Domain walls in confining theories, holography and extended hydrodynamics

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RJ, M. Järvinen, J. Sonnenschein 2105.XXXX

Outline

Motivation
- Long-term goal
- Some recent developments
- Goal for this work

Witten model

The Aharony-Minwalla-Weisman domain wall solution

Modeling the energy-momentum tensor

Formulation in terms of an action

Application: Thermodynamic nucleation probability

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Understand passage through phase transitions during real time evolution

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  2. The high temperature phase is described by an AdS black hole
- These are two distinct (euclidean) backgrounds, the phase transition occurs when equating the free energies..
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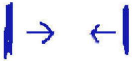

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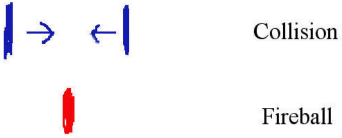

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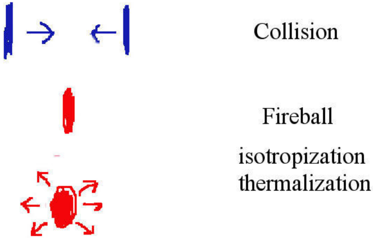

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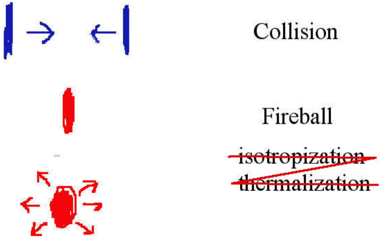

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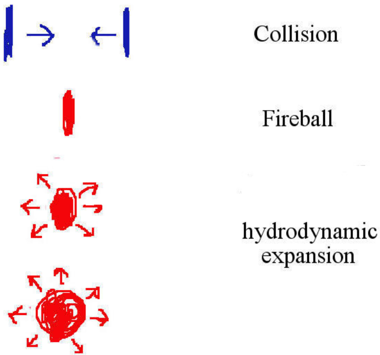

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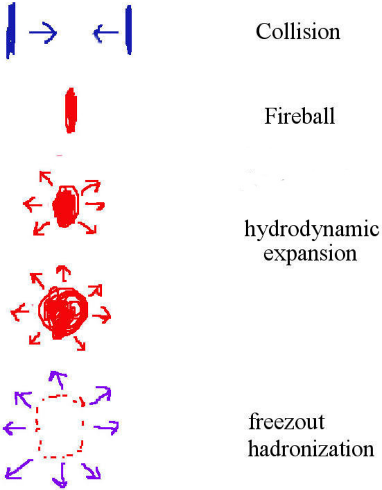

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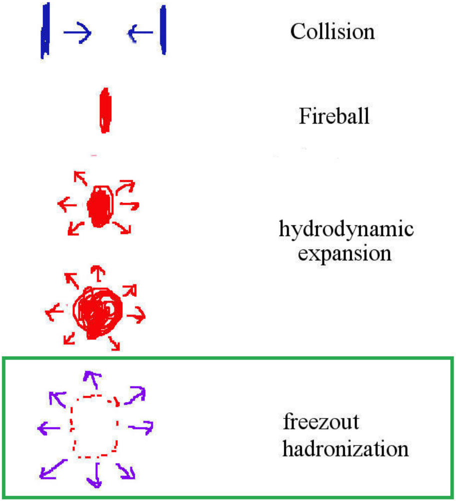

Some recent (motivating) developments...

Dynamics in a holographic theory with a 1^{st} order phase transition...

RJ, Jankowski, Soltanpanahi, Belladuono

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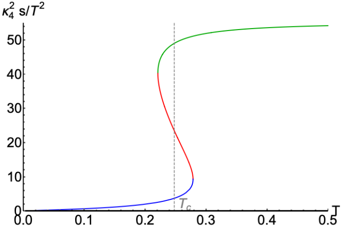

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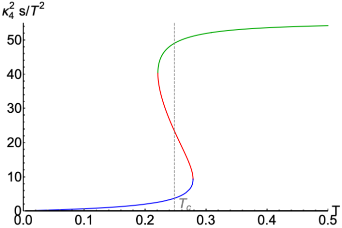

