

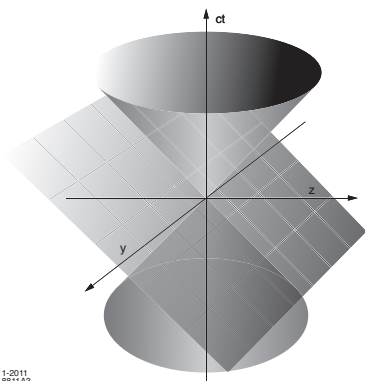
Light-front holographic QCD and superconformal algebra: An overview

Guy F. de Téramond

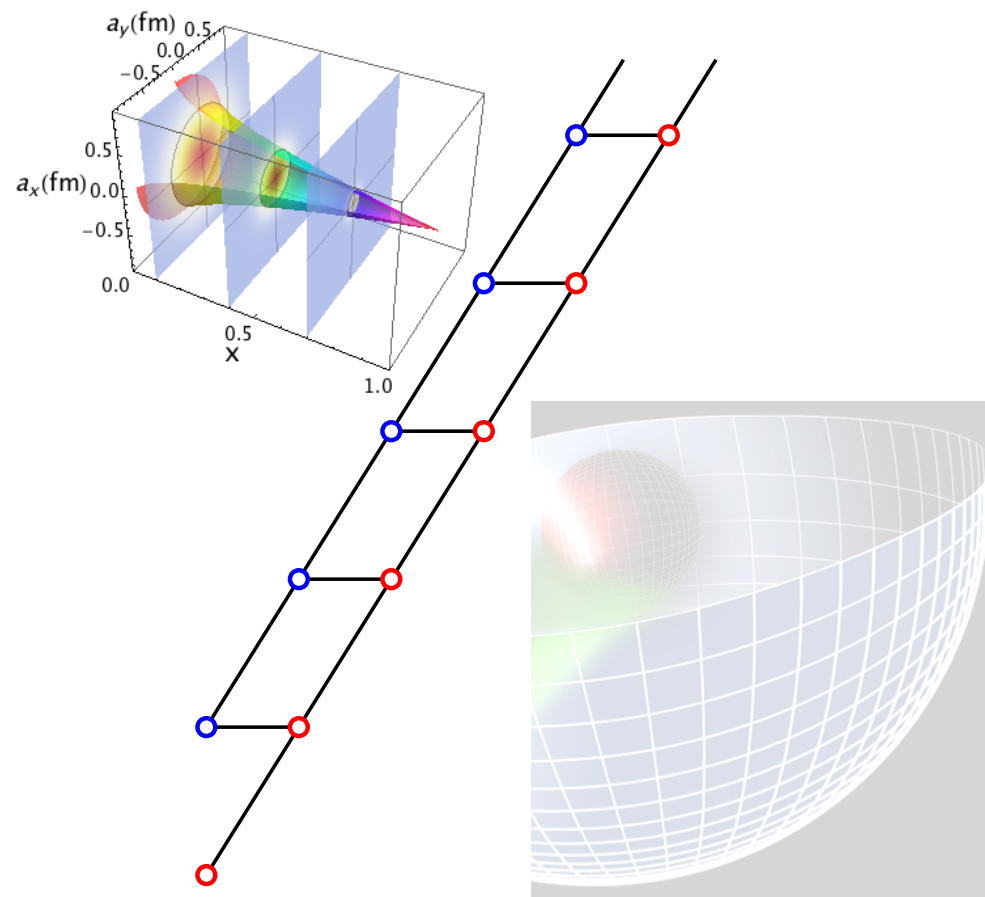
UCR

ILCAC Seminar Series

28 October 2020



1-2011
8811A3



In collaboration with Stan Brodsky, Hans G. Dosch, Alexandre Deur, Tianbo Liu and Raza Sabbir Sufian

A quest for nonperturbative analytic structures in QCD

- Recent insights into the nonperturbative structure of QCD based on light-front quantization and its holographic embedding have lead to effective semiclassical bound-state equations for arbitrary spin
- The confinement potential for baryons and mesons is determined by an underlying superconformal algebraic structure leading to unsuspected connections across the entire mass spectrum of hadrons
- The extension of this framework incorporates the structure of Veneziano amplitudes useful to describe form factors and parton distributions including the sea quark component of the proton

Review in:

S. J. Brodsky, GdT, H.G. Dosch, J. Erlich, *Phys. Rept.* **584**, 1 (2015) [[hep-ph/9705477](#)]

L. Zou and H. G. Dosch, [arXiv:1801.00607](#) [[hep-ph](#)]

R. Sandapen, [arXiv:2001.03479](#) [[hep-ph](#)].

S. J. Brodsky, GdT, H. G. Dosch, [arXiv:2004.07756](#) [[hep-ph](#)]

Contents

1	Semiclassical approximation to QCD in the light front	4
2	Integer-spin wave equations in AdS and LF holographic embedding	6
3	Half-integer-spin wave equations in AdS and LF holographic embedding	9
4	Superconformal algebraic structure in LFHQCD	11
5	Light-front mapping and baryons	13
6	Superconformal meson-baryon-tetraquark symmetry	14
7	Form factors, parton distributions and intrinsic quark sea	18
8	Outlook	26

1 Semiclassical approximation to QCD in the light front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}$$

- Express the hadron 4-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, in terms of dynamical fields $\psi_+ = \Lambda_+ \psi$ and \mathbf{A}_\perp ($\Lambda_\pm = \gamma^0 \gamma^\pm$) quantized in the null plane $x^+ = x^0 + x^3 = 0$

$$P^- = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi}_+ \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi_+ + \text{interactions}$$

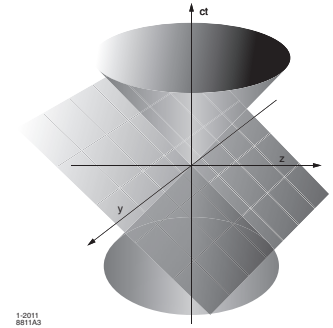
$$P^+ = \int dx^- d^2 \mathbf{x}_\perp \bar{\psi}_+ \gamma^+ i\partial^+ \psi_+$$

$$\mathbf{P}_\perp = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi}_+ \gamma^+ i\nabla_\perp \psi_+$$

- LF Hamiltonian from Poincaré group invariant $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle, \quad |\psi\rangle = \sum_n \psi_n |n\rangle$$

- Simple structure of LF vacuum allows a quantum-mechanical probabilistic interpretation of hadronic states in terms of wave functions, $\psi_n = \langle n | \psi \rangle$, similar to usual Schrödinger equation

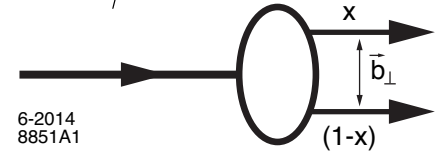


- The mass spectrum for a two-parton bound state is computed from the hadron matrix element

$$\langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = M^2 \langle \psi(P') | \psi(P) \rangle$$

- We factor out the longitudinal $X(x)$ and orbital $e^{iL\varphi}$ dependence from the LFWF ψ

$$\psi(x, \zeta, \varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$



with invariant impact “radial” LF variable $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$ and $L = \max|L^z|$

- Ultra relativistic limit $m_q \rightarrow 0$ longitudinal modes $X(x)$ decouple

$$M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the effective potential U includes all interactions, including those from higher Fock states

