Light-front wavefunctions, covariant Bethe–Salpeter amplitudes, and meson elastic and transition form factors

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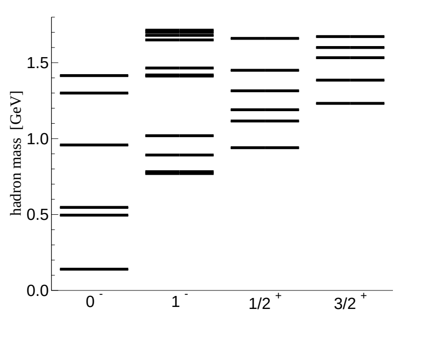
Outline

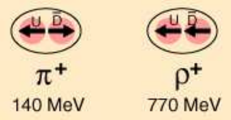
Hadron Physics

Where do hadrons get there mass from if the quark masses are only a few MeV?



PDG: at scale $\mu = 2\ \mathrm{GeV}$
$2\ \mathrm{MeV} < m_{u,d} < 8\ \mathrm{MeV}$

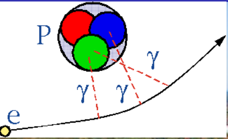
Why are pions so much lighter than all other hadrons ?

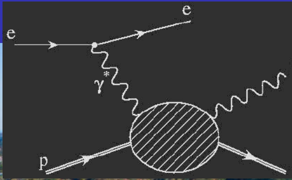


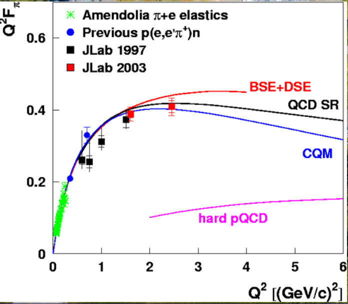
Delta	↑↑↑	1.23 GeV
proton	↑↓↑	0.94 GeV
rho	↑↑	0.77 GeV
pi	↑↓	0.14 GeV

hyperfine splitting??

Scattering observables







Quantum Chromo Dynamics

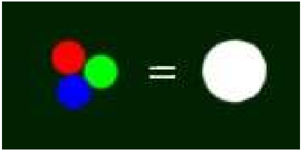
- Relativistic Quantum Field Theory

$$\mathcal{L}(\psi,\bar{\psi},A) = \bar{\psi}\Big(i\gamma^\mu(\partial_\mu + ig\tfrac{\lambda}{2}A_\mu) - m\Big)\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{ gauge fixing}$$

- Dynamical chiral symmetry breaking
- Quarks and gluons are confined
  - they have never been observed in isolation
  - only colorless bound states of quarks and gluons are observed

mesons

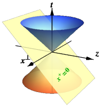
baryons

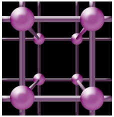


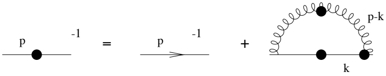
- Need nonperturbative methods

Nonperturbative methods

- Light-Front QCD
- AdS/QCD holography
- Lattice QCD
- Dyson–Schwinger Equations
- QCD sum rules
- Chiral Effective Field Theory
- Heavy-quark EFT (nonrelativistic)
- ...







Light-Front QCD vs Dyson–Schwinger Equations

- Based on Minkowsky formulation
- LF QCD Hamiltonian
  - LF wfns eigenstates of $\hat{H}$
  - only few-body interactions
- Form factors
  - obtained from LF wfn
- Fock space truncation
- Advantages
  - direct access to LF wfn
- Challenges
  - chiral symmetry breaking
  - restoring rotational symmetry
  - higher Fock spaces

- Based on QCD Lagrangian in Euclidean formulation
- Hadrons: poles in n-point fns
  - Bethe–Salpeter eqn
- Form factors
  - obtained from BSAs
- Rainbow-Ladder truncation
- Advantages
  - explicitly covariant
  - chiral symmetry breaking
- Challenges
  - obtaining LF wfn
  - beyond RL truncation

Open question: Manifestation of Confinement ?

Light-Front Holography and Confinement

- Holographic variable $\vec{\zeta}_\perp = \sqrt{x(1-x)}\ \vec{r}_\perp$
- Effective confining interaction in transverse direction

$$V_{\perp\ \text{conf}} = \kappa^4 \zeta_\perp^2 = \kappa^4\ x(1-x)\ r_\perp^2$$

Brodsky, de Teramond, Dosch, Erlich, Phys. Rept. 584, 1 (2015)

- Effective longitudinal confinement

$$V_{x\ \text{conf}} = -\frac{\kappa^4}{m_q + m_{\bar{q}}} \partial_x \left[x(1-x)\partial_x\right]$$

Yang Li, Maris, Zhao, Vary, PLB758, 118 (2016)

  - combines, in nonrelativistic limit, with transverse confinement into 3-D harmonic oscillator confinement
  - distribution amplitudes match pQCD asymptotics
  - exactly solvable

Basis Light-Front Quantization

- Effective Hamiltonian

transverse confinement — longitudinal confinement

$$H_{\text{eff}} = \underbrace{\frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \kappa^4 \vec{\zeta}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x \left(x(1-x) \partial_x \right) + V_g$$

LF kinetic energy — one-gluon exchange

- Basis representation: Eigenfunctions of **LF kinetic energy**, **transverse confinement** and **longitudinal confinement**

$$\psi(x, k_\perp) = \sum_{n,m,l} c_{nmlss'} \, \phi_{nm}\left(k_\perp / \sqrt{x(1-x)}\right) \, \chi_l(x)$$

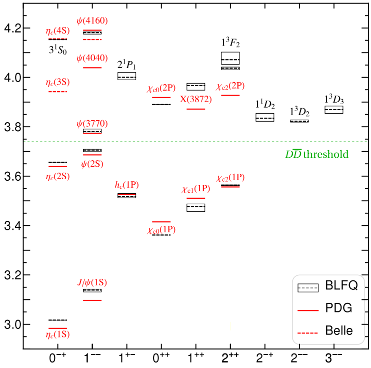
  - transverse direction: 2-D harmonic oscillator functions
  - longitudinal direction: Jacobi polynomials weighted by $x^\alpha (1-x)^\beta$

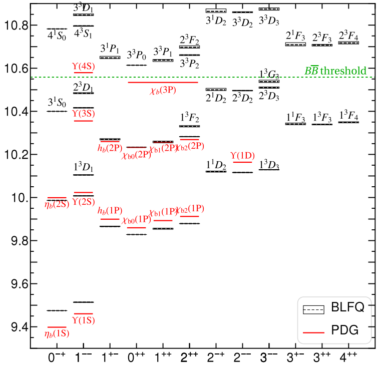
- **One-gluon exchange** with running coupling

$$V_g = -\frac{4}{3} \times \frac{4\pi \alpha_s(Q^2)}{Q^2} \bar{u}_{\sigma'} \gamma^\mu u_\sigma \bar{v}_s \gamma_\mu v_{s'}$$

Quarkonium Spectroscopy

Yang Li, Maris, Vary, PRD96, 016022 (2017)





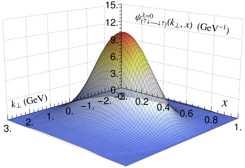
	κ (GeV)	m_q (GeV)	fitted states	rms dev. (MeV)	$\overline{\delta_J M}$ (MeV)	truncation $N_{\max}$	basis dim.
$c\bar{c}$	0.966	1.603	8	31	17	32	1812
$b\bar{b}$	1.389	4.902	14	38	8	32	1812

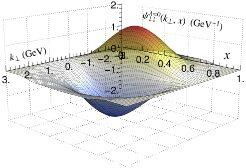
fitted value for κ follows expected trajectory $\kappa_h \propto \sqrt{M_h}$

LF Wave Functions

available at Yang Li (2019), Mendeley Data, v2; DOI: 10.17632/cjs4ykv8cv.2

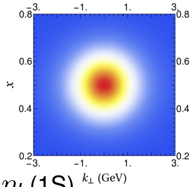
- Pseudoscalar mesons: two spin structures, $\psi(x, k_\perp)_{(\uparrow\downarrow-\downarrow\uparrow)}$ and $\psi(x, k_\perp)_{\downarrow\downarrow} = \psi(x, k_\perp)^*_{\uparrow\uparrow}$



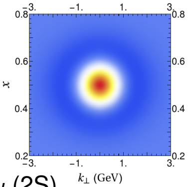


η_c

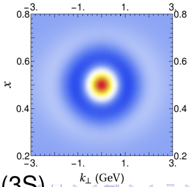
- Vector mesons: 6 different Dirac structures
- Heavy quarkonia: non-relativistic configurations dominate
- Radial excitations more spread out in coordinate space