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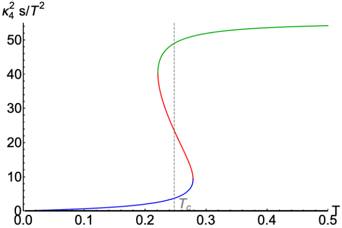

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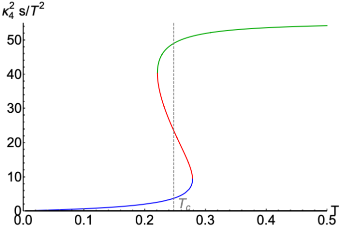

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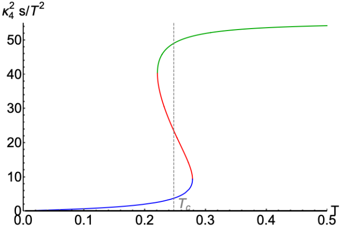

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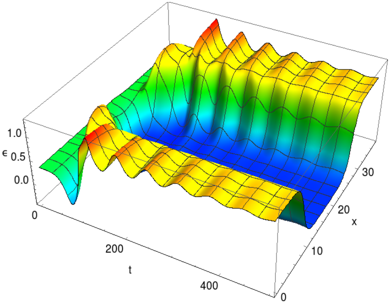

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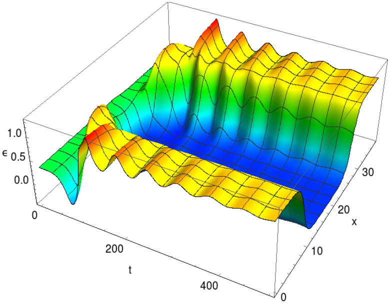

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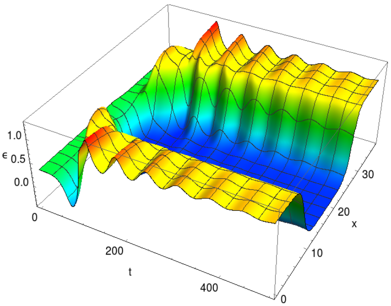

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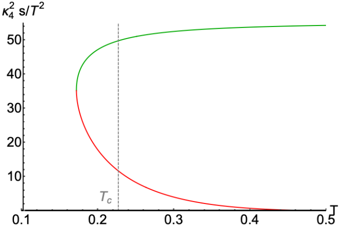

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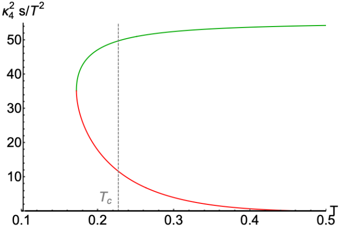

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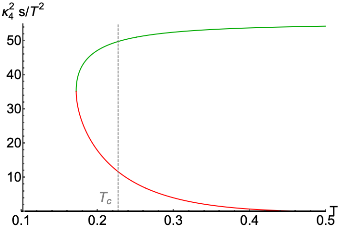

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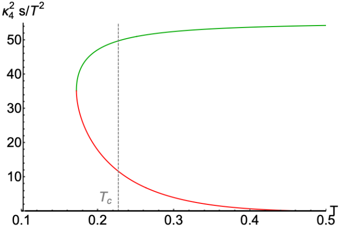

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- On the boundary one compactifies a coordinate (ϕ) on a circle and imposes anti-periodic boundary conditions for the fermions.
- At low temperatures the bulk geometry of the ϕ circle closes off into a cigar, generating confinement
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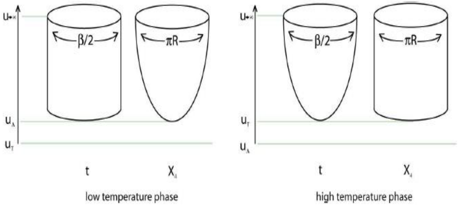

