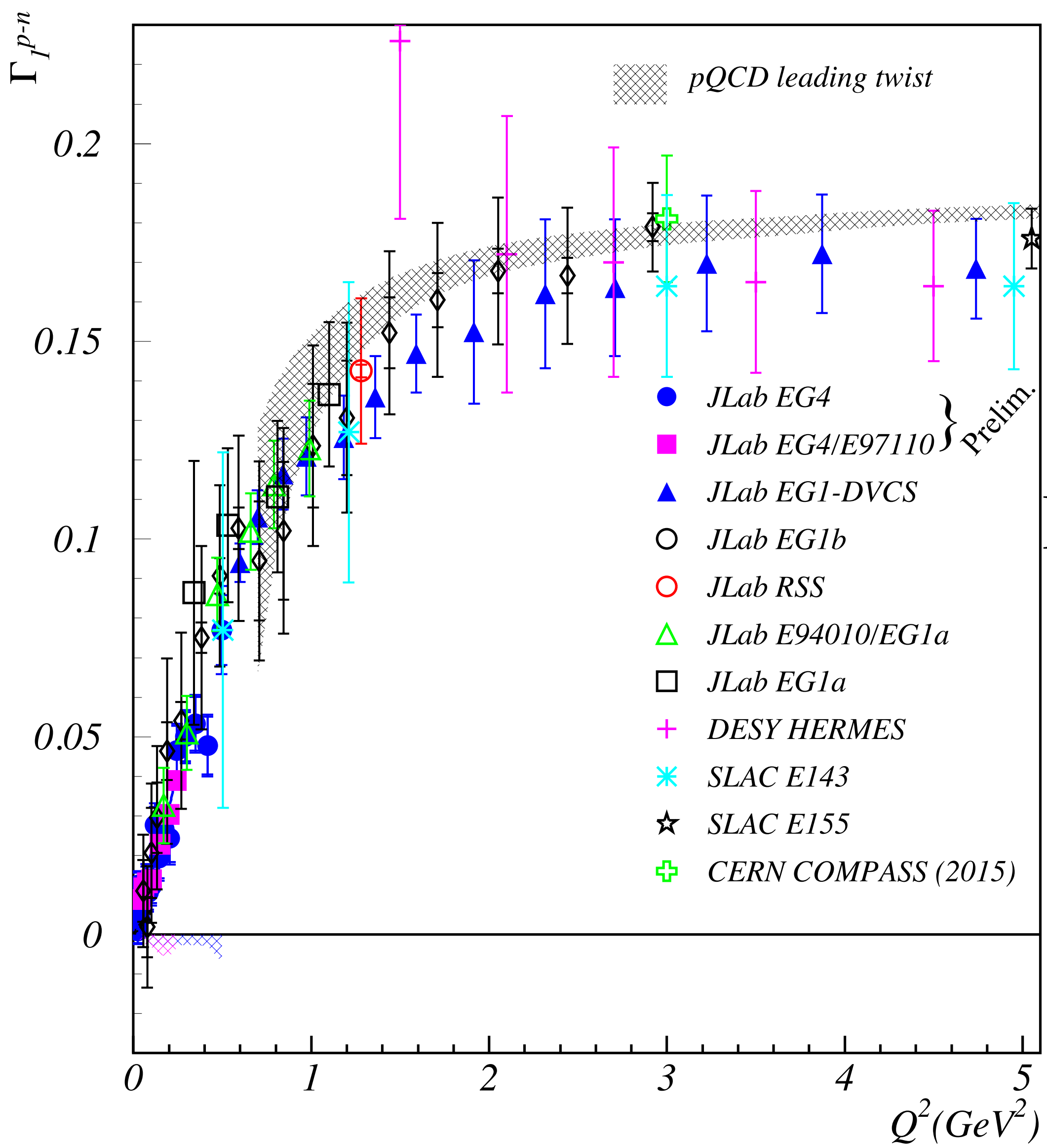

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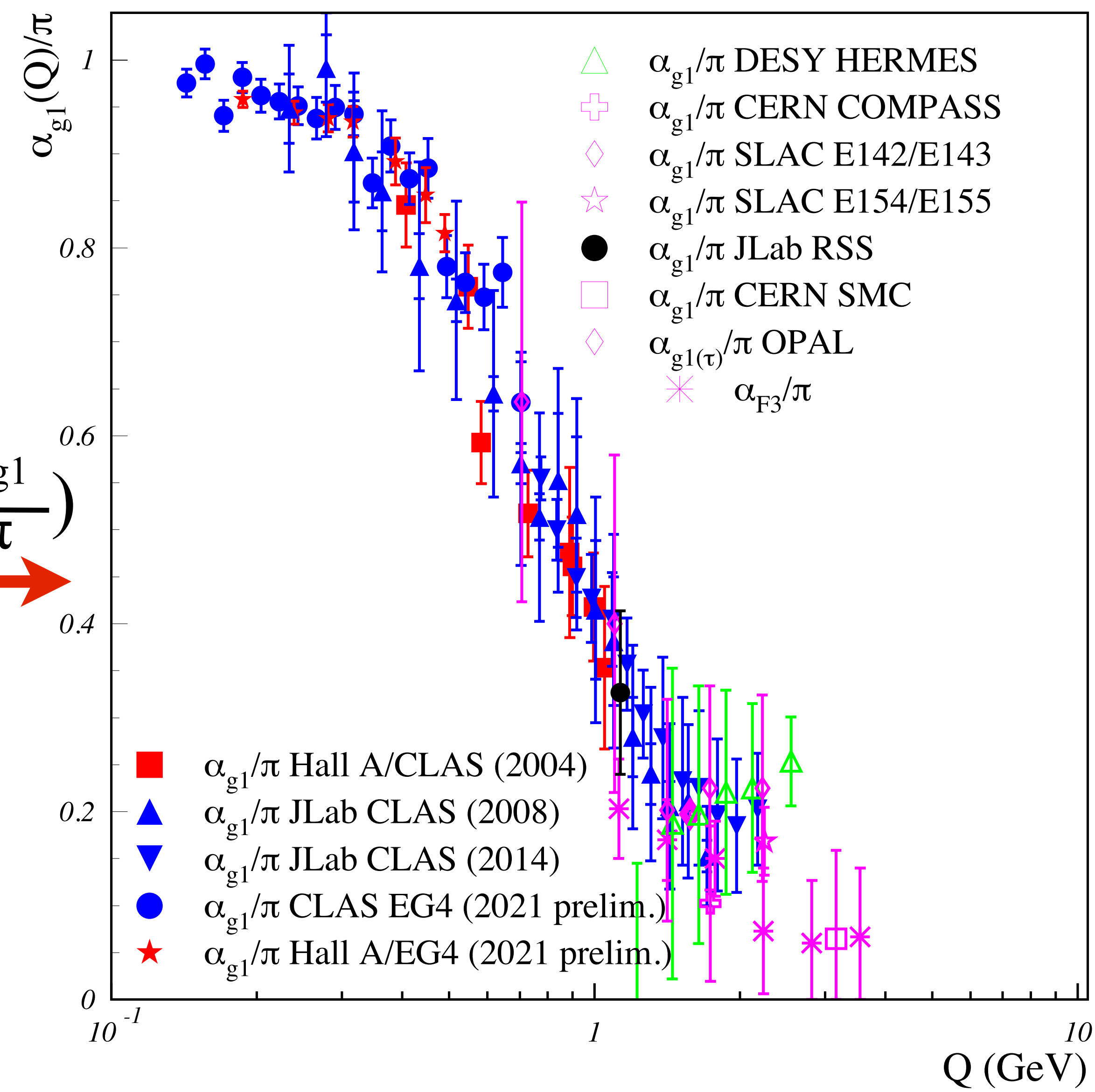


Low Q^2 limit

At $Q^2 = 0$, a sum rule related to the Bjorken sum rule exists: the Gerasimov-Drell-Hearn (GDH) sum rule:

At $Q^2 = 0$, GDH sum rule:

$$\Gamma_1 = \frac{-\kappa^2 Q^2}{8M^2}$$

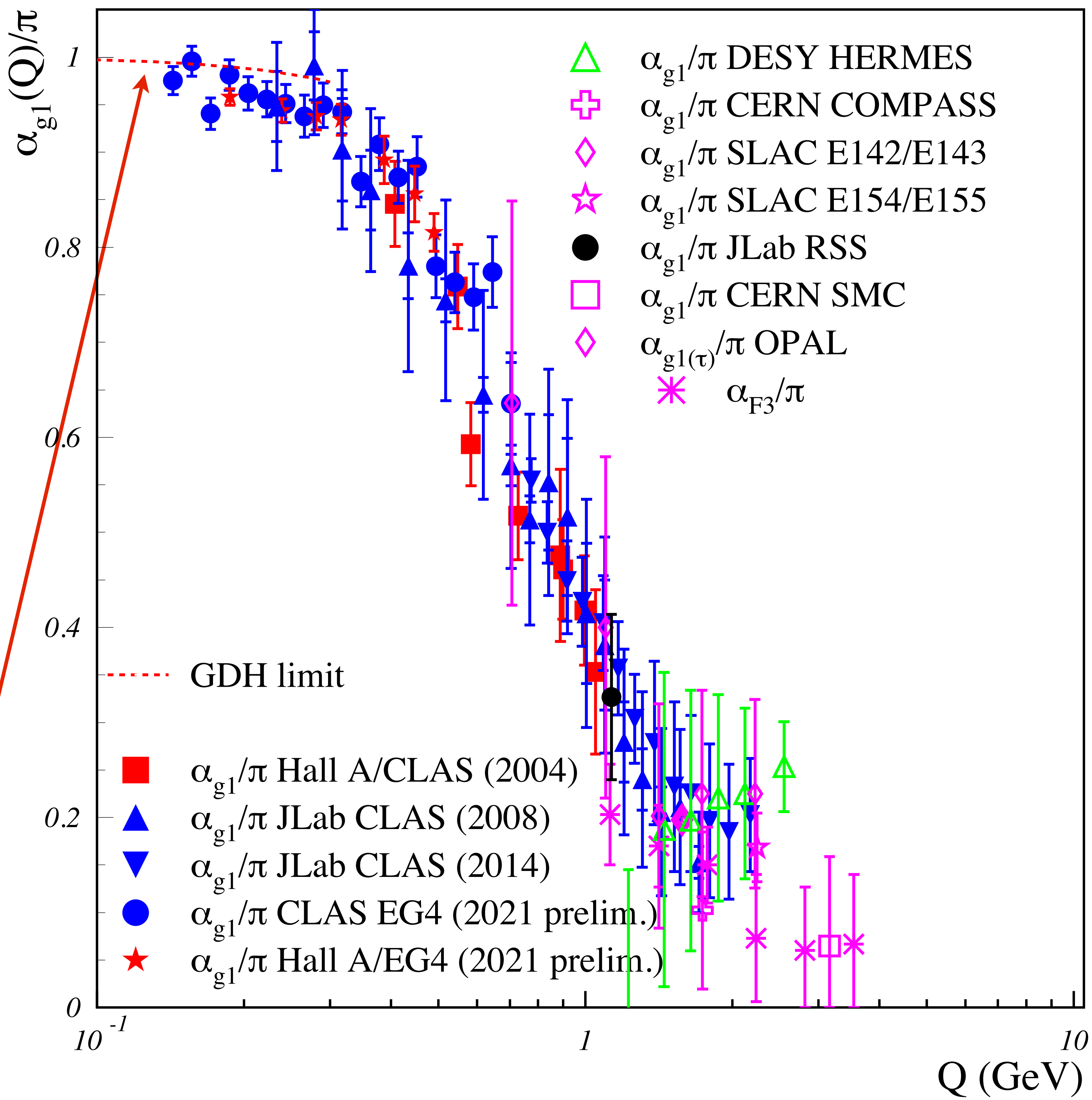
anomalous magnetic moment

Nucleon mass

$\Rightarrow$ $Q^2 = 0$ constraints:

$$\Rightarrow \begin{cases} \alpha_{g1} = \pi \\ \dfrac{d\alpha_{g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^2}{M_n^2} - \dfrac{\kappa_p^2}{M_p^2}\right) \end{cases}$$