$\eta_b(1S)$



$\eta_b(2S)$



$\eta_b(3S)$

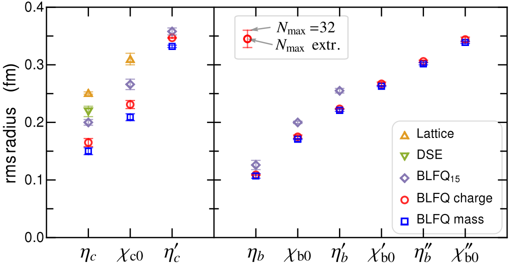
Charge and Gravitational radii

Yang Li, Maris, Vary, PRD96, 016022 (2017)

Scalar and pseudoscalar states

$$\langle r_{\rm c}^2\rangle = \frac{3}{2}\langle \vec{b}_\perp^2\rangle \equiv \frac{3}{2}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp\, (1-x)^2\, \vec{r}_\perp^2\; \widetilde{\psi}^*_{s\bar{s}}(\vec{r}_\perp, x)\widetilde{\psi}_{s\bar{s}}(\vec{r}_\perp, x)$$

$$\langle r_{\rm m}^2\rangle = \frac{3}{2}\langle \vec{\zeta}_\perp^2\rangle \equiv \frac{3}{2}\sum_{s,\bar{s}}\int_0^1 \frac{dx}{4\pi}\int d^2 r_\perp\, x(1-x)\, \vec{r}_\perp^2\; \widetilde{\psi}^*_{s\bar{s}}(\vec{r}_\perp, x)\widetilde{\psi}_{s\bar{s}}(\vec{r}_\perp, x)$$



'Charge' radii:
couple photon to quark,
but not to anti-quark

- 'Charge' radii slightly larger than mass radii for charmonia, but nearly equal for bottomonia
  - splitting is relativistic effect

Electromagnetic form factors on the Light-Front

Consider (pseudo)scalar elastic form factor F on the light-front

- Longitudinal or transverse components related by kinematic boost

$$\langle\psi_h(p'^+,\vec{p}'_\perp + p'^+\vec{\beta}_\perp;\omega)|\vec{J}_\perp|\psi_h(p^+,\vec{p}_\perp + p^+\vec{\beta}_\perp;\omega)\rangle \; = \\ = \; \langle\psi_h(p'^+,\vec{p}'_\perp;\omega)|\vec{J}_\perp|\psi_h(p^+,\vec{p}_\perp;\omega)\rangle + \vec{\beta}_\perp\langle\psi_h(p'^+,\vec{p}'_\perp;\omega)|J^+|\psi_h(p^+,\vec{p}_\perp;\omega)\rangle$$

- Evaluated from either transverse or longitudinal component

$$\begin{aligned}\langle\psi_h(p')|J^+(0)|\psi_h(p)\rangle &= (p^+ + p'^+)\; F(z,Q^2)\\ \langle\psi_h(p')|J_\perp(0)|\psi_h(p)\rangle &= (p_\perp + p'_\perp)\; F(z,Q^2)\end{aligned}$$

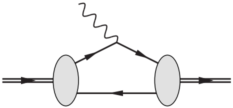
  but F depends on momentum transfer $Q^2 = -q^2$ and $z = \frac{q^+}{p'^+}$

- Becomes indepent of z once Poincaré invariance is restored
- In practice, Drell–Yan frame, $q^+ = 0$ (in combination with longitudinal current component J^+) is preferred on the light-front, because vacuum pair production/annihilation is suppressed

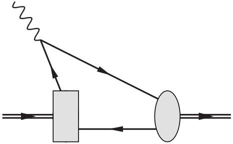
$$\langle\psi_h(p')|J^+(0)|\psi_h(p)\rangle \;=\; (p^+ + p'^+)\; F(z=0,Q^2)$$

Light-Front Time ordering vs. Explicitly covariant

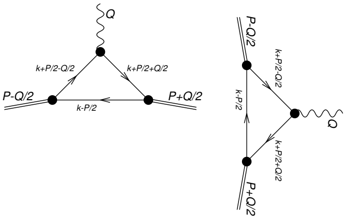
- Leading Fock space



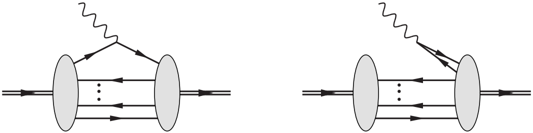
- Higher Fock space



Covariant triangle diagrams have no time ordering; orientation is irrelevant



Time ordering matters on the Light-Front



Frame dependence of form factors

Yang Li, Maris, Vary, PRD97, 054034 (2018)

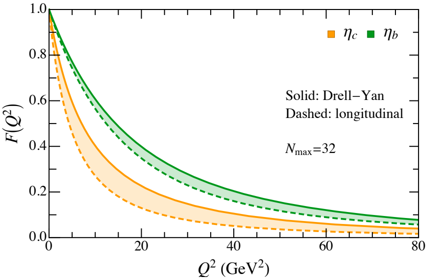
Define frames in terms of boost invariants $z = \frac{q^+}{p'^+}$ and $\vec{\Delta}_\perp = \vec{q}_\perp - z\vec{p}'_\perp = p^+ \left(\frac{\vec{p}'_\perp}{p'^+} - \frac{\vec{p}_\perp}{p^+} \right)$ with mom. transfer $Q^2 = \frac{z^2 M_h^2 + \Delta_\perp^2}{1-z}$

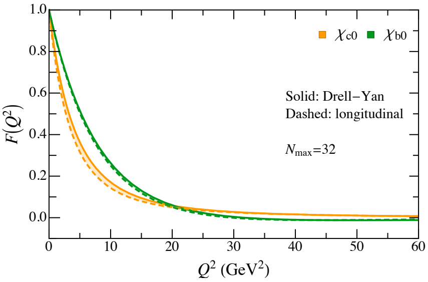
- Transverse (i.e. Drell–Yan) frame: $z = 0$ and $Q^2 = q_\perp^2 \geq 0$
- Longitudinal frame: $\Delta_\perp = 0$ and thus $Q^2 = \frac{z^2 M_h^2}{(1-z)}$

Form factor of (pseudo)scalar mesons in leading Fock sector

$$F(z, Q^2) = \frac{\sqrt{1-z}}{1 - \frac{1}{2}z} \sum_{s,\bar{s}} \int_0^1 \frac{\mathrm{d}x}{2x(1-x)} \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^3} \sqrt{\frac{x}{x + z(1-x)}} \times \psi^*_{s\bar{s}/h}\left(x + z(1-x), \vec{k}_\perp + (1-x)\vec{\Delta}_\perp\right) \psi_{s\bar{s}/h}\left(x, \vec{k}_\perp\right)$$

Frame dependence of quarkonia charge form factors





Yang Li, Maris, Vary, PRD97, 054034 (2018)

Frame dependence

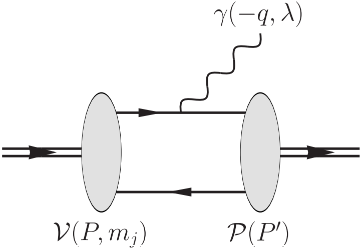
- stronger for charmonia than for bottomonia
- stronger for η_c than for χ_c

Sources Lorentz symmetry violation

- Fock space truncation
- Effective interaction
  - confining potential in transverse and longitudinal direction
  - one-gluon exchange (transverse)

Radiative decays

Electromagnetic transitions between quarkonium states via photon emission offers insight into the internal structure of quark-antiquark bound states



vector(1^{--}) $\leftrightarrow$ pseudoscalar (0^{-+})
e.g. $J/\Psi \to \eta_c(1S)\gamma$,
$\eta_c(2S) \to J/\Psi\gamma$,
$\Upsilon(3S) \to \eta_b(1S)\gamma$

Hadron matrix elements parametrized by trantision form factor $V(Q^2)$

$$I^{\mu}_{m_j}(P,P') \equiv \langle\mathcal{P}(P')|J^{\mu}|\mathcal{V}(P,m_j)\rangle = \frac{2V(Q^2)}{m_{\mathcal{P}}+m_{\mathcal{V}}}\epsilon^{\mu\alpha\beta\sigma}P'_{\alpha}P_{\beta}e_{\sigma}(P,m_j)$$

where $q^{\mu} = P'^{\mu} - P^{\mu}$ with $Q^2 \equiv -q^2 > 0$ spacelike
and $e_{\sigma}(P,m_j)$ the vector meson polarization

Radiative decay width

Physical decay:
transition amplidute with on-shell photon

$$\mathcal{M}_{m_j,\lambda} = \langle\mathcal{P}(P')J^{\mu}(0)\rangle\mathcal{V}(P,m_j)\epsilon^{*}_{\mu,\lambda}(q)Big|_{Q^2=0}$$

where $\epsilon_{\mu,\lambda}$ is the photon polarization vector

On-shell decay width

$$\Gamma(\mathcal{V}\to\mathcal{P}+\gamma) = \int \mathrm{d}\Omega_q \frac{1}{32\pi^2}\frac{|\vec{q}|}{m_{\mathcal{V}}^2}\frac{1}{2J_{\mathcal{V}}+1}\sum_{m_j,\lambda}|\mathcal{M}_{m_j,\lambda}|^2$$

$$= \frac{(m_{\mathcal{V}}^2 - m_{\mathcal{P}}^2)^3}{(2m_{\mathcal{V}})^3(m_{\mathcal{P}}+m_{\mathcal{V}})^2}\frac{|V(0)|^2}{(2J_{\mathcal{V}}+1)\pi}$$

