Lightfront field theory and intense laser physics

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LightCone seminar series

EPSRC
Engineering and Physical Sciences
Research Council

LEVERHULME TRUST

UNIVERSITY OF PLYMOUTH ⟶ Higgs Centre for Theoretical Physics THE UNIVERSITY of EDINBURGH

PRD 92 (2015) 025009, PRL 118 (2017) 11, PRX 7 (2017) 041003, PLB 782 (2018) 22, PRX 8 (2018) 011020, PRD 102 (2020) 076013, JHEP 09 (2020) 200

Outline

1. Overview: lightfront field theory & intense lasers.
2. Non-perturbative pair production & lightfront zero modes.

(3. Yang-Mills and Gravity.)

Intense lasers & strong fields

- Laser-electron. 
- Laser parameters: Energy $\omega \sim 1$ eV Duration $\sim 10^{-15}$ s
  Spot size $\sim \mu$m^2
- Intensity $\sim 10^{22}$ W/cm^2: huge photon flux.

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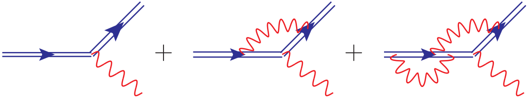
- a_0 = coupling to laser $\implies$ non-perturbative physics.

Strong-field QED

- QED + background field A_{bg} describing laser:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\partial\!\!\!/ - m)\psi - e\bar{\psi}A\!\!\!/\psi - e\bar{\psi}A\!\!\!/_{\text{bg}}\psi \\ &= \bar{\psi}[i\partial\!\!\!/ - eA\!\!\!/_{\text{bg}} - m]\psi - e\bar{\psi}A\!\!\!/\psi\end{aligned}$$

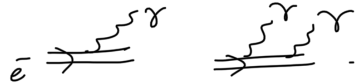
- Calculate amplitudes in Furry expansion. Furry PR 81 (1951) 115
  (background field perturbation theory).



- Coupling $a_0 \sim eA_{\text{bg}}$: treated exactly.
- QED coupling α: perturbation theory.
- Double line: all-orders interaction with background.

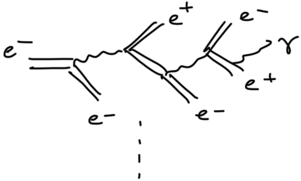
Laser-particle collisions

- Emission of radiation

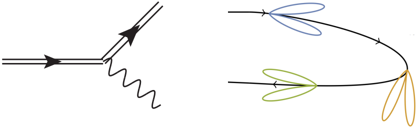


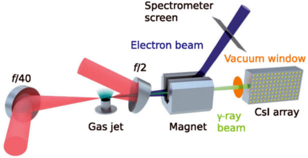
- Pair production

- Cascades



Experiment

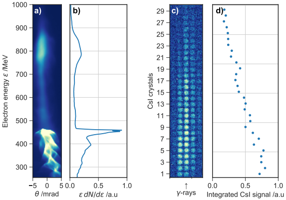




- Radiation & back-reaction in laser-particle collisions.
- At $a_0 \sim 10$

Cole et al, PRX 8 (2018) 011020

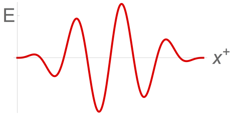
- Simultaneous measurement of e^- & γ.

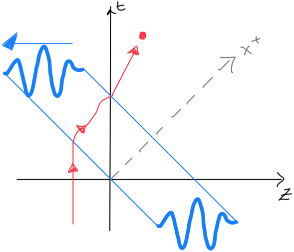


- Consistent with quantum models.
- Radiation reaction.

Laser → plane wave

- Plane waves in QED, YM and gravity.





  - ‘Lightfront time’ $n \cdot x = x^+ \equiv t + z$
  - Univariate: $A_\mu \equiv A_\mu(n \cdot x)$
  - Transverse: $n \cdot A = 0$.

- Captures laser strength a_0, frequency & temporal profile.
- Laser: a coherent state of zero modes, $p^+ = 0$!

Heinzl, **AI**, Seipt PRD 98 (2018) 016002

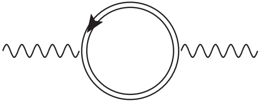
- Beyond plane waves: hot topic.