$Q^2=0$



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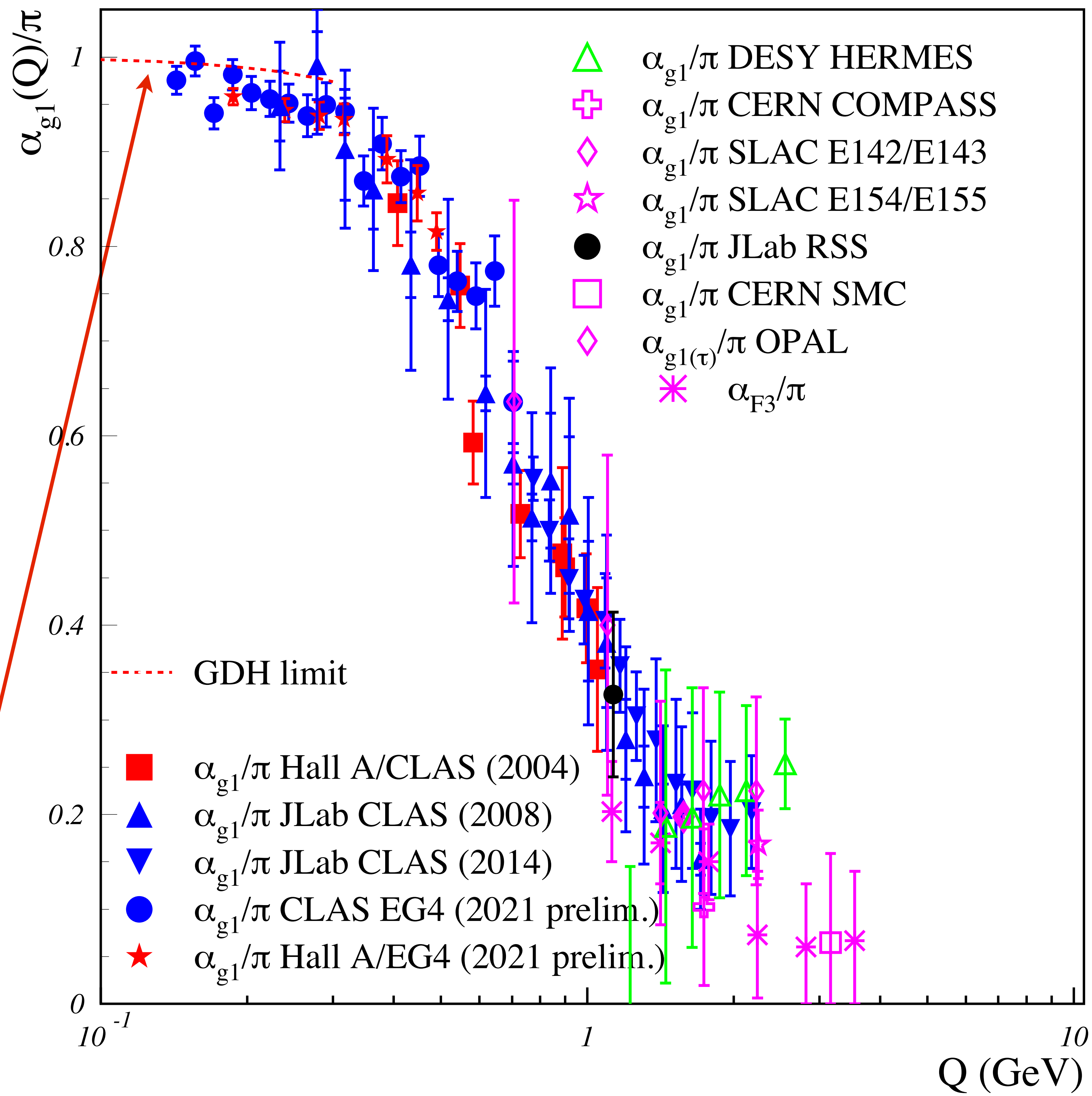
Nucleon mass (M)

$\Rightarrow Q^2 = 0$ constraints:

$$\Rightarrow \begin{cases} \alpha_{g1} = \pi \\ \dfrac{d\alpha_{g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^2}{M_n^2} - \dfrac{\kappa_p^2}{M_p^2}\right) \end{cases}$$

κ_n^2: 3.66 $\quad$ κ_p^2: 3.20



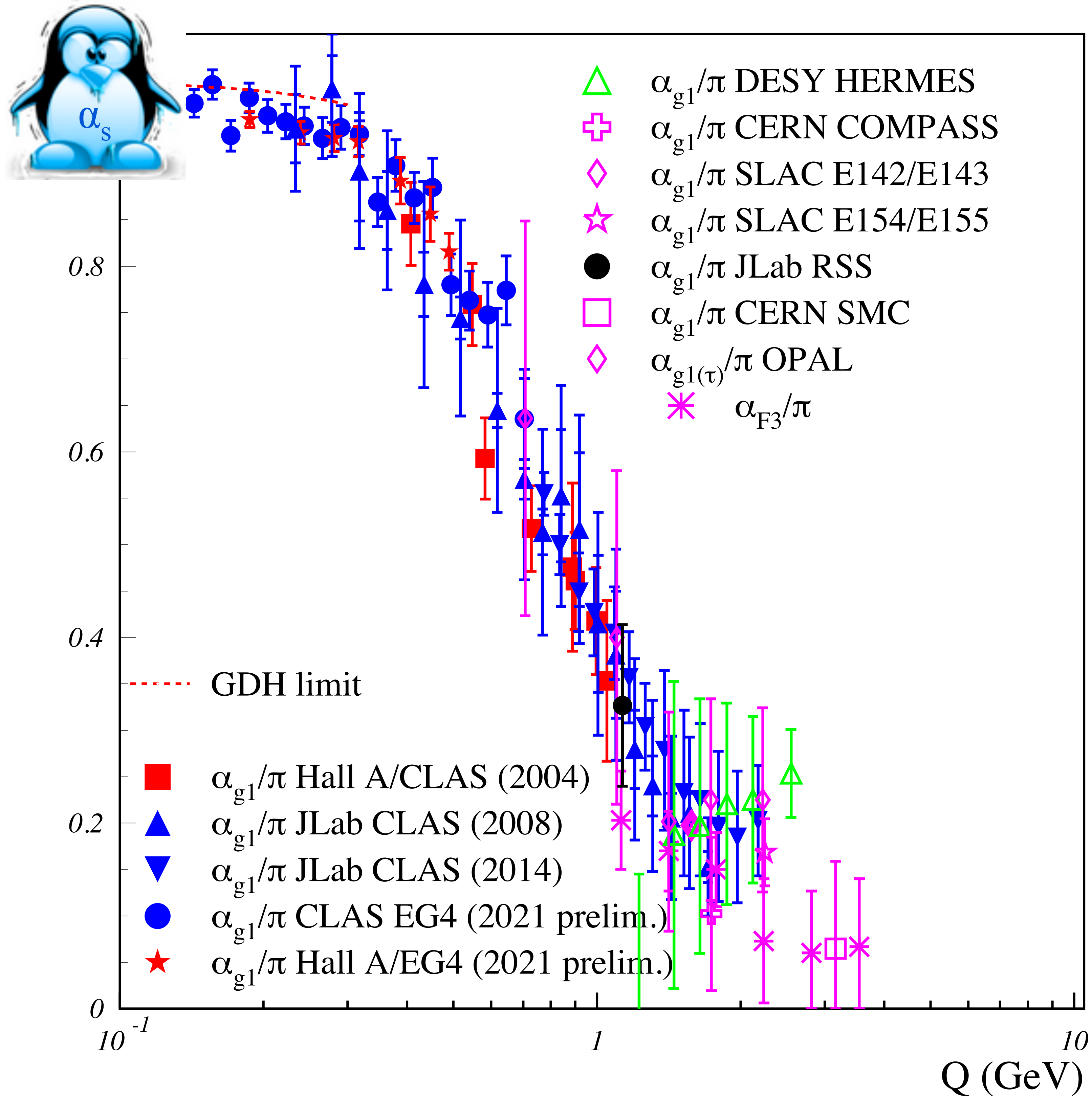
First experimental evidence of nearly *conformal behavior* (i.e. no Q^2-dependence) of QCD at low Q^2.

Low Q² limit

Jargon:

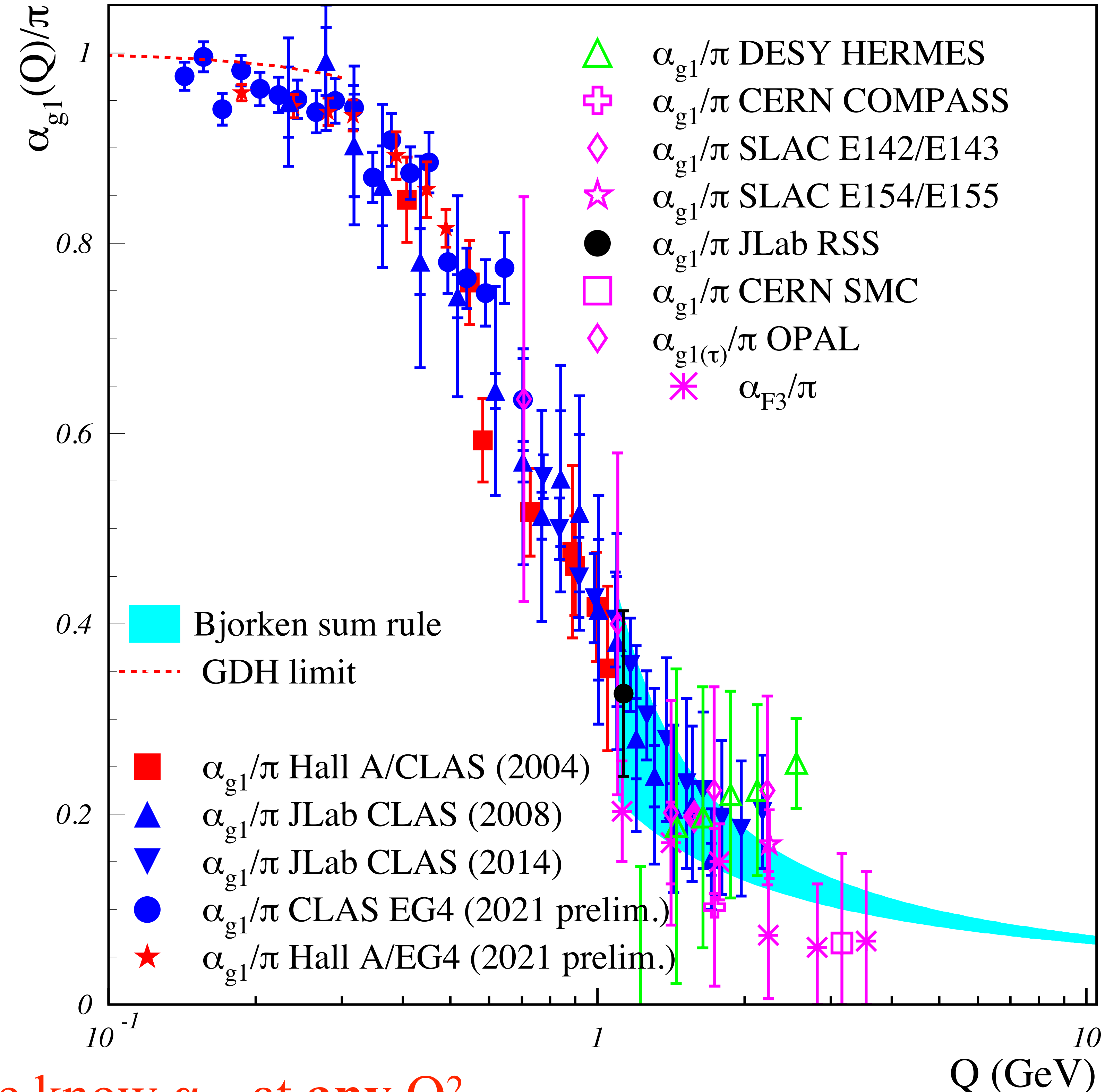
α_s freezes.
or
QCD is conformal

at low Q².



Large Q^2 limit

$$\int(g^p_1-g^n_1)dx \triangleq \Gamma_1^{p-n} = \frac{1}{6}g_A\left(1-\frac{\alpha_{\overline{MS}}}{\pi} -3.58\left(\frac{\alpha_{\overline{MS}}}{\pi}\right)^2-...\right) \triangleq g_A\left(1-\frac{\alpha_{g1}}{\pi}\right)$$



⇒ We know α_{g1} at **any** Q^2.

