Proton Mass Decompositions

(A. Metz, Temple University)

- Motivation
- Energy momentum tensor (EMT) and its renormalization
- Mass decompositions (sum rules)
- Numerics for proton mass sum rules
- Proton mass and the EIC: measuring the trace anomaly
- Summary

Based on: S. Rodini, A. Metz, B. Pasquini, JHEP 09 (2020) 067, arXiv:2004.03704
A. Metz, B. Pasquini, S. Rodini, PRD 102 (2020) 114042, arXiv:2006.11171

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Motivation

- For decades, community has studied proton spin sum rule
(Jaffe, Manohar, 1989 / Ji, 1996)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G^{\rm can} + L_q^{\rm can} + L_g^{\rm can} = \frac{1}{2}\Delta\Sigma + L_q^{\rm kin} + J_g^{\rm kin}$$

- In comparison, less work has been done for mass sum rule

- Yet, different mass sum rules exist

- How do mass sum rules compare to each other ?
(proton mass largely due to trace anomaly, parton energies, or both ?)

- What is impact of recent developments concerning renormalization of EMT ?
(Hatta, Rajan, Tanaka, 2018 / Tanaka, 2018)

- Disclaimer: No discussion of proton mass for constituent-quark-type picture
(see, e.g., Roberts, Schmidt, arXiv:2006.08782 and references therein)

EMT: Definition

- Canonical EMT: Noether current of space-time translational invariance $\longrightarrow$ conserved

$$\partial_\mu T_C^{\mu\nu}(x) = 0$$

- Symmetric (gauge invariant) EMT: definition (QCD)

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T_q^{\mu\nu} = \frac{i}{4}\,\bar{\psi}\,\gamma^{\{\mu}\overset{\leftrightarrow}{D}{}^{\nu\}}\,\psi \qquad \left(\gamma^{\{\mu}\overset{\leftrightarrow}{D}{}^{\nu\}} = \gamma^\mu\overset{\leftrightarrow}{D}{}^\nu + \gamma^\nu\overset{\leftrightarrow}{D}{}^\mu\right)$$

$$T_g^{\mu\nu} = -F^{\mu\alpha}F^\nu{}_\alpha + \frac{g^{\mu\nu}}{4}\,F^2$$

  - summation over quark flavors and gluon colors understood
  - renormalization of parameters of QCD Lagrangian implied
  - $T_q^{\mu\nu}$ contains gluon field due to covariant derivative

$$\overset{\leftrightarrow}{D}{}^\mu = \overset{\rightarrow}{\partial}{}^\mu - \overset{\leftarrow}{\partial}{}^\mu - 2igA_a^\mu\,T_a$$

EMT: Renormalization

(Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)

- Total EMT not renormalized, but individual terms $T_i^{\mu\nu}$ require (extra) renormalization
- Operators that mix under renormalization

$$\mathcal{O}_1 = -F^{\mu\alpha} F^{\nu}{}_{\alpha} \qquad \mathcal{O}_2 = g^{\mu\nu} F^2$$

$$\mathcal{O}_3 = \frac{i}{4} \bar{\psi} \gamma^{\{\mu} \overleftrightarrow{D}^{\nu\}} \psi \qquad \mathcal{O}_4 = g^{\mu\nu} m \bar{\psi} \psi$$

$$T^{\mu\nu} = \mathcal{O}_1 + \frac{\mathcal{O}_2}{4} + \mathcal{O}_3$$

- Mixing equations

$$\mathcal{O}_{1,R} = Z_T \mathcal{O}_1 + Z_M \mathcal{O}_2 + Z_L \mathcal{O}_3 + Z_S \mathcal{O}_4$$

$$\mathcal{O}_{2,R} = Z_F \mathcal{O}_2 + Z_C \mathcal{O}_4$$

$$\mathcal{O}_{3,R} = Z_\psi \mathcal{O}_3 + Z_K \mathcal{O}_4 + Z_Q \mathcal{O}_1 + Z_B \mathcal{O}_2$$

$$\mathcal{O}_{4,R} = \mathcal{O}_4$$

- Trace (anomaly) of EMT
  (Adler, Collins, Duncan, 1977 / Nielsen, 1977 / Collins, Duncan, Joglekar, 1977 / ...)