- The Lorentz invariant equation $P_\mu P^\mu |\psi\rangle = M^2 |\psi\rangle$ becomes a LF wave equation for ϕ

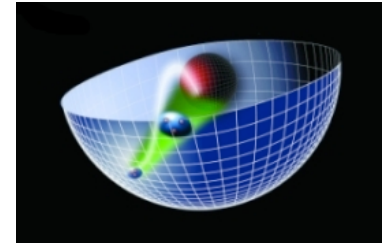
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$

- Critical value $L = 0$ corresponds to the lowest possible stable solution
- Relativistic and frame-independent semiclassical WE: It has identical structure of AdS WE

2 Integer-spin wave equations in AdS and LF holographic embedding

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- AdS₅ is a 5-dim space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space
- Isomorphism of $SO(4, 2)$ conformal group with the group of isometries of AdS₅
- Physical interpretation: holographic variable z inverse of energy scale of a probe
- We start from effective AdS action for a rank- J tensor field $\Phi_{N_1 \dots N_J}$ with AdS mass μ and a dilaton profile φ which breaks the maximal symmetry of AdS (the conformality of the dual theory)

$$ds^2 = \frac{R^2}{z^2} (dx_\mu dx^\mu - dz^2)$$

$$S_{eff} = \int d^d x dz \sqrt{g} e^{\varphi(z)} g^{N_1 N'_1} \dots g^{N_J N'_J} \left(g^{MM'} D_M \Phi_{N_1 \dots N_J}^* D_{M'} \Phi_{N'_1 \dots N'_J} - \mu_{eff}^2(z) \Phi_{N_1 \dots N_J}^* \Phi_{N'_1 \dots N'_J} \right)$$

where $\sqrt{g} = (R/z)^{d+1}$ and the covariant derivative D_M includes the affine connection

- Effective mass $\mu_{eff}(z)$ is determined by precise mapping to light-front physics

- In holographic QCD a hadron is described by a z -dependent wave function $\Phi_J(z)$ and a plane wave in physical spacetime with polarization indices ν along Minkowski coordinates

$$\Phi_{\nu_1 \dots \nu_J}(x, z) = e^{iP \cdot x} \epsilon_{\nu_1 \dots \nu_J}(P) \Phi_J(z)$$

with invariant mass $P_\mu P^\mu = M^2$

- From $\delta S / \delta \Phi = 0$ follows the eigenvalue equation ($m = m(\mu, \varphi) = \text{const}$)

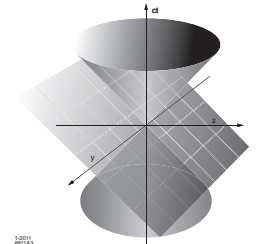
$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left(\frac{mR}{z} \right)^2 \right] \Phi_J = M^2 \Phi_J$$

- Upon the substitution

$$\Phi_J(z) = z^{(d-1)/2-J} e^{-\varphi(z)/2} \phi_J(z)$$

we find the semiclassical QCD LFWE for $d = 4$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)$$



with the holographic variable z

$$z^2 \rightarrow \zeta^2 = x(1-x)b_\perp^2$$

identified with the LF invariant separation between two quarks and $(mR)^2 = -(2 - J)^2 + L^2$

- The effective LF potential U

$$U(\zeta, J) = \frac{1}{2}\varphi''(\zeta) + \frac{1}{4}\varphi'(\zeta)^2 + \frac{2J-3}{2\zeta}\varphi'(\zeta)$$

is determined in terms of the AdS dilaton profile φ (the IR modification of AdS space)

- Non-trivial geometry of AdS encodes the higher-spin kinematic constraints

$$\eta^{\mu\nu}\partial_\mu\Phi_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\Phi_{\mu\nu\nu_3\cdots\nu_J} = 0$$

which in AdS follow from the variation of the AdS action

- Additional IR deformations of AdS encode the dynamics, including confinement
- AdS Breitenlohner-Freedman bound $(mR)^2 \geq -4$ is equivalent to LF QM stability condition $L^2 \geq 0$
- Question: how can we determine the effective LF potential U , equivalently, the dilaton field φ ?
- Important clues from the description of baryons in AdS ...

3 Half-integer-spin wave equations in AdS and LF holographic embedding

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

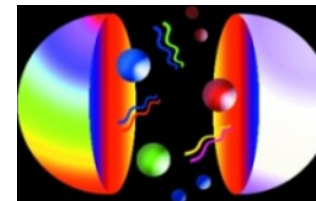


Image credit: N. Evans

- Extension of holographic ideas to higher half-integral spin- J hadrons by considering wave equations for Rarita-Schwinger (RS) spinor fields in AdS space $\Psi_{N_1 \dots N_{J-1/2}}$ and their mapping to LF physics
- From the RS action in AdS with effective potential V follows the coupled equations for the chiral components $\psi_{\pm}(z)$

$$-\frac{d}{dz}\psi_- - \frac{\nu + \frac{1}{2}}{z}\psi_- - V(z)\psi_- = M\psi_+$$

$$\frac{d}{dz}\psi_+ - \frac{\nu + \frac{1}{2}}{z}\psi_+ - V(z)\psi_+ = M\psi_-$$

with $|\mu R| = \nu + \frac{1}{2}$ (μ is the AdS mass)

- Mapping to the light front $z \rightarrow \zeta$, system of linear eqs in AdS is equivalent to second order eqs:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U^+(\zeta) \right) \psi_+ = M^2 \psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4(L+1)^2}{4\zeta^2} + U^-(\zeta) \right) \psi_- = M^2 \psi_-$$

the semiclassical QCD LF WE with ψ_+ and ψ_- corresponding to LF orbital L and $L + 1$ with

$$U^\pm(\zeta) = V^2(\zeta) \pm V'(\zeta) + \frac{1 + 2L}{\zeta} V(\zeta), \quad L = \nu,$$

and $L = \nu$

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Example: For internal spin $S = \frac{3}{2}$ and $L = 2$ the mass of the $\Delta^{\frac{1}{2}}$, $\Delta^{\frac{3}{2}}$, $\Delta^{\frac{5}{2}}$ and $\Delta^{\frac{7}{2}}$ quartet is fully degenerate for the different J -values, consistent with the LF-AdS structure independently of the specific form of $V(\zeta)$: It should only depend on S and L
- How can we determine the form of $V(\zeta)$? It is completely fixed by superconformal QM

4 Superconformal algebraic structure in LFHQCD

[V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

[S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

[S. J. Brodsky, GdT and H. G. Dosch, Phys. Lett. B **729**, 3 (2014)]