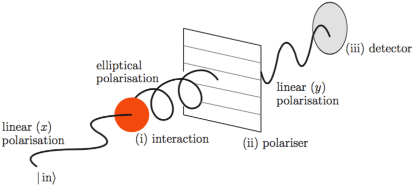
Heinzl, **AI** PRL 118 (2017) 11

Hu, **AI**, Zhao PRD 102 (2020) 016017

Laser-laser collisions

- Collide: intense optical laser & x-ray 'probe'.





- Probe polarisation: linear ⟶ elliptic. Heinzl et al Opt.Comm. 267 (2006)

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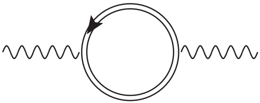
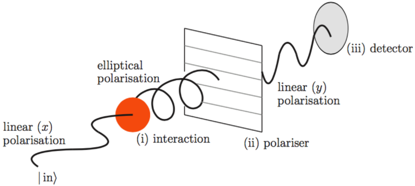




- Probe polarisation: linear ⟶ elliptic. Heinzl et al Opt.Comm. 267 (2006)

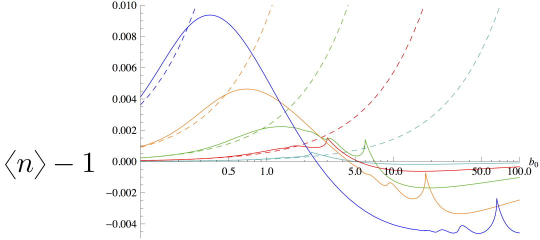
- The quantum vacuum, exposed to intense light, is birefringent.

Toll, PhD thesis, 1952

(Vacuum) birefringence

- ‘Vacuum’ refractive indices:

Dinu, Heinzl, **AI**, Marklund, Torgrimsson PRD 90 (2014) 045025

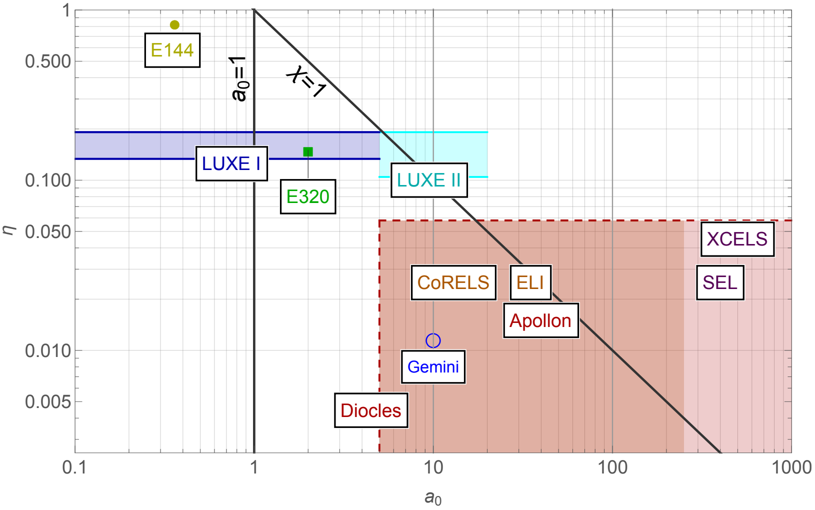


- $a_0 = 0.5, 1, \ \ldots 8 \qquad b_0 = k^+_{\text{probe}} \omega_{\text{laser}} / m^2$
- Peaks: resonances at threshold lightfront momentum for pairs.
- Flagship experiment HIBEF @ EU-XFEL.

Schlengvoit, Heinzl, et al Phys.Scr. 91 (2016) 023010

Experiment

LUXE experiment @DESY arXiv: 2102.02032 [hep-ex]



Intensity: a_0. Energy: $\eta = p^+\omega_{\text{laser}}/m^2$. $\chi = a_0\eta$

Part 2

Non-perturbative pair production

- A sufficiently strong electric field E can collapse into pairs:

$$\frac{\mathbb{P}}{VT} \sim (eE)^2 \exp\left[-\pi\frac{m^2}{eE}\right]$$

- Non-perturbative effect.

Sauter, Schwinger

- Exponential supression below $E_S := m^2/e$

  Would require multiple laser pulses.

Bulanov et al PRL 104 (2010) 220404, Gonoskov et al PRL 111 (2013) 060404

! Assumes a constant & homogeneous field.

? How to go beyond constant fields?