Evaluation on the Light-Front (Drell–Yan frame)

Amplitudes depend on current component and vector meson polarization

$$I^+_{m_j} = \frac{2V(Q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} \begin{cases} 0, & m_j = 0 \\ \frac{i}{\sqrt{2}} P^+ \Delta^R, & m_j = 1 \\ -\frac{i}{\sqrt{2}} P^+ \Delta^L, & m_j = -1 \end{cases}$$

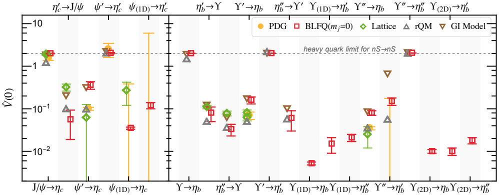
$$I^R_{m_j} = \frac{2V(Q^2)}{m_{\mathcal{P}} + m_{\mathcal{V}}} \begin{cases} -i m_{\mathcal{V}} \Delta^R, & m_j = 0 \\ \frac{i}{\sqrt{2}} P^R \Delta^R, & m_j = 1 \\ \frac{i}{\sqrt{2} z} (z^2 m^2_{\mathcal{V}} - m^2_{\mathcal{P}} - P'^R \Delta^L), & m_j = -1 \end{cases}$$

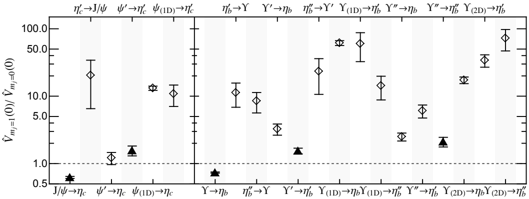
using $z \equiv P'^+ / P^+$ and $\vec{\Delta}_\perp = \vec{P}'_\perp - z \vec{P}_\perp$, and notation $k^{R,L} = kx \pm iky$

- Transverse and longitudinal current components give same results for vector meson spin polarization $m_j = \pm 1$

Radiative decay quarkonia

Meijian Li, Yang Li, Maris, Vary, PRD98, 034024 (2018)



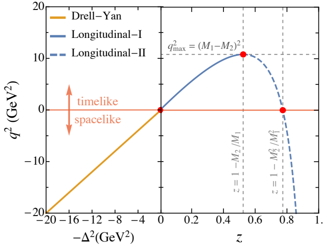


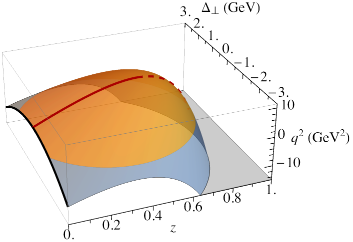
- Drell–Yan frame
  - Transverse current component with spin component $m_j = 0$ probes dominant components of vector meson LFWF
  - Longitudinal current J^+ w. spin component $m_j = 1$ probes small components vector meson LFWF which vanish in non-relativistic limit

Transverse current $J_\perp$, with vector meson $m_j = 0$, more robust because it involves dominant (non-relativistic) LFWF components

Frame dependence transition form factors

Meijian Li, Yang Li, Maris, Vary, PRD100, 036006 (2019); Tang, Jia, Maris, Vary, arxiv:2002.06489 (2020)

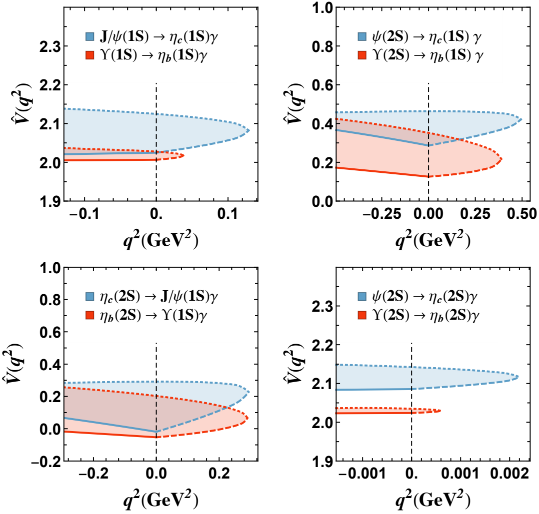




- Boost invariants $z = \frac{q^+}{p_A^+}$ and $\vec{\Delta}_\perp = \vec{q}_\perp - z\vec{p}_{A\perp}$
- Momentum transfer $q^2 = z\left(M_A^2 - \frac{M_B^2}{1-z}\right) - \frac{\Delta_\perp^2}{1-z}$
- Drell–Yan frame: $z = 0$ with $q^2 = -\vec{\Delta}_\perp^2$ limited to spacelike momentum transfers
- Longitudinal frame: $\vec{\Delta}_\perp = 0$ with $q^2 = z\,M_A^2 - \frac{z\,M_B^2}{1-z}$ spacelike and timelike up to $q^2_{\max} = (M_A - M_B)^2$

Transition form factors

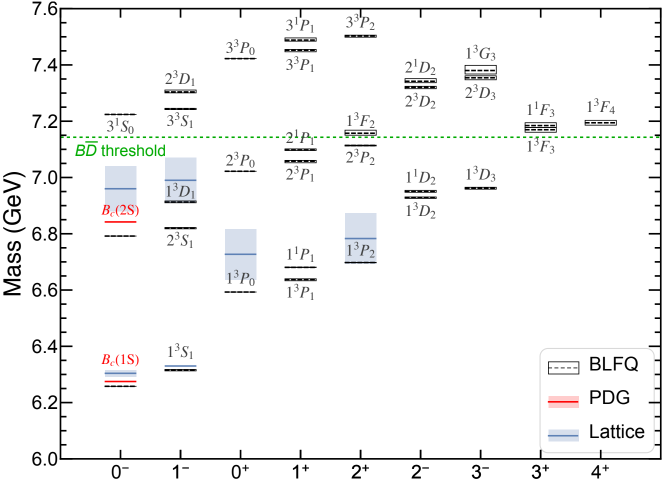
Meijian Li, Yang Li, Maris, Vary, PRD100, 036006 (2019)



- Drell–Yan and Longitudinal frame connect continuously at $q^2 = 0$, but derivative is dicontinuous
- Transitions between states with same radial quantum number $\mathcal{V}(nS) \to \mathcal{P}(nS)$
  - depend weakly on frame
  - dependence decreases with increasing mass
- Transitions between states with different radial quantum number depend strongly on choice of frame

Unequal quark masses: B_c mesons Tang, Li, Maris, Vary, PRD98, 114038 (2018)

Use parameters as fitted for charmonium and bottomonium



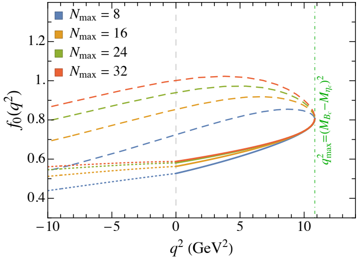
Semi-leptonic decay $B_c \to \eta_c$

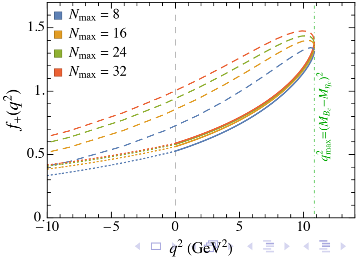
Tang, Jia, Maris, Vary, arxiv:2002.06489 (2020)

$$\begin{aligned}\langle P'_{\eta_c}|V^\mu|P_{B_c}\rangle &= f_+(q^2)P^\mu + f_-(q^2)q^\mu \\ &= f_+(q^2)\left(P^\mu - \frac{M^2_{B_c}-M^2_{\eta_c}}{q^2}q^\mu\right) + f_0(q^2)\frac{M^2_{B_c}-M^2_{\eta_c}}{q^2}q^\mu\end{aligned}$$

Use longitudinal and transverse current components to calculate $f_\pm$

$$\begin{aligned}f_+(q^2) &= \frac{(\Delta^R + zP^R_{B_c})\mathcal{M}^+ - zP^+_{B_c}\mathcal{M}^R}{2\Delta^R P^+_{B_c}} \\ f_-(q^2) &= \frac{\left[\Delta^R - (2-z)P^R_{B_c}\right]\mathcal{M}^+ + (2-z)P^+_{B_c}\mathcal{M}^R}{2\Delta^R P^+_{B_c}}\end{aligned}$$

