Rapid Communications

Frame-independent spatial coordinate $\tilde{z}$: Implications for light-front wave functions, deep inelastic scattering, light-front holography, and lattice QCD calculations

Gerald A. Miller, UW, Stanley J. Brodsky, SLAC

Motivation 1-curiosity

Three dimensional structure of proton presented in terms of transverse spatial coordinates but longitudinal momentum coordinate x
What about 3 spatial dimensions?

Everyone knows the Bjorken variable x.

Parton model x: ratio of quark k^+ to proton P^+ momentum $x = \frac{k^+}{P^+}$

What is the longitudinal spatial variable canonically conjugate to x?
$0 \leq x \leq 1$ compact range, but k^+ goes up to P^+

The variable is $\tilde{z} = x^- P^+$ $\qquad x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$

Similar thoughts - Glazek, Hoyer,….

Motivation 2: Lattice calculations of q(x)

- Old way compute a few moments and reconstruct
- Now quasi-pdfs in longitudinal coordinate space

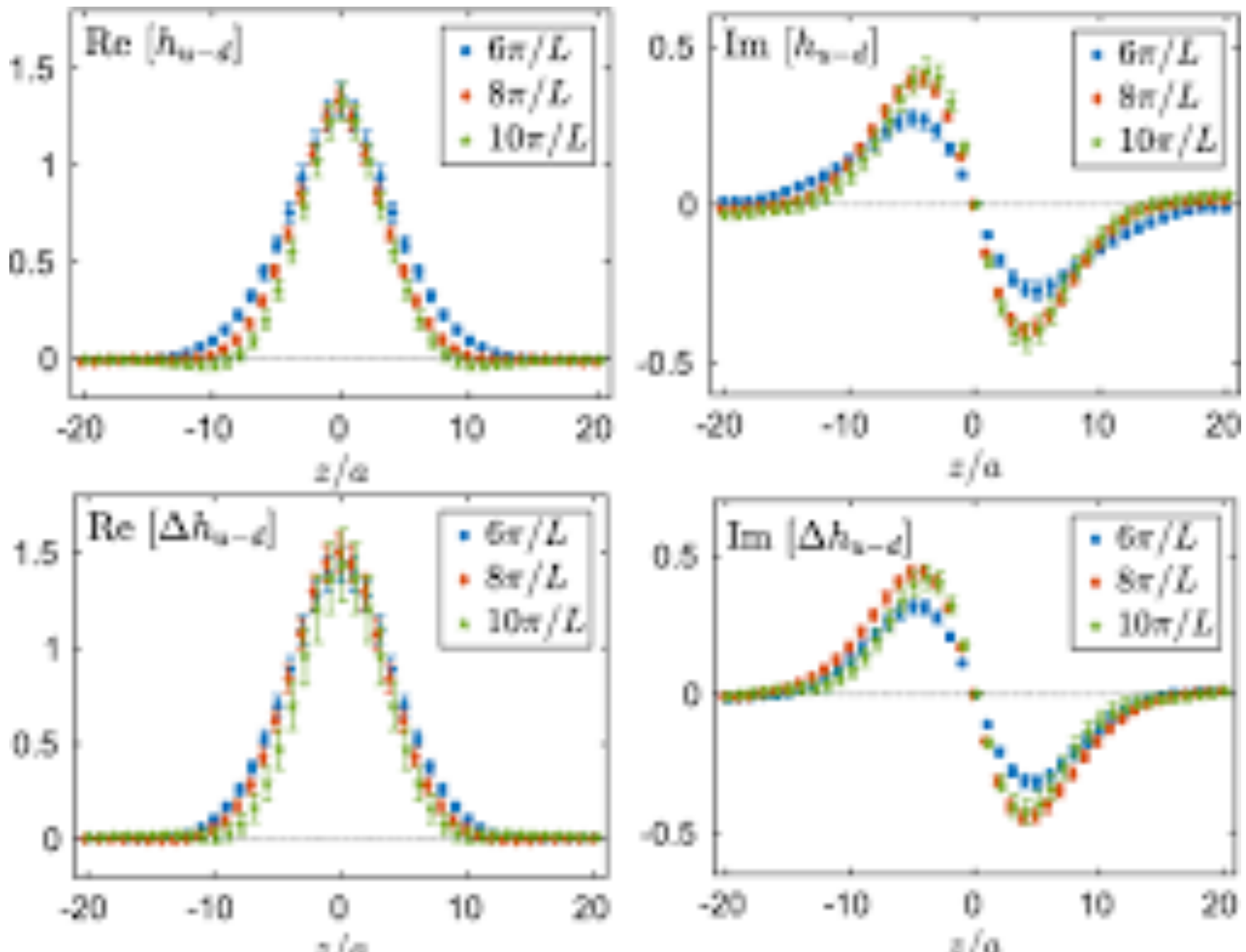
PHYSICAL REVIEW LETTERS **121**, 112001 (2018)

$$\tilde{q}(x, \Lambda, P) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-ixP_3 z} h_\Gamma(P, z),$$

Light-Cone Parton Distribution Functions from Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Karl Jansen,[5] Aurora Scapellato,[1,6] and Fernanda Steffens[7]

$$h_\Gamma(P, z) = \langle P|\bar{\psi}(0, z)\Gamma W(z)\psi(0, 0)|P\rangle$$



Can we understand h_Γ in terms of light front wave functions?

Motivation 2: Lattice calculations of q(x)