Schwinger effect in non-constant fields

- **L**ocally **C**onstant **F**ield **A**pproximation?

$$\mathbb{P}_{\text{pairs}} \sim VTE^2 \exp\Big[-\frac{\pi E_S}{E}\Big]$$

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$$\mathbb{P}_{\text{pairs}} \sim VTE^2 \exp\left[-\frac{\pi E_S}{E}\right] \xrightarrow{?} \int \mathrm{d}^4x\, E^2(x) \exp\left[-\frac{\pi E_S}{E(x)}\right]$$

Q. When does this work?

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Q. When does this work?

A1. Often a good approximation.

Schneider et al PRD 98 (2018)

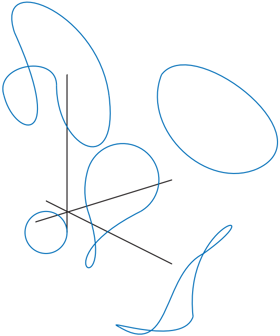
A2. Not exact in general.

Nikishov NPB 21 (1970)
Narozhnyi & Nikishov Sov.J.Nucl.Phys. 11 (1970)
Dunne et al PRD73 (2006)

A3. Only exact for $E = E_z(x^+)$ . . . why?

Tomaras et al PRD 62 (2000)

The effective action



- Probability of pair production:

$$\mathbb{P}_{\text{pairs}} = 1 - e^{-2\text{Im}\,\Gamma} \simeq 2\text{Im}\,\Gamma$$

- One-loop worldline representation[1]:

- $$\Gamma = \int_0^\infty \frac{\mathrm{d}T}{T} \oint_{\text{loops}} \mathcal{D}^4x \; e^{-iS}$$

- S: classical particle action, $x^\mu(\tau)$ in background $F_{\mu\nu}(x)$.
- Our background: $E_z(x^+)$

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[1] Feynman PR 80 (1950) 440, Schwinger PR 82 (1951) 664, Affleck et al., NPB 197 (1982) 509, Bern & Kosower NPB 379 (1992) 451, Strassler NPB 385 (1992) 145, Schubert APPB 27 (1996) 3965

Localisation

- Vacuum trivial up to lightfront zero modes: $p^+ = 0$.
- Start evaluating path integrals. Perp components straightforward.

$$\oint \mathcal{D}x^+ \mathcal{D}x^- e^{iS} = \oint \mathcal{D}x^+ \, \delta[\dot{x}^+(\tau)] \cdots = \int \mathrm{d}x^+ \cdots$$

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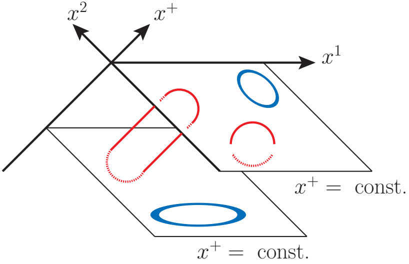
- $p^+ \leftrightarrow m\dot{x}^+$!
- Only zero modes contribute.

→ localisation.

- …why this simplicity?



Semiclassical approximation

- Semiclassical approx: $\Gamma \sim \exp\Big[-iS(x_{\text{cl}})\Big]$

$$m\ddot{x}^{\mu}_{\text{cl}} = eF^{\mu}{}_{\nu}(x_{\text{cl}})\dot{x}^{\nu}_{\text{cl}}$$

- Classical periodic path: Instanton. Complex-valued.[2]

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[2] Lavrelashvili et al NPB 329 (1990), Kim & Page PRD 75 (2007), Bender et al PRL 104 (2010), Dumlu & Dunne PRD 84 (2011), AI Torgrimsson Wårdh PRD 92 (2015)

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$$q = \lambda\, t + \sqrt{1-\lambda^2}\; z\;, \qquad \lambda = 1 \downarrow 1/\sqrt{2}$$

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- Example: $\boxed{E \equiv E_0\, \mathsf{sech}^2(\omega q)}$

$$-iS(x_{\mathsf{cl}}) \sim \oint \mathrm{d}q\; \dot{q} \qquad \dot{q}^2 = \lambda + a_0^2 \tanh^2(\omega q)$$

- Contour integral over the instanton in the complex plane.