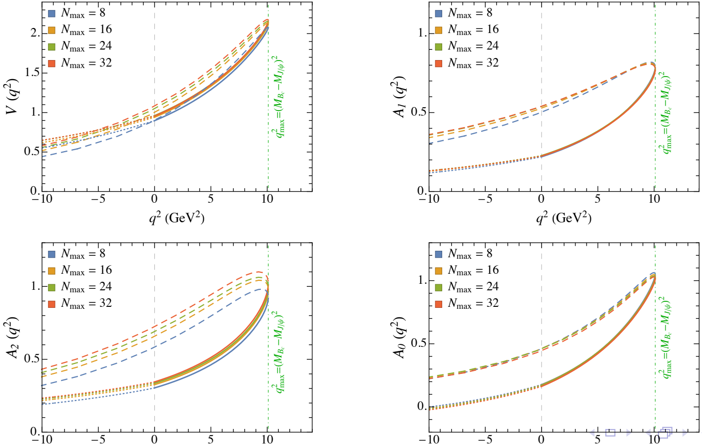




Semi-leptonic decay $B_c \to J/\psi$

Tang, Jia, Maris, Vary, arxiv:2002.06489 (2020)

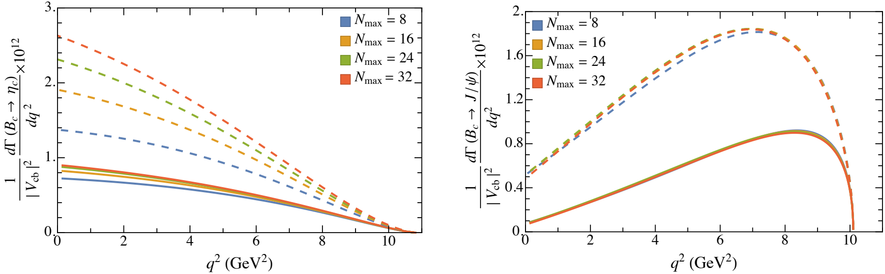
- Vector form factor (analogous to electromagnetic transition):
  use $m_j = 0$ and transverse current
- Axial form factors:
  use both $m_j = 0$ and $m_j = 1$, as well as both transverse and longitudinal current



Semi-leptonic decay width

Tang, Jia, Maris, Vary, arxiv:2002.06489 (2020)

Differential decay width for $B_c \to \eta_c\, e\, \bar{\nu}$ and $B_c \to J/\psi\, e\, \bar{\nu}$

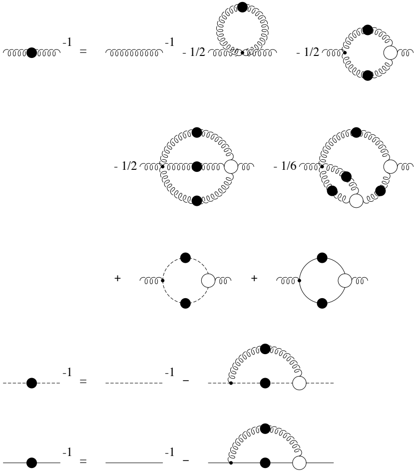


- Significant dependence of differential decay width on choice of frame for both decay to η_c and decay to J/ψ
- Decay to J/ψ almost independent of computational details, but decay to η_c not well converged

Conclusions on the Light-Front

- Quarkonium forms an ideal system to develop and validate methods to compute Light-Front Wave Functions
  - even simpler: scalar Yukawa model
- Basis Light-Front Quantization
  - obtain LFWF as eigenfunctions of effective LF Hamiltonian
  - limited to minimal Fock space
  - fit model parameters to reproduce spectrum
  - use LFWF to evaluate form factors
- Open questions (at least to me)
  - Fock space convergence
  - restoration of Lorentz invariance
  - dynamical symmetry breaking
  - zero-modes

Nonperturbative QCD: Dyson–Schwinger Equations



- Infinite hierarchy of coupled integral eqns for Green's functions of QCD
- Reduce to pQCD in weak coupling limit
- Nonperturbative

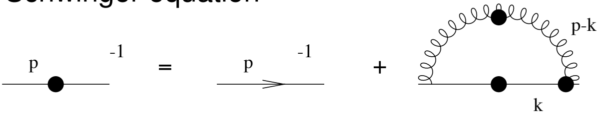
- Truncation needed
- Constraints on truncation
  - Preserve symmetries
  - Self-consistency

- aka Dyson–Schwinger Eqns

Nonperturbative quark propagator (Euclidean Metric)

$$S_0(p) = \frac{1}{i\ \not p + m_q} \quad \longrightarrow \quad S(p) = \frac{Z(p^2)}{i\ \not p + M(p^2)} = \frac{1}{i\ \not p A(p^2) + B(p^2)}$$

- Satisfies Dyson–Schwinger equation



$$S(p)^{-1} \quad = \quad i\ \not p\, Z_2 + m_q(\mu)\, Z_4 + Z_1^g \int \frac{d^4k}{(2\pi)^4}\, g^2 D_{\mu\nu}(q)\gamma_\mu \tfrac{\lambda^a}{2} S(k)\Gamma_\nu(k,p)\tfrac{\lambda^a}{2}$$

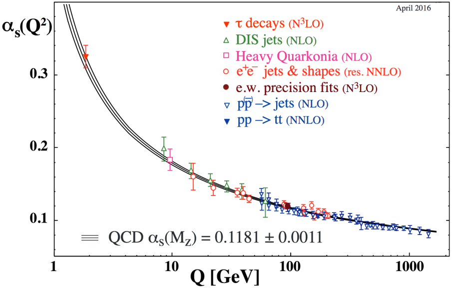
- Nonlinear integral equation for the quark propagator
  - coupled nonlinear integral equations for $M(p^2)$ and $Z(p^2)$
- Allows for a nontrivial solution $M(p^2) \neq 0$ even if $m_q = 0$ provided the long-range part of the interaction is sufficiently strong
  - dynamical chiral symmetry breaking
  - pions are the (near) massless Goldstone bosons

Model for effective interaction

- Rainbow truncation for quark DSE

$$Z_1^g \, g^2 D_{\mu\nu}(q) \, \Gamma_\nu(k,p) \;\longrightarrow\; 4\pi\alpha_{\text{model}}(q^2) \, D_{\mu\nu}^{\text{free}}(q) \, \gamma_\nu$$

- Assume dressed vertex times dressed gluon has the same tensor structure as bare vertex times free gluon propagator
- Use Landau gauge
  - in principle we could use any covariant gauge
- Use pQCD for UV behavior
- Model for IR behavior of $\alpha(q^2)$ fitted to give chiral condensate $\langle \bar{q}q \rangle = -(240\ \text{MeV})^3$

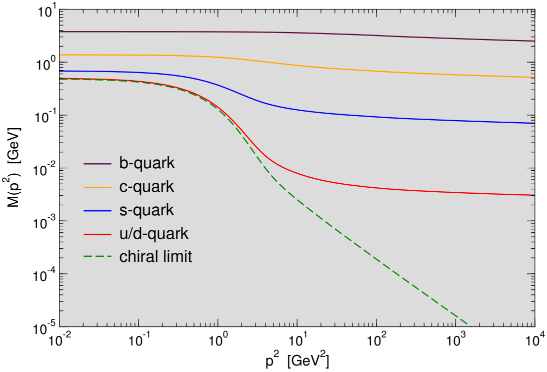


Model results for nonperturbative quark propagator

- Rainbow truncation for quark DSE

$$Z_1^g \, g^2 D_{\mu\nu}(q) \, \Gamma_\nu(k,p) \;\longrightarrow\; 4\pi\alpha_{\text{model}}(q^2)\, D^{\text{free}}_{\mu\nu}(q)\, \gamma_\nu$$

- Evolution from constituent quark mass to current quark mass
- $M(p^2)$ connects constituent mass with perturbative QCD



Maris, Roberts, PRC56, 3369 (1997)

Nonzero $m_q(\mu)$

$$M(p^2) \overset{\text{large } p^2}{\longrightarrow} \frac{m_q(\mu)}{(\ln(p/\Lambda_{QCD}))^{\gamma_m}}$$

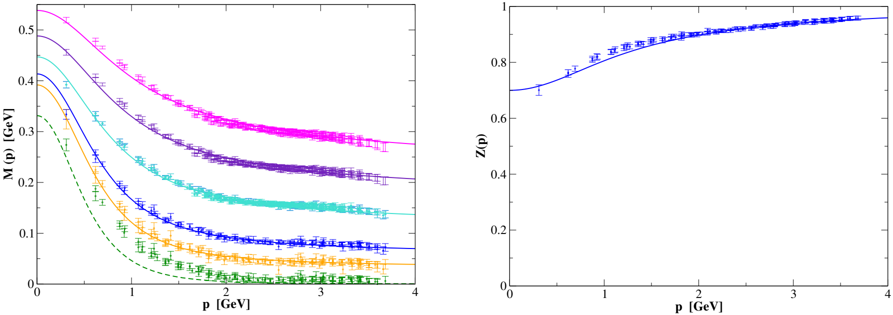
Dynamical χSB

$$M(p^2) \overset{\text{large } p^2}{\propto} \frac{\langle \bar{q}q \rangle^0}{p^2\,(\ln(p/\Lambda_{QCD}))^{1-\gamma_m}}$$