$$T^{\mu}{}_{\mu} = \underbrace{(m\bar{\psi}\psi)_R}_{\text{classical trace}} + \underbrace{\gamma_m\,(m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}(F^2)_R}_{\text{trace anomaly}}$$

- Quark and gluon contribution to trace of EMT

$$T^{\mu}{}_{\mu} = (T_{q,R})^{\mu}{}_{\mu} + (T_{g,R})^{\mu}{}_{\mu}$$

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \Big(\tfrac{\beta}{2g} - x\Big)(F^2)_R$$

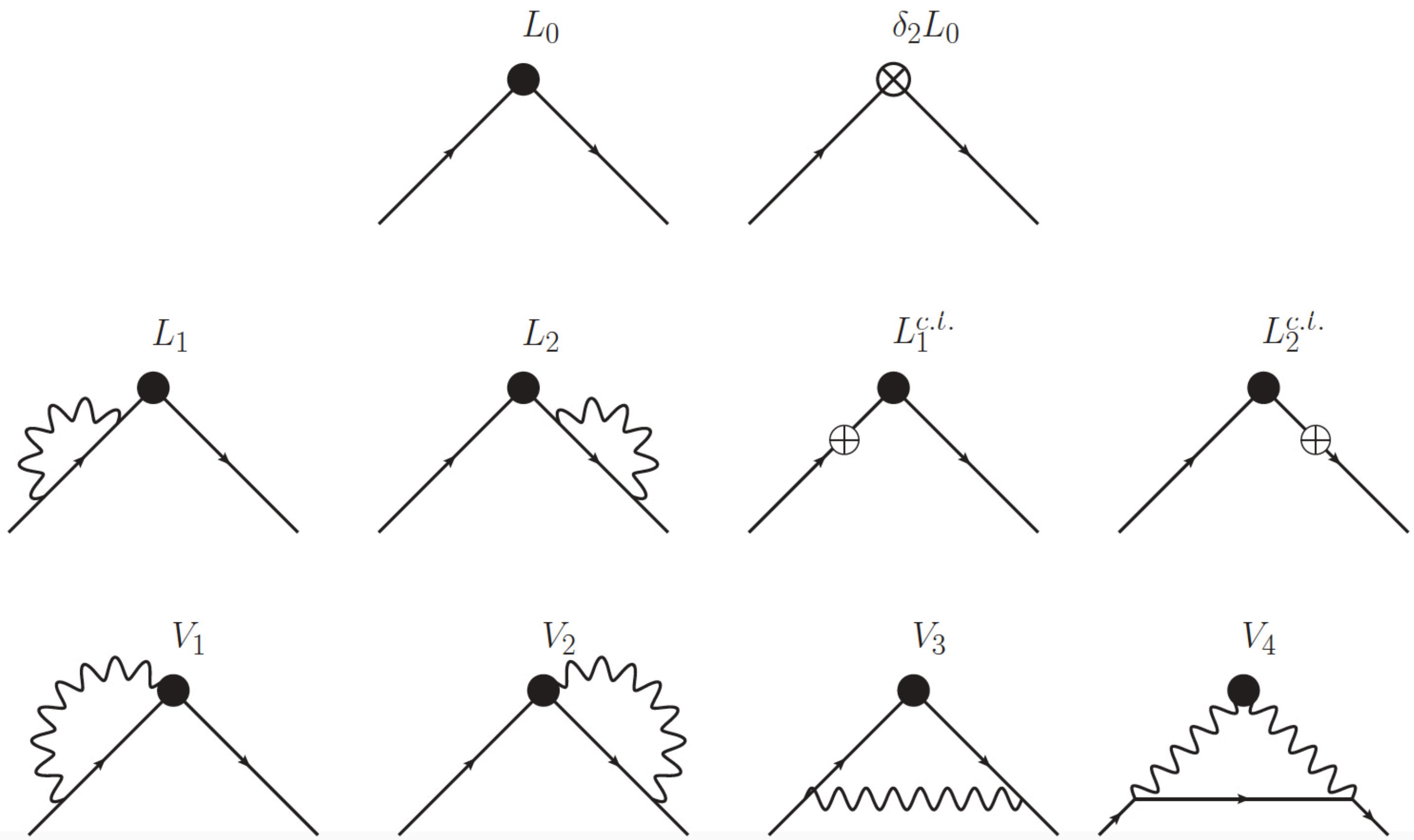
  x and y related to finite parts of renormalization constants $\rightarrow$ choose scheme