Low Q² limit

Jargon:

α_s freezes.
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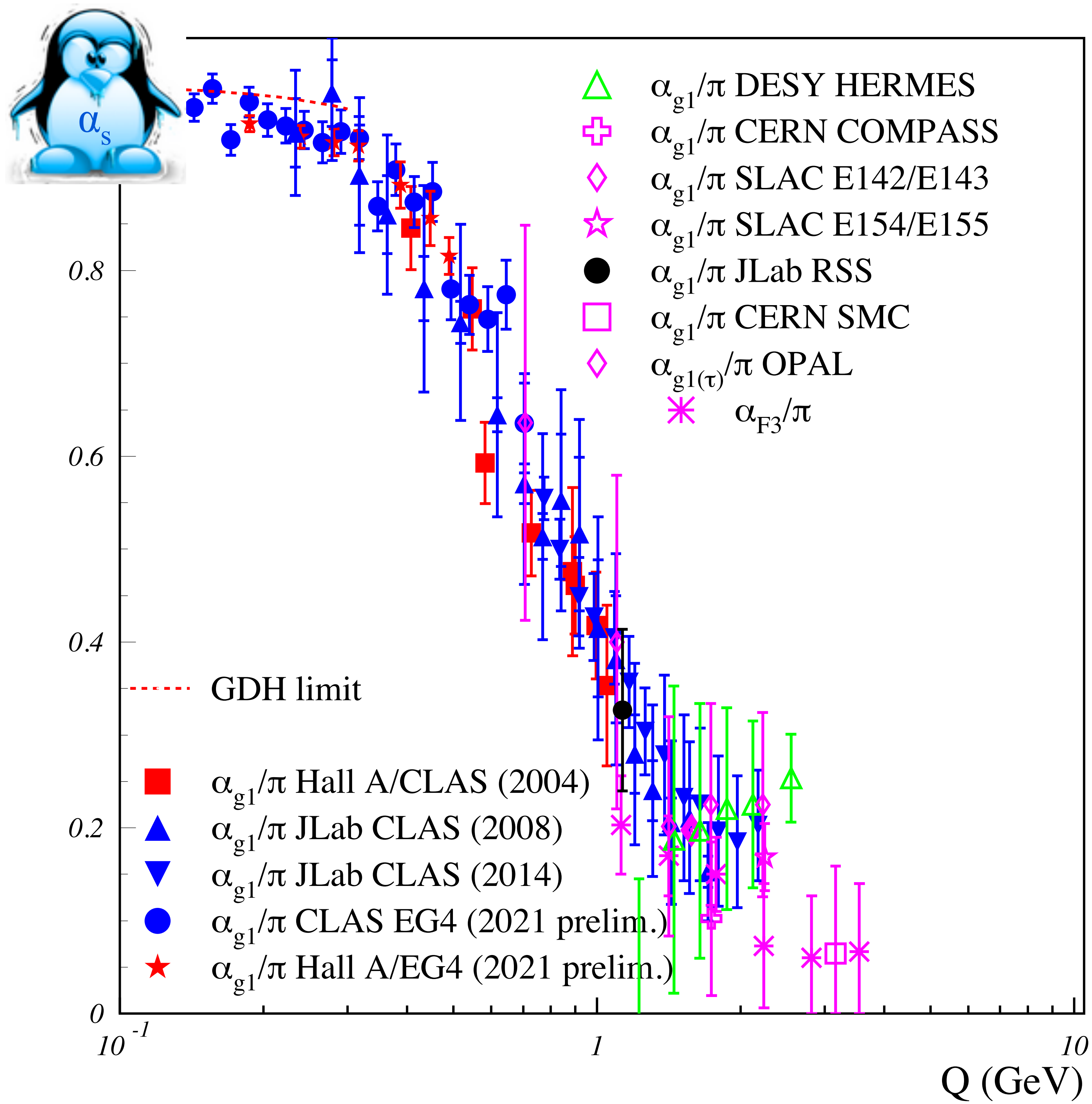

⇒ One can use tools associated with conformal theories to study non-perturbative QCD.

The Light-Front holography approximation (LFHQCD)

Review: Brodsky, de Teramond, Dosch, Erlich, Phys. Rep. 05 (2015) 001. arXiv:1407.8131

•Light-front QCD: Rigorous and exact formulation of non-perturbative QCD. Yields a relativistic Schrödinger-like equation for hadrons. Confining potential calculable in principle but not tractable in 3+1 dimensions.

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LFHQCD: semi-classical model for QCD (no short-distance quantum fluctuations)

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- To produce the harmonic oscillator potential (i.e. the confinement forces), the AdS space is deformed. This done by distorting the AdS metric.

Harmonic oscillator on light front $\Rightarrow$ in AdS space, $ds^2 \to \exp(\kappa^2 z^2) ds^2$

z is the 5th dimension of AdS space. z^2 is the scale at which the hadron is probed, i.e. $1/Q^2$.

κ is the universal scale factor of LFHQCD.

α_s from LFHQCD

Perturbative QCD:

pQCD effective coupling $\alpha_s(Q^2)$: small distance QCD effect are folded into the definition of the coupling constant α_s.

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General Relativity Action: $S \propto \int d^4x \sqrt{g} \frac{1}{G_N} R$, with R the Ricci scalar and $g=\det(g_{\mu\nu})$

AdS Action: $S \propto \int d^5x \sqrt{g} \frac{1}{g^2_5} F^2$, with F the gauge field and g_5 the coupling

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Deformed AdS Action: $S \propto \int d^5x \sqrt{g}\, e^{\kappa^2 z^2} \frac{1}{g_5^2} F^2$

Confinement potential

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Effective coupling at large distance

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Effective coupling at large distance

Transforming to momentum space:

$$\alpha_s^{LFH}(Q^2)=\alpha_s^{LFH}(Q^2=0)e^{(-Q^2/4\kappa^2)}$$

Brodsky, de Téramond, Deur.
Phys. Rev. D 81, 096010 (2010)

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Effective coupling at large distance

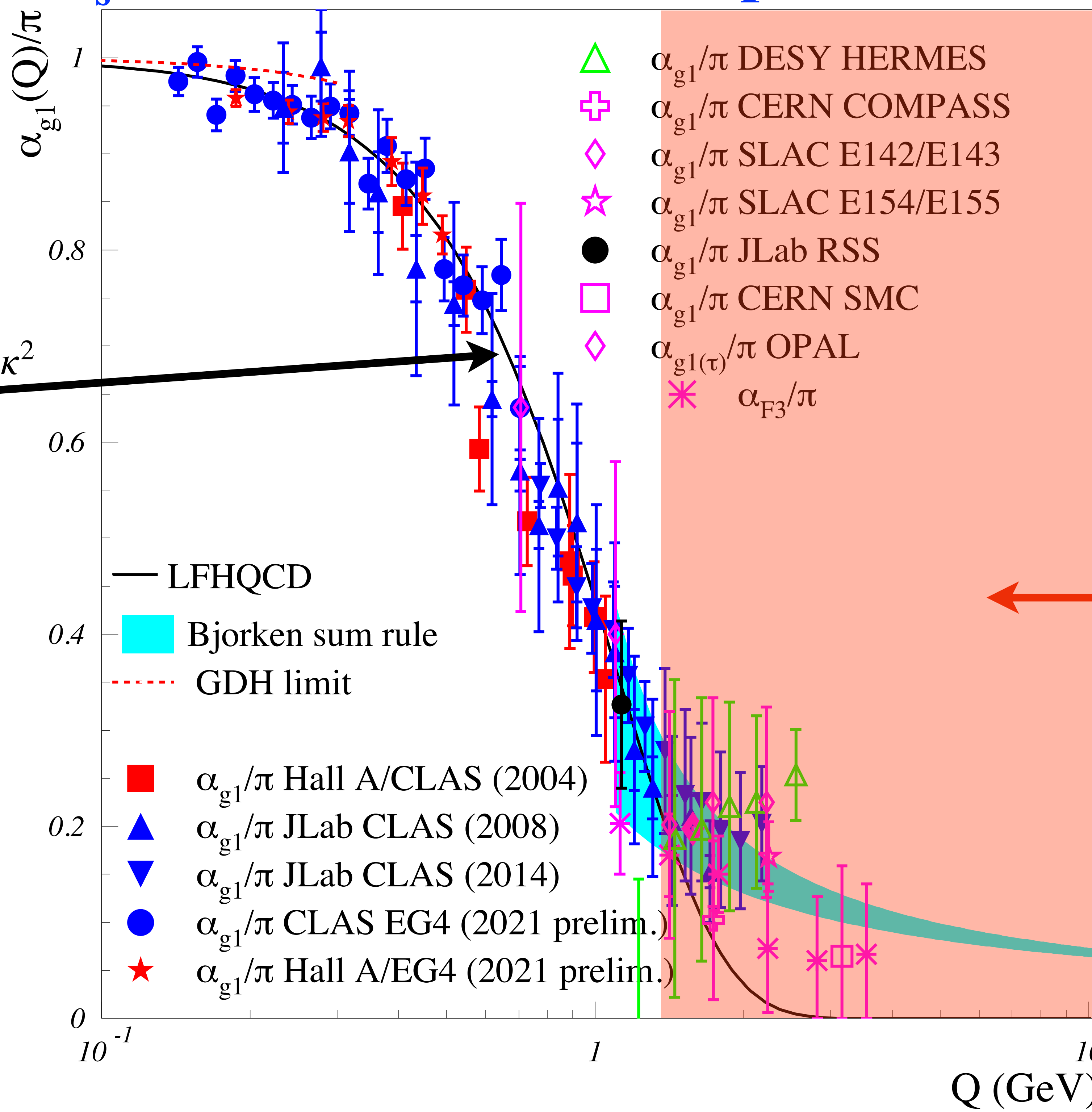
Transforming to momentum space: $\alpha_s^{LFH}(Q^2)=\alpha_s^{LFH}(Q^2=0)e^{(-Q^2/4\kappa^2)}$

$\alpha_s^{LFH}(0)\equiv\pi$: $\alpha_s^{LFH}(Q^2)$ in the g_1 scheme.

α_s and LFHQCD: Comparison with data

$$\frac{\alpha_s^{LFH}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$

$\kappa = M_\rho/\sqrt{2}$



LFHQCD not valid

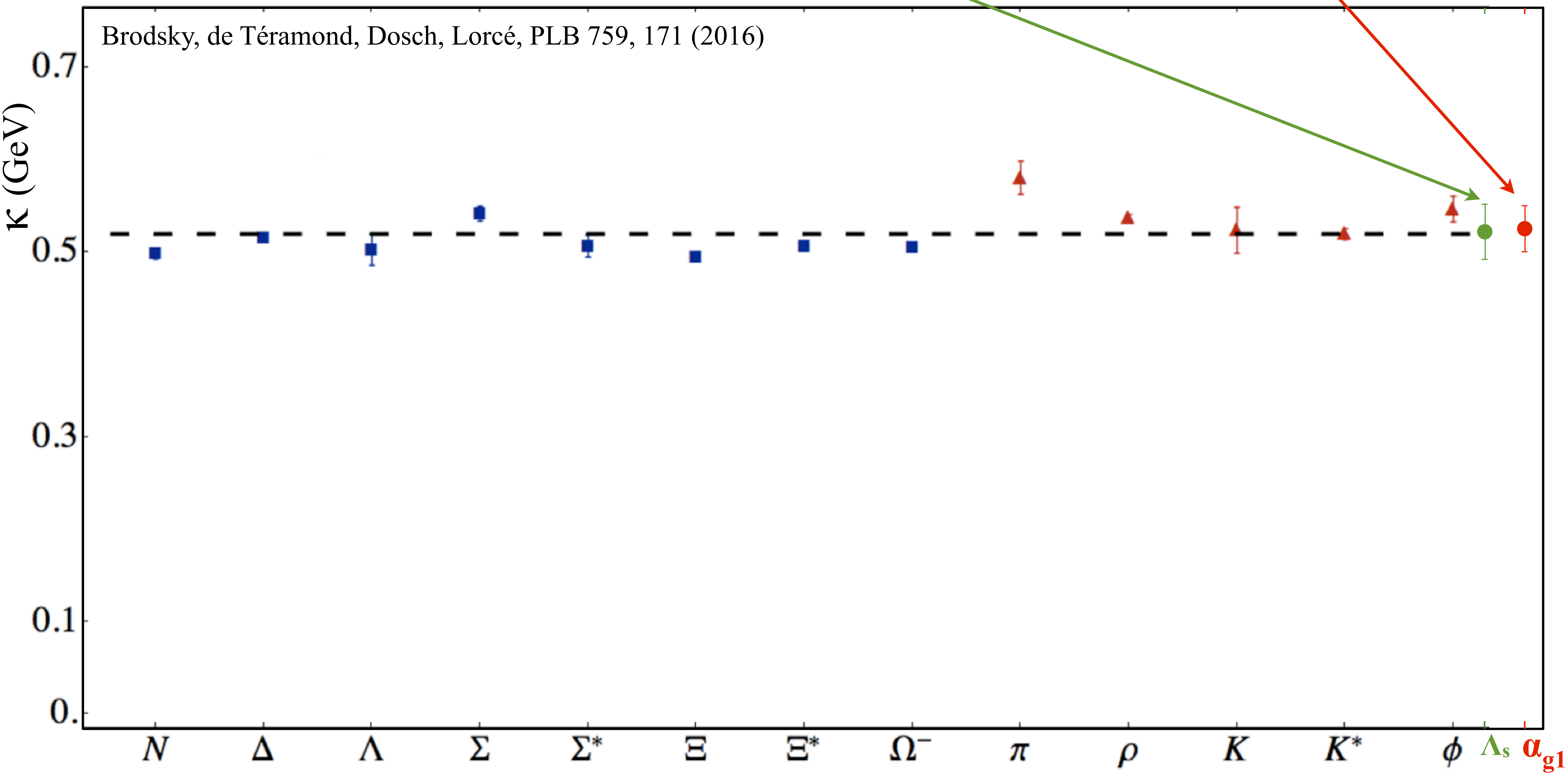
⇒ **Prediction for α_s at long distances. No free parameters ($\kappa=M_\rho/\sqrt{2}$).**
Agrees very well with the α_s extracted from JLab's Bjorken sum data.