- Old way compute a few moments and reconstruct

- Now quasi-pdfs in longitudinal coordinate space

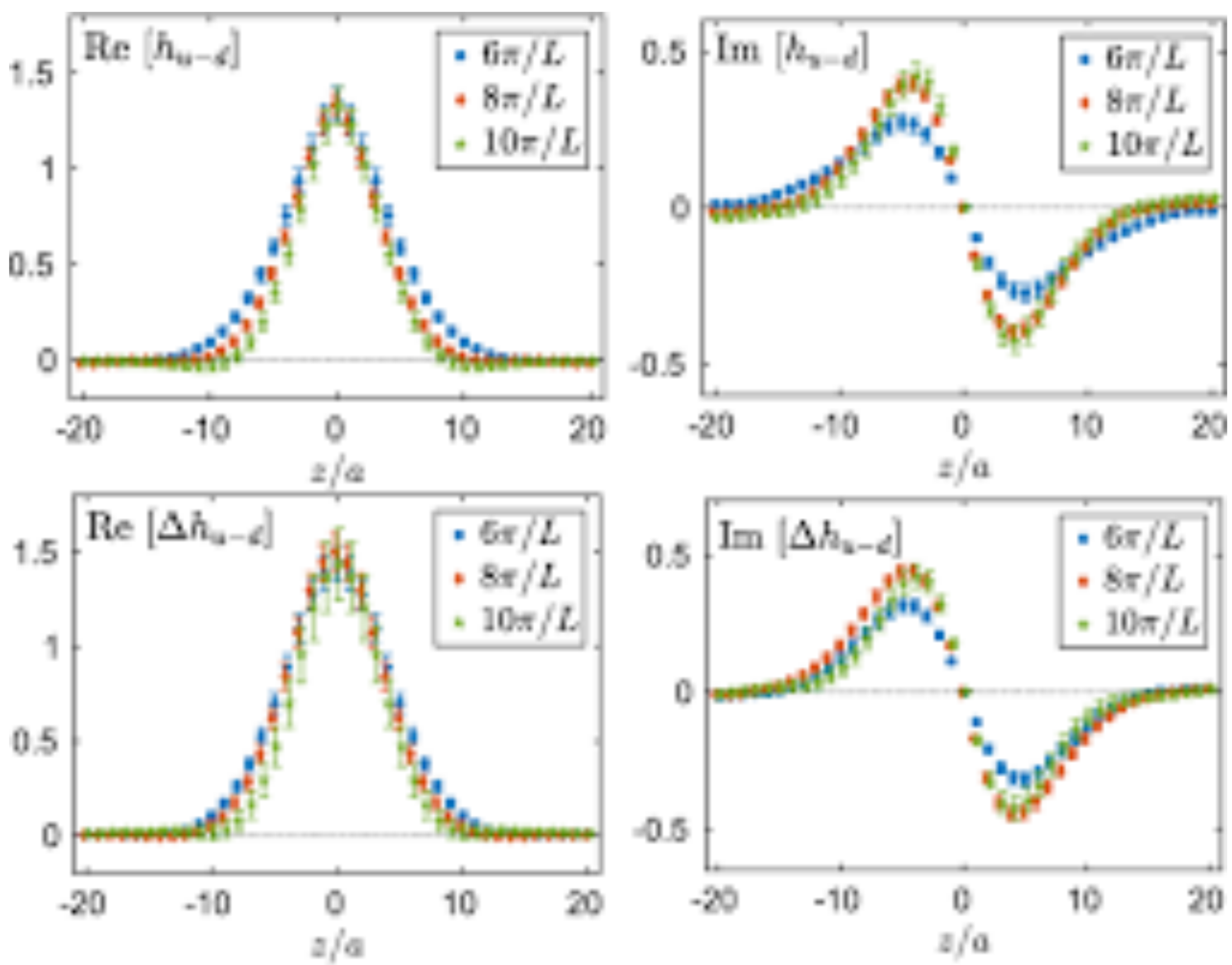
PHYSICAL REVIEW LETTERS **121**, 112001 (2018)

Light-Cone Parton Distribution Functions from Lattice QCD

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Karl Jansen,[5] Aurora Scapellato,[1,6] and Fernanda Steffens[7]

$$\tilde{q}(x,\Lambda,P)=\int_{-\infty}^{+\infty}\frac{dz}{4\pi}e^{-ixP_3 z}h_\Gamma(P,z),$$

$$h_\Gamma(P,z)=\langle P|\bar{\psi}(0,z)\Gamma W(z)\psi(0,0)|P\rangle$$

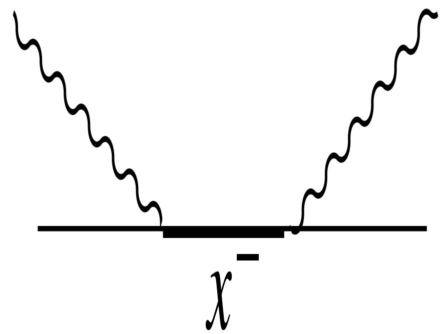


Can we relate z to light front wave functions?

Can we understand h_Γ in terms of light front wave functions?

Quark distributions & coordinate space

$$F_q(X) = \frac{1}{2}\int \frac{dx^-}{2\pi} e^{iXP^+x^-} \underline{\langle P|\overline{\psi}_+(-\tfrac{x^-}{2})\gamma^+\psi_+(\tfrac{x^-}{2})|P\rangle},$$



Recent lattice calculations compute the matrix element, then Fourier transform to get distributions

Insert complete set of states between the field operators matrix elements are light-front wave functions

$$q_n(x) = \int \frac{d^2k}{(2\pi)^2}\,|\psi_n(x,\mathbf{k})|^2$$

Contribution of Fock space component (n) to F(X)

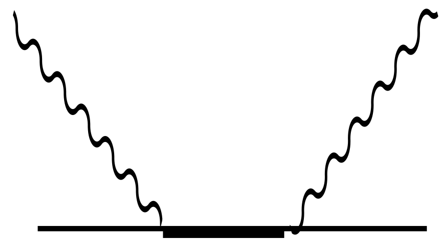
There has been much focus on coordinate-space versions -GPDs, in transverse coordinate space Burkardt

Longitudinal canonical variable??

$$x = \frac{k^+}{P^+}, \quad \tilde{z} = x^- P^+ \quad \text{Frame Independent}$$

Quark distributions & coordinate space

$$F_q(X) = \frac{1}{2}\int \frac{dx^-}{2\pi} e^{iXP^+x^-} \langle P|\overline{\psi}_+(-\frac{x^-}{2})\gamma^+\psi_+(\frac{x^-}{2})|P\rangle,$$



x^-

Recent lattice calculations compute the matrix element, then Fourier transform to get distributions

Insert complete set of states between the field operators matrix elements are light-front wave functions

$$q_n(x) = \int \frac{d^2k}{(2\pi)^2} \left|\psi_n(x,\mathbf{k})\right|^2$$

Contribution of Fock space component (n) to F(X)

There has been much focus on coordinate-space versions -GPDs, in transverse coordinate space Burkardt

Longitudinal canonical variable??

$$x = \frac{k^+}{P^+}, \quad \tilde{z} = x^- P^+ \quad \text{Frame Independent}$$

We are familiar with models of $\psi_n(x, \mathbf{k})$

Next step -get coordinate-space wave functions

Light front wave functions $x_i = k_i^+/P^+$, $\mathbf{k}_i$ for i'th constituent.

Relative coordinates: $x_i = k_i^+/P^+$, $\mathbf{k}_i$ for i'th constituent.

Canonically-conjugate spatial coordinates $\tilde{z}_i, \mathbf{b}_i$, $\tilde{z}_i = P^+ x_i^-$

$\tilde{z}_i$ is for each spatial coordinate unlike so-called Ioffe time

In the following use $\tilde{z}$ instead of $\tilde{z}_i$ for the struck quark, simplify notation

$$\psi_n(\tilde{z}, \mathbf{k}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{k}) e^{i\tilde{z}x}. \qquad \psi_n(\tilde{z}, \mathbf{b}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{i\tilde{z}x}.$$

Next question why $\tilde{z}$ instead of x^-?