- Different scheme choices
  - MS scheme / $\overline{\text{MS}}$ scheme (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)
  - D1 scheme: $x = 0$, $y = \gamma_m$
  - D2 scheme: $x = y = 0$

  D-type schemes look natural

Renormalization of EMT in QED

- 1-loop diagrams



- Performed renormalization procedure to obtain finite $T_e^{\mu\nu}$ and $T_\gamma^{\mu\nu}$
- Studied mass decompositions of electron (see also, Ji, Lu, 1998)

EMT and Proton Mass

- Forward matrix element of total EMT (for spin-0 and spin-$\frac{1}{2}$)

$$\langle\, T^{\mu\nu}\,\rangle \equiv \langle P|\, T^{\mu\nu}\,|P\rangle = 2P^\mu P^\nu$$

  - $\langle\, T^{\mu\nu}(x)\,\rangle$ does not depend on space-time point x

- Forward matrix element of $T^{\mu\nu}_{i,R}$

$$\langle\, T^{\mu\nu}_{i,R}\,\rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu}\bar{C}_i(0)$$

  - form factors A_i and $\bar{C}_i$ satisfy

$$A_q(0) + A_g(0) = 1 \qquad\qquad \bar{C}_q(0) + \bar{C}_g(0) = 0$$

  - in forward limit, matrix elements of EMT fully determined by two form factors
  - any mass sum rule for the proton related to at most two independent form factors (emphasized in Lorcé, 2017)

- Trace of EMT and proton mass (here $n = \frac{1}{2M}$, depends on normalization of state)

$$n \, \langle \, T^{\mu}{}_{\mu} \, \rangle = M$$

- T^{00} and proton mass (in rest frame)

$$n \, \langle \, T^{00} \, \rangle = M$$

- Working with QCD Hamiltonian

$$\int d^3\mathbf{x} \, T^{00} = \int d^3\mathbf{x} \, \mathcal{H}_{\mathrm{QCD}} = H_{\mathrm{QCD}}$$

$$\left. \frac{\langle \, H_{\mathrm{QCD}} \, \rangle}{\langle P | P \rangle} \right|_{\mathbf{P}=0} = M$$

- Mass sum rules discussed below based on decomposition of $\langle \, T^{\mu}{}_{\mu} \, \rangle$ or $\langle \, T^{00} \, \rangle$ into quark and gluon parts

Two-Term Sum Rule by Hatta, Rajan, Tanaka

(Hatta, Rajan, Tanaka, JHEP 12, 008 (2018) / Tanaka, JHEP 01, 120 (2019))

- Sum rule based on decomposition of $T^{\mu}{}_{\mu}$

$$M = \bar{M}_q + \bar{M}_g = n \left(\langle\, (T_{q,R})^{\mu}{}_{\mu} \,\rangle + \langle\, (T_{g,R})^{\mu}{}_{\mu} \,\rangle \right)$$

- Recall operators

$$(T_{q,R})^{\mu}{}_{\mu} = (1 + y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \left(\frac{\beta}{2g} - x\right)(F^2)_R$$

- Using D-type schemes

$$(T_{q,R})^{\mu}{}_{\mu}\big|_{\mathrm{D1}} = (1 + \gamma_m)(m\bar{\psi}\psi)_R \qquad (T_{g,R})^{\mu}{}_{\mu}\big|_{\mathrm{D1}} = \frac{\beta}{2g}\,(F^2)_R$$

$$(T_{q,R})^{\mu}{}_{\mu}\big|_{\mathrm{D2}} = (m\bar{\psi}\psi)_R \qquad (T_{g,R})^{\mu}{}_{\mu}\big|_{\mathrm{D2}} = \gamma_m(m\bar{\psi}\psi)_R + \frac{\beta}{2g}\,(F^2)_R$$