α_s and LFHQCD: Comparison with data

One can also fit the $\alpha_{g1}(Q^2)$ data to get κ: κ=0.513±0.025 GeV

Or use the relation between κ and Λ_{QCD} (latter slides):

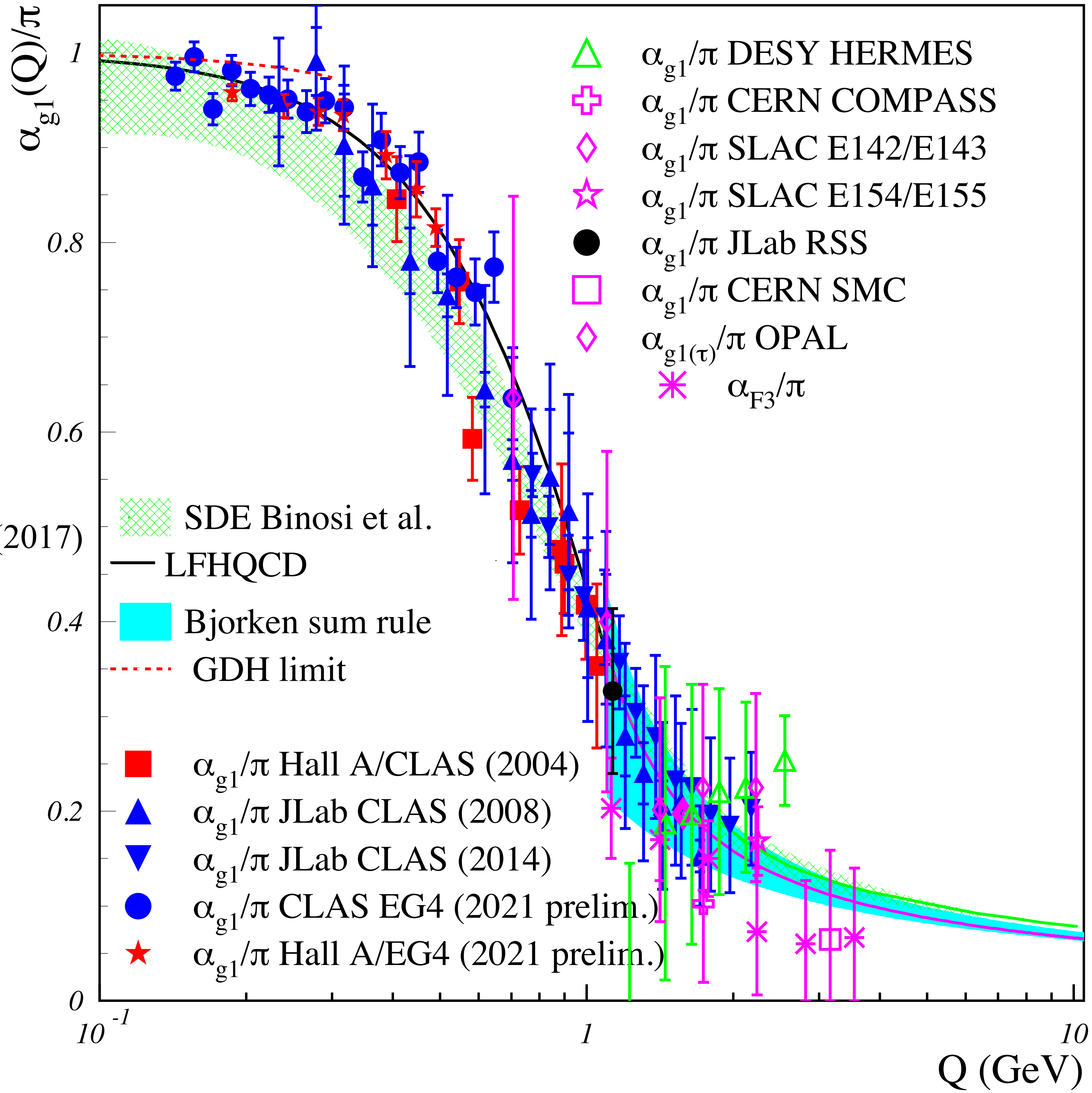
PDG value for Λ_{QCD} yields κ=0.512±0.030 GeV



Agree with other determinations of κ. ~10% universality of κ confirms that LFHQCD is a good model for QCD. (Also, nucleon or pion Form Factors provide compatible κ.)

Comparison with process-independent α_s from SDE calculation

Binosi et al. PRD 96, 054026 (2017)

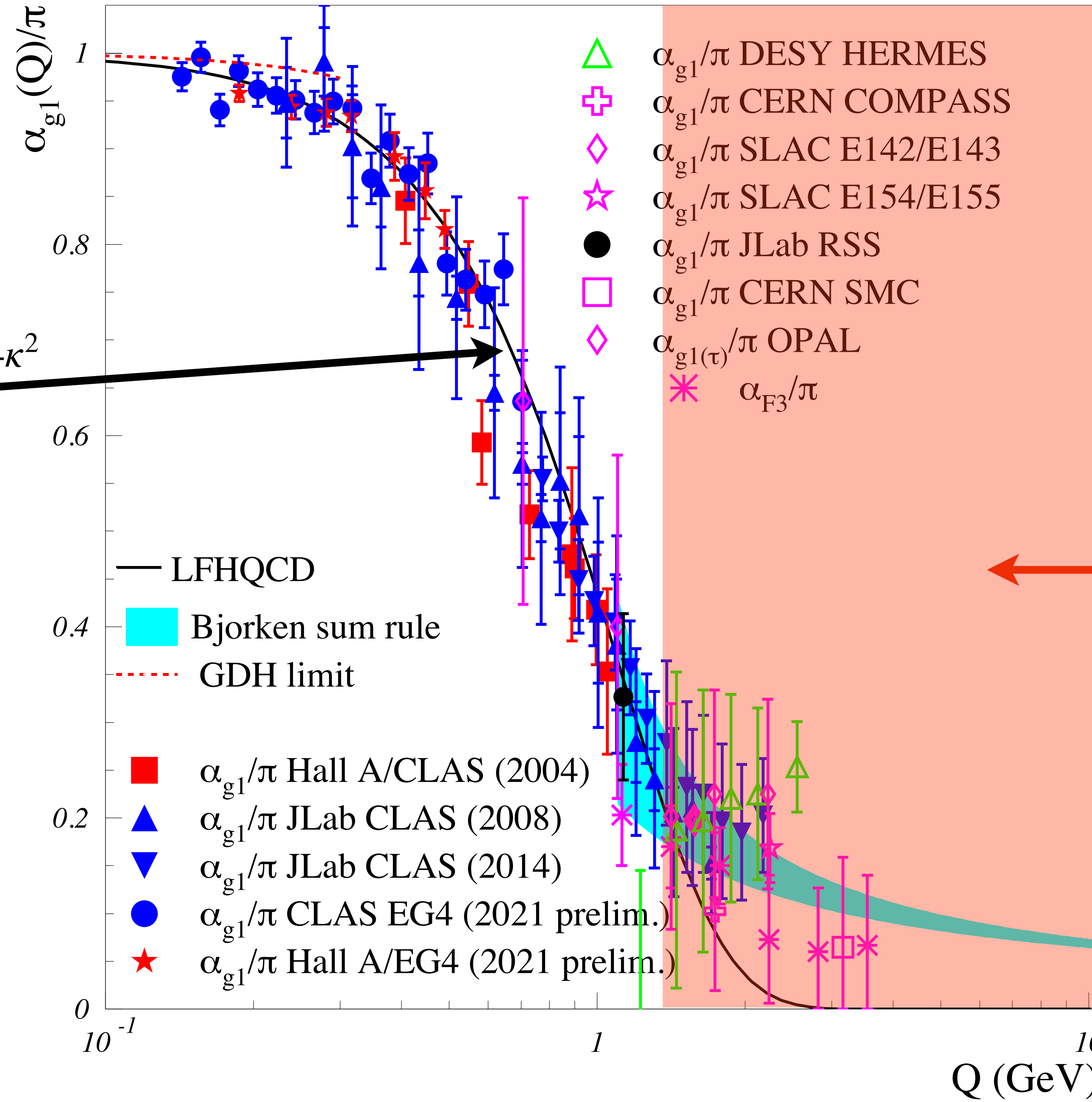


⇒ SDE, LFHQCD and data agree very well.