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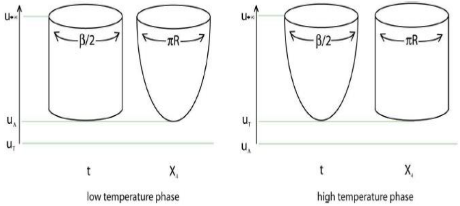

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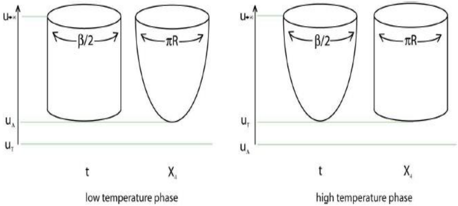

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- AMW constructed numerically a static planar domain wall solution interpolating between confined and deconfined phases
- The numerical relativity setup is very nontrivial due to the different topologies of the geometries corresponding to the different phases

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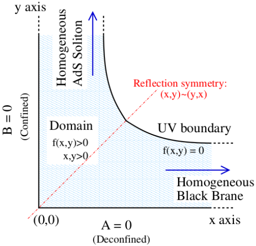

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Aharony, Minwalla, Weisman '05

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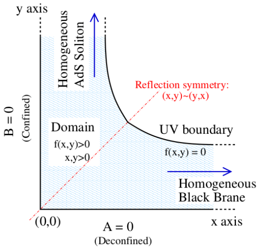
- The numerical relativity setup is very nontrivial due to the different topologies of the geometries corresponding to the different phases



- It turns out that **the physical content** of the solution looks extremely simple from the point of view of the boundary field theory...

What is the physical content of a given geometry?

- From a given geometry we can extract the profile of the energy-momentum tensor of the boundary theory
- For a 5D bulk we have

$$ds^2 = \frac{g_{\mu\nu}(x,z)dx^\mu dx^\nu + dz^2}{z^2}$$

with

$$g_{\mu\nu}(x,z) = \eta_{\mu\nu} + \langle T_{\mu\nu}(x)\rangle z^4 + \ldots$$

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$\langle T_{\mu\nu}(x) \rangle$ for the AMW domain wall solution:

- $\frac{1}{2}(T_{tt} + T_{\phi\phi})$ from the numerical holographic AMW solution
- Excellent fit by

$$\frac{1}{2}\left(1 + \tanh \frac{q_* x}{2}\right)$$

- $T_{xx} - T_{yy}$ looks more nontrivial
- Can be fit by

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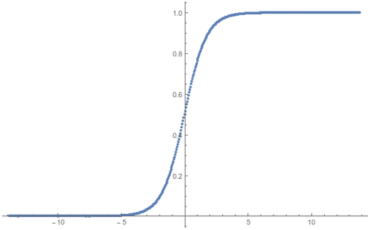

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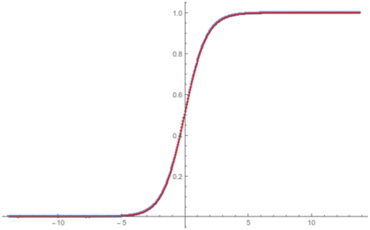

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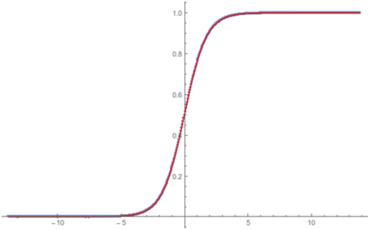

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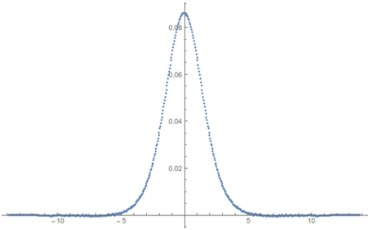

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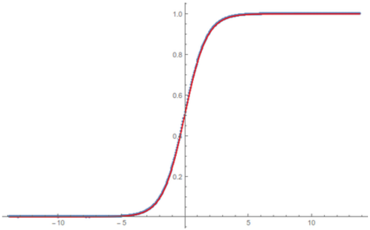