Nonperturbatively dressed quark propagator

- Predictions from solution of the quark DSE
  have been confirmed by lattice simulations of QCD
- Detailed comparison lattice simulations and DSE soln possible

Maris, Raya, Roberts, & Schmidt, EPJA18, 231 (2003)



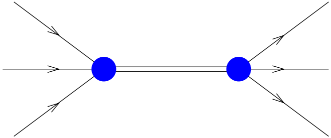
Lattice-inspired DSE model: Bhagwat, Pichowsky, Roberts, Tandy, PRC68, 015203 (2003)
Quenched lattice data: Bowman, Heller, Leinweber, Williams, NP Proc.Suppl.119, 323 (2003)

Hadrons

- Bound states of nonperturbatively dressed quarks
- Pole in color-singlet *n*-point functions of QCD
- Bound state amplitudes Γ describe coupling between
  - meson and quark-antiquark pair

$$G^{(4)} \sim \frac{\Gamma(p_1, p_2; P)\bar{\Gamma}(k_1, k_2; P)}{P^2 + M^2_{\text{meson}}}$$

  - baryon and three quarks



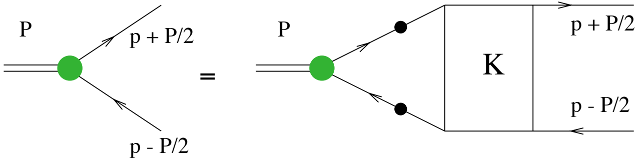
$$G^{(6)} \sim \frac{\Gamma(p_1, p_2, p_3; P)\bar{\Gamma}(k_1, k_2, k_3; P)}{P^2 + M^2_{\text{baryon}}}$$

- Euclidean metric: mass poles at $P^2 = -M^2_{\text{hadron}}$

Mesons

Quark-antiquark bound states satisfy homogeneous
Bethe–Salpeter equation at mass pole $P^2 = -M^2_{\rm meson}$

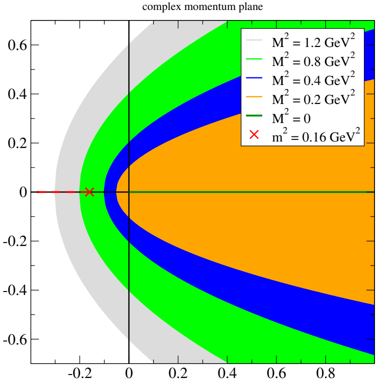
$$\Gamma_H(p;P) \quad = \quad \int \frac{d^4k}{(2\pi)^4}\, K(p,k;P)\; S(k+P/2)\,\Gamma_H(k;P)\,S(k-P/2)$$



- $K(p,k;P)$: amputated $q\bar{q}$ scattering kernel
- Quark propagators nonperturbatively dressed

Euclidean formulation

- Meson BSA functions of two independent variables: p^2 and $p \cdot P$
- Rest-frame $P = (iM, 0, 0, 0)$
- Relative momentum p Euclidean
  - p^2 space-like
  - $p \cdot P$ imaginary in rest-frame
- Integration variable k Euclidean
- Quark propagator arguments $k^2 \pm k \cdot P + M^2/4$ become complex
- Constituent propagators: no problem for bound states, that is, for $M < 2m$
  - e.g. constituent quark mass of 400 MeV fine for π and ρ



Analytic continuation of dressed quark propagator

$$A(p^2) = 1 + \int \frac{d^4k}{4\pi^3} \frac{\alpha(q^2)}{q^2} \frac{A(k^2)\, K^A(p^2, k^2, p\cdot k)}{k^2\, A^2(k^2) + B^2(k^2)}$$

$$B(p^2) = m_q(\mu) + \int \frac{d^4k}{4\pi^3} \frac{\alpha(q^2)}{q^2} \frac{4\, B(k^2)}{k^2\, A^2(k^2) + B^2(k^2)}$$

- Fit Euclidean solution with your favorite function
  - results will (strongly) depend on choice of functional form
- Use Taylor expansion of Euclidean solution
  - limited range, but should be okay for light systems
- Calculate $A(p^2)$ and $B(p^2)$ at complex momenta p^2 after solving quark DSE on real Euclidean axis
  - only correct if effective interaction α vanishes at $q^2 = 0$, otherwise, pinch-singularity forces integration path dk through $k = p$
- Analytic continuation of quark DSE into complex plane
  - can be done, but is nontrivial to avoid branch-cuts

Lightest quark-antiquark states: Pions

$$\Gamma_{\rm PS}(p;P) \;=\; \int \frac{d^4k}{(2\pi)^4}\, K(p,k;P)\, S(k+P/2)\,\Gamma_{\rm PS}(k;P)\, S(k-P/2)$$

Decompose Bethe–Salpeter amplitude $\Gamma_{\rm PS}(k;P)$ in Lorentz invariants

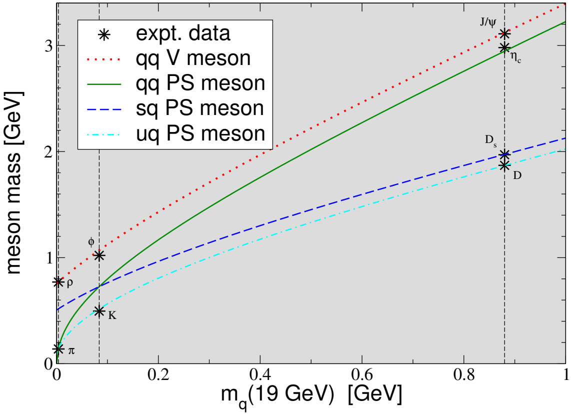
$$\gamma_5\left[iE_\pi(k^2,k\cdot P) + \not{P} F_\pi(k^2,k\cdot P) + \not{k}\, G_\pi(k^2,k\cdot P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k^2,k\cdot P)\right]$$

Axial-Vector WTI

$$-iP_\mu\, \Gamma_{5\mu}(k;P) \;=\; S^{-1}(k_+)\,\gamma_5 + \gamma_5\, S^{-1}(k_-) - 2\, m_q(\mu)\,\Gamma_5(k;P)$$

- Pions are Goldstone bosons: massless in chiral limit
- Dominant pion BS amplitude E_π in chiral limit $E_\pi(p^2) = B(p^2)/f_\pi$
- Gell-Mann–Oakes–Renner relation $f_\pi^2\, m_\pi^2 = -2\, m_q(\mu)\, \langle \bar{q}q\rangle_\mu$
- Decay constant of excited pions vanishes in chiral limit

Rainbow-Ladder truncation with Maris–Tandy model



Fitted to π and K mesons, applied to ρ and K^* in Maris, Tandy, PRC60, 055214 (1999)

Maris, Tandy, nucl-th/0511017

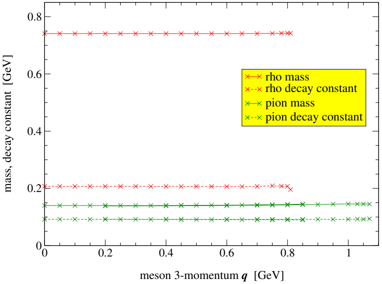
- Beyond RL corrections small for pseudoscalar and vector mesons
- Significant corrections for scalar and axial-vectors
- Have to couple to 4 quark amplitudes (Eichmann, Fischer)
  - meson-loop effects (width) for quark-antiquark states
  - meson-molecules, diquark-antidiquark, ...

Frame independence

- Meson BSE: discrete solutions at $P^2 = -M^2$

$$\Gamma_H(p;P) = \frac{-4}{3}\int \frac{d^4k}{(2\pi)^4} 4\pi\,\alpha((p-k)^2) D_{\mu\nu}(p-k)\,\gamma_\mu\, S(k+P/2)\Gamma_H(k;P)S(k-P/2)\,\gamma_\nu$$

  - rest frame: $P = (iM, 0, 0, 0)$
  - moving meson: $P = (iE, q, 0, 0)$ with $E^2 = M^2 + q^2$
- Taylor expansion from rest frame to moving frame: ok for small q^2
- Solve BSE in moving frame



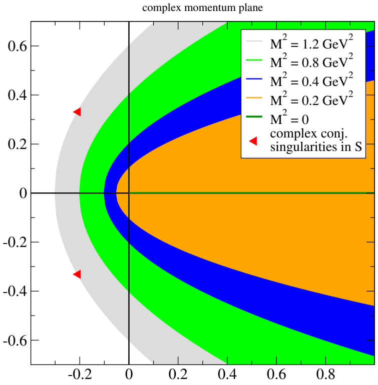
Maris, Tandy, nucl-th/0511017

- Numerically more expensive
  - more independent variables
  - $p \cdot P$ complex, instead of imaginary
- Limited by analytic structure of quark propagators