Two-Term Sum Rule by Lorcé

(Lorcé, EPJC 78, 120 (2018))

- Sum rule based on decomposition of T^{00}

$$M = U_q + U_g = n\left(\langle\, T^{00}_{q,R}\,\rangle + \langle\, T^{00}_{g,R}\,\rangle\right)$$

- Renormalized operators discussed below

- Relation to EMT form factors for two-term sum rules

$$U_i = M\big(A_i(0) + \bar{C}_i(0)\big)$$

$$\bar{M}_i = M\big(A_i(0) + 4\bar{C}_i(0)\big)$$

  - $U_i \neq \bar{M}_i$ obviously
  - $U_q + U_g = \bar{M}_q + \bar{M}_g$ because $\bar{C}_q(0) + \bar{C}_g(0) = 0$

- Two-term sum rules have one independent term

(Modified) Four-Term Sum Rule by Ji

(Ji, PRL 74, 1071 (1995) and PRD 52, 271 (1995) / our papers)

- Sum rule based on decomposition of T^{00} into traceless part and trace part

$$T^{\mu\nu} = \underbrace{\left(T^{\mu\nu} - \hat{T}^{\mu\nu}\right)}_{\text{traceless part}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{trace part}}$$

$$\hat{T}^{\mu\nu} = \frac{1}{4}\, g^{\mu\nu}\, T^{\alpha}_{\ \ \alpha}$$

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \hat{T}^{\mu\nu}$$

  - main difference between Ji's and our work is calculation of $\bar{T}^{\mu\nu}$
  - we use same $\hat{T}^{\mu\nu}$ for trace part and for defining traceless part $\bar{T}^{\mu\nu}$ (otherwise $\bar{T}^{\mu\nu}$ actually not traceless)

- Decomposition into quark and gluon parts

$$T_i^{\mu\nu} = \bar{T}_i^{\mu\nu} + \hat{T}_i^{\mu\nu}$$

$$\mathcal{H}'_q = \bar{T}^{00}_{q,R} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + (m\bar{\psi}\psi)_R - \frac{1+y}{4}(m\bar{\psi}\psi)_R - \frac{x}{4}(F^2)_R$$

$$\mathcal{H}'_m = \hat{T}^{00}_{q,R} = \frac{1+y}{4}(m\bar{\psi}\psi)_R + \frac{x}{4}(F^2)_R$$

$$\mathcal{H}'_g = \bar{T}^{00}_{g,R} = \frac{1}{2}(E^2+B^2)_R + \frac{y-\gamma_m}{4}(m\bar{\psi}\psi)_R - \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^2)_R$$

$$\mathcal{H}'_a = \hat{T}^{00}_{g,R} = \frac{\gamma_m - y}{4}(m\bar{\psi}\psi)_R + \frac{1}{4}\left(\frac{\beta}{2g} - x\right)(F^2)_R$$

  – summing four terms provides mass

$$M = n\left(\langle\mathcal{H}'_q\rangle + \langle\mathcal{H}'_m\rangle + \langle\mathcal{H}'_g\rangle + \langle\mathcal{H}'_a\rangle\right)$$

- Form suitable linear combinations of $\mathcal{H}'_{q,m,g,a}$ to obtain "nice" terms $\mathcal{H}_{q,m,g,a}$
  - one must satisfy

$$\mathcal{H}_q + \mathcal{H}_m + \mathcal{H}_g + \mathcal{H}_a = \mathcal{H}'_q + \mathcal{H}'_m + \mathcal{H}'_g + \mathcal{H}'_a$$

  - M expressed in terms of linear combinations $\mathcal{H}_{q,m,g,a}$

$$M = n\Big(\langle\mathcal{H}_q\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_g\rangle + \langle\mathcal{H}_a\rangle\Big) = M_q + M_m + M_g + M_a$$

- Final form of sum rule

$$\mathcal{H}_q = (\psi^\dagger \, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \qquad \text{quark (kinetic plus potential) energy}$$

$$\mathcal{H}_m = (m\bar{\psi}\psi)_R \qquad \text{quark mass term}$$

$$\mathcal{H}_g = \tfrac{1}{2}(E^2 + B^2)_R \qquad \text{gluon energy}$$

  - three (instead of four) nontrivial terms only
  - sum rule has two independent terms