[GdT, H. G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

[H. G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

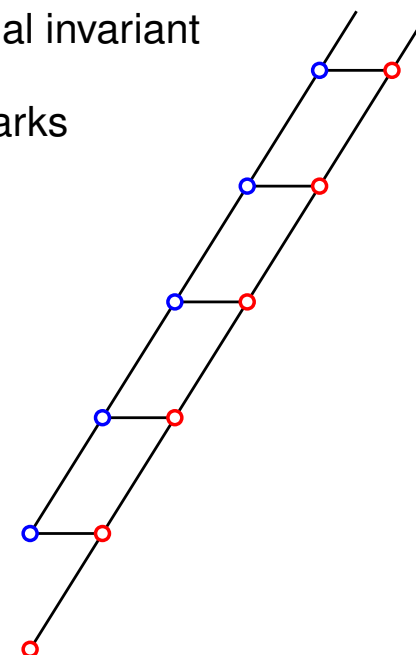
- Superconformal algebra underlies in LFHQCD the scale invariance of the QCD Lagrangian. It leads to the introduction of a scale in the Hamiltonian maintaining the action conformal invariant
- It also leads to a specific connection between mesons, baryons and tetraquarks underlying the $SU(3)_C$ representation properties: $\bar{3} \rightarrow 3 \times 3$
- SUSY QM contains two fermionic generators Q and Q^\dagger , and a bosonic generator, the Hamiltonian H [E. Witten, NPB **188**, 513 (1981)]

$$\frac{1}{2}\{Q, Q^\dagger\} = H$$

$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \quad [Q, H] = [Q^\dagger, H] = 0$$

which closes under the graded algebra $sl(1/1)$

- Since $[Q^\dagger, H] = 0$, the states $|E\rangle$ and $Q^\dagger|E\rangle$ for $E \neq 0$ are degenerate, but for $E = 0$ we can have the trivial solution $Q^\dagger|E = 0\rangle = 0$ (the pion ?)



- Conformal graded-Lie algebra has in addition to the Hamiltonian H and supercharges Q and Q^\dagger , a new operator S related to the generator of conformal transformations $\frac{1}{2}\{S, S^\dagger\} = K$
- Following Fubini and Rabinovici we define the fermionic generator $R = Q + \lambda S$, $[\lambda] = \text{GeV}^2$,

$$\{R_\lambda, R_\lambda^\dagger\} = G_\lambda$$

$$\{R_\lambda, R_\lambda\} = \{R_\lambda^\dagger, R_\lambda^\dagger\} = 0, \quad [R_\lambda, G_\lambda] = [R_\lambda^\dagger, G_\lambda] = 0$$

which also closes under the graded algebra $sl(1/1)$:

- In a 2×2 matrix representation the Hamiltonian equation $G|\phi\rangle = E|\phi\rangle$ leads to the equations

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \right) \phi_1 = E \phi_1$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \right) \phi_2 = E \phi_2$$

5 Light-front mapping and baryons

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

- Upon LF mapping in the superconformal eigenvalue equations (slide 12)

$$x \mapsto \zeta, \quad E \mapsto M^2, \quad f \mapsto L + \frac{1}{2}$$

$$\phi_1 \mapsto \psi_-, \quad \phi_2 \mapsto \psi_+$$

we recover the baryon bound-state equations (slide 10)

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L + 1) \right) \psi_+ = M^2\psi_+$$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4(L + 1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda L \right) \psi_- = M^2\psi_-$$

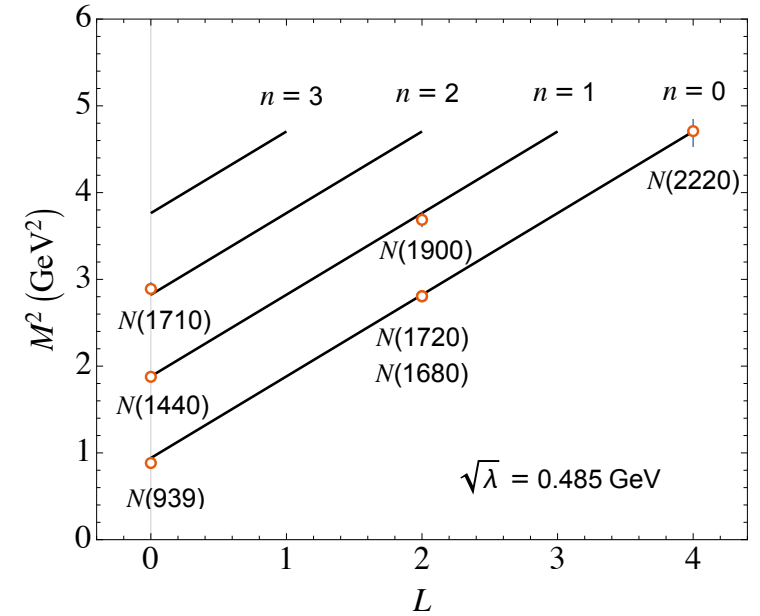
with $U^+ = \lambda^2\zeta^2 + 2\lambda(L + 1)$ and $U^- = \lambda^2\zeta^2 + 2\lambda L$

- Eigenvalues

$$M^2 = 4\lambda(n + L + 1)$$

- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+L} e^{-\lambda\zeta^2/2} L_n^L(\lambda\zeta^2), \quad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+L} e^{-\lambda\zeta^2/2} L_n^{L+1}(\lambda\zeta^2)$$



6 Superconformal meson-baryon-tetraquark symmetry

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

- Upon substitution in the superconformal equations (slide 12)

$$x \mapsto \zeta, \quad E \mapsto M^2,$$

$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

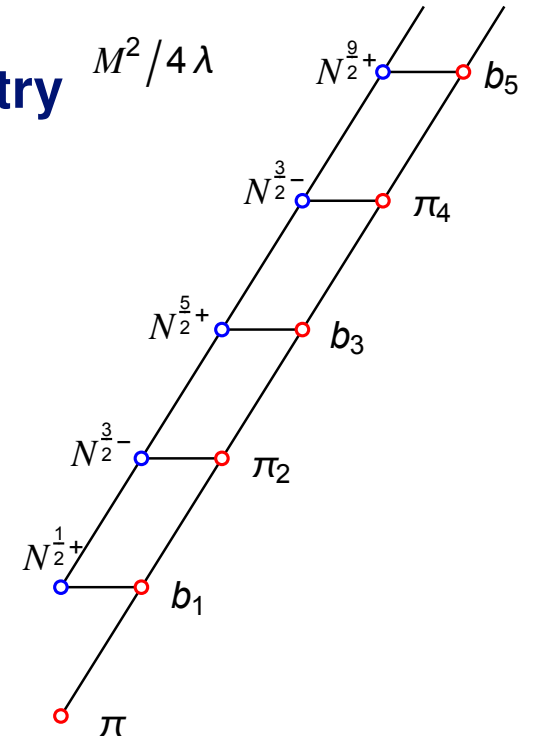
$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) \right) \phi_M = M^2 \phi_M$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) \right) \phi_B = M^2 \phi_B$$

- Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $L_M = L_B + 1$
- L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator cluster in the baryon



- Special role of the pion as a unique state of zero energy

$$R^\dagger |M, L\rangle = |B, L - 1\rangle, \quad R^\dagger |M, L = 0\rangle = 0$$

- Hadron quantum numbers determined from the pion
- Spin-dependent Hamiltonian for mesons and baryons with internal spin S