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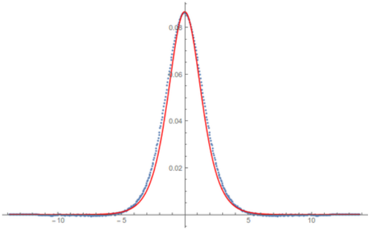

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- A gravity+scalar system with a phase transition between two types of plasma (3D theory) data from [RJ, Jankowski, Soltanpanahi]

- Independently observed for various holographic 4D theories (with phase transitions between deconfined phases)
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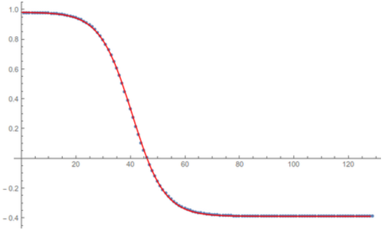

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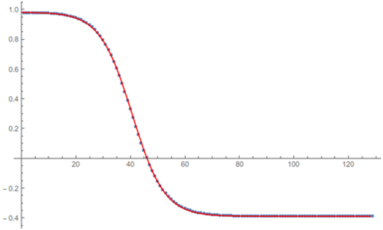

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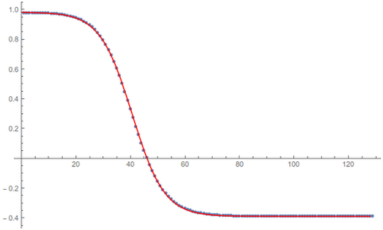

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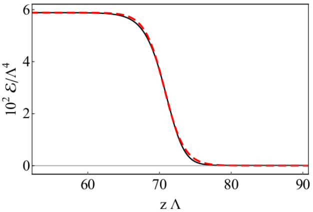

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- The surprising simplicity of $\langle T_{\mu\nu}(x) \rangle$, suggests that perhaps one can find a description purely in terms of the field theory energy-momentum tensor...
- For cases with **both deconfined phases**, Mateos et.al. proposed a hydrodynamic description...
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Construct a model directly for the energy-momentum tensor...

The deconfined phase

- In the deconfined phase we model the system by **hydrodynamics**
- We neglect dissipative terms and just keep the leading perfect-fluid part

$$T_{\mu\nu}^{deconf} = p_{hydro}(T)\,(\eta_{\mu\nu} + 4u_\mu u_\nu)$$

(here we used tracelessness, valid for the $d = 3$ Witten model, consequently $p_{hydro}(T) \propto T^4$)

- In the Witten model, on the boundary we have the auxiliary ϕ circle:
  1. We assume no dependence on ϕ
  2. We assume that no flow occurs in the ϕ direction

  $$u_\mu n^\mu = 0$$

  where n^μ is a unit vector in the ϕ direction

- Using fluid/gravity duality, the dual geometry looks like a locally boosted (in the direction of flow velocity u^μ) black hole

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Confined phase

- The confined phase energy-momentum tensor can be read off from the gravitational solution

$$T^{conf}_{\mu\nu} = \eta_{\mu\nu} - 4 n_\mu n_\nu$$

- In the physical 3D space (i.e. excluding the auxiliary ϕ circle) we have full Lorentz symmetry
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- The domain wall configuration should interpolate between the two energy-momentum tensors (here $p_{hydro}(T_c) = 1$)

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- The two energy-momentum tensors have quite a different form...
- Introduce a **new degree of freedom** $\gamma(x)$

$$T^{mix}_{\mu\nu}(x) = \gamma(x)\, T^{conf}_{\mu\nu}(x) + (1 - \gamma(x))\, T^{deconf}_{\mu\nu}(x)$$

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Does introducing $\gamma(x)$ make sense??