- Comparison with Ji's original work (Ji, 1995)

$$\big(\mathcal{H}_q\big)_{[\mathrm{Ji}]} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$$

$$\big(\mathcal{H}_m\big)_{[\mathrm{Ji}]} = \Big(1+\frac{\gamma_m}{4}\Big)(m\bar{\psi}\psi)_R$$

$$\big(\mathcal{H}_g\big)_{[\mathrm{Ji}]} = \tfrac{1}{2}(\boldsymbol{E}^2+\boldsymbol{B}^2)_R$$

$$\big(\mathcal{H}_a\big)_{[\mathrm{Ji}]} = \frac{\beta}{8g}(F^2)_R$$

  - sum rules differ by terms in red ($\frac{1}{4}$ of trace anomaly at operator level)
  - difference apparently due to difference in traceless part $\bar{T}^{00}$

- Comparison with Lorcé's two-term decomposition (Lorcé, 2017)

$$M = U_q + U_g = n\Big(\langle\, T^{00}_{q,R}\,\rangle + \langle\, T^{00}_{g,R}\,\rangle\Big)$$

$$T^{00}_{q,R} = (m\bar{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$$

$$T^{00}_{g,R} = \tfrac{1}{2}(\boldsymbol{E}^2+\boldsymbol{B}^2)_R$$

  - modified Ji sum rule can be considered refinement of two-term sum rule by Lorcé

Overview: Comparison of Sum Rules

- Two-term decomposition of $\langle\, T^{\mu}{}_{\mu} \,\rangle$ (in D2 scheme)

$$M = \bar{M}_q + \bar{M}_g = n\left(\left\langle (m\bar{\psi}\psi)_R \right\rangle + \left\langle \gamma_m (m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}(F^2)_R \right\rangle\right)$$

  – two scale-independent terms

- Two-term decomposition of $\langle\, T^{00} \,\rangle$

$$M = U_q + U_g = n\left(\left\langle (m\bar{\psi}\psi)_R + (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \right\rangle + \left\langle \tfrac{1}{2}(E^2 + B^2)_R \right\rangle\right)$$

  – two scale-dependent terms

- Three-term decomposition of $\langle\, T^{00} \,\rangle$

$$M = M_q + M_m + M_g = n\left(\left\langle (m\bar{\psi}\psi)_R \right\rangle + \left\langle (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R \right\rangle + \left\langle \tfrac{1}{2}(E^2 + B^2)_R \right\rangle\right)$$

  – one scale-independent term, and two scale-dependent terms

- Relation between matrix elements

$$\Big\langle (\psi^\dagger \, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R + \tfrac{1}{2}(E^2+B^2)_R \Big\rangle = \Big\langle \gamma_m (m\bar{\psi}\psi)_R + \tfrac{\beta}{2g}(F^2)_R \Big\rangle$$

  - one can speak about contribution from trace anomaly or from parton energies
  - a sum rule with contributions from trace anomaly and parton energies does not appear naturally
  - relation between matrix elements confirmed in recent calculation for hydrogen atom (Sun, Sun, Zhou, 2020)
  - relation between matrix elements, not between operators
  - further meaningful and unambiguous separation of parton energies into "classical" and "quantum" contributions ? (Ji, Liu, 2021)

Numerical Results

- First input: parton momentum fractions a_i, related to traceless parton operators

$$\frac{3}{2} M^2 a_q = \langle \bar{T}^{00}_{q,R} \rangle \qquad \frac{3}{2} M^2 a_g = \langle \bar{T}^{00}_{g,R} \rangle \qquad (a_q + a_g = 1)$$

- Second input: quark mass term

$$2M^2 b = (1+\gamma_m) \langle (m\bar{\psi}\psi)_R \rangle \;\rightarrow\; 2M^2 (1-b) = \frac{\beta}{2g} \langle (F^2)_R \rangle$$