Predictions of the hadronic mass spectrum

$$\frac{\alpha_s^{LFH}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$

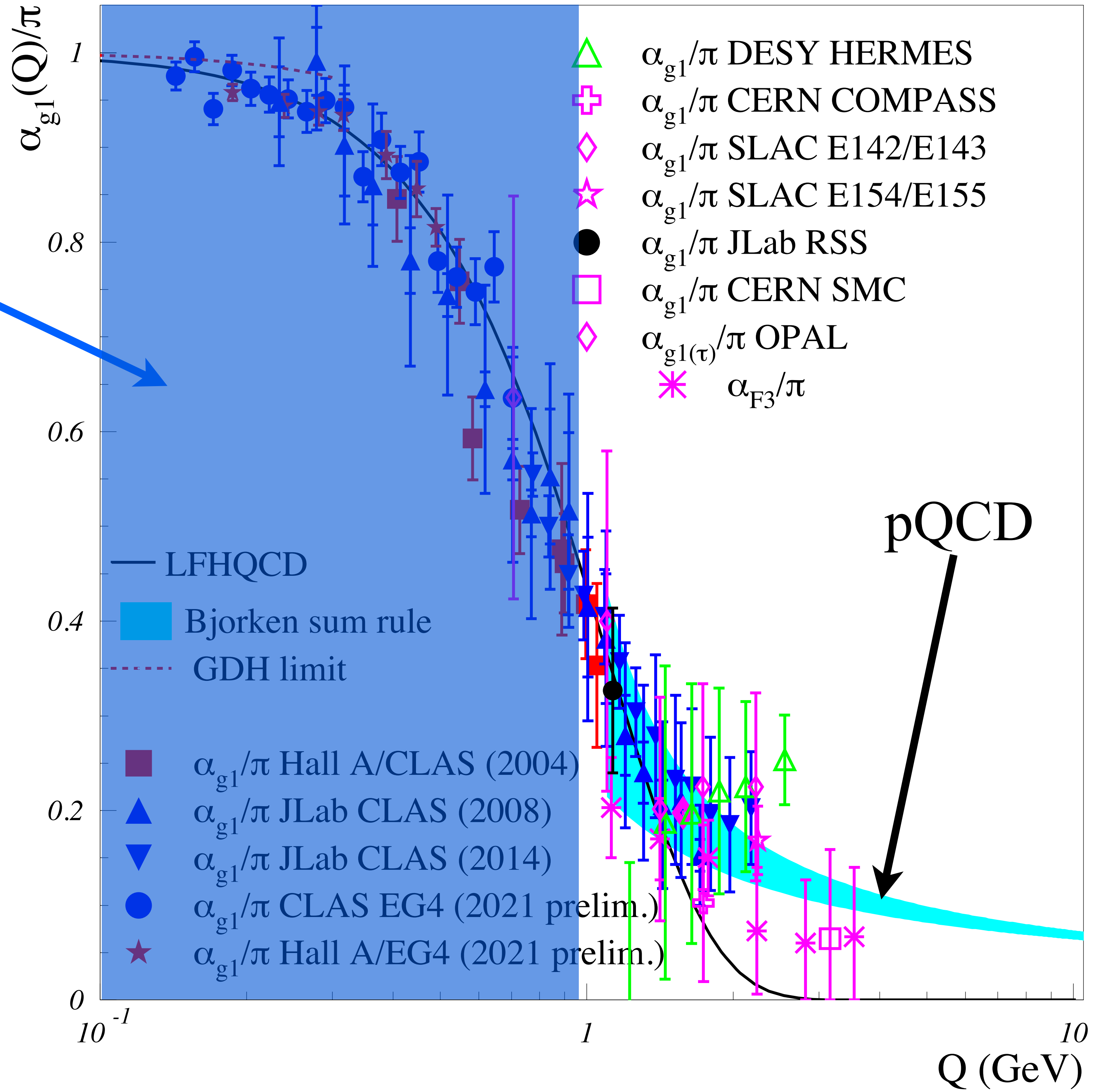
$$\kappa = M_\rho/\sqrt{2}$$



LFHQCD not valid

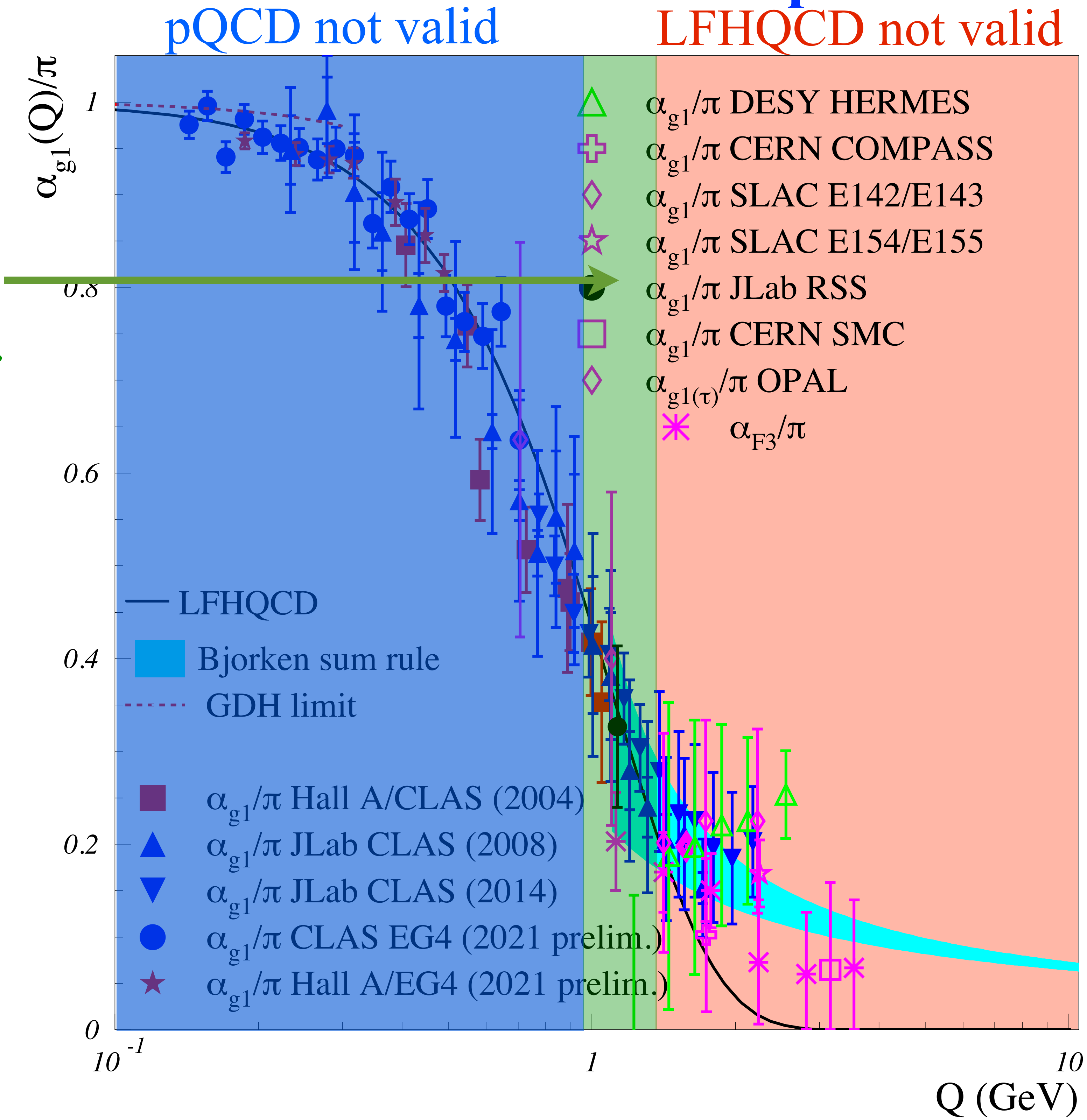
Predictions of the hadronic mass spectrum

pQCD not valid



Predictions of the hadronic mass spectrum

pQCD and LFHQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)



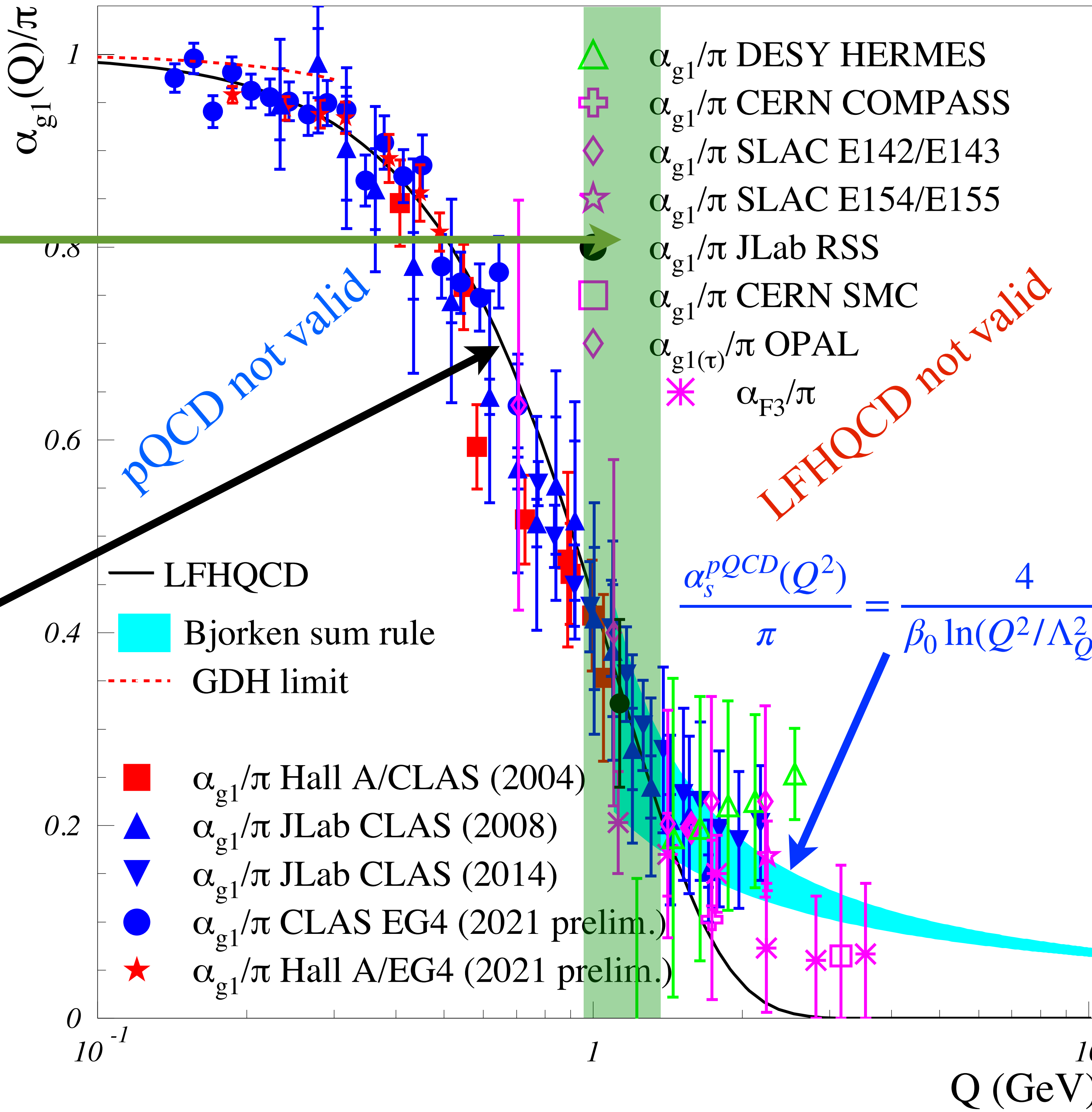
Match LFHQCD and pQCD expressions of α_{g1} and its β-function:

$\Rightarrow$ Relate **hadronic masses** to fundamental QCD parameter Λ_{QCD}.

Connecting κ to Λ_{QCD}

pQCD and LFHQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)

$$\frac{\alpha_s^{LFH}(Q^2)}{\pi} = e^{-Q^2/4\kappa^2}$$



$$\frac{\alpha_s^{pQCD}(Q^2)}{\pi} = \frac{4}{\beta_0 \ln(Q^2/\Lambda^2_{QCD})}(1+\text{higher orders})$$

$\kappa = \Lambda_{QCD}\, e^{(a+1)}(a/2)^{1/2}$ At LO

$a=4(\sqrt{\ln(2)^2-1+\beta_0/4}-\ln(2))/\beta_0$

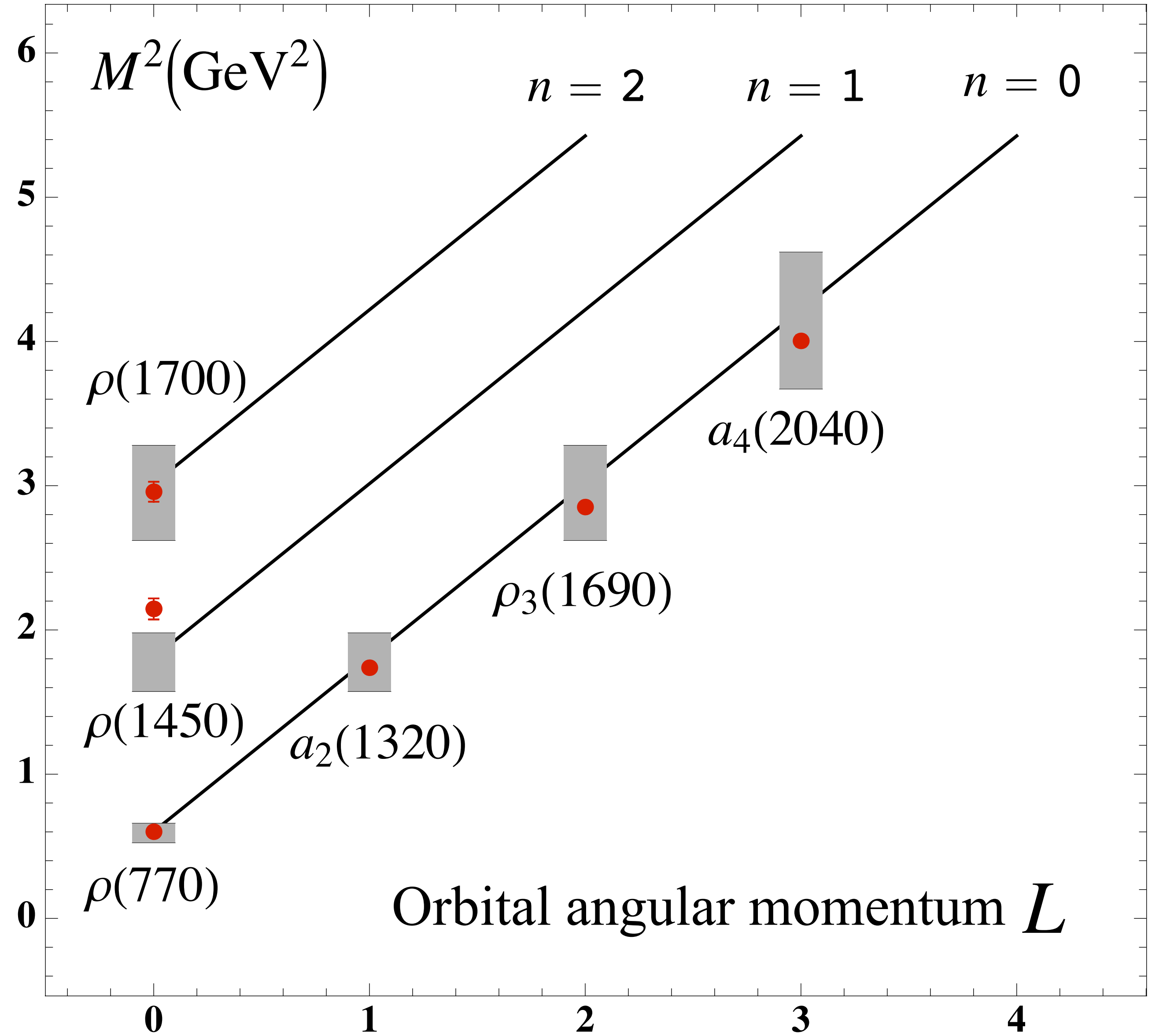
$\kappa = 1.607\Lambda_{\overline{MS}}$ At N3LO

Deur, Brodsky, de Téramond, PLB 750, 528 (2015)

Predictions of the hadronic mass spectrum

$\kappa = 1.607\Lambda_{\overline{MS}}$

$\kappa = M_\rho/\sqrt{2}$



: LFHQCD predictions with Λ_{QCD} from Part. Data Group as only input.

: Slopes predicted by LFHQCD.

: Measurements.

Baryon spectrum obtained from hadronic supersymmetry or from proton mass.