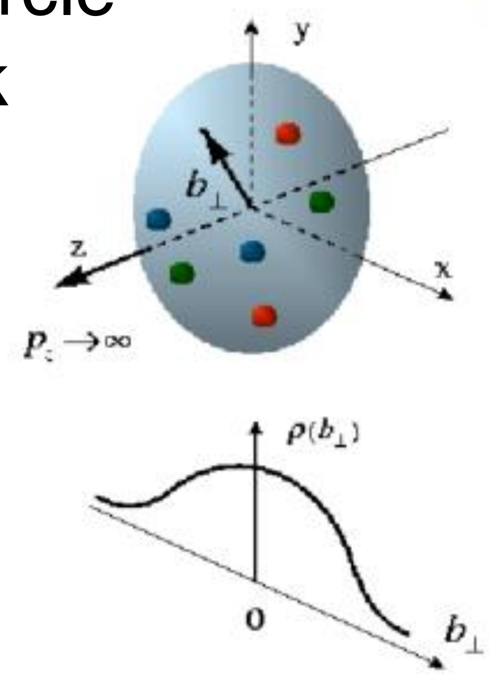
x^- is frame dependent

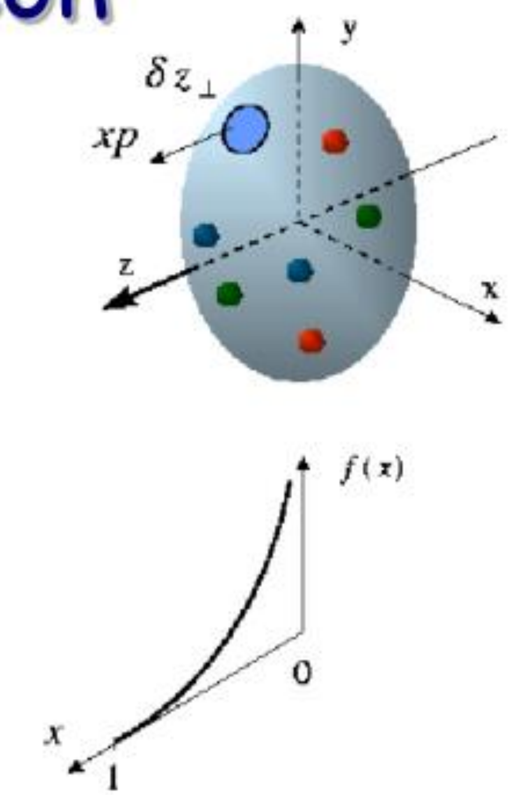
Pancakes in the infinite momentum frame?

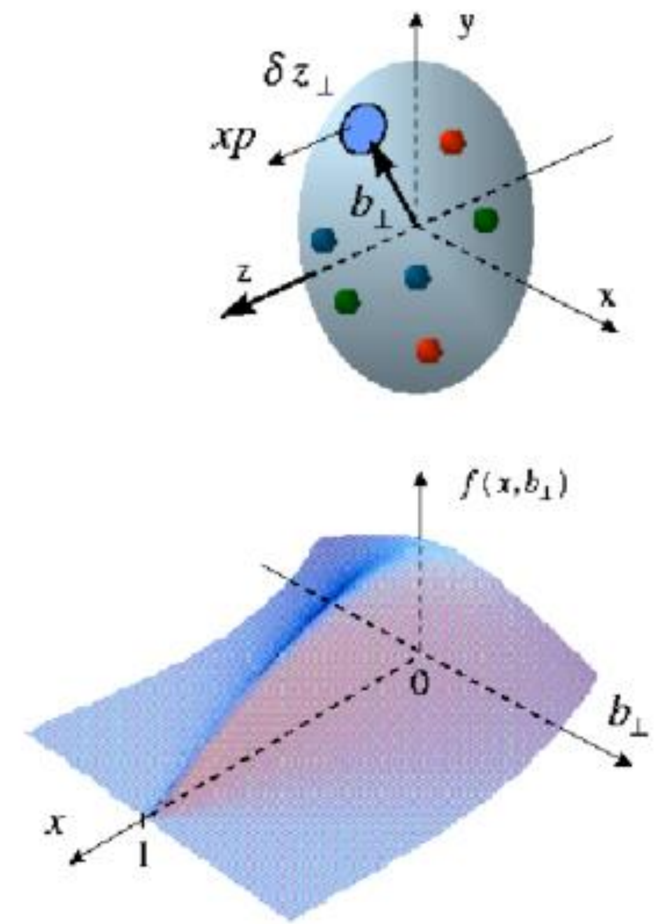
Generalized Parton Distributions : yield 3-dim quark structure of nucleon

Burkardt (2000,2003)
Belitsky,Ji,Yuan (2004)

The blue circle
is a disk







Elastic Scattering
transverse quark distribution in coordinate space

DIS
longitudinal quark distribution in momentum space

DES (GPDs)
fully-correlated quark distribution in both coordinate and momentum space

Yes, disks/pancakes

$$\psi_n(\tilde{z}, \mathbf{b}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{i\tilde{z}x},$$

We use $\tilde{z}$

Another choice uses x^-:

$$\chi_P^+(x^-, b) = \sqrt{\frac{P^+}{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{ix^- P^+ x}.$$

$$\rho_{P^+}(x^-, b) = |\chi_{P^+}(x^-, b)|^2$$

Densities contract to a disk in the infinite momentum frame IF function of x^- and there is a γ^+

Next- back to frame independent $\tilde{z}$

Yes, disks/pancakes

We use $\tilde{z}$

$$\psi_n(\tilde{z}, \mathbf{b}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{i\tilde{z}x},$$

Another choice uses x^-:

$$\chi_P^+(x^-, b) = \sqrt{\frac{P^+}{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{ix^- P^+ x}.$$

Limit $P^+ \to \infty$ $\quad \rho_{P^+}(x^-, b) = |\chi_{P^+}(x^-, b)|^2$

Densities contract to a disk in the infinite momentum frame IF function of x^- and there is a γ^+

Next- back to frame independent $\tilde{z}$

Yes, disks/pancakes

We use $\tilde{z}$

$$\psi_n(\tilde{z}, \mathbf{b}) = \frac{1}{\sqrt{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{i\tilde{z}x},$$

Another choice uses x^-:

$$\chi_P^+(x^-, b) = \sqrt{\frac{P^+}{2\pi}} \int_0^1 dx\, \psi_n(x, \mathbf{b}) e^{ix^- P^+ x}.$$

Limit $P^+ \to \infty$

$$\rho_{P^+}(x^-, b) = |\chi_{P^+}(x^-, b)|^2$$
$$= \int dx\, |\psi_n(x, b)|^2 \delta(x^-)$$

Densities contract to a disk in the infinite momentum frame IF function of x^- and there is a γ^+

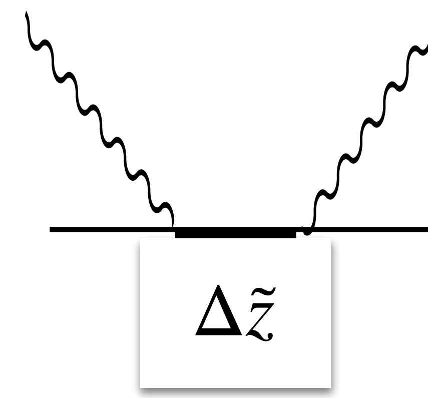
Next- back to frame independent $\tilde{z}$