  - direct input on trace anomaly (from experiment and/or LQCD) would be useful

- Example: modified Ji sum rule in terms of a_i and b

$$M_q = \frac{3}{4} M a_q + \frac{1}{4} M \left(\frac{(y-3)\, b}{1+\gamma_m} + x(1-b)\frac{2g}{\beta} \right)$$

$$M_m = M \frac{b}{1+\gamma_m}$$

$$M_g = \frac{3}{4} M a_g + \frac{1}{4} M \left[\frac{(\gamma_m - y)\, b}{1+\gamma_m} + \left(1 - x\frac{2g}{\beta}\right)(1-b) \right]$$

- Momentum fractions from CT18NNLO parameterization (at $\mu = 2\,\mathrm{GeV}$)

$$a_q = 0.586 \pm 0.013 \qquad a_g = 1 - a_q = 0.414 \pm 0.013$$

- Quark mass term from sigma terms

$$\sigma_u + \sigma_d = \sigma_{\pi N} = \frac{\langle P|\, \hat{m}\, (\bar{u}u + \bar{d}d)\, |P\rangle}{2M} \quad \sigma_s = \frac{\langle P|\, m_s\, \bar{s}s\, |P\rangle}{2M} \quad \sigma_c = \frac{\langle P|\, m_c\, \bar{c}c\, |P\rangle}{2M}$$

  - Scenario A: sigma terms from phenomenology
    (Alarcon et al, 2011, 2012 / Hoferichter et al, 2015)

$$\sigma_{\pi N}\big|_{\mathrm{ChPT}} = (59 \pm 7)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{ChPT}} = (16 \pm 80)\,\mathrm{MeV}$$

  - Scenario B: sigma terms from lattice QCD
    (Alexandrou et al, 2019)

$$\sigma_{\pi N}\big|_{\mathrm{LQCD}} = (41.6 \pm 3.8)\,\mathrm{MeV} \qquad \sigma_s\big|_{\mathrm{LQCD}} = (39.8 \pm 5.5)\,\mathrm{MeV}$$

$$\sigma_c\big|_{\mathrm{LQCD}} = (107 \pm 22)\,\mathrm{MeV}$$

  - main difference between scenarios: including or not σ_c

- Scheme dependence, for modified Ji sum rule (at $\mu = 2\,\mathrm{GeV}$)

		MS	$\overline{\mathrm{MS}}_1$	$\overline{\mathrm{MS}}_2$	D1	D2
Scenario A	M_q	0.309 ± 0.044	0.194 ± 0.033	0.178 ± 0.032	0.362 ± 0.045	0.357 ± 0.051
	M_m	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080	0.075 ± 0.080
	M_g	0.555 ± 0.036	0.669 ± 0.047	0.686 ± 0.048	0.502 ± 0.035	0.507 ± 0.029
Scenario B	M_q	0.234 ± 0.006	0.135 ± 0.003	0.120 ± 0.003	0.286 ± 0.006	0.272 ± 0.008
	M_m	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023	0.187 ± 0.023
	M_g	0.517 ± 0.017	0.617 ± 0.020	0.631 ± 0.020	0.465 ± 0.017	0.479 ± 0.015

  - considerable numerical scheme dependence
  - qualitatively, similar results for other sum rules
  - scheme dependence no new phenomenon

- Numerics for sum rule by Hatta, Raban, Tanaka (MS scheme)

		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	$\bar{M}_q$	-0.113 ± 0.102	-0.120 $\pm$0.105	-0.115 ± 0.107
	$\bar{M}_g$	1.051 ± 0.102	1.057 ± 0.105	1.053 ± 0.107
Scenario B	$\bar{M}_q$	0.032 ± 0.030	0.030 ± 0.031	0.035 ± 0.030
	$\bar{M}_g$	0.906 ± 0.030	0.908 ± 0.030	0.903 ± 0.030

  - perturbative expansion very stable (applies for all sum rules, and for all schemes)
  - $\bar{M}_q$ can become negative

- Numerics for sum rule by Lorcé (MS scheme)

		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
Scenario A	U_q	0.384 ± 0.035	0.383 ± 0.036	0.384 ± 0.036
	U_g	0.554 ± 0.035	0.556 ± 0.036	0.555 ± 0.036
Scenario B	U_q	0.420 ± 0.016	0.420 ± 0.017	0.421 ± 0.017
	U_g	0.518 ± 0.016	0.518 ± 0.017	0.517 ± 0.017