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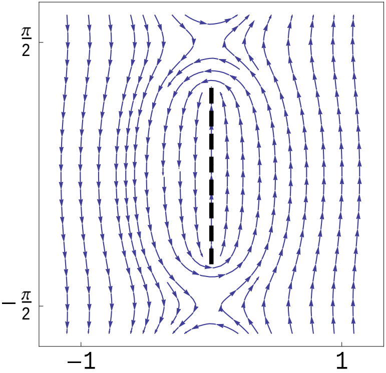
Interpolating instantons

Complex $q(\tau)$ plane

$$\boxed{E \equiv E(q)\ , \quad q = t + 0z}$$

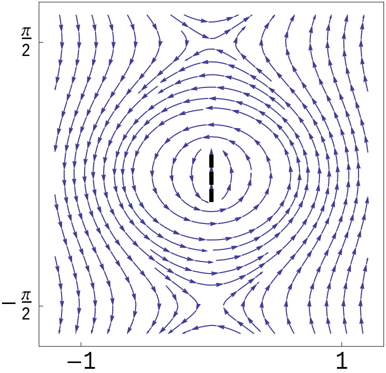


- Time-dependent.
- Instanton circles a branch.
- Extended / nonlocal.
- Exact results in $E(t)$:
  not LCFA.

Interpolating instantons

Complex $q(\tau)$ plane

$$\boxed{E \equiv E(q), \quad q = 0.9t + 0.4z}$$

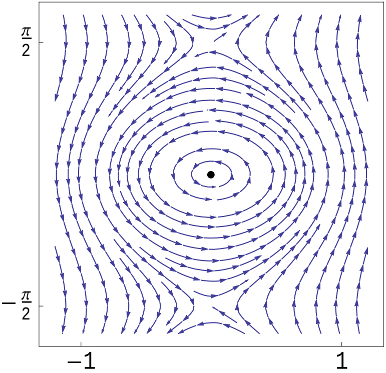


- Time & space dependent.
- Branch points move closer.
- Extended / nonlocal

Interpolating instantons

Complex $q(\tau)$ plane

$$\boxed{E\equiv E(q)\ ,\quad q=(t+z)/\sqrt{2}}$$



- q: lightfront time.
- Branch cut → pole!
- Instantons → points (Cauchy)

  AI Torgrimsson Wårdh PRD 92 (2015) 025009
- Localisation: LCFA exact.

  AI Torgrimsson Wårdh PRD 92 (2015) 065001

Double copy

- ‘Double copy’: colour → kinematics.

YM amplitudes $\boxed{\mathcal{M}^2 = \mathcal{A}}$ gravity amplitudes

- Beyond vacuum / flat backgrounds?

- Amplitudes: YM plane waves → plane wave spacetimes

$$\begin{aligned} \mathrm{d}s^2 &= \mathrm{d}x^+\mathrm{d}x^- - \mathrm{d}x^j\mathrm{d}x^j - H_{ij}(x^+)x^i x^j \mathrm{d}x^+\mathrm{d}x^+ \\ \mathrm{d}A &= -E_j(x^+)x^j\mathrm{d}x^+ \\ i,j &\in \{1,2\} \end{aligned}$$

- E_j: (colour) electric field. H_{ij}: curvature.

Double copy of classical & quantum radiation

- Classical colour radiation in YM.

Goldberger & Ridgway, PRD 95 (2017) 125010, Luna et al, JHEP 04 (2017) 069

- Generalise classical double copy to background fields.

Adamo & AI, JHEP 09 (2020) 200

- Consistency with quantum double copy.

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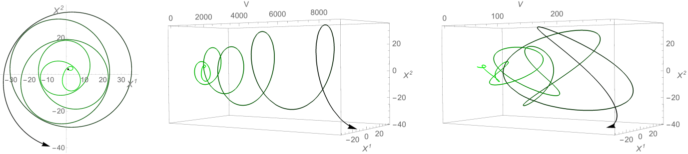
- Double copy of focussed laser backgrounds . . .
 . . . gravitational waves.

AI, PLB 782 (2018) 22



Conclusions

- Lightfront field theory essential for laser-matter interactions.
- Zero mode physics: non-perturbative pair production.

Other topics:

- Numerical approaches.... **tBLFQ** Vary, Zhao, many others!. . .
- Strong-field physics in heavy-ion collisions, QGP, astro. . .
- Ritus-Narozhny conjecture.

Review: Fedotov, J.Phys.Conf.Ser. 826 (2017) 012027