Quark distributions and $\tilde{z}$

$$q_n(x) = \int \frac{d^2k}{(2\pi)^2} \left|\psi_n(x,\mathbf{k})\right|^2$$

Fourier transform each of the two wave functions

$$q_n(x) = \int \frac{d\tilde{z}\,d\tilde{z}'}{2\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \psi_n^*(\tilde{z}',\mathbf{k})\psi_n(\tilde{z},\mathbf{k}) e^{i(\tilde{z}-\tilde{z}')x}.$$



$$\tilde{Z} \equiv (\tilde{z}+\tilde{z}')/2, \;\; \Delta\tilde{z} = \tilde{z}-\tilde{z}'$$

Integrate on $\tilde{Z}$

$$\boxed{\begin{aligned} q_n(x) &= \int_{-\infty}^{\infty} d(\Delta\tilde{z}) g_n(\Delta\tilde{z},x), \\ g_n(\Delta\tilde{z},x) &= \frac{1}{2\pi}\int_0^1 dy\, q_n(y) \cos\, \Delta\tilde{z}(y-x). \end{aligned}}$$

$g_n(\Delta\tilde{z},x)$ measures contribution to quark (anti-quark) distributions occurring at a separation $\Delta\tilde{Z}$

Variables $\Delta\tilde{Z}, x$ canonically related, so g_n is a longitudinal Wigner distribution

7/17

I. Pseudoscalar meson, massless quarks, $q\bar{q}$ LF holographic model

$$\psi_M(x,\mathbf{b}) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}e^{-\frac{\mathbf{b}^2\kappa^2 x(1-x)}{2}}$$

b transverse dist between quark and cm

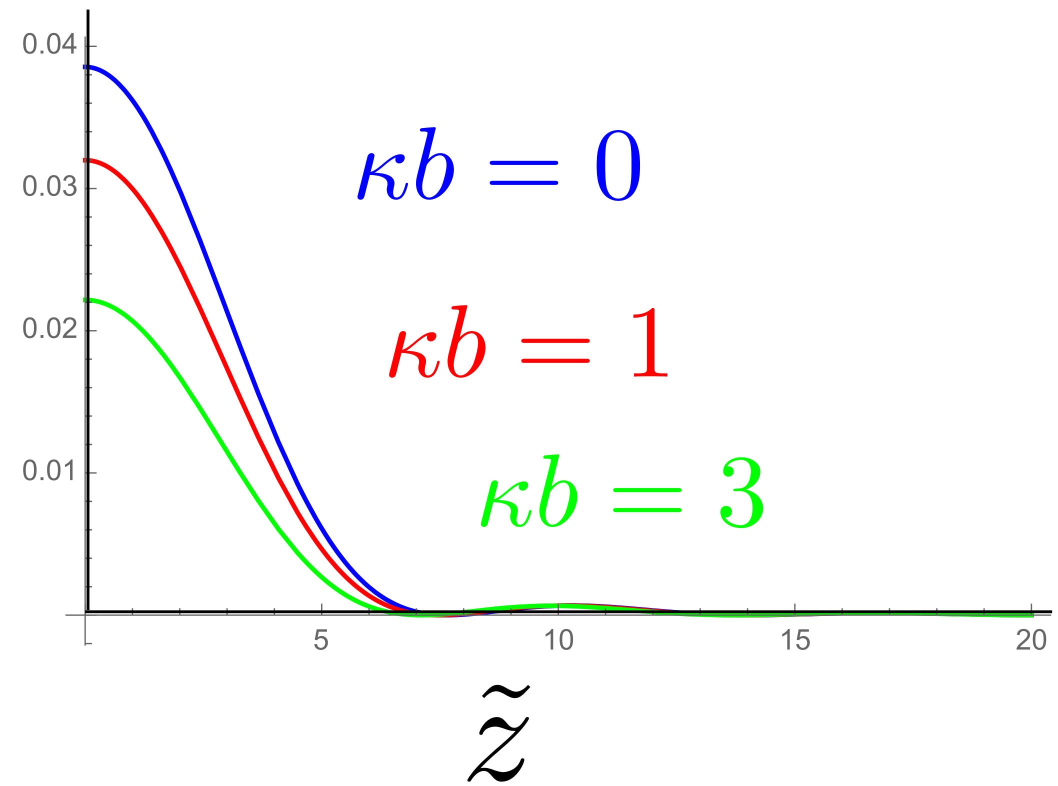
Fourier transform. $\frac{\pi}{\sqrt{2}\kappa}\psi_M^S(\tilde{z},b) \approx \frac{\pi}{4}e^{i\tilde{z}/2}e^{-b^2\kappa^2/8}\frac{J_1(\tilde{z}/2)}{\tilde{z}}$

$\tilde{z} = x^- P^+$ $\quad$ x$^-$: quark spectator longitudinal distance

Very long tail

$$\rho_M(\tilde{z},b) \equiv \left|\psi_M^S(\tilde{z},b)\right|^2$$

Slow fall off with large $\tilde{z}$, $\rho_M \sim \frac{1}{\tilde{z}^3}$



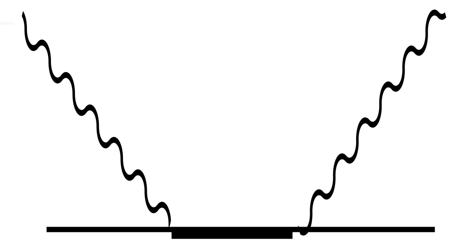
Models to understand $g(\Delta\tilde{z}, x)$

I. Pseudoscalar meson, massless quarks, $q\bar{q}$ LF holographic model

$\psi_M(x, \mathbf{b}) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}e^{-\frac{\mathbf{b}^2\kappa^2 x(1-x)}{2}}$ b transverse dist between quark and cm

Take Ψ_M square it, integrate over $\mathbf{b}$ to get $q_M(y)$. Use result to get g_M