- It is instructive to first consider $\gamma(x)$ to be very small...

$$T^{mix}_{\mu\nu}(x) = \gamma(x)\, T^{conf}_{\mu\nu}(x) + (1-\gamma(x))\, T^{deconf}_{\mu\nu}(x)$$

- The dual geometry will be a black hole with a small perturbation $\propto \gamma(x)$...
- At the linearized level, this will be a *slightly non-standard* quasi-normal mode...
- Since the domain wall builds up exponentially

$$\gamma(x) \sim e^{q_* x}$$

  the QNM will have purely imaginary momentum and vanishing frequency (static configuration)
- Such QNM's were first introduced by Sonner in the context of domain walls...
- Since this gravitational degree of freedom is very much relevant for the transition between the two phases, we should build it in into the desired effective description

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- At the linearized level, this will be a *slightly non-standard* quasi-normal mode...
- Since the domain wall builds up exponentially

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Is $T^{mix}_{\mu\nu}(x)$ enough to describe the domain wall?

$$T^{mix}_{\mu\nu}(x) = \gamma(x)\, T^{conf}_{\mu\nu}(x) + (1-\gamma(x))\, T^{deconf}_{\mu\nu}(x)$$

Answer: No

We are missing contributions localized at the domain wall...

- $T_{xx} - T_{yy}$ is an example ($x \perp$, $y \parallel$ domain wall)
- Responsible for the **surface tension** of the domain wall

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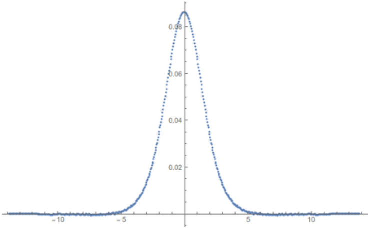

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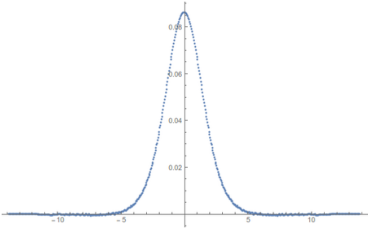

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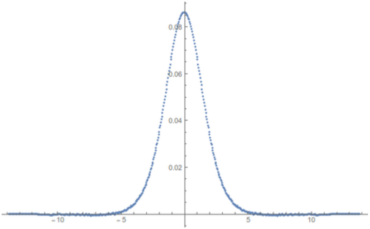

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How to determine $T^{\Sigma}_{\mu\nu}(x)$?

- We do not have any guidance from known phases...
- We have an additional unit vector perpendicular to the domain wall v^{μ}...
- Build up the most general expression from elementary tensors

$$\eta_{\mu\nu}, \qquad u_\mu u_\nu, \qquad v_\mu v_\nu, \qquad n_\mu n_\nu$$

- We know from the AMW solution that nondiagonal combinations do not appear (unless proportional to $(u \cdot v)$)
- We get

$$T^{\Sigma}_{\mu\nu} = \Sigma \left(-\eta_{\mu\nu} + A v_\mu v_\nu - B u_\mu u_\nu - C n_\mu n_\nu \right)$$

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- For the planar AMW domain wall in the x direction, energy-momentum conservation implies that

$$T_{xx} = const \qquad \Longrightarrow \qquad T^{\Sigma}_{xx} = 0$$

- It follows that $A = 1$
- From tracelessness, we get $C = B - 3$
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- Σ can be obtained by comparing with $T_{xx} - T_{yy}$

$$\Sigma = c \cdot \frac{\gamma'^2}{\gamma(1-\gamma)}$$

- The integral of Σ is the domain wall **surface tension**
- Subsequently B can be obtained from any other component e.g. T_{tt} It turns out that

$$B = 1 + \gamma$$

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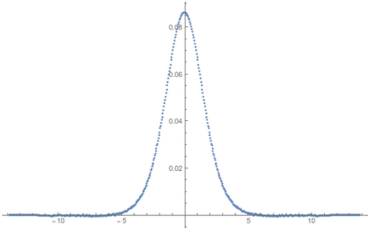

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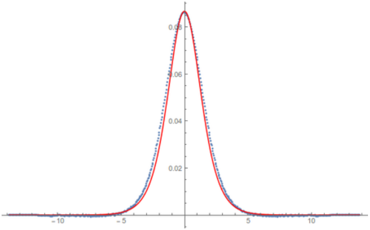