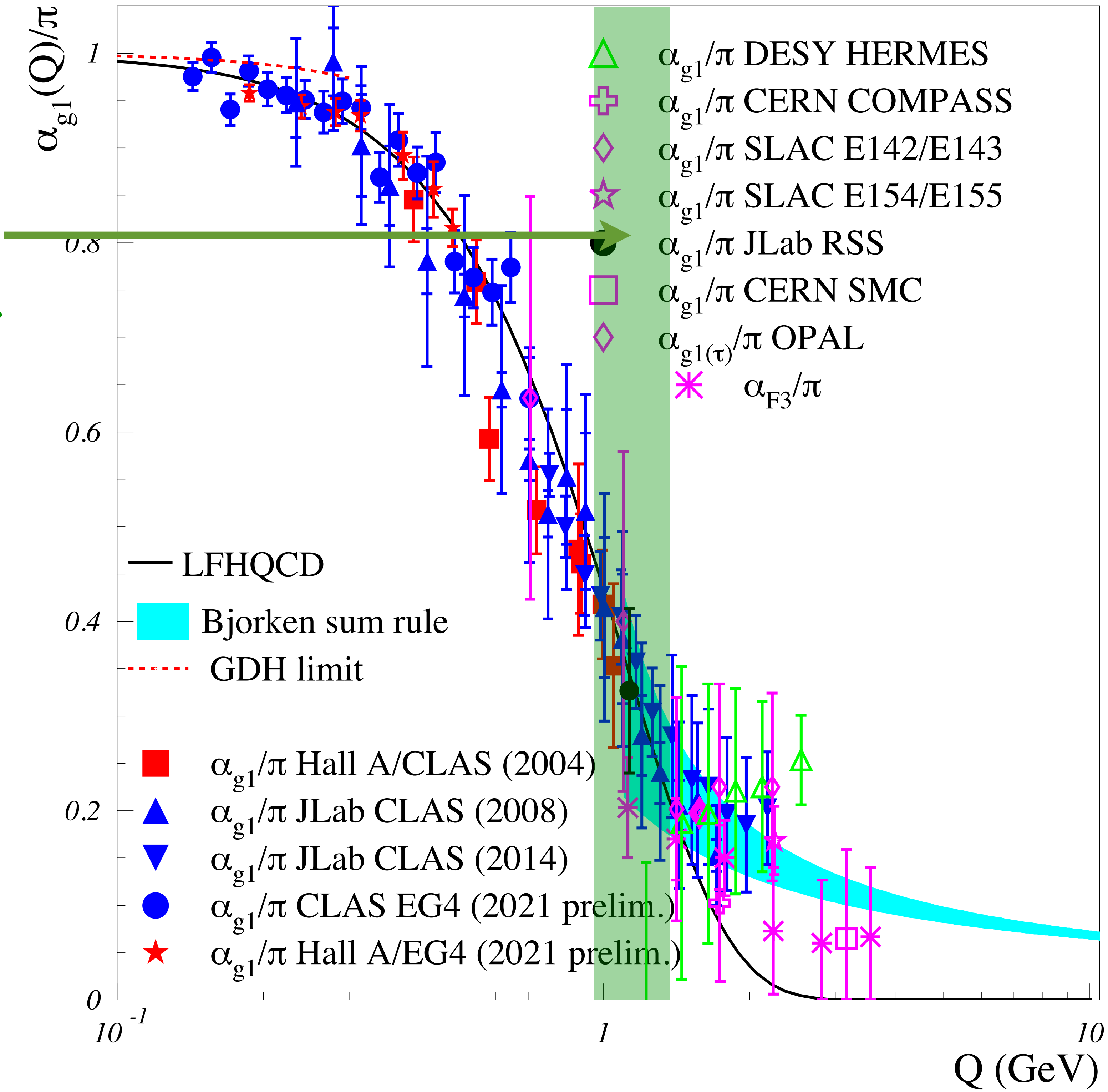
Brodsky, de Téramond, Dosch and Lorcé, Int. J. Mod. Phys. A 31, 1630029 (2016)

For hadrons with heavy valence quarks, heavy mass quarks are also needed as input.

Analytic determination of hadron spectrum with Λ_{QCD} as only input (+heavy quark mass if needed)

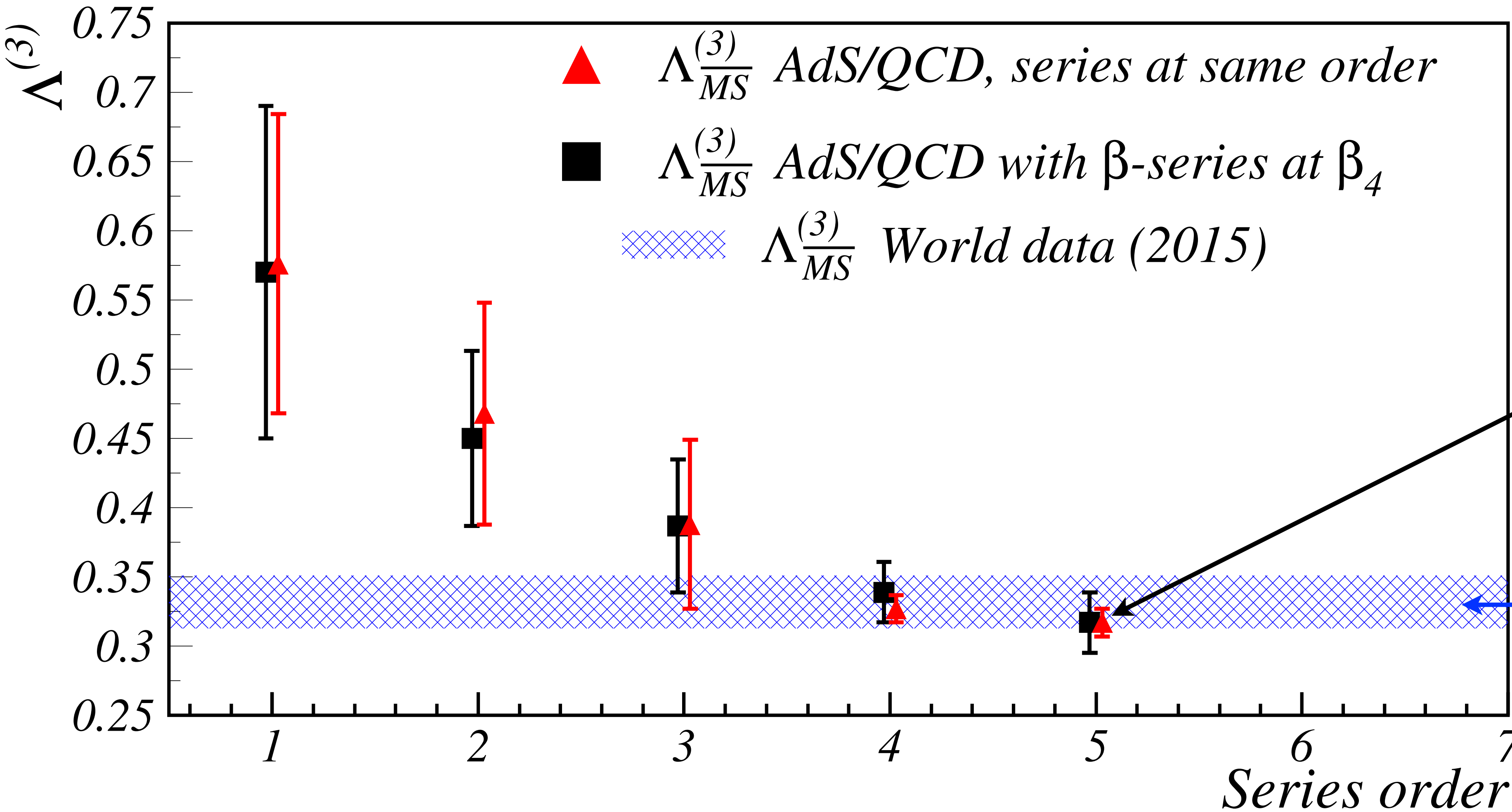
Predictions of the hadronic mass spectrum

pQCD and LFHQCD both provide a good description of α_{g1} (i.e. the Bjorken sum)



Conversely, one can use κ, from hadron masses or form factors and apply the same matching procedure to predict QCD's fundamental parameter Λ_{QCD}.

Prediction of Λ_{QCD} from hadronic observable



Λ^{AdS}_{QCD} =0.339(19) GeV

Deur, Brodsky, de Téramond, J. Phys. G 44,10, 105005 (2017)

Λ^{PDG}_{QCD} =0.332(19) GeV

Particle Data Group, 2015

n_f=3,
5-loop α_s calculation,
κ from hadron masses.

Determination of Λ_{QCD} in excellent agreement with PDG world average and with similar uncertainty.

Summary

- α_s is the fundamental parameter of QCD.
- Meaning of coupling constant and running coupling (including in non-pert. domain).
- Bjorken Sum Rule is advantageous to define an effective coupling α_{g1}.
- Data and sum rules allow us to know α_{g1} at all Q^2.
- α_{g1} freezes at low Q^2 $\Rightarrow$ Application of AdS/CFT to non-perturbative QCD.
- α_s obtained with LFHQCD.
  - Form imposed by respecting QCD's basic (approximate) symmetries: either conformal symmetry of QCD Lagrangian (mass scale emerging in QCD's Action: dAFF mechanism), or chiral symmetry (massless pion). Arises also from observed hadronic supersymmetry.
  - No free parameters (uses only one parameter, κ, known from very different phenomenology).
  - Remarkable agreement with α_{g1} data and recent SDE calculation.
  - Analytic determination of hadron spectrum with Λ_{QCD} as the only input.
  - High precision determination of Λ_{QCD}.

Coupling constants

When charges are quantized: (coupling constant)$^{1/2}$ normalizes the unit charge to 1 (e.g. $\alpha_s = g^2/4\pi$; $\alpha = e^2/4\pi$).

$\Rightarrow$ set the magnitude of the force (classical domain) or the probability amplitude to emit a quantum force vector (QFT).

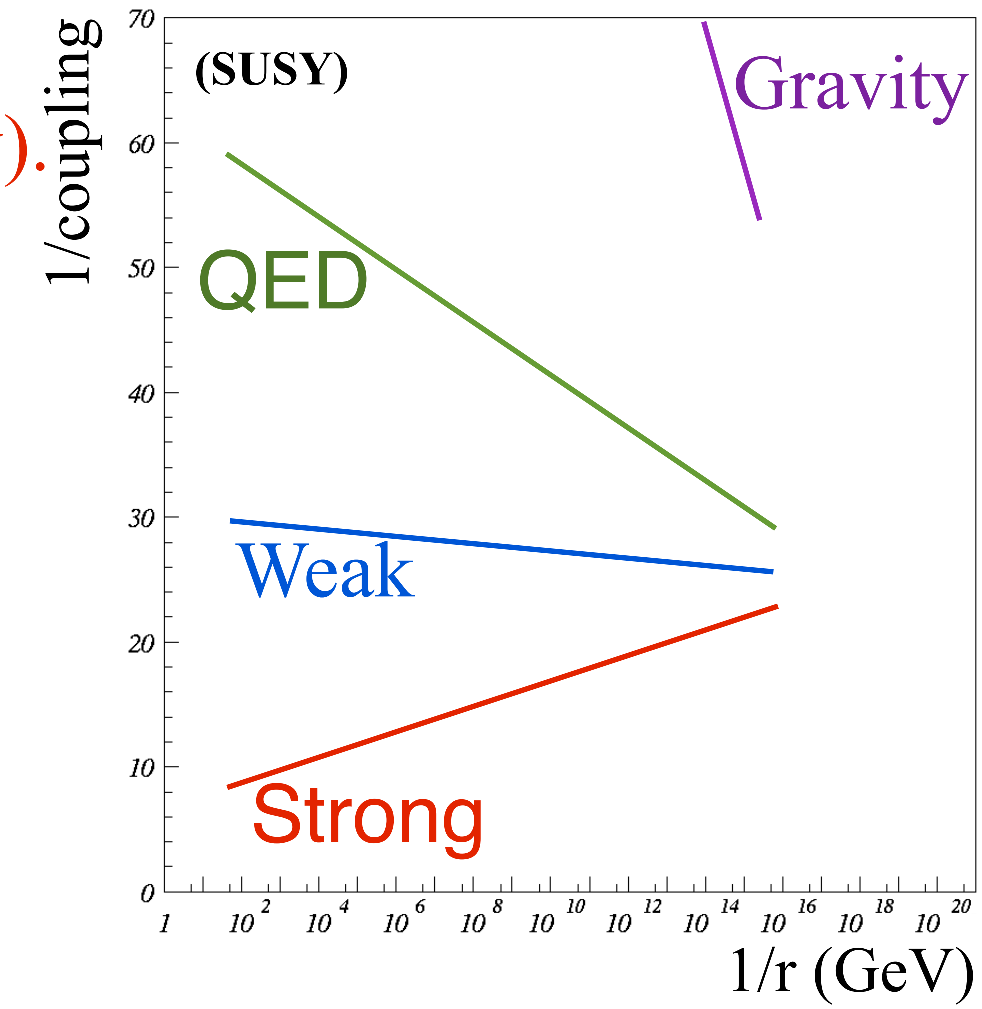
Force=coupling constant×charge$_1$×charge$_2$×f(r)

(static case)