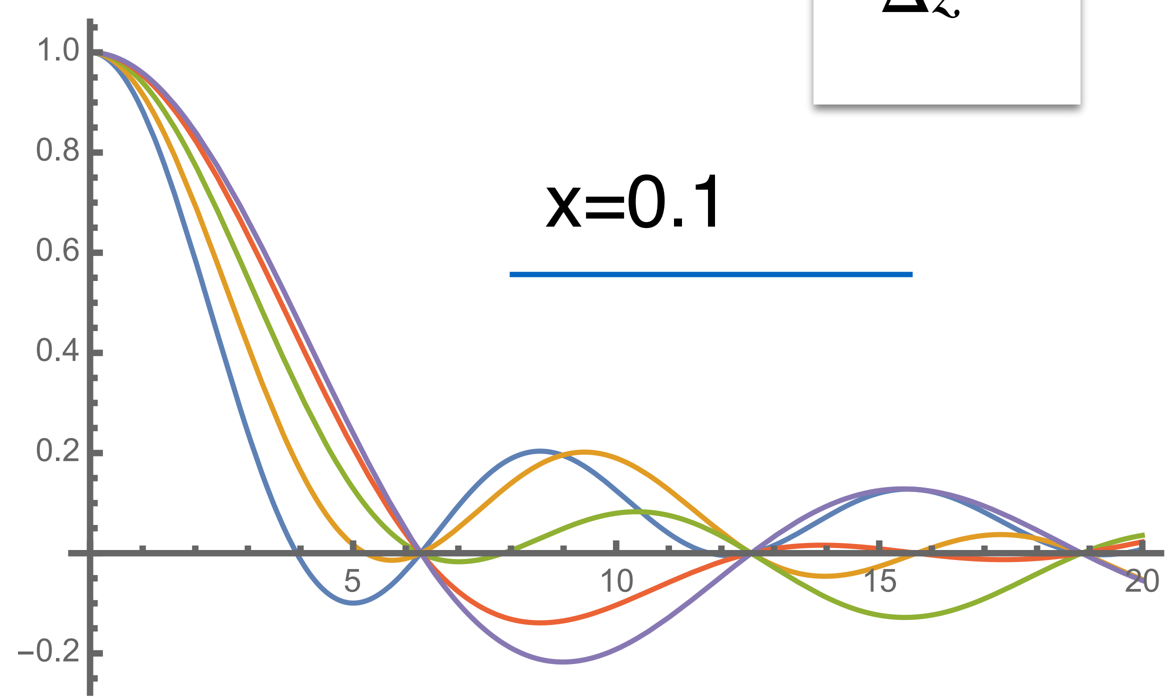
$$g_M(\Delta\tilde{z}, x) = \frac{1}{2\pi}\left(\frac{\sin\Delta\tilde{z}(1-x) + \sin\Delta\tilde{z}x}{\Delta\tilde{z}}\right)$$



$\Delta\tilde{z}$

Very long tail

$g_M(\Delta\tilde{z}, x)$



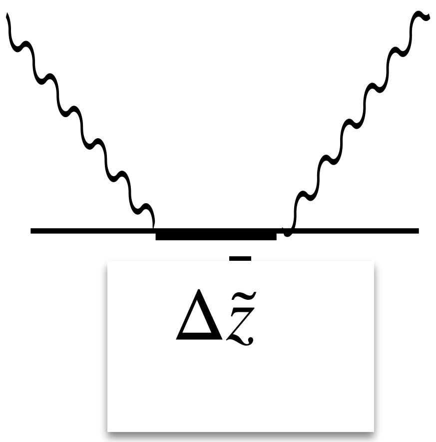
Simple wf, no Regge behavior
Absorption-emission distance can be large

In lab frame, if $\Delta\tilde{z} = 16, \Delta x^- \approx 5$ fm

$\Delta\tilde{z}$

Intermediate summary

- Using the frame-independent variable $\tilde{z}$ gives a way to study light front wave functions in three dimensions
- Simplest two particle wave function has a large spatial longitudinal extent
- Deep inelastic scattering may occur at large values of $\Delta\tilde{z}$
- Will such effects occur in your model?
- $q_M(x) = 1\,!$ need to have more realistic wave function



Remainder of talk is concerned with one different model

arXiv:1801.09154

PRL 120 (2018) 18, 182001

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2], Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

PRL 120 (2018) 18, 182001

arXiv:1801.09154

GPD Model incorporates Regge behavior small x, inclusive counting rules high x precise descriptions of nucleon and pion q(x)

$\tau = 2, 3, 4 =$ number of constituents in Fock-space wavefunction

$$q_\tau(x) = \frac{1}{N_\tau}[1 - w(x)]^{\tau-2} w(x)^{-(1/2)} w'(x), \qquad (18)$$

$$f(x) = \frac{1}{4\lambda}\left[(1-x)\ln\left(\frac{1}{x}\right) + a(1-x)^2\right], \qquad (19)$$

and $w(x) = x^{1-x} e^{-a(1-x)^2}$.

The value of the universal scale λ is fixed from the ρ mass: $\sqrt{\lambda} = \kappa = m_\rho/\sqrt{2} = 0.548$ GeV [37,39]. The flavor-independent parameter $a = 0.531 \pm 0.037$. The u and d quark distributions of the proton are given by a linear superposition of q_3 and q_4 whereas those of the pion are obtained from q_2 and q_4.

GPD $H_\tau(x,t) = q_\tau(x) e^{t f(x)}$

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2], Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