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé (2016)]

$$G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda S \quad S = 0, 1$$

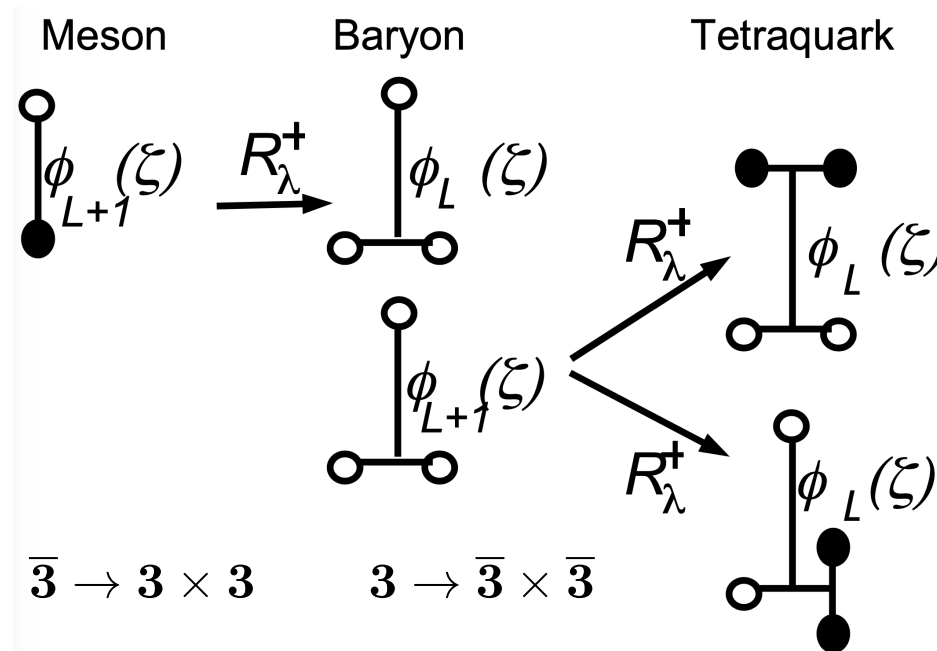
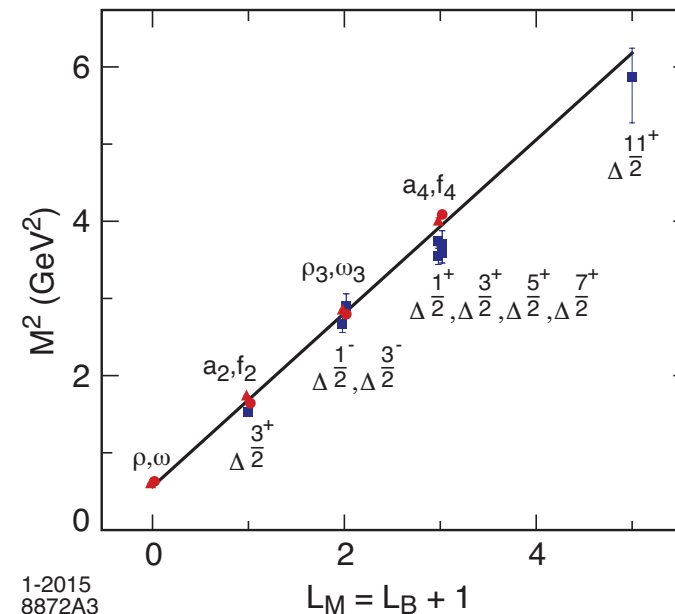
- Supersymmetric 4-plet:

$$M_M^2 = 4\lambda(n + L_M) + 2\lambda S$$

$$M_B^2 = 4\lambda(n + L_B + 1) + 2\lambda S$$

$$M_T^2 = 4\lambda(n + L_T + 1) + 2\lambda S$$

- Expected accuracy $1/N_C^2 \sim 10\%$



Introduction of light quark masses

- Quark masses treated as effective masses in LF Hamiltonian ($m_{u,d} = 46 \text{ MeV}$, $m_s = 357 \text{ MeV}$)

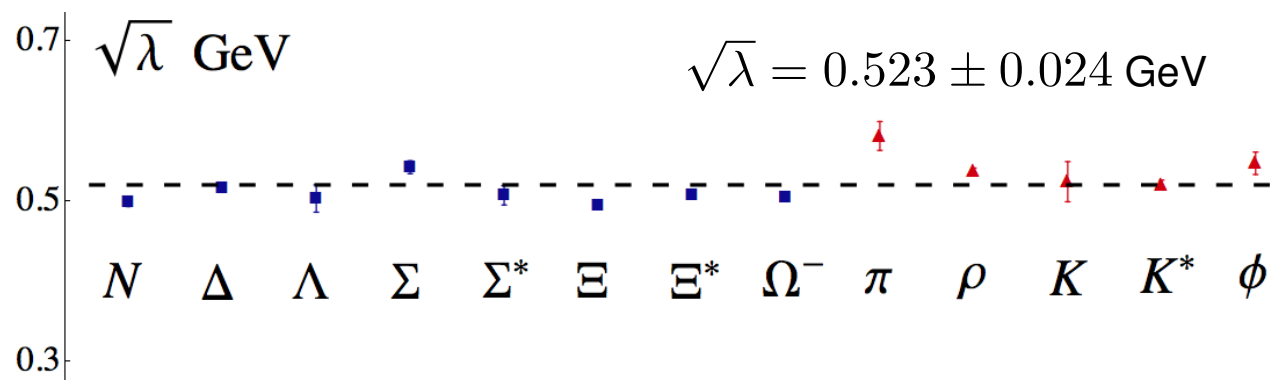
$$\Delta M^2[m_1, \dots, m_n] = \frac{\lambda^2}{F} \frac{dF}{d\lambda}$$

with

$$F[\lambda] = \int_0^1 \cdots \int_0^1 dx_1 \cdots dx_n e^{-\frac{1}{\lambda} \left(\sum_{i=1}^n \frac{m_i^2}{x_i} \right)} \delta\left(\sum_{i=1}^n x_i - 1\right)$$