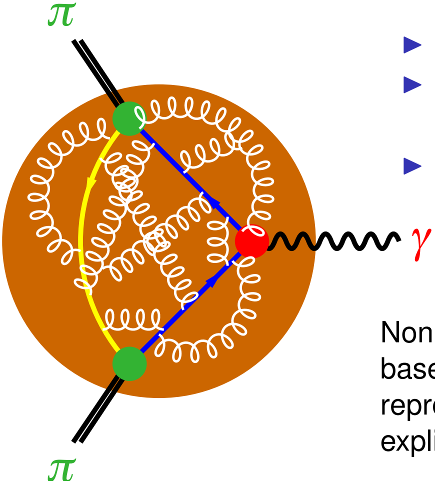
Analytic structure of rainbow DSE solution

- Landau gauge, bare vertex, pQCD for UV behavior of coupling
- Model for IR behavior of $\alpha(q^2)$ fitted to give chiral condensate $\langle \bar{q}q \rangle = -(240\ \text{MeV})^3$

- Solution appears to have pair of complex-conjugate singularities rather than real mass-like pole
- Allows for bound state calculations of light mesons up to masses of about $\sim 1.2\ \text{GeV}^2$
- Similar structure found for a wide range of models, as well as from lattice QCD



Meson electromagnetic form factors



- Quark propagator
- Meson BS amplitude
  - in moving frame
- Quark-photon coupling

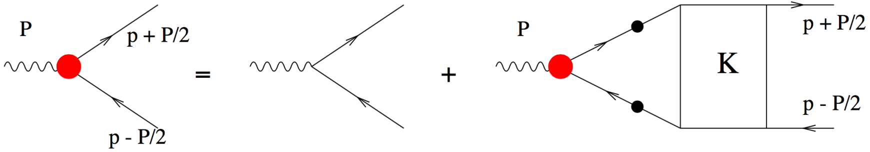
Nonperturbative QFT approach
based on QCD dynamics
reproduces pQCD results
explicitly relativistic, Poincaré invariant

Quark-photon coupling

- Electromagnetic current conservation $\partial_\mu J^\mu = 0$
- Vector Ward–Takahashi identity

$$i\,P_\mu\,\Gamma_\mu(k_+,k_-;P) \;=\; S^{-1}(k+P/2) - S^{-1}(k-P/2)$$

- Inhomogeneous Bethe–Salpeter equation for the quark-photon vertex



- Same kernel K as meson bound state eqn
- Solve for

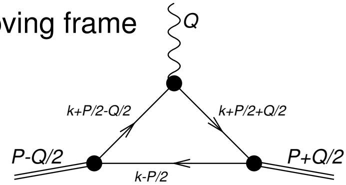
$$\Gamma^T_\mu(k_+,k_-;P) \;=\; \sum_{i=0}^{8} T^i_\mu(k,P) F_i(k^2, k\cdot P; P^2)$$

- Guarantees electromagnetic current conservation

Pion electromagnetic form factor

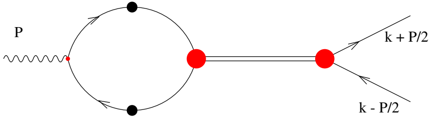
$$\Lambda_\mu(P,Q) = 2\,P_\mu\,F_\pi(Q^2) = N_c \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\bar{\Gamma}^\pi\,S\,i\Gamma_\mu\,S\,\Gamma^\pi\,S\right]$$

- Need at least one BS amplitude in moving frame
- Within numerical accuracy, results independent of
  - choice integration variables
  - form factor frame



- Note: Form factor has pole at vector meson masses
  - quark-photon vertex BSE has poles at $Q^2 = -M_{\rho,\omega,\phi,\ldots}$

$$\Gamma_\mu(k;Q) \simeq \frac{f_\rho\,M_\rho}{Q^2+M_\rho^2}\,\Gamma^\rho_\mu(k;Q)$$

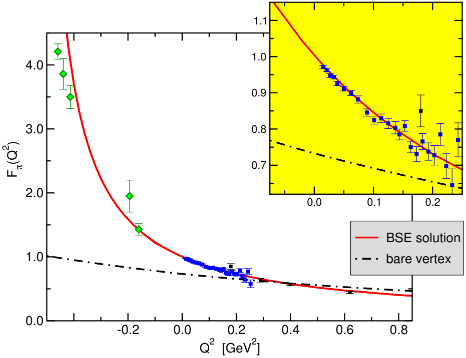


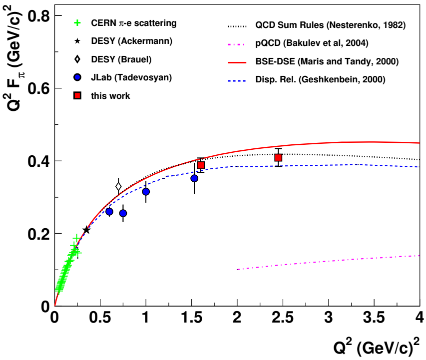
Pion form factor in RL truncation

Tadevosyan *et al.* [Fpi2 Collaboration], nucl-ex/0607007;

Maris, Tandy, PRC62,055204 (2000) [nucl-th/0005015]

Horn *et al.* [Fpi2 Collaboration], nucl-ex/0607005





- Calculation with MT model straightforward up to $Q^2 = 4$ GeV^2 using consistently dressed propagators and vertices without nontrivial deformations of integration contours

Semi-leptonic decays

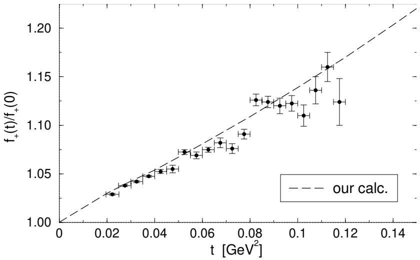
C-R. Ji, Maris, PRD64, 014032 (2001)

Kaon semi-leptonic decay

$$J_\mu^{K^0}(P,Q) = \langle \pi^-(p)|\bar{s}\gamma_\mu u|K^0(k)\rangle = f_+(-Q^2)P_\mu + f_-(-Q^2)Q_\mu$$
$$= N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\big[S^d\,\Gamma_K^{d\bar{s}}\,S^s\,i\Gamma_\mu^{s\bar{u}W}\,S^u\,\bar{\Gamma}_\pi^{u\bar{d}}\big]$$

with $P_\mu = (p+k)_\mu$ and $Q_\mu = (k-p)_\mu$, and $P \cdot Q = m_\pi^2 - m_k^2$

- Kaon and pion BS amplitudes
- Nonperturbatively dressed
  - propagators
  - dressed W vertex

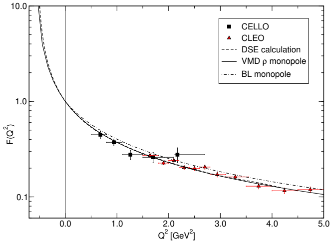


Obtained partial decay width in $10^6 s^{-1}$
$\Gamma(K_{e3}) = 7.38$ (expt. 7.50)
$\Gamma(K_{\mu3}) = 4.90$ (expt. 5.26)

PVV transition form factors

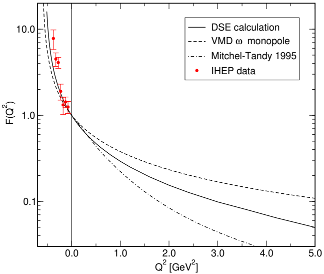
Maris, Tandy, PRC65, 045211 (2002)

$$\Lambda^{PVV}_{\mu\nu}(P;Q) \;=\; N_c \int \mathrm{Tr}\big[S\,\Gamma_P\,S\,\Gamma_\mu\,S\,\Gamma_\nu\big] \;=\; F_{PVV}\Big((P+Q)^2, P^2, Q^2\Big)\,\epsilon_{\mu\nu\rho\sigma}\,P_\rho\,Q_\sigma$$



$\pi\gamma\gamma$ transition

- constraint by axial anomaly: $g_{\pi\gamma\gamma} = 0$ in chiral limit
- anomaly perfectly reprocuced
- form factor agrees with data



$\rho\pi\gamma$ and $\omega\pi\gamma$ transition

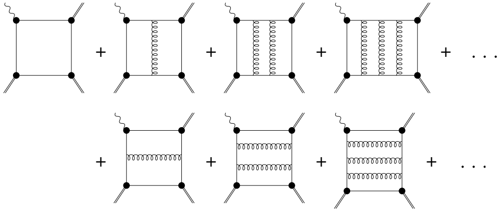
	calc. g/m	expt. g/m
$\rho^0 \to \pi^0\gamma$	0.68 GeV^{-1}	$0.9 \pm .2$
$\rho^\pm \to \pi^\pm\gamma$	0.68	$0.74 \pm .05$
$\omega^0 \to \pi^0\gamma$	2.07	$2.31 \pm .08$

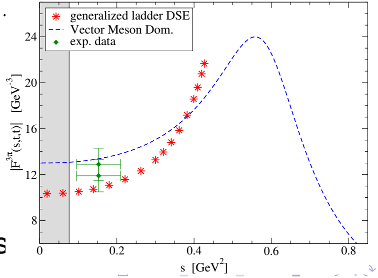
form factor in agreement with available data

Beyond triangles: γ-3π form factor

- Use ladder kernel not only for propagators and vertices, but also inside box diagrams in order to preserve symmetries
- Results for $\pi\pi$ scattering agree with dynamical χSB

Bicudo, Cotanch, Llanes-Estrada, Maris, Ribeiro and Szczepaniak, PRD65, 076008 (2002)





- Results for $\gamma 3\pi$ agree with χSB and current conservation

Cotanch, Maris, PRD68, 036006 (2003)

- Need to include 4 quark amplitudes in order to incorporate pion loop effects

Challenge: From Euclidean BSAs to LFWFs

Can we extract the LFWFs from the Euclidean BSAs ?