PRL 120 (2018) 18, 182001
arXiv:1801.09154

Proton

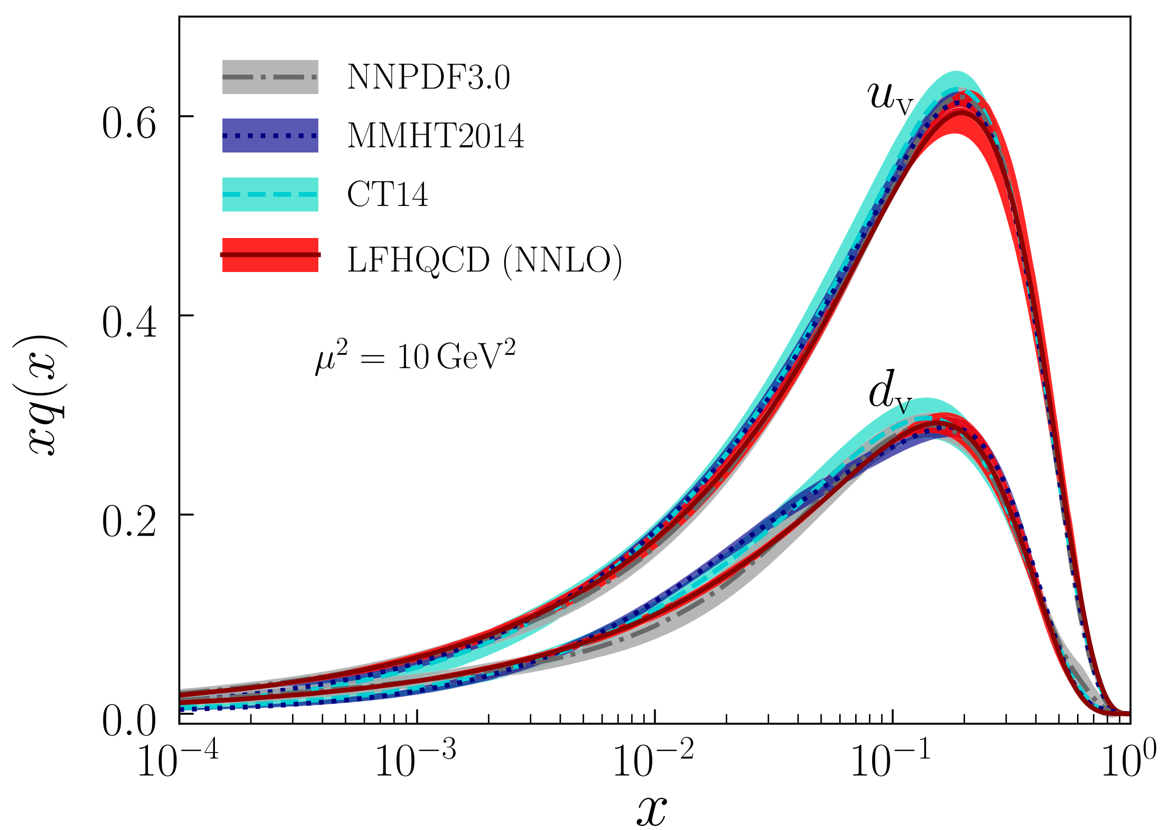


FIG. 1. Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

pion

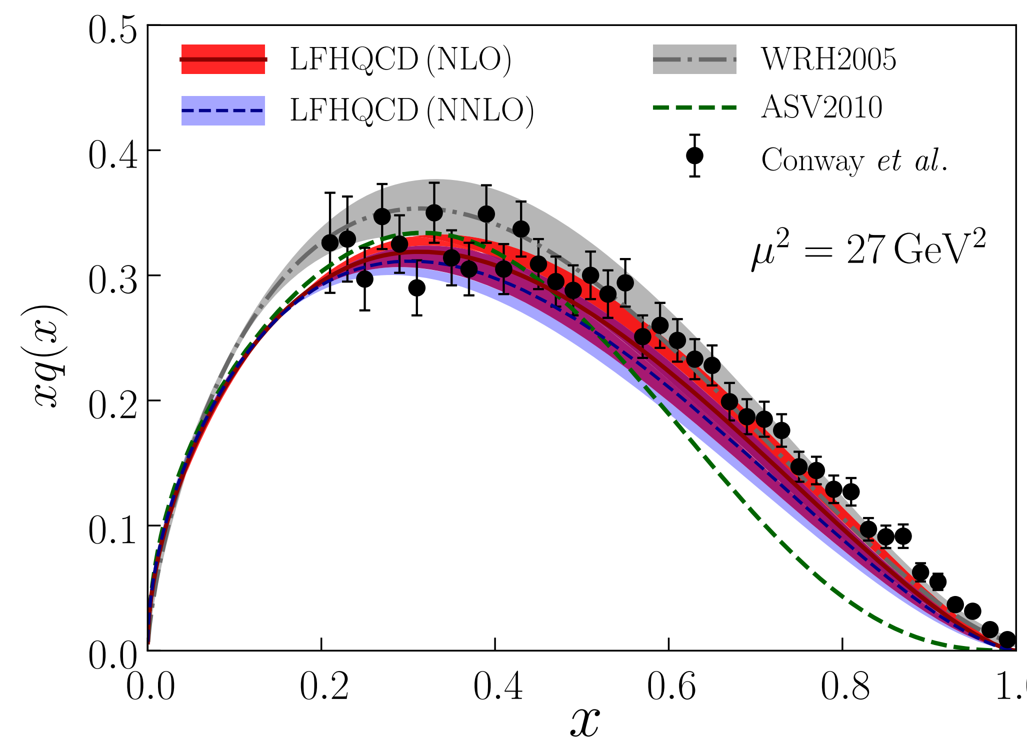


FIG. 4. Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2$ GeV at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2],
Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

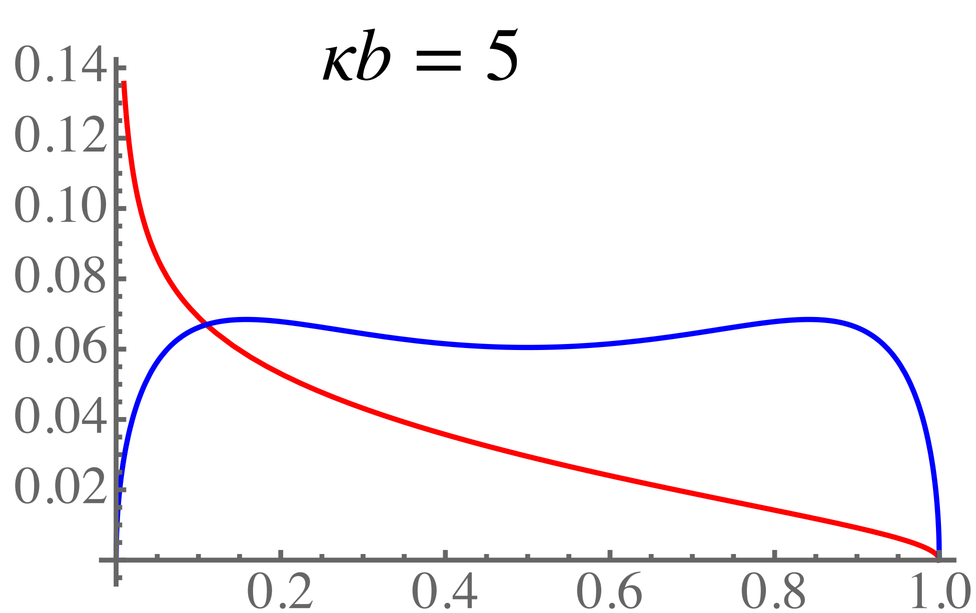
arXiv:1801.09154
PRL 120 (2018) 18, 182001

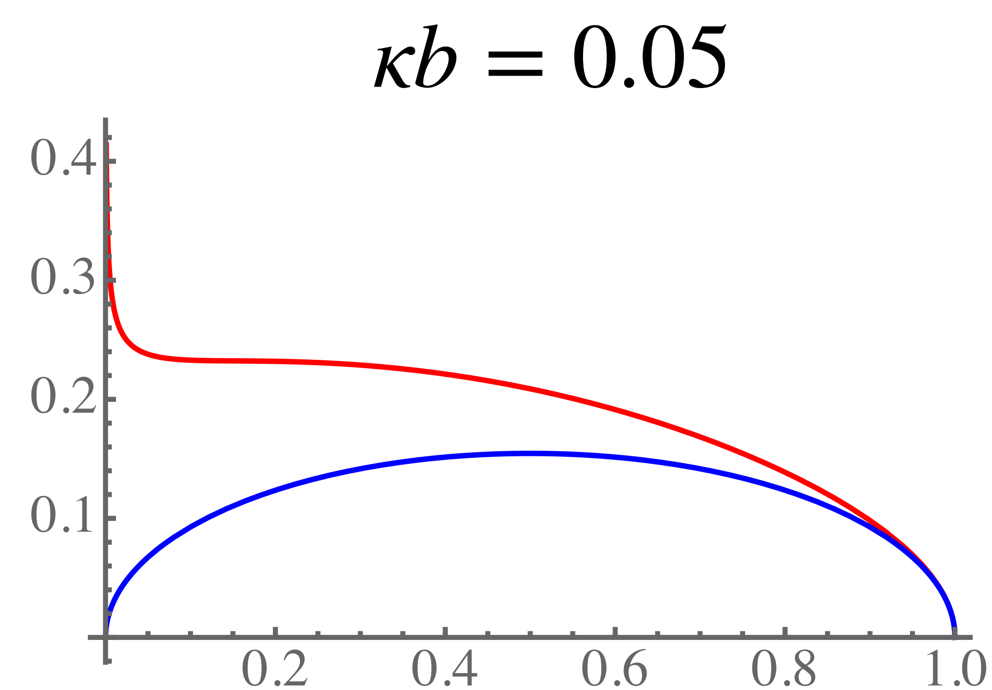
$\tau = 2, 3, 4 =$ number of constituents in Fock-space wavefunction