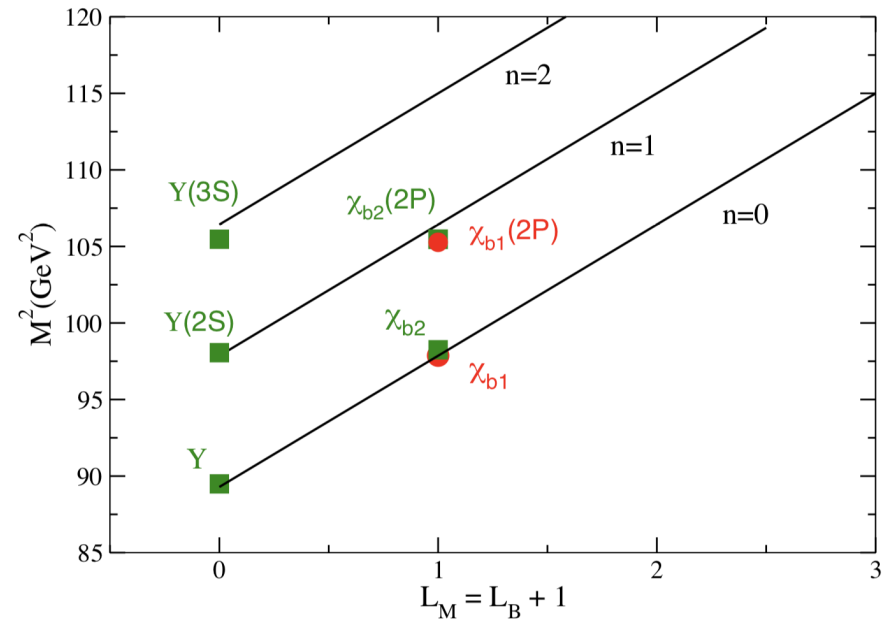
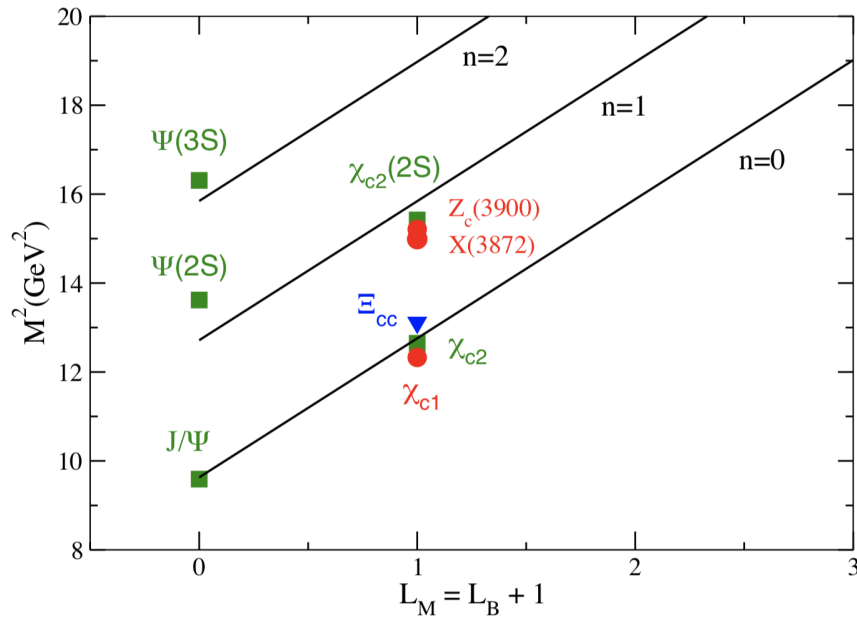
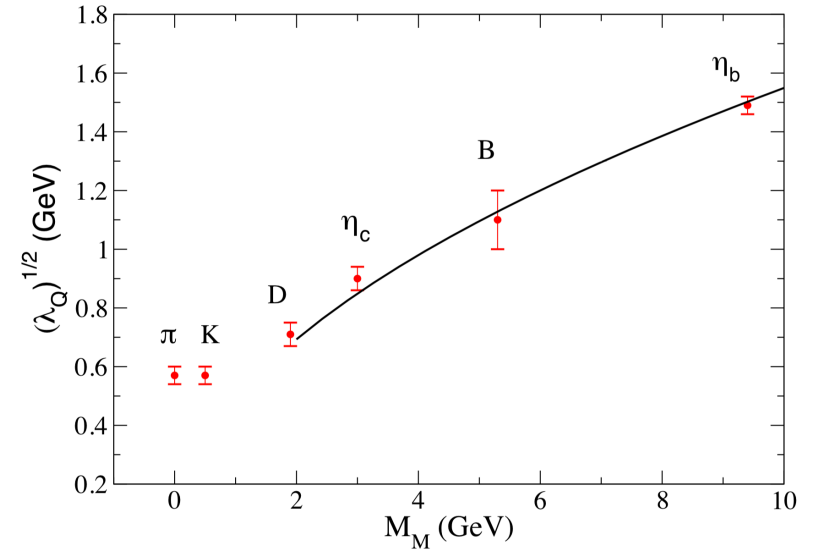
[See also: C. Mondal, S. Xu, J. Lan, X. Zhao, Y. Li, D. Chakrabarti and J. P. Vary (BLFQ Collaboration), Phys. Rev. D **102**, 016008 (2020), and references therein]

- Hadron scale determined from different light hadron channels including all radial and orbital excitations



Extension to heavy-light and heavy-heavy sectors

- Scale dependence of hadronic scale λ from HQET
- Extension to the heavy-light hadronic sector:
[H. G. Dosch, GdT, S. J. Brodsky (2015, 2017)]
- Extension to the double-heavy hadronic sector:
[M. Nielsen, S. J. Brodsky, GdT, H. G. Dosch *et al.* (2018)]
- Extension to the isoscalar hadronic sector:
[L. Zou, H. G. Dosch, GdT, S. J. Brodsky (2018)]



7 Form factors, parton distributions and intrinsic quark sea

- Form factors and parton distributions from extension of LF holographic framework to arbitrary Regge trajectories incorporating the analytic structure of Veneziano amplitudes
- Study of strange and charm quark-sea in the proton from combined LQCD and holographic methods
- Nucleon Form Factors:
[R. S. Sufian, GdT, S. J. Brodsky, A. Deur, H. G. Dosch (2017)]
- Generalized quark distributions:
[GdT, T. Liu, R. S. Sufian, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS Collaboration, 2018)]
- Strange-quark sea in the nucleon
[R. S. Sufian, T. Liu, GdT, H. G. Dosch, S. J. Brodsky, A. Deur, M. T. Islam, B-Q. Ma (2018)]
- Unified description of polarized and unpolarized quark distributions:
[T. Liu, R. S. Sufian, GdT, H. G. Dosch, S. J. Brodsky, A. Deur (HLFHS Collaboration, 2019)]
- Intrinsic-charm content of the proton:
[R. S. Sufian, T. Liu, A. Alexandru, S. J. Brodsky, GdT, H. G. Dosch, T. Draper, K. F. Liu and Y. B. Yang (2020)]

Form factors

- Hadron form factor expressed as a sum from the Fock expansion of states

$$F(t) = \sum_{\tau} c_{\tau} F_{\tau}(t)$$

where the c_{τ} are spin-flavor expansion coefficients for different twist

- $F_{\tau}(t)$ in LFHQCD has the Euler's Beta function structure

$$F_{\tau}(t) = \frac{1}{N_{\tau}} B(\tau - 1, 1 - \alpha(t))$$

found by Ademollo and Del Giudice and Landshoff and Polkinghorne in the pre-QCD era, extending the Veneziano duality model (1968)