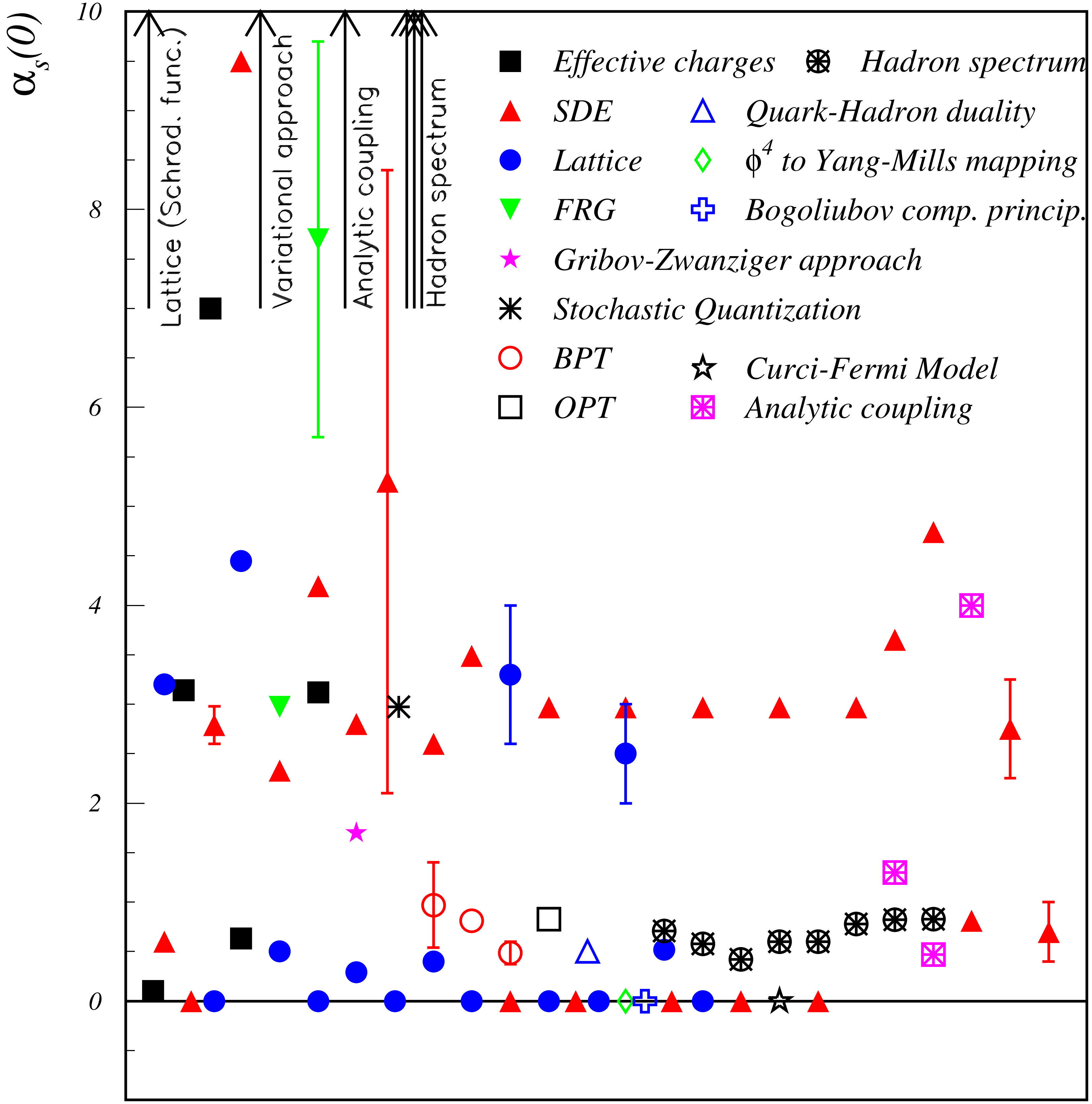
α (QED), α_s (QCD), G_F (Weak Force), G_N (gravity).

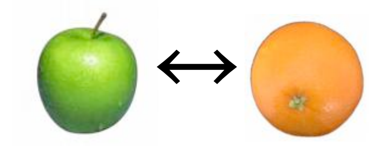
Quantum effects induce an energy dependence.

(effective couplings: the couplings are "running")



The many values of $\alpha_s(0)$ (from literature)





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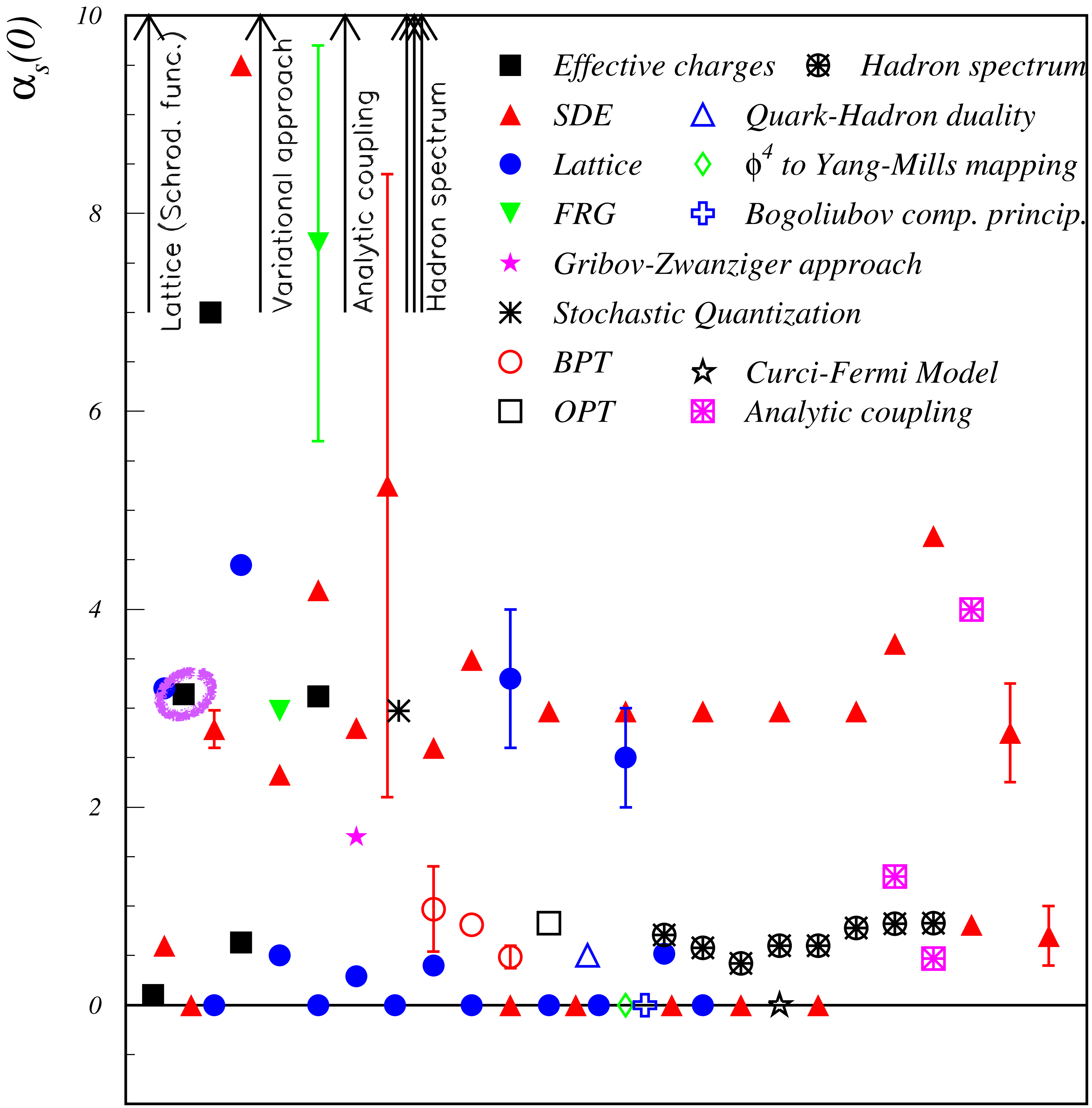
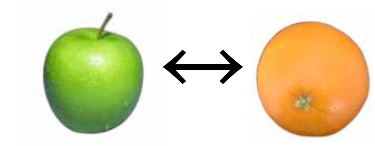
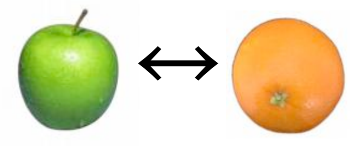
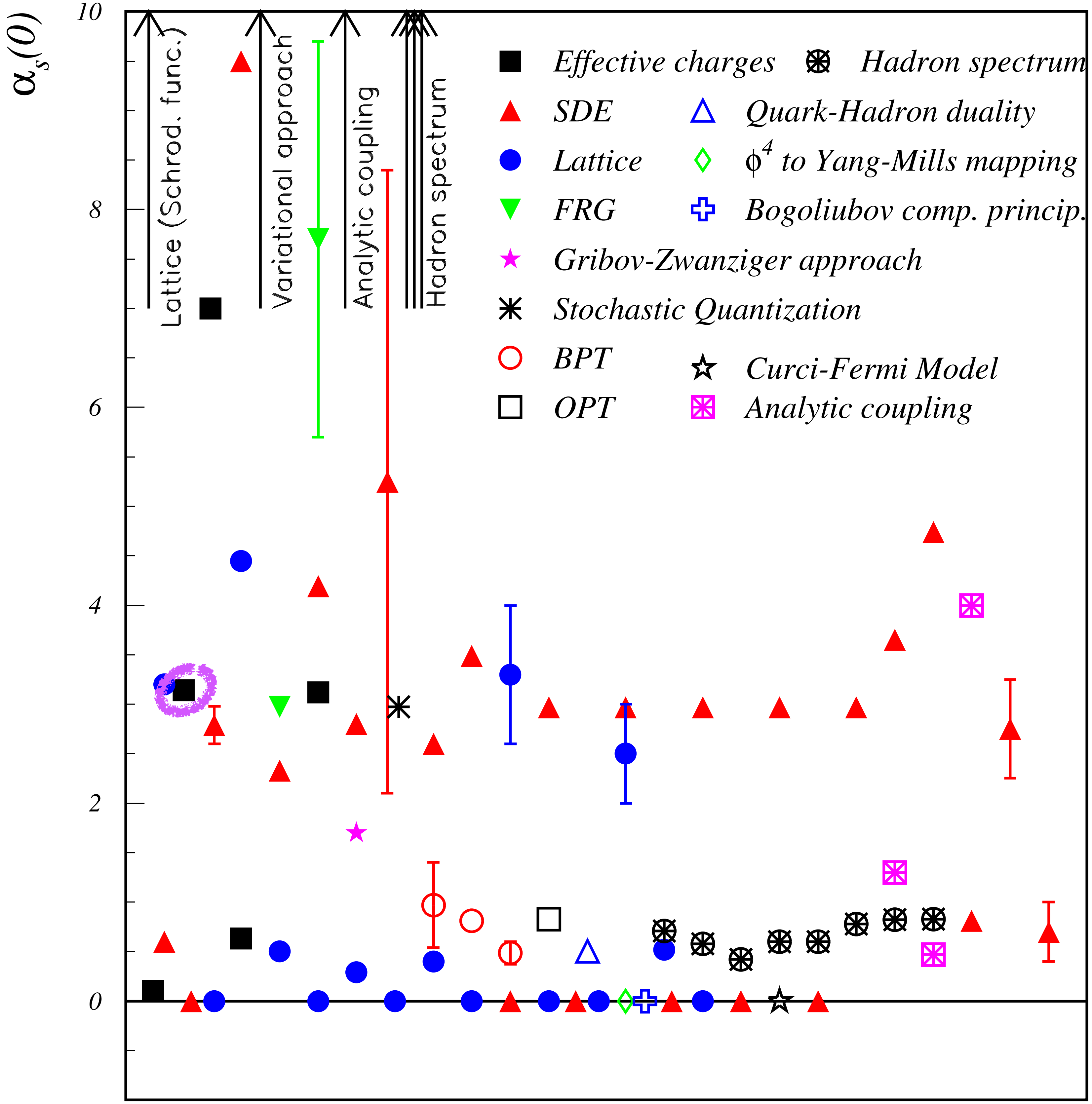


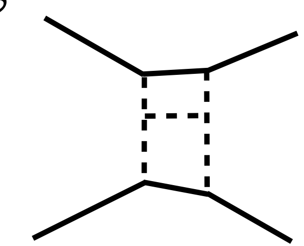






∞

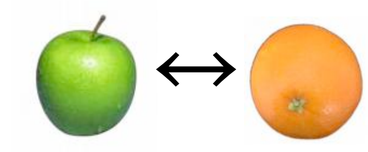
Mostly calculations using V scheme. (problematic because of multi-gluon H-diagram divergences)



Calculations mostly using MOM scheme.

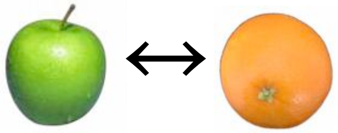
Calculations mostly using $\overline{\text{MS}}$ scheme.

(Separate and coexistent solution of SDE and Lattice. Unphysical?)



AdS/QCD results can be used to obtain $\alpha_s(0)$ in a given scheme: Use $\alpha_s(0)$ and Q_0 as free parameters rather than Λ_s (or κ or M_ρ) and Q_0.