  - very roughly, quark and gluon energies contribute equally to proton mass
  - in $\overline{\mathrm{MS}}$ scheme, contribution from gluon energy somewhat larger

- Numerics for modified Ji sum rule (MS scheme)

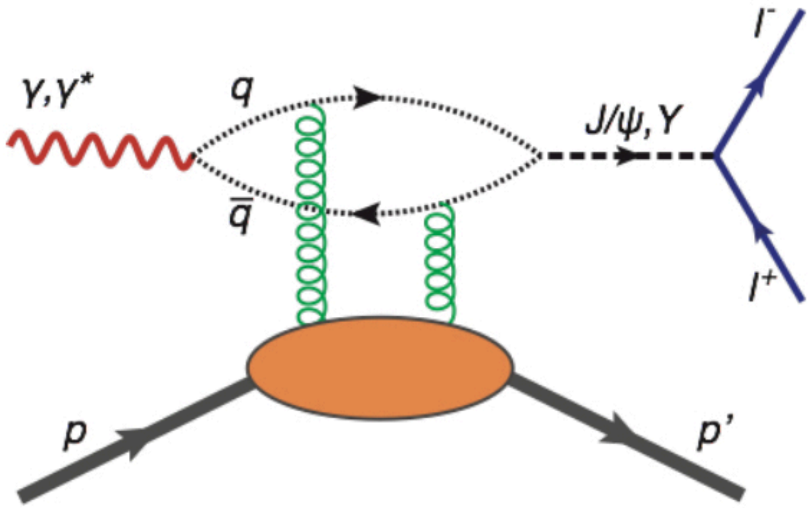
		$O(\alpha_s^1)$	$O(\alpha_s^2)$	$O(\alpha_s^3)$
	M_q	0.311 ± 0.043	0.310 ± 0.043	0.309 ± 0.044
Scenario A	M_m	0.073 ± 0.080	0.073 ± 0.079	0.074 ± 0.080
	M_g	0.554 ± 0.035	0.556 ± 0.036	0.555 ± 0.036
	M_q	0.237 ± 0.006	0.235 ± 0.006	0.234 ± 0.006
Scenario B	M_m	0.183 ± 0.023	0.184 ± 0.022	0.187 ± 0.023
	M_g	0.518 ± 0.016	0.518 ± 0.017	0.517 ± 0.017

  - $M_g = U_g$ $\rightarrow$ discussion for gluon part like for sum rule by Lorcé
  - M_q dominates over M_m, but feature less significant if σ_c included
  - precise determination of M_m important for proton mass decomposition
  - contribution of M_m is $\sim 8\%$ for Scenario A, $\sim 20\%$ for Scenario B
    $\rightarrow$ (much) larger than $\sim 1\%$ which is frequently attributed to Higgs mechanism

Proton Mass and the EIC

- Hunting gluon part of trace anomaly $\frac{\beta}{2g}F^2$ through threshold production of J/ψ or Υ
  (Kharzeev, 1996 / Hatta, Yang, 2018 / Mamo, Zahed, 2019 / Boussarie, Hatta, 2020 / Gryniuk, Joosten, Meziani, Vanderhaeghen, 2020 / ...)