Light front wave function $\tau = 2$

First HLFQCD wave function

$$\psi_M(x, \mathbf{b}) = \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-[\mathbf{b}^2 \kappa^2 x(1-x)]/2}.$$





Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2],
Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

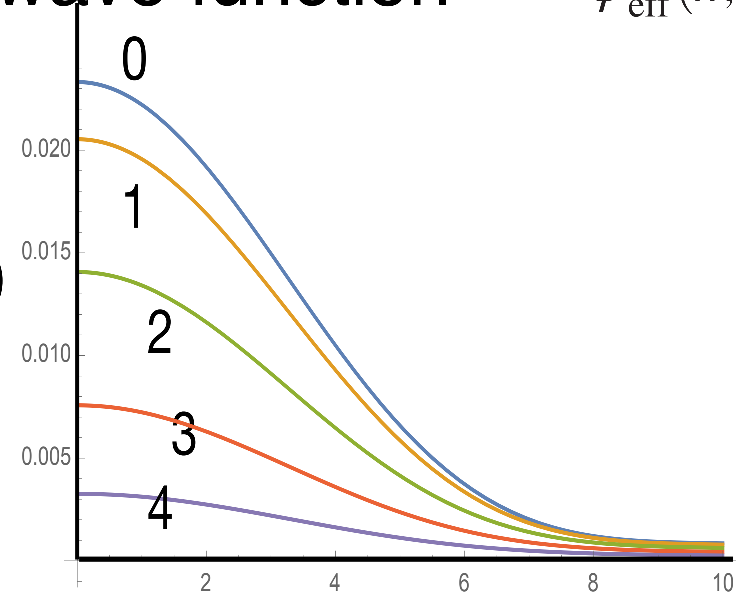
$\tilde{z}$ dependence

$\tau = 2, 3, 4 =$ number of constituents in Fock-space wavefunction

Light-front wave function²

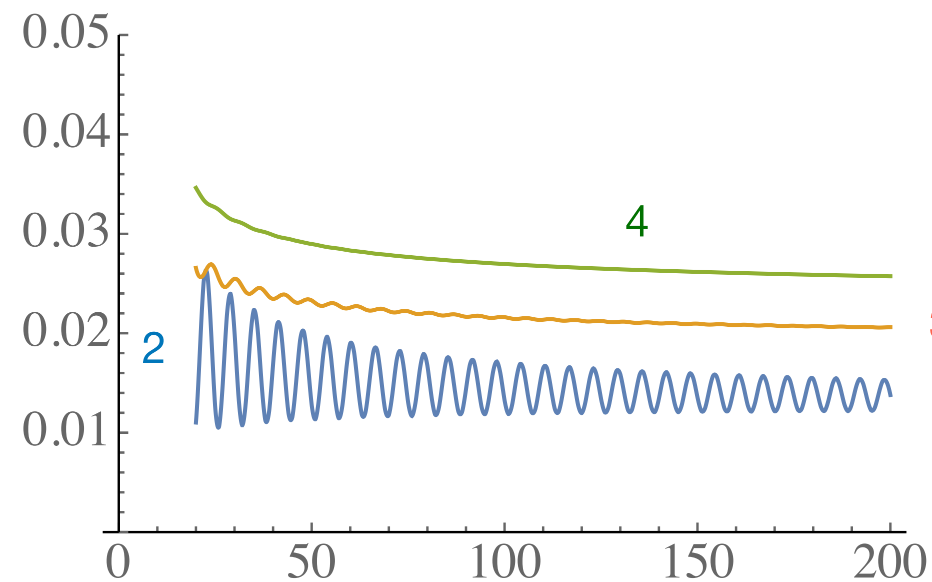
$$\psi^{\tau}_{\rm eff}(x, \mathbf{b}_\perp) = \frac{1}{2\sqrt{\pi}} \sqrt{\frac{q_\tau(x)}{f(x)}} (1-x) \exp\left(-\frac{(1-x)^2}{8 f(x)} \mathbf{b}_\perp^2\right)$$

$\rho(\tilde{z}, b)$



$\tilde{z}$

$\tilde{z}^{1.6}\rho(\tilde{z},0)$



$\tilde{z}$

$$\rho_M(\tilde{z}, b) \equiv \left|\psi^S_M(\tilde{z}, b)\right|^2$$

The numbers refer to b in units of 1/GeV

FIG. 5. $\tilde{z}^{1.6}\rho(\tilde{z}, 0)$. The numbers refer to the value of τ, the number of constituents in the Fock state.

At large $\Delta\tilde{z}$ falloff is very slow - long range

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2],
Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

$\tau = 2, 3, 4 =$ number of constituents in Fock-space wavefunction

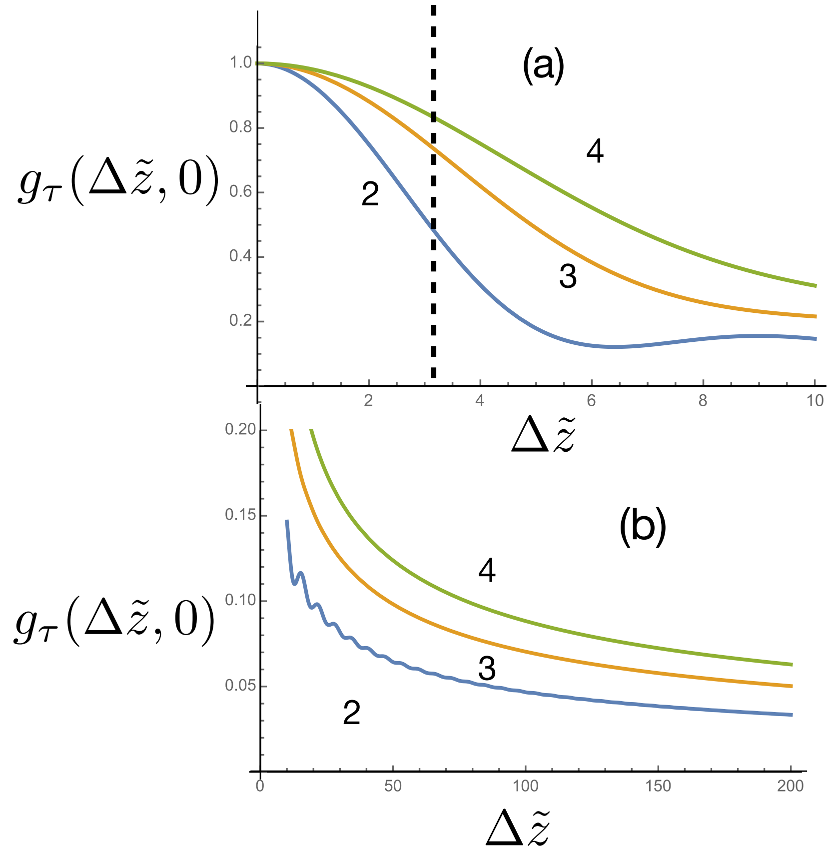
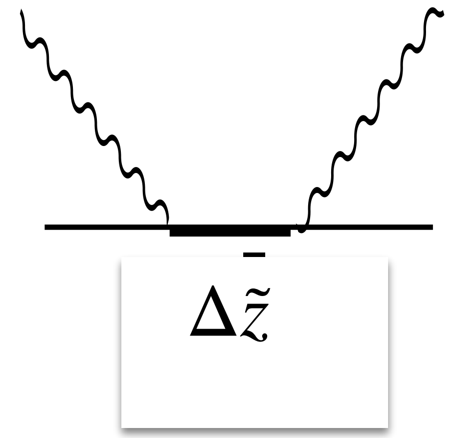
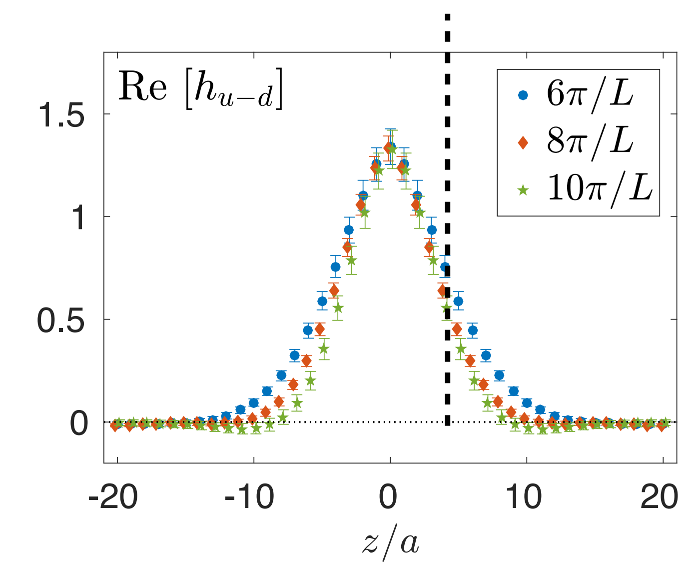