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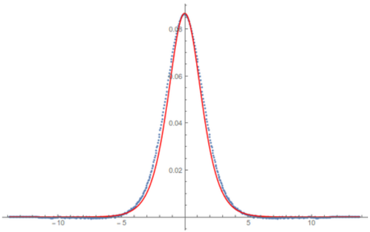

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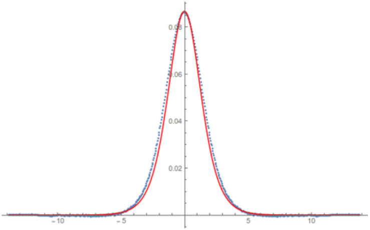

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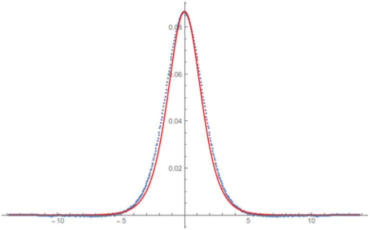

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with

$$\gamma(x) = \frac{1}{2}\left(1 + \tanh\frac{q_* x}{2}\right)$$

Questions:

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2. Can we write an **action** for γ so that $T_{\mu\nu}^{\Sigma}$ will arise as the corresponding energy-momentum tensor?

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$$\underbrace{\gamma T_{\mu\nu}^{conf} + (1-\gamma) T_{\mu\nu}^{deconf}}_{T_{\mu\nu}^{mix}} + \underbrace{\frac{c\gamma'^2}{\gamma(1-\gamma)} \left(-\eta_{\mu\nu} + v_\mu v_\nu - (1+\gamma) u_\mu u_\nu + (2-\gamma) n_\mu n_\nu\right)}_{T_{\mu\nu}^{\Sigma}}$$

with

$$\gamma(x) = \frac{1}{2}\left(1 + \tanh\frac{q_* x}{2}\right)$$

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$$\gamma(x) = \frac{1}{2}\left(1 + \tanh\frac{q_* x}{2}\right)$$

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- This is a solution of the equations of motion for an action

$$\mathcal{L}_\gamma = -a(\gamma)\left(\frac{1}{2}(\partial\gamma)^2 + V(\gamma)\right)$$

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- Dubovsky, Hui, Nicolis, Son considered an action formulation for hydrodynamics, however it convenient to use a reformulation by Haehl, Loganayagam, and Rangamani which reproduces the holographic Euclidean on-shell action...
- Recall the hydrodynamic energy-momentum tensor

$$T_{\mu\nu}^{deconf} = p(T)\left(\eta_{\mu\nu} + 4u_\mu u_\nu\right)$$

- The degrees of freedom are T and the flow velocity u^μ (normalized as $u^2 = -1$)
- In the action formulation, one uses instead an *unnormalized* vector field β^μ whose length is related to the temperature

$$T = \frac{1}{\sqrt{-g_{\mu\nu}\beta^\mu\beta^\nu}} \qquad u^\mu = T\beta^\mu$$

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- The linear combination of the confining and deconfined energy-momentum tensors

$$T_{\mu\nu}^{mix} = \gamma T_{\mu\nu}^{conf} + (1-\gamma) T_{\mu\nu}^{deconf}$$

follows from the Lagrangian

$$\mathcal{L} = (1-\gamma)p(T) + \gamma$$

- In order to couple the scalar field action for γ to hydrodynamic degrees of freedom, it is enough to add T dependence (recall $T \equiv 1/\sqrt{-g_{\mu\nu}\beta^\mu\beta^\nu}$)

$$\mathcal{L}_\gamma = -a(\gamma, T)\left(\frac{1}{2}(\partial\gamma)^2 + V(\gamma, T)\right)$$

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leads to the energy momentum tensor (evaluated at $T = T_c = 1$)

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With

$$a(\gamma) = \frac{const}{\gamma(1-\gamma)} \qquad\qquad \frac{3}{2}\alpha + \frac{1}{2}\beta = 1 + \gamma$$

we reproduce the expression fit to the AMW numerical domain wall solution...