- Solve Bethe–Salpeter Eqn 'near' Minkowski space
  - explicit Wick rotation back to Minkowski space, starting from converged solution in Euclidean space
- Project (approximate) Minkowski BSA onto the Light-Front

work in progress

- Nakanishi formulation
  - can be implemented for BSE, using constituent propagators
  - implementation for fermion DSE more difficult   work in progress
  - applicability to confined states unclear to me

Nakanishi integral representation

Nakanishi, Phys.Rev. 130, 1230 (1963); Prog.Theor.Phys.Suppl. 43, 1 (1969)

- For propagators (2-point functions) of asymptotic states

$$S(p) \quad = \quad -i \int_0^\infty d\gamma \frac{\rho(\gamma)}{(\gamma + m^2 - p^2 - i\epsilon)^n}$$

$n = 1$ gives usual Källen–Lehmann representation

- For two-body BSA for bound state with mass $M^2 = P^2$

$$\Gamma(p; P) \quad = \quad -i \int_{-1}^1 dz \int_0^\infty d\gamma \frac{g(\gamma, z)}{(\gamma + m^2 - M^2/4 - p^2 - p \cdot P\, z - i\epsilon)^n}$$

- Used for
  - 2-body scalar BSE — Kusaka *et al*, PRD56, 5071 (1997)
  - fermion DSE and BSE — Sauli, JHEP 0303, 1 (2003)
  - recent work: Carbonell, Karmanov, Frederico, Salmè, ...

Un-Wick rotating from Euclidean to Minkowski metric

$$\Gamma(p;P) \;=\; g^2 \int_{-\infty}^{\infty} dk_0 \int \frac{d^3\vec{k}}{(2\pi)^4}\, K(p,k;P)\, S(k+p/2)\,\Gamma(k;P)\, S(k-p/2)$$

- Un-Wick rotate p_0 and k_0 from Euclidean metric in decrements θ starting from $\theta = \pi/2$

$$\begin{aligned} p_4 &\to \exp\left(-i(\pi/2-\theta)\right)\; p_4 \;=\; \exp(i\theta)\; p_0 \\ k_4 &\to \exp\left(-i(\pi/2-\theta)\right)\; k_4 \;=\; \exp(i\theta)\; k_0 \end{aligned}$$

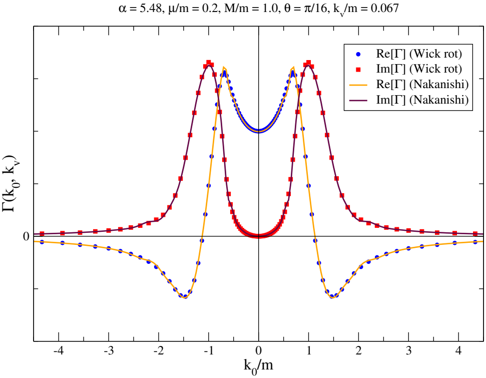
- Solve BSE iteratively as function of p_0 and $\vec{p}^2$ along rotated p_0 axis, starting with solution at previous value of θ, to obtain Green's functions as function of $p_0\,\mathrm{e}^{i\theta}$ and $\vec{p}^2$, instead of as function of Lorentz scalar p^2
- Use Pauli–Villars regulator to remove UV divergences
- Approach Minkowski space for $\theta \to 0$
  - space-like region $p_0^2 = 0$ with $\vec{p}^2 > 0$
  - time-like region $p_0^2 > 0$ with $\vec{p}^2 = 0$
- Manifestly covariant BSA for space- and time-like momenta

Example: scalar model in ladder truncation

Castro *et al*, JPCS 1291, 012006 (2019)

Use Nakanishi representation for $\chi(k;P)$ at $P^2 = M^2$

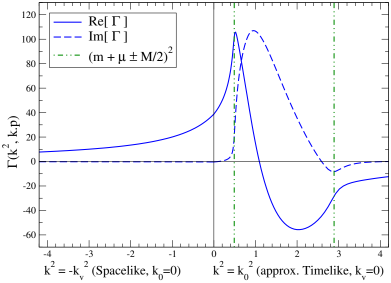
$$\chi(k;P) \equiv \Delta(k+P/2)\,\Gamma(k;P)\,\Delta(k-P/2)$$
$$= -i\int_{-1}^{1} dz \int_0^\infty d\gamma \frac{g(\gamma,z)}{(\gamma + m^2 - P^2/4 - k^2 - k\cdot P\,z - i\epsilon)^3}$$

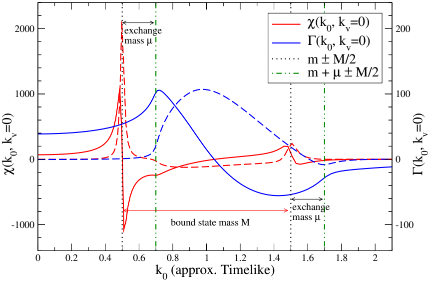


- Calculate Γ using $\Delta^{-1}\,\chi\,\Delta^{-1}$
- Γ has singularities at $k_0^{\pm} = \sqrt{(m+\mu)^2 + \vec{k}^2} \pm \frac{M}{2}$
- $\chi(k;P)$ contains constituent poles at $k\cdot p = \pm(k^2 - m^2 + M^2/4)$, as well as above singularities

Spacelike and (almost) timelike BS Amplitudes

Castro *et al*, JPCS 1291, 012006 (2019)





- $\Gamma(k_0, \vec{k})$ has singularities at $k_0^\pm = \sqrt{(m+\mu)^2 + \vec{k}^2} \pm \frac{M}{2}$
- $\chi = \Delta\,\Gamma\,\Delta$ has additional singularities
  due to the mass poles in the constituents $\Delta(P/2 \pm k)$

LFWF from Covariant Bethe–Salpeter Amplitude

work in progress, w. Ydrefors, de Paula, Frederico, Jia, ...

Project BSA $\chi(k;P) = \Delta(k+P/2)\,\Gamma(k;P)\,\Delta(-k+P/2)$ onto the light-front to obtain the LFWF $\psi(x,k_\perp)$

$$\psi(x,k_\perp) \quad = \quad i\,P^+\,x\,(1-x)\int_{-\infty}^{\infty}\frac{dk^-}{2\pi}\,\chi(k;P)$$

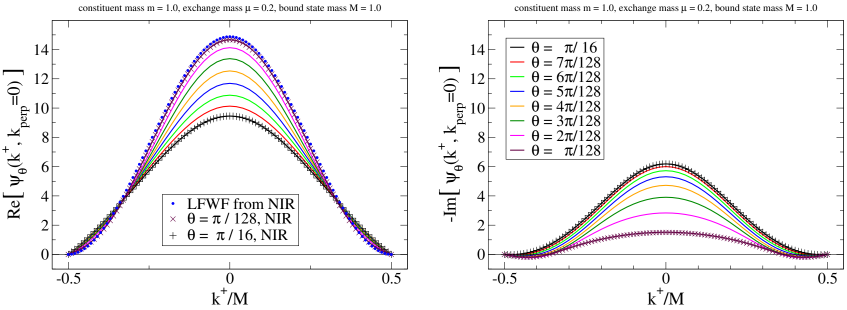
- Can be done with Nakanishi representation for χ
- Can be approximated by un-Wick rotating the BSE from the spacelike region and project

$$\psi_\theta(k^+,k_\perp) \quad = \quad i\,M\left(\frac{1}{2}+\frac{k^+}{M}\right)\left(\frac{1}{2}-\frac{k^+}{M}\right)\int\frac{dk^-}{2\pi}\,\chi(k_\theta;p)$$

  where $k_\theta = (k_0\exp(i\theta),\vec{k})$, and $k^\pm = k_0 \pm k_3$
- In the limit $\theta \to 0$, the 'quasi' LFWF $\psi_\theta(k^+,k_\perp)$ becomes the LFWF $\psi(x,k_\perp)$ with $x = \frac{1}{2}+\frac{k^+}{M}$

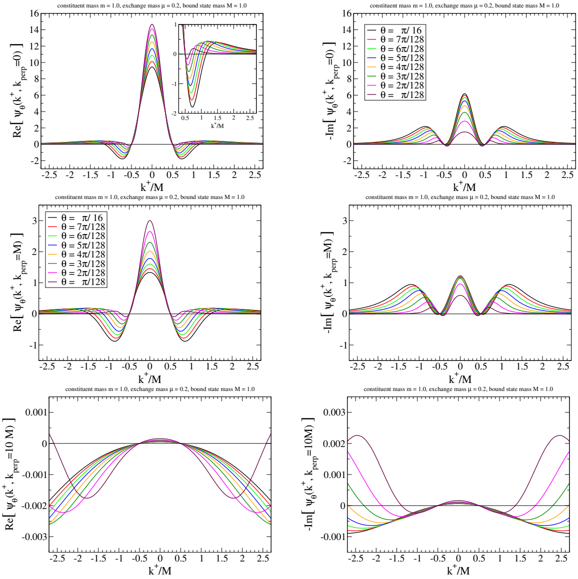
Example: LFWF for scalar model

work in progress, w. Ydrefors, de Paula, Frederico, Jia, . . .