FIG. 3. $g_\tau(\Delta\tilde{z}, 0)$. The numbers refer to the value of τ, the number of constituents in the Fock state. (a) $0 \leqslant \Delta\tilde{z} \leqslant 10$, and (b) $10 \leqslant \Delta\tilde{z} \leqslant 200$.





PHYSICAL REVIEW LETTERS **121,** 112001 (2018)

$$g_n(\Delta\tilde{z}, x) = \frac{1}{2\pi}\int_0^1 dy\, q_n(y) \cos\, \Delta\tilde{z}(y - x).$$

$$q_n(x) = \int_{-\infty}^{\infty} d(\Delta\tilde{z}) g_n(\Delta\tilde{z}, x),$$

Significant contributions at large $\Delta\tilde{z}$

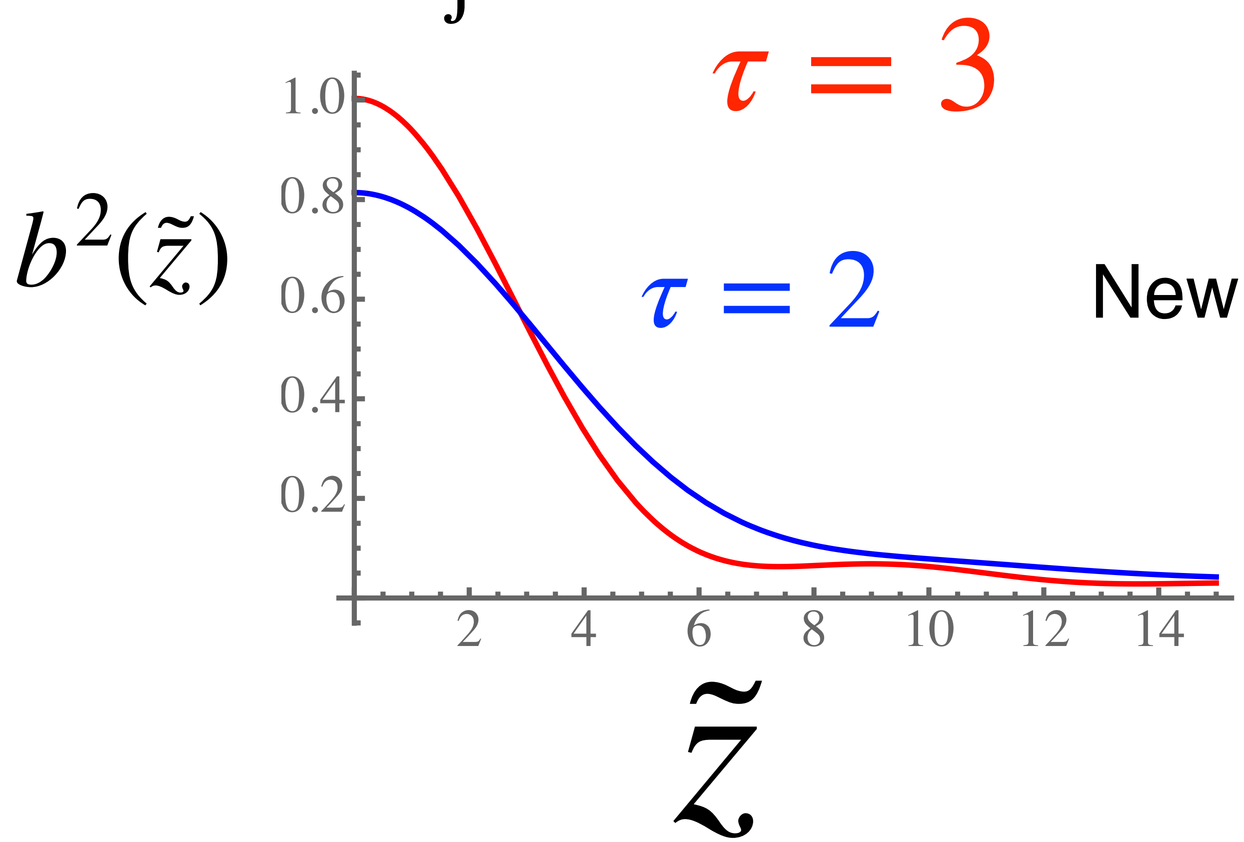
Guy F. de Téramond[1], Tianbo Liu[2,3], Raza Sabbir Sufian[2], Hans Günter Dosch[4], Stanley J. Brodsky[5], Alexandre Deur[2]

Average value of b^2

$\tau = 2, 3, 4 =$ number of constituents in Fock-space wavefunction

$$b^2(\tilde{z}) \equiv \int d^2b \, b^2 \, |\psi(\tilde{z}, b)|^2$$

Different than in paper



New feature -long tail

Summary

- Frame-independent longitudinal spatial variable, canonically conjugate to x introduced $\tilde{z} = x^{-} P^{+}$
- Square of wave functions have long tail in $\tilde{z}$
- Distance between absorption and emission of virtual photons in DIS can be very large, especially so at small x
- The function g($\Delta\tilde{z}$, x) may serve as a bridge between light front wave functions and lattice QCD calculations of GPDs
- What is the $\tilde{z}$ dependence of your wave functions?