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leading to the energy momentum tensor

$$T_{\mu\nu} = (1-\gamma)\,T_{\mu\nu}^{deconf} + \gamma\,T_{\mu\nu}^{conf} + T_{\mu\nu}^{\Sigma}$$

- The mixing terms in square brackets lead to an effectively asymmetric potential away from $T = T_c$
- We believe that the overall structure is very generic and should be applicable to numerous other contexts with a 1^{st} order phase transition

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Thermodynamic nucleation probability

- From equilibrium thermodynamics one can compute the probability of nucleation of a bubble of a different phase

  (c.f. Landau, *Statistical Physics*)

- The probability is given by a difference of thermodynamic potentials, which include a contribution of the surface tension of the interface...

$$\Omega_{before} = -P(V + V_{droplet}) \qquad \Omega_{after} = -PV - P_{droplet} V_{droplet} + \Sigma A$$

then

$$probability \propto e^{-\frac{1}{T}(\Omega_{after} - \Omega_{before})} = e^{-\frac{1}{T}(-(P_{droplet} - P)V_{droplet} + \Sigma A)}$$

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How to reproduce this in our framework?

Thermodynamic nucleation probability

- Coleman introduced an Euclidean picture of tunneling as instanton/bounce solutions in QFT...
- This was generalized by Linde '81 to the finite temperature case

- In case d) we need the Euclidean action of a static bubble with thermal periodicity
- The action density can be read off from the coefficient of $\eta_{\mu\nu}$ in $T_{\mu\nu}$

$$\mathcal{L}_E(x) = -P(x) + \Sigma(x)$$

- In the thin wall approximation we reproduce Landau's result e.g.

$$S_{after}^{on-shell} = \int \mathcal{L}_E(x) d\tau d^2x d\phi = -\frac{1}{T}\left(PV + P_{droplet}V_{droplet} - \Sigma A\right)$$

see also Bigazzi, Caddeo, Cotrone, Paredes for a gravitational perspective

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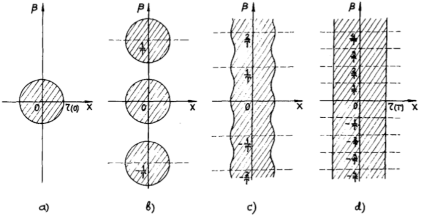

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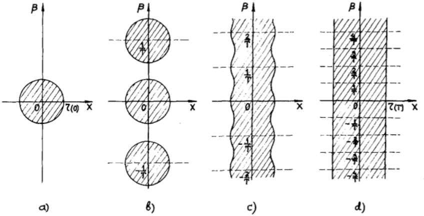

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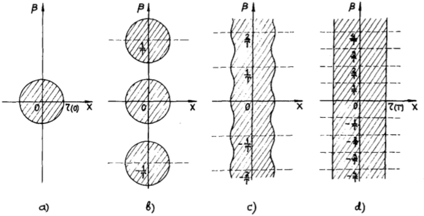

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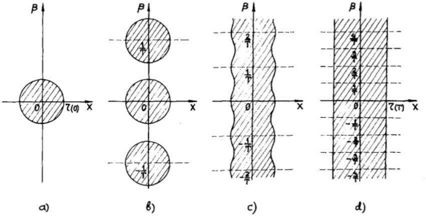

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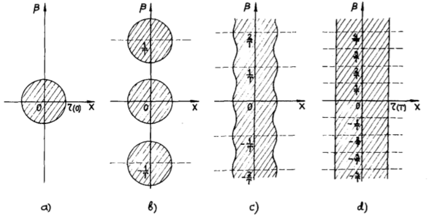

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Conclusions

- The structure of domain walls is much simpler than one could expect from the complicated numerical gravitational backgrounds
- This suggests that one could model them directly on the level of the boundary field theory energy-momentum tensor
- The incorporation of **confining** phases necessitates the introduction of an additional degree of freedom γ
- We proposed a structure of the energy-momentum tensor describing domains of both phases separated by domain walls
- We proposed an action for the scalar field γ coupled to hydrodynamic degrees of freedom
- The tanh profiles are obtained analytically from this model
- One needs more holographic solutions for going away from $T = T_c$ and taking into accounts effects of flow (terms like $u^\mu \partial_\mu \gamma$)
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