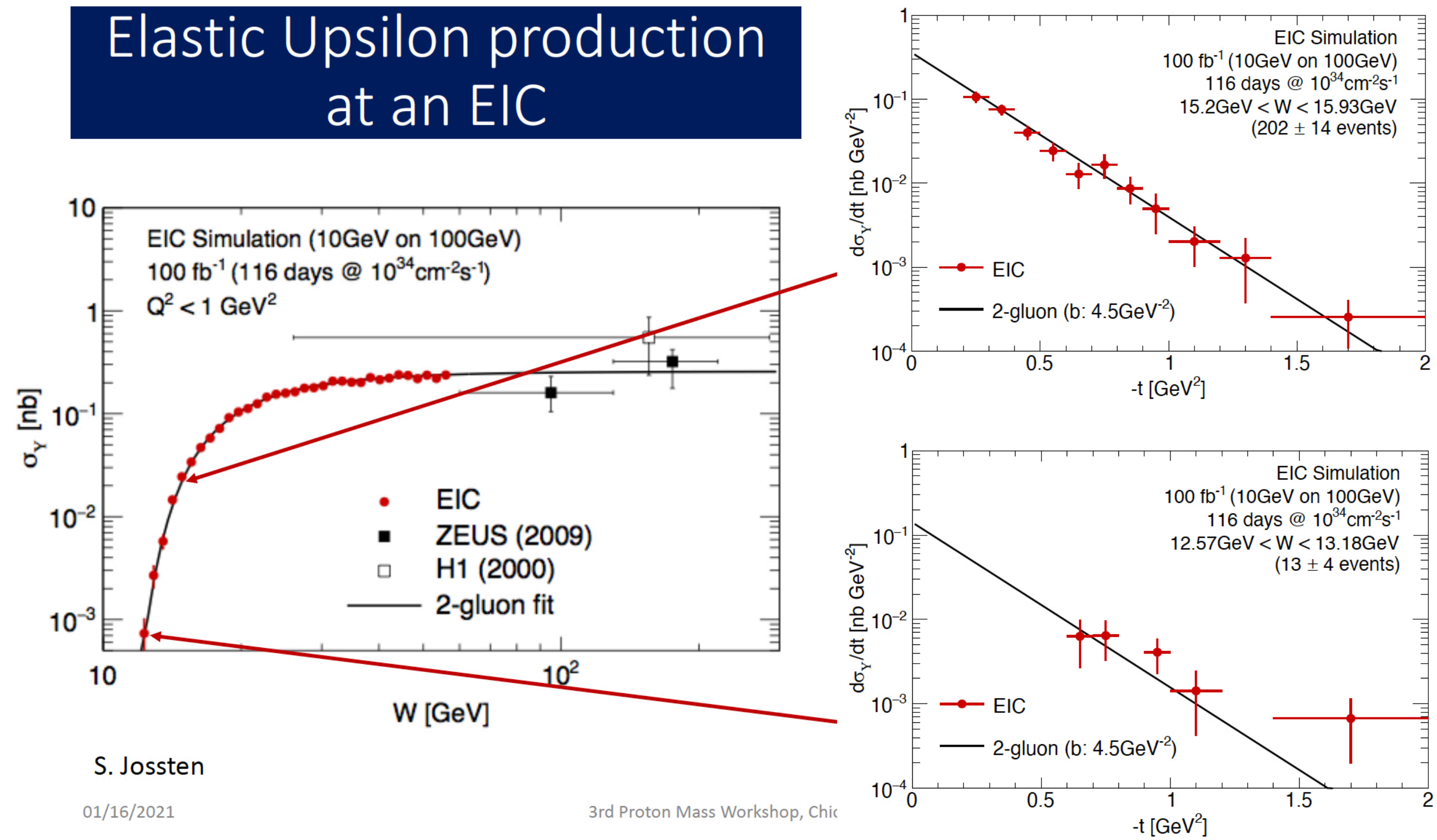
- General idea
  - quarkonium production probes gluon content of proton



(figure from talk by Meziani, 3$^{\mathrm{rd}}$ Proton Mass Workshop, 2021)

  - for photo-production, connection to trace anomaly requires simplifying assumptions
  - electro-production theoretically cleaner, but count rates go down
  - production of J/ψ has been (will be) studied at Jefferson Lab

- Prospects for the (theoretically cleaner) Υ production at EIC



(slide from talk by Meziani, 3$^{\mathrm{rd}}$ Proton Mass Workshop, 2021)

  - EIC will allow for measurement of Υ production with unprecedented precision
  - also electro-production can be explored at EIC
  - valuable direct experimental information about trace anomaly expected

Summary

- Several sum rules for proton mass exist that are based on EMT ($T^{\mu}_{\ \mu}$ or T^{00})

- Proton mass can be considered
  - largely due to the trace anomaly
  - or largely due to parton energies

- Numerics for sum rules
  - considerable scheme dependence
  - very mild dependence on order of perturbation theory
  - contributions from mass terms for heavy quarks considerable
  - attributing $\sim 1\%$ of proton mass to Higgs mechanism is misleading

- Direct measurement of (gluon part) of trace anomaly may be possible at EIC