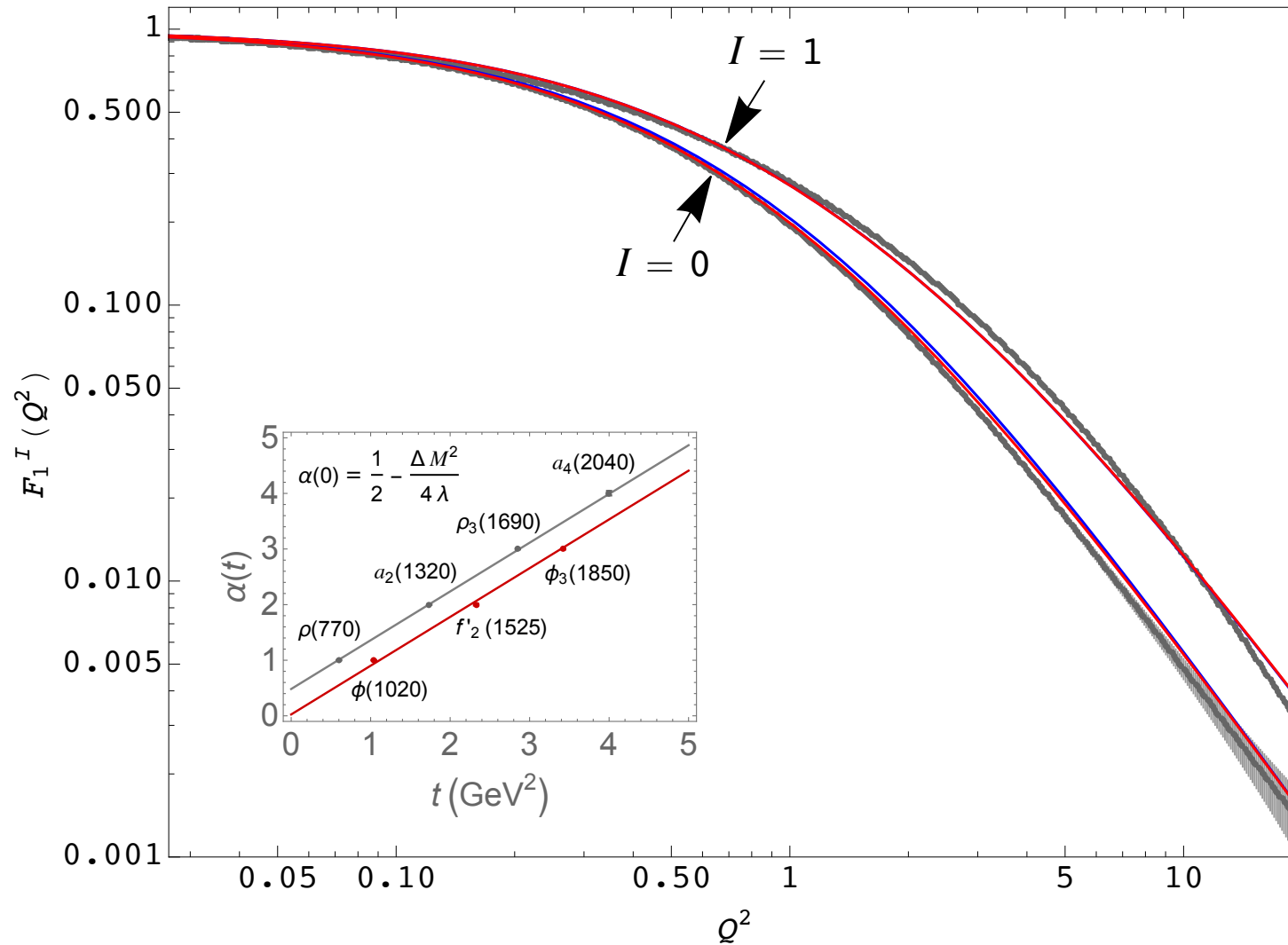
- $\alpha(t)$ is the Regge trajectory of the VM which couples to the quark EM current in the hadron
- For $\tau = N$, the number of constituents in a Fock component, the FF is an $N - 1$ product of poles

$$F_{\tau}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{M_{n=0}^2}\right) \left(1 + \frac{Q^2}{M_{n=1}^2}\right) \cdots \left(1 + \frac{Q^2}{M_{n=\tau-2}^2}\right)}$$

located at

$$-Q^2 = M_n^2 = \frac{1}{\alpha'} (n + 1 - \alpha(0))$$

It generates the radial excitation spectrum of the exchanged VM particles in the t -channel



Nucleon isospin form factors $F^{I=0,1}(t) = F_p(t) \pm F_n(t)$

HLFHS (2019): — Valence contribution only

HLFHS (2019): — Including $u\bar{u}$ and $d\bar{d}$

Ye et al. (2018): — z -expansion data analysis

Quark distributions

- Using integral representation of Beta function FF is expressed in a reparametrization invariant form

$$F(t)_\tau = \frac{1}{N_\tau} \int_0^1 dx w'(x) w(x)^{-\alpha(t)} [1 - w(x)]^{\tau-2}$$

with $w(0) = 0$, $w(1) = 1$, $w'(x) \geq 0$

- Flavor FF is given in terms of the valence GPD $H_\tau^q(x, \xi = 0, t)$ at zero skewness

$$F_\tau^q(t) = \int_0^1 dx H_\tau^q(x, t) = \int_0^1 dx q_\tau(x) \exp[tf(x)]$$

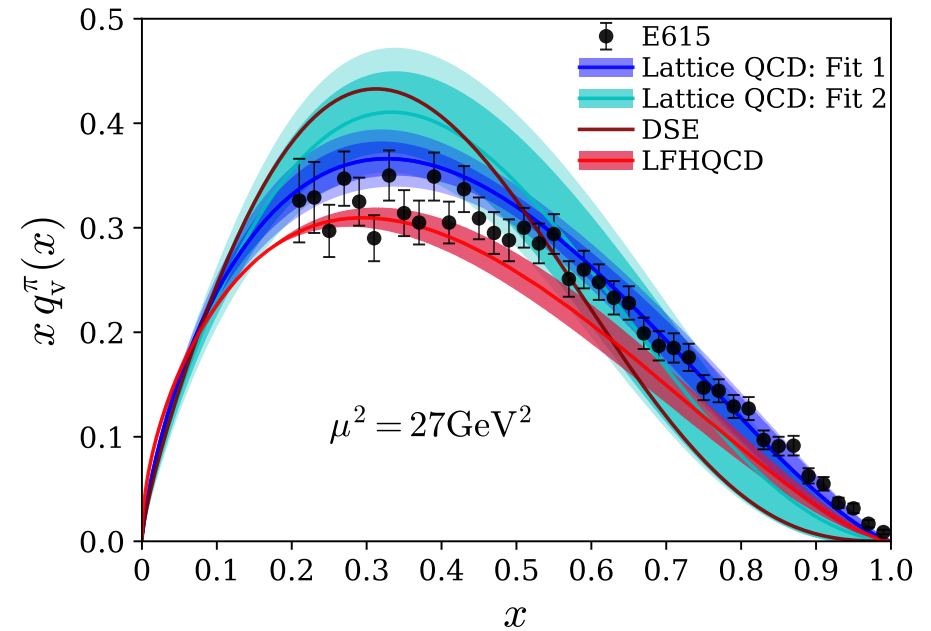
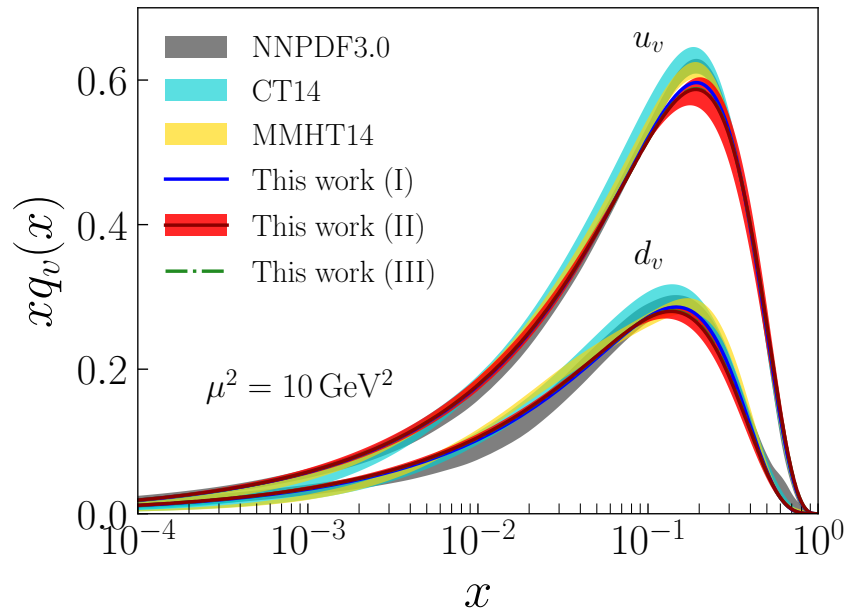
with the profile function $f(x)$ and PDF $q(x)$ determined by $w(x)$