- Perfect agreement for 'quasi' LFWF $\psi_\theta(k^+, k_\perp)$ at $\theta > 0$ between independent calculations using the Nakanishi Intergral Representation and by un-Wick rotating the BSE from the spacelike region

LFWF from covariant Bethe–Salpeter Eqn



- Finite domain $0 < x < 1$ arises naturally as θ decreases
- No need to constrain range on k^+
- However, as $k_\perp$ increases, one needs very small values of θ
- Can take the limit $\theta \to 0$ with NIR

Valence probability

- Valence probability from projected BSA

$$\mathcal{P} \; = \; \int_0^1 \frac{dx}{x(1-x)} \int \frac{d^2k_\perp}{2(2\pi)^3} \; |\psi(x,k_\perp)|^2$$

  - $\mathcal{P} \sim 0.65$ to 0.8 for moderate and strong binding
  - $\mathcal{P} \to 1$ in the limit of zero binding

Frederico, Salmè, Viviani, PRD89 016010 (2014)

- BSA also contains contributions from $|q\bar{q}g\rangle$ Fock sectors as is also evident from the singularities at $k_0^\pm = \sqrt{(m+\mu)^2 + \vec{k}^2} \pm \frac{M}{2}$

Castro *et al*, JPCS 1291, 012006 (2019)

- Calculate (light-front) observables directly from BSA instead of projecting BSA on the LFWF, and computing observables from LFWF

Conclusion BSAs and outlook

- Nonperturbatively dressed quark propagator connects current quark mass and pQCD with constituent quark models
- Preserve relevant symmetries and properties
  - Poincaré invariance
  - axial-vector WTI for chiral symmetry & chiral symmetry breaking
  - vector WTI for electromagnetic observables
- Mesons can be described accurately as $q\bar{q}$ bound states
  - form factors in good agreement with available data
- Challenge
  - beyond rainbow-ladder
  - incorporating open decay channels
  - observables defined on the light-front (GPDs, ...)

Outlook

- project meson BSA onto LF wavefunction
- use BSA directly to calculate LF observables

Electromagnetic form factors in BLFQ

- Frame dependence of form factors in light-front dynamics,
  Yang Li, Maris, Vary, PRD97, 054034 (2018)
- Radiative transitions between 0^{-+} and 1^{--} heavy quarkonia on the light front,
  Meijian Li, Yang Li, Maris, Vary, PRD98, 034024 (2018)
- Form factors and generalized parton distributions of heavy quarkonia in basis light front quantization,
  Adhikari, Yang Li, Meijian Li, Vary, PRD99, 035208 (2019)
- Frame dependence of transition form factors in light-front dynamics,
  Meijian Li, Yang Li, Maris, Vary, PRD100, 036006 (2019)
- Semileptonic Decay of B_c to η_c and J/ψ on the Light Front,
  Tang, Jia, Maris, Vary, arxiv:2002.06489 (2020), submitted for publication

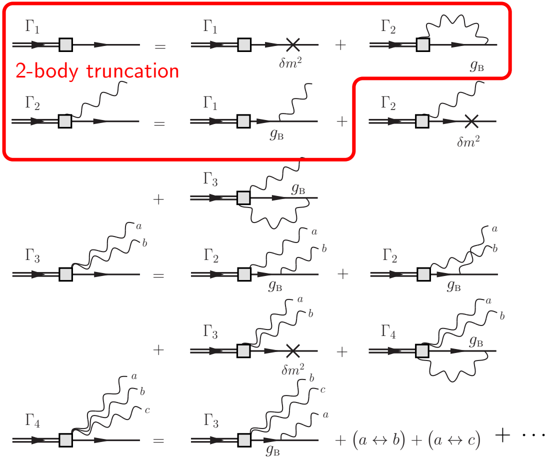
Electromagnetic form factors in DSE approach

- The pi, K+, and K0 electromagnetic form factors,
  Maris, Tandy, PRC62,055204 (2000)
- Electromagnetic transition form factors of light mesons,
  Maris, Tandy, PRC65, 045211 (2002)
- K(l3) transition form factors, C-R. Ji, Maris, PRD64, 014032 (2001)
- Ladder Dyson–Schwinger calculation of the anomalous gamma-3pi form factor, Cotanch, Maris, PRD68 036006 (2003)
- Electromagnetic properties of ground and excited state pseudoscalar mesons, Höll, Krassnigg, Maris, Roberts, PRC71, 065204 (2005)
- QCD modeling of hadron physics, Maris, Tandy, NPB Proc.Suppl. 161, 136 (2006)
- Hadron Physics and the Dyson-Schwinger Equations of QCD,
  Maris, AIP Conf. Proc. 892, 65 (2007)
- Vector meson form factors and their quark-mass dependence,
  Bhagwat, Maris, PRC77, 025203 (2008)

Fock-space convergence in scalar Yukawa theory

$$|\chi_{\text{ph}}\rangle = |\chi\rangle + |\chi\varphi\rangle + |\chi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\rangle + |\chi\varphi\varphi\varphi\varphi\rangle + \cdots$$

Diagrams in the one-body sector:



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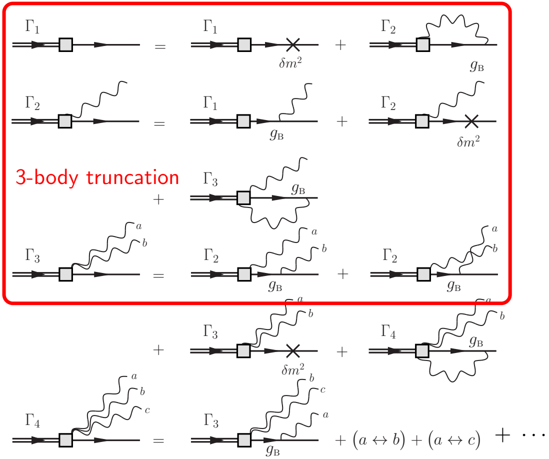

Fock-space convergence in scalar Yukawa theory

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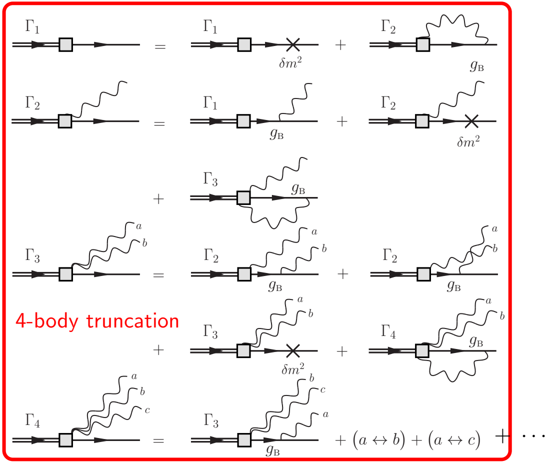


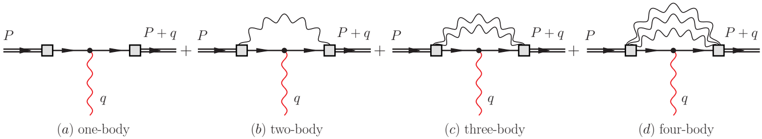
Convergence of form factor with Fock space expansion

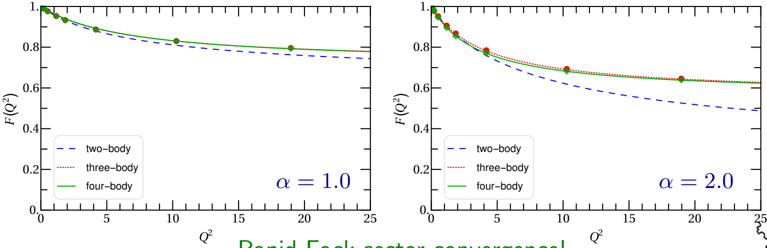
Yang Li, Karmanov, Maris, Vary, PLB748, 278 (2015)

Obtain solutions of the charge-one sector up to four-body: $\chi + \varphi\varphi\varphi$

Study the convergence of Fock sector expansion by comparing different Fock sector truncations

Fock sector convergence of the electromagnetic form factor:





Rapid Fock sector convergence!