$\Rightarrow$ Quantify scheme-dependence of $\alpha_s(0)$ in the non-perturbative domain.



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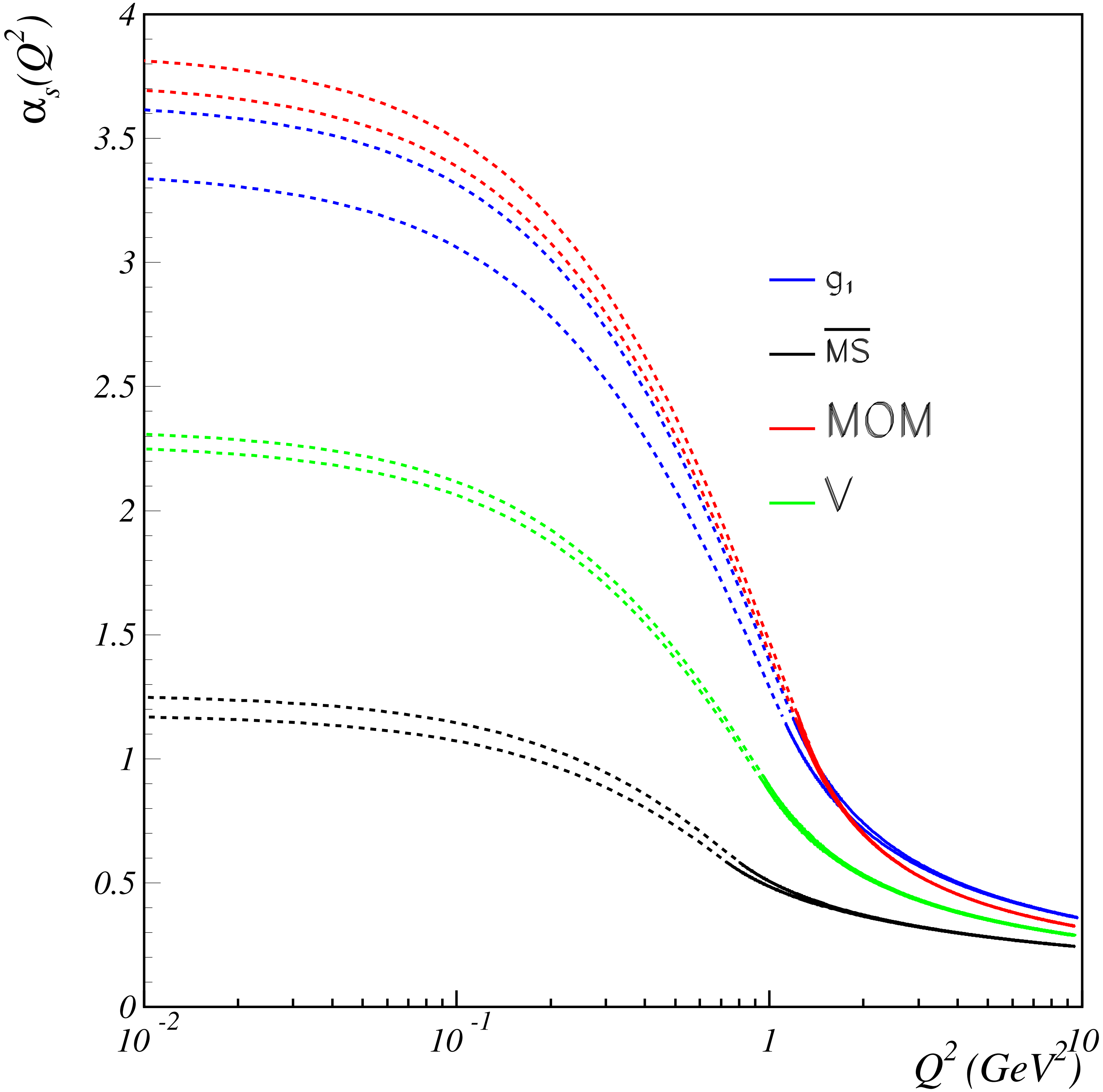
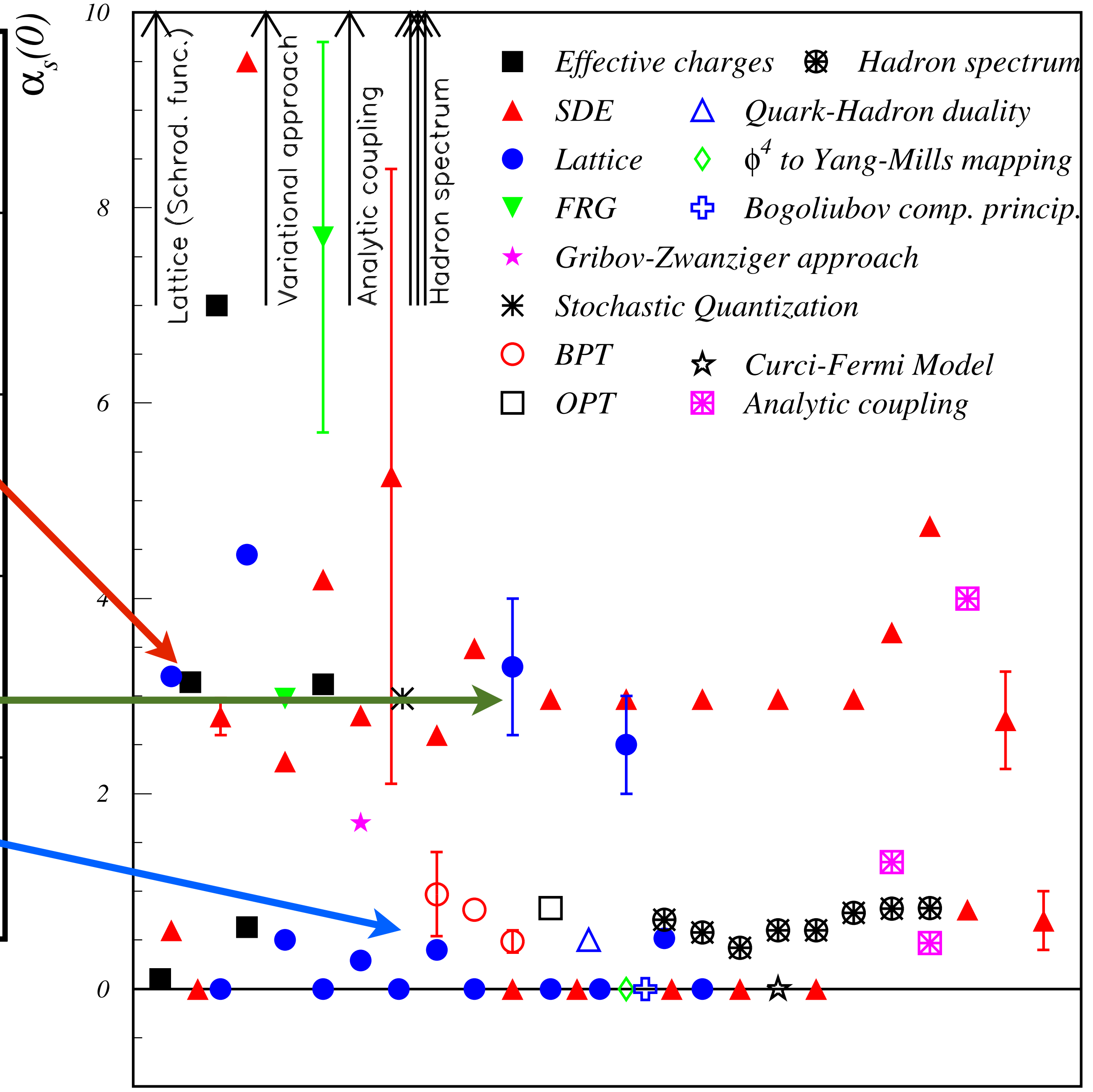


Comparison with literature

$\alpha_s(0)$	Scheme	Expectation from Literature
3.51±0.62 (n_f=3 at Q_0)	g_1	π
2.30±0.35 (n_f=3 at Q_0)	V	-
2.84±0.60 (n_f=0 at Q=0)	MOM	2.97 (n_f=0)
0.80±0.10 (n_f=0 at Q=0)	$\overline{\text{MS}}$	~0.6 (n_f=0)

Deur, Brodsky, de Teramond
arXiv:1601.06568



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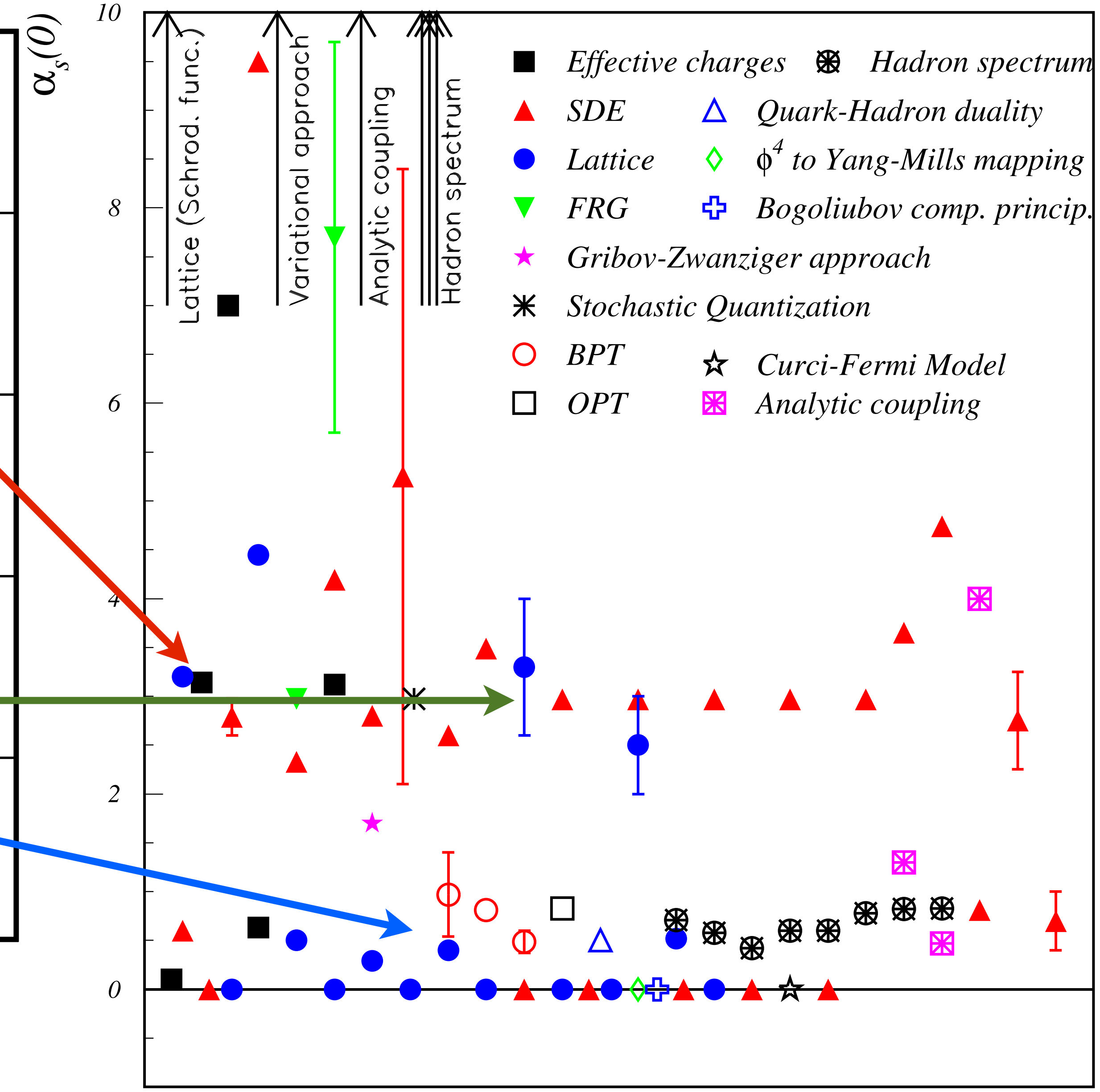
3.51±0.62 (n_f=3 at Q_0)	g_1	π
2.30±0.35 (n_f=3 at Q_0)	V	-
2.84±0.60 (n_f=0 at Q=0)	MOM	2.97 (n_f=0)
0.80±0.10 (n_f=0 at Q=0)	$\overline{\text{MS}}$	~0.6 (n_f=0)



Deur, Brodsky, de Teramond
arXiv:1601.06568

Also compatible with $\alpha_s(0)\rightarrow\infty$ results: based on $V\propto r$: linear static quark-quark potential. AdS/QCD harmonic oscillator potential on Light-Front form equivalent to linear potential in usual frame (Instant-Front form).

$\Rightarrow$ Discrepancy in non-perturbative α_s behavior seen in literature can be explained by scheme-dependence, mismatch in coordinate system used.