$$f(x) = \frac{1}{4\lambda} \log\left(\frac{1}{w(x)}\right)$$

$$q_\tau(x) = \frac{1}{N_\tau} [1 - w(x)]^{\tau-2} w(x)^{-\alpha(0)} w'(x)$$

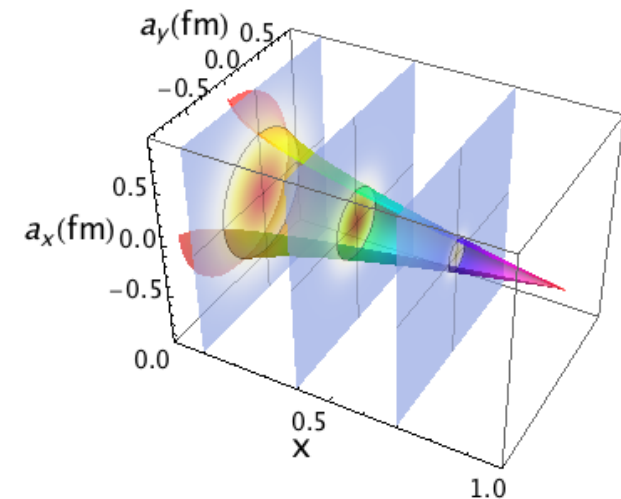
- Boundary conditions: At $x \rightarrow 0$, $w(x) \sim x$ from Regge behavior, $q(x) \sim x^{-\alpha(0)}$, and $w'(1) = 0$ to recover counting rules at $x \rightarrow 1$, $q_\tau(x) \sim (1 - x)^{2\tau-3}$ (inclusive-exclusive connection)
- If $w(x)$ fixed by nucleon PDFs then pion PDF is a prediction. Example: $w(x) = x^{1-x} e^{-a(1-x)^2}$

Unpolarized GPDs and PDFs (HLFHS Collaboration, 2018)



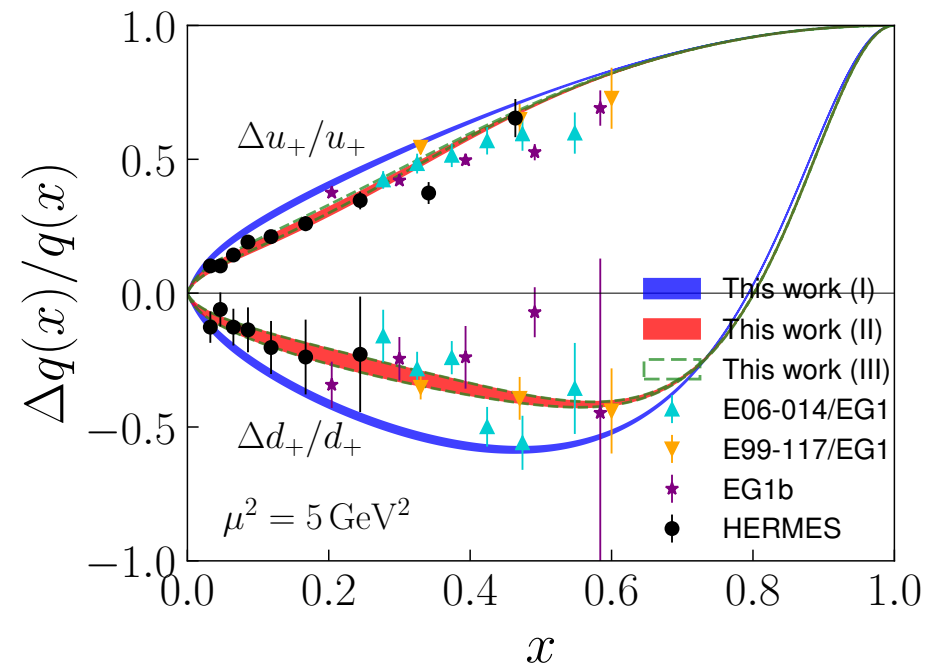
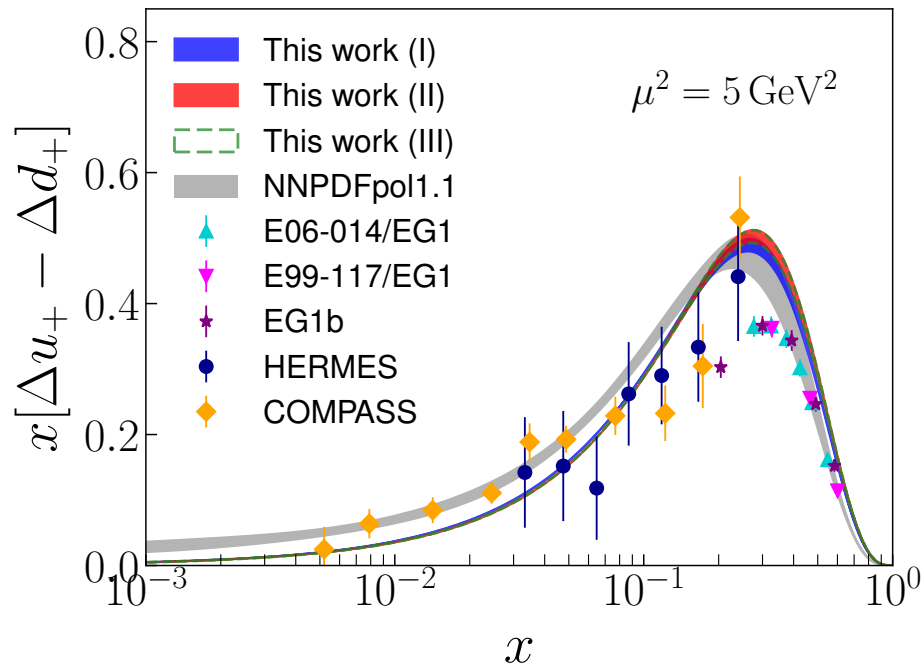
- Transverse impact parameter quark distribution

$$u(x, \mathbf{a}_\perp) = \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} e^{-i\mathbf{a}_\perp \cdot \mathbf{q}_\perp} H^u(x, \mathbf{q}_\perp^2)$$



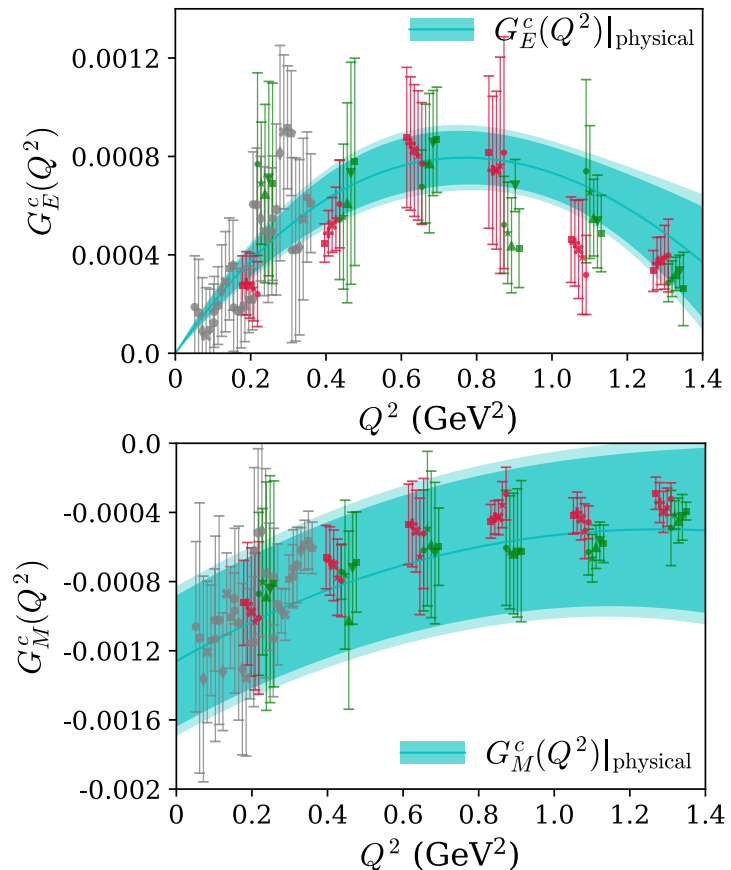
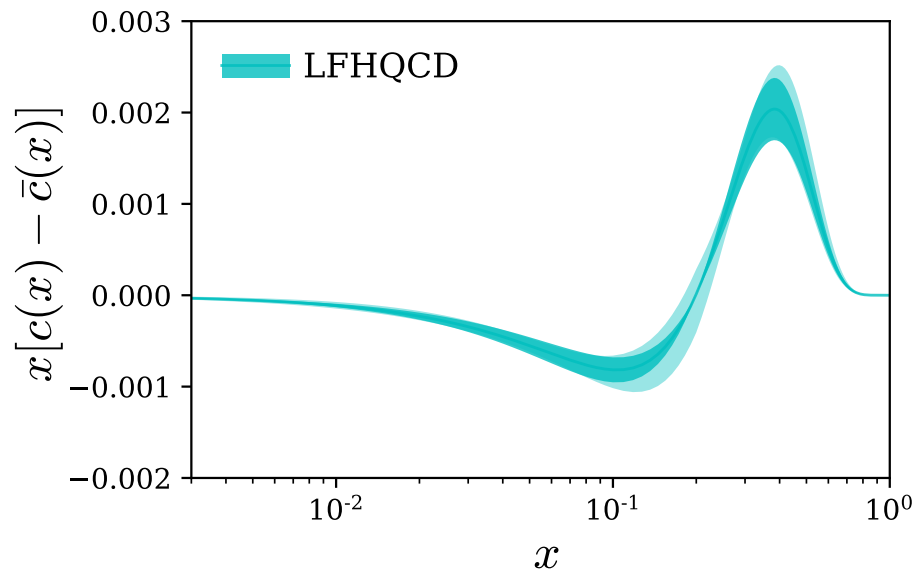
Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_T are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction): $\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron: $\lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$



Intrinsic charm in the proton (2020)

- S. J. Brodsky, P. Hoyer, C. Peterson, N. Sakai,
The intrinsic charm of the proton (1980)
- First lattice QCD computation of the charm quark
EM form factors with three gauge ensembles
(one at the physical pion mass)
- Nonperturbative intrinsic charm asymmetry $c(x) - \bar{c}(x)$
determined from LQCD and LFHQCD analysis



32I ($a = 0.08\text{fm}$)	\bullet	\times	\times	$+$	\square
	$m_{0.261}$	$m_{0.293}$	$m_{0.313}$	$m_{0.349}$	$m_{0.403}$
24I ($a = 0.11\text{fm}$)	\bullet	\times	\blacktriangleleft	\blacktriangleright	\square
	$m_{0.251}$	$m_{0.278}$	$m_{0.315}$	$m_{0.341}$	$m_{0.382}$
48I ($a = 0.11\text{fm}$)	\bullet	\square	\blacktriangleleft	\blacktriangleright	\square
	$m_{0.135}$	$m_{0.150}$	$m_{0.182}$	$m_{0.207}$	$m_{0.267}$
					$m_{0.331}$

Unpolarized gluon distribution (HLFHS Collaboration)

- Gravitational FF expressed in terms of Euler Beta function

$$A_\tau(t) \sim B(\tau - 1, 2 - \alpha(t))$$

- Pomeron-proton vertex described by Regge trajectory for a soft Donnachie-Landshof Pomeron

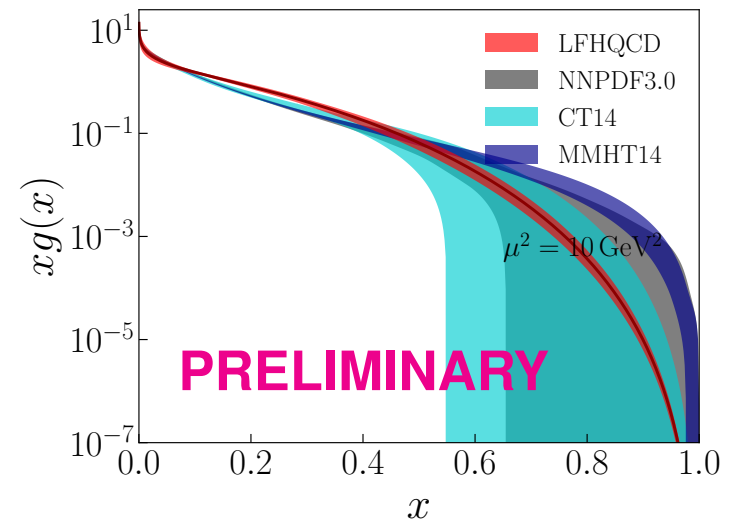
$$\alpha(t) = \alpha_P(0) + \alpha'_P t$$

interpreted in QCD as a bound state of two gluons with $\alpha'_P(0) \simeq 1.08$ and $\alpha' \simeq 0.25 \text{ GeV}^{-2}$

- Gluon PDF normalization determined from the sum rule:

$$\sum_q \langle x \rangle_q + \sum_{\bar{q}} \langle x \rangle_{\bar{q}} + \langle x \rangle_g = 1$$

- If coefficients c_τ fixed in the quark sector there are no adjustable parameters for prediction of $g(x)$
- Red error band in the fig. from initial evolution scale, uncertainty from sum rule not included



8 Outlook

- Classical equations of motion derived from the 5-dim theory have identical form of the semiclassical bound-state equations for massless constituents in LF quantization
- Implementation of superconformal algebra determines uniquely the form of the confining interaction and thus the modification of the AdS action, both for mesons and nucleons
- Approach incorporates basic nonperturbative properties which are not apparent from the chiral QCD Lagrangian, such as the emergence of a mass scale and the connection between mesons and baryons
- Prediction of massless pion in chiral limit is a consequence of the superconformal algebraic structure and not of the Goldstone mechanism: vacuum chosen *ab initio*
- Structural framework of LFHQCD also provides nontrivial connection between the structure of form factors and polarized and unpolarized quark distributions with pre-QCD nonperturbative results such as Regge theory and the